Asia Pacific Journal of Mathematics, Vol. 1, No. 2 (2014), 182-196 ISSN 2357-2205

A TWO STAGE SUPPLY CHAIN MODEL WITH SELLING PRICE DEPENDENT DEMAND AND INVESTMENT FOR QUALITY IMPROVEMENT

M. VIJAYASHREE [1*] AND R. UTHAYAKUMAR[2]

Department of Mathematics, Gandhigram Rural Institute - Deemed University, India

*Corresponding author

ABSTRACT. This paper presents the problem of a vendor-buyer integrated production inventory model for two stage supply chain under investment for quality improvement. The buyer faces as a linear demand, which is assumed to be function of its unit selling price. Total profit is the supply chain performance measure and it is computed as the difference between revenue from sales and total cost, where the latter is sum of the vendor's and buyer's setup/order and inventory holding costs, and opportunity investment cost . Moreover, a capital investment, which is necessary to improve the quality of the product, is also considered in the total profit function. Main focus for this paper is the investment for quality improvement for joint optimization. In this paper we can use logarithmic function to obtain the investment in quality improvement. The model is based on the integrated total profits of both buyer and vendor which find out the optimal value of order quantity, opportunity investment cost for quality improvement.In this study first, we developed mathematical model and procedure of finding the optimal solution is developed. Also the solution procedure is developed in order to find the total profit of the vendor and the buyer which is to be maximized. Some numerical examples are also used to analyze the effect of the price-sensitivity of demand on the improvements in total profit. A computer code using the software Matlab is developed to derive the optimal solution and numerical example is presented to illustrate the proposed models. The result are illustrate with the help of numerical example. Graphical representation is also presented to illustrate the proposed model.

2010 Mathematics Subject Classification. 90B05.

Key words and phrases. Supply Chain Model, Vendor buyer coordination, Total profit, Shipment policy, Investment for quality improvement.

1. INTRODUCTION

Supply chain management often requires the integration of inter and intra organizational relationships and coordination of different types of flows within the entire supply chain structure. Supply chain management (SCM) helps firms in integrating their business by collaborating with other value chain partners to meet the unpredictable demand of the end user. Supply chain management (SCM) seems to be a growing area of interest amongst researchers and practitioners from varied disciplines. Supply chain Management (SCM) has evolved from the age when issues related to materials flow (Forrester, 1961) was introduced, which later on become part of Supply Chain Management (SCM).

The Just in time (JIT) system plays an important role in present supply chain management. One of the major tasks of maintaining the competitive advantages of just in time production is to compress the lead time needed which is associated with delivering high quality products to customers. In the dynamic, competitive environment, successful companies have devoted considerable attention to reduce inventory cost and lead time. In the past, most of the inventory model researchers considered only the independent view point. However, in supply chain environmental, the co-ordination of all the partners is the key to efficient management of a supply chain to achieve global optimality. Research on coordinating supply chains is currently very popular. During the last few years, the concept of integrated vendor-buyer inventory management has attracted considerable attention, accompanying the growth of Supply Chain Management (SCM).

A supply chain is defined as a network of facilities and distribution options that perform the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers. Managing such functions along the whole chain; that is, from the supplier's supplier to the customer's customer; requires a great deal of coordination among the players in the chain. The effectiveness of coordination in supply chains could be measured in two ways: reduction in total supply chain costs and enhanced coordination services provided to the end customer and to all players in the supply chain.

A supply chain encompasses all activities in fulfilling customer demands and requests. These activities are associated with the flow and transformation of goods from the raw materials stage, through to the end user, as well as the associated information and funds flows. There are four stages in a supply chain: the supply network, the internal supply chain (which are manufacturing plants), distribution systems, and the end users. Moving up and down the stages are the four flows: material flow, service flow, information flow and funds flow. The supply chain begins with a need for a computer. In this example, a customer places an order for a Dell computer through the internet. Since Dell does not have distribution centers or distributors, this order triggers the production at Dell's manufacturing center, which is the next stage in the supply chain. Microprocessors used in the computer may come from AMD (Advanced micro divisor) and a complementary product like a monitor may come from Sony.

Dell receives such parts and components from these suppliers, who belong to the up- stream stage in the supply chain. After completing the order according to the customer's Specification, Dell then sends the computer directly to the users through UPS (Uniform power supply), a third party logistics provider. Supply Chain Management is a set of synchronized decisions and activities utilized to efficiently integrate suppliers, manufacturers, warehouses, transporters, retailers, at the right quantities, to the right locations, and at the right time, in order to minimize system- wide costs while satisfying customer service level requirements. The objective of Supply Chain Management (SCM) is to achieve sustainable competitive advantage.

In many practical situations, setup cost can be controlled and reduced through various effects such as worker training, procedural changes and specialized equipment acquisition. If the ordering cost per order could be reduced effectively, the total relevant cost per unit time could be automatically improved. Through the Japanese experience of using Just in time (JIT) production and benefits associated with efforts to reduce the ordering cost can be clearly perceived. In recent years, several authors have studied inventory models with controllable setup cost and lead time.

The integrated vendor-buyer problem is called the joint economic lot sizing (JELS) problem. The global supply chain is very complex, and link-by-link understanding of joint policies can be very useful Ben-Daya et al.[14] and Khan et al.[12] proposed a two-stage integrated inventory system to incorporate the just in time (JIT) concept in the conventional joint batch- supplier problem. The integrated model between supplier and purchaser for improving the performance of inventory control has attracted a great deal of attention from researchers. Goyal[21] suggested a joint economic lot-size model where the objective was to minimize the total relevant costs for both the vendor and the buyer. Goyal[21] and Goyal[20] first introduced the idea of a joint total cost for a single-vendor and a single-buyer scenario assuming an infinite production rate for the vendor and lot- for- lot policy for the shipments from the vendor to the buyer.

Banerjee[1]relaxed the infinite production rate assumption. Then Goyal[19] contributed to the efforts of generalizing the problem by relaxing the assumption of lot-for-lot. He assumed that the production lot is shipped in a number of equal-size shipments. Viswanathan et.al[23] later developed a model of single-vendor, single-retailer distribution channel, where the retailer faces a price sensitive deterministic demand.

Another recent paper in this area is by Ray et al.[22] who introduced an integrated marketing-inventory model for two pricing policies, price as a decision variable and mark- up pricing. Recently, Bakal et al.[10] presented two inventory models with price-sensitive demand. Two different pricing strategies were also investigated, where (i) the firm chooses to offer a single price in all markets selected, and (ii) a different price is set for each market. Recently, the lot sizing problem has received considerable attention. But the majority of analysis have always assumed implicitly perfect quality of products. Product quality, however, is not always perfect, and is usually a function of the state of the production process. When the production process is in control, the items produced would be of high or perfect quality. As time goes on, the process may deteriorate and begin to produce defective items. Thus, the relationship between production lot size and the quality of the product may be significant.

As global market became more competitive, supply chain coordination became a key component for enhancing its profitability and responsiveness. When there is no coordination, the supply chain members act independently to maximize their own profit, which does not ensure that the parties as a whole reach on optimal result Sajadieh et al.[17]both form economic and environmental points of view. When there is coordination, the total supply chain profit/cost is maximized / minimized, but the saving from coordination shifts to the side of the vendor and buyer is the one who will be operation off its optimal policy.

In the cases where no coordination exists between supply chain members, the vendor and the buyer will act independently to maximize their own profit. This independent decision behavior usually cannot assure that the two parties, as whole, reach the optimal state. In traditional inventory management, the optimal inventory and shipment policies for manufacturer and buyer in a two echelon supply chain are managed independently. As a result the optimal lot size for the purchaser may not result in an optimal policy for the vendor and vice versa. To overcome this difficulty, the integrated vendor-buyer model is developed, where the joint total relevant cost for the purchaser as well as the vendor is maximized. Consequently, determining the optimal policies, based on integrated total cost function rather than buyer or supplier individual cost function, result in a reduction of the total inventory cost of the system.

Quality has been highly emphasized in modern production/inventory management systems. Also, it has been evidenced that the success of Just-In-Time (JIT) production is partly based on the belief that quality is a controllable factor, which can be improved through various efforts such as worker training and specialized equipment acquisition. In the classical economic order quantity (EOQ) model, the quality-related issue is often neglected; it implicitly assumes that quality is fixed at an optimal level (i.e., all items are assumed with perfect quality) and not controllable. However, this may not be true. In a real production environment, we can often observe that there are defective items being produced. These defective items must be rejected, repaired, reworked, or, if they have reached the customer, refunded; and in all cases, substantial costs are incurred.

Porteus[5] and Rosenblatt et al.[16]were the first to explicitly elaborate on the significant relationship between quality imperfection and lot size. Specifically, Porteus et al.[5] extended the economic order quantity EOQ model to include a situation where the production process is imperfect, and based on this model he further studied the effects of investment in quality improvement by introducing the additional investing options. Since Porteus[5]several authors proposed the quality improvement models under various settings, see e.g. (Keller et al.[7]; Hwang et al.[8]; Moon[9]; Hong et al.[11]; and Ouyang et al.[13]). We note that in the body of literature (Hong et al.[11]; Hwang et al.[8]; Keller et al.[7]; Lau et al.[2]; Nasri et al.[6]; Moon[9]) a common approach utilized to develop the total cost of quality improvement model is adding the investment cost required for quality improvement to the system operating costs, where the investment amount is further charged a fixed opportunity cost instead of modeling the system with discounted costs. However, in practice, the opportunity cost rate (e.g., interest rate) may not be fixed; it may slightly change from time to time, particularly, in an unstable environment.

We assume that the relationship between setup cost reduction (or process quality improvement) and capital investment can be described by the logarithmic investment function. This logarithmic investment function which has been used in previous researches by (Paknejad et al.[15]; Nasri et al.[6]; Sarker et al.[3]; Hofmann[4]) is consistent with the Japanese experience as reported in Hall[18]. In addition, a procedure is provided to find the optimal production runs time and then to find the optimal setup cost and process quality improvement level.

In this paper, we develop an integrated production-inventory system consisting of a single vendor and single buyer, under investment in quality improvement. On the other hand, it can be considered as an integrated operations-marketing model which is developed for a two stages supply chain model, under investment in quality improvement where production optimization incorporated. The objective of this paper is to maximize the total profit for the single vendor and single buyer.

The paper is organized as follows; the problem is defined in section 2. Notations and assumptions are discussed in section 3. Section 4 is discussed with joint optimization for the buyer and the vendor integrated model. In section 4.1 investments for quality improvement. In section 5, Solution procedure for joint optimization model is presented. In section 6, an algorithm is developed to find the optimal solution to the integrated model. In section 7, numerical examples are presented. Finally, we draw conclusion and further researches are summarized in Section 8.

2. Problem definition

Consider a two supply chain model for a product which consists of a single vendor and single buyer. The final demand for this product is assumed to be deterministic but price sensitive. The objective is to determine the number of shipments m , the selling price δ as well as the optimal order quantity Q, so that the total profits of the vendor and the buyer total profit TP are maximized.

3. Notations and Assumptions

To develop the proposed model, we adopt the following notations and assumptions which are similar to those used

3.1. Notations.

To develop the proposed model, we adopt the following notations

P	Production rate of the vendor , $P > D$
Q	Optimal order quantity.
A_v	Vendor's setup cost.
A_b	Buyer's ordering cost.
δ	The buyer unit selling price.
$D(\delta)$	Demand rate as a function of unit selling price.
h_v	Inventory holding cost for the vendor per year.
h_b	Inventory holding cost for the buyer per year.
m	The number of deliveries in which the product is delivered from the vendor to the buyer in one production cycle, a positive integer.
g	Vendor unit defective cost per defective item.
θ_0	Original percentage of defective products produced once the system is in the out of control state prior to investment.
θ	Percentage of defective products produced once the system is in the out of control state.
$q(\theta)$	Vendor's capital investments require reducing the out of control probability from θ_0 toθ
i	Vendor's fractional opportunity cost of capital per unit time.

ε	Percentage decrease in θ per dollar increase investment in $q(\theta)$
TP	Total Profit for both the vendor and the buyer.

3.2. **Assumptions.** To develop the proposed model, we adopt the following assumptions

1. The integrated system of single-vendor and single-buyer for a single product is considered.
2. The buyer faces a linear demand $D(\delta) = a - b\delta (a > b > 0)$ as a function of its unit selling price.
3. The inventory is continuously reviewed and replenished.
4. Shortage is not allowed.
5. Shipments from the vendor to the buyer use a lot-for-lot policy
6. The relationship between lot size and quality is formulated as follows: while vendor is producing a lot, the process can go out of control with a given probabilityθ each time another unit is produced. The process is assumed to be in control in the beginning of the production process. Once out of control, the process produces defective items and continues to do so until the entire lot is produced. (This assumption is in line with Porteus[5]).
7. The out of control probability is a continuous decision variable, and is described by a logarithmic investment function. The quality improvement and capital investment is represented by$q(\theta) = q_1 ln\left(\frac{\theta_0}{\theta}\right)$ for$(0 < \theta \leq \theta_0)$,where θ_0 is the current probability that the production process can go out of control, and $q_1\left(\frac{1}{\varepsilon}\right)$, with denoting the percentage decrease in θ per dollar increase in $q(\theta)$. The application of the logarithmic function on capital investment and quality improvement has been proposed by many authors, for example, Porteus[5]; Keller et al.[7] and Hong et al.[11].
8. The inventory holding cost at the buyer is higher than that at the vendor, i.e. $h_b > h_v$.
9. All defective items produced are detected after the production cycle is over, and rework cost for defective items will be incurred.
10. Defective item rework cost per unit time, the expected number of defective items in a run ofmQ size with a given probability of θ that the process can go out of control is $\frac{m^2Q^2\theta}{2}$. Thus, the defective cost per unit time is given $\frac{gmDQ\theta}{2}$.
11. Opportunity cost of quality improvement investment $q_1 ln\left(\frac{\theta_0}{\theta}\right)$.

4. Model formulation

In this section, the optimal policy of the integrated system is derived. However, for comparative purposes, we first obtain the buyer and the vendor policies if each party optimizes its profit independently. The policies and profits are then compared to the case of integrated system when they cooperate; particularly in information sharing. We assume that the buyer faces a linear demand $D(\delta) = a - b\delta (a > b > 0)$ as a function of its unit selling price. As $D(\delta) > 0$. The maximum selling price is $\frac{a}{b}$ i.e. $\delta < \frac{a}{b}$. The total profit (TP) is equal to the gross revenue minus the sum of purchasing, order processing for both vendor and buyer, and inventory holding costs for both vendor and buyer costs. The total profit (TP)

$$\begin{aligned}\max_{Q,m} TP(Q,m) &= a\delta - b\delta^2 - \frac{(a-b\delta)(A_v + mA_b)}{mQ} - \frac{h_v Q}{2}\left[m\left(1 - \frac{a-b\delta}{P}\right) - 1\right. \\ &\left. + \left(\frac{2(a-b\delta)}{P}\right)\right] \end{aligned} \tag{1}$$

s.t. $\delta < \frac{a}{b}, Q > 0, m$ is an integer

4.1. **Investment for Quality Improvement.**

Based on (1), we wish to study the effect of investment for quality improvement. Consequently, the objective of the integrated model is to maximize the profit for the optimal ordering, shipment, and investment for quality improvement by simultaneously determining the optimal values of Q, m, θ subject to $(0 < \theta \leq \theta_0)$ Thus, the total profit per year that is,

$$\begin{aligned} TP(Q,m,\theta) &= a\delta - b\delta^2 - \frac{(a-b\delta)(A_b + \frac{A_v}{m})}{Q} - \frac{Q}{2}\left\{h_b + gm\theta(a-b\delta) + \right. \\ &\left. h_v\left[m\left(1 - \frac{a-b\delta}{P}\right) - 1 + \left(\frac{2(a-b\delta)}{P}\right)\right]\right\} - iq_1 ln\left(\frac{\theta_0}{\theta}\right) \end{aligned} \tag{2}$$

for $0 < \theta_0 < \theta$ where i is the vendor's fractional opportunity cost of capital per unit time

5. Solution procedure for total profit model

The system for total profit is concave in Q for given values of the buyer's selling price δ and the number of shipments m . The optimal order quantity and process quality can then be obtained as

$$\begin{aligned} \frac{\partial TP}{\partial Q} &= \frac{(a-b\delta)(A_b + \frac{A_v}{m})}{Q^2} - \frac{1}{2}\left\{h_b + gm\theta(a-b\delta)\right. \\ &\left. + h_v\left[m\left(1 - \frac{a-b\delta}{P}\right) - 1 + \left(\frac{2(a-b\delta)}{P}\right)\right]\right\} \end{aligned}$$

$$\frac{\partial TP}{\partial Q} = 0$$

(3) $$Q^* = \sqrt{\frac{2(a-b\delta)(A_b + \frac{A_v}{m})}{\left\{h_b + gm\theta(a-b\delta) + h_v\left[m\left(1 - \frac{a-b\delta}{P}\right) - 1 + \left(\frac{2(a-b\delta)}{P}\right)\right]\right\}}}$$

$$\frac{\partial TP}{\partial \theta} = \frac{-Qgm(a-b\delta)}{2} + \frac{iq_1}{\theta}$$

$$\frac{\partial TP}{\partial \theta} = 0$$

(4) $$\theta^* = \frac{2iq_1}{Qgm(a-b\delta)}$$

Substituting expression (3) into the cost function (2), we obtain

(5) $$TP(Q,m,\theta) = a\delta - b\delta^2 - \sqrt{2(a-b\delta)(A_b + \frac{A_v}{m})\left\{G + G_1\right\}} + iq_1\left(\frac{\theta_0}{\theta}\right)$$

where $G = h_b + gm\theta(a-b\delta)\text{and}G_1 = \left\{h_v\left[m\left(1 - \frac{a-b\delta}{P}\right) - 1 + \left(\frac{2(a-b\delta)}{P}\right)\right]\right\}$

For a given value of δ ,maximizing TP is equivalent to minimizing the following expression

$$TP = 2(a-b\delta)\left(A_b + \frac{A_v}{m}\right)\left\{h_b + gm\theta(a-b\delta)\left[h_v\left[m\left(1 - \frac{a-b\delta}{P}\right) - 1 + \left(\frac{2(a-b\delta)}{P}\right)\right]\right]\right\}$$

We first assume that m is a continuous variable. Taking the first and second partial derivatives of TP with respect to m,we obtain

$$\frac{\partial TP}{\partial m} = 2\left(a-b\delta\right)\left[\frac{m^2A_bh_v(P-a+b\delta) - 2A_vh_v(a-b\delta) - A_v(h_b-h_v)P + m^2PA_bg(a-b\delta)Q}{m^2P}\right]$$

$$\frac{\partial TP}{\partial m^2} = 4(a-b\delta)A_v\left[\frac{2h_v(a-b\delta) + (h_b-h_v)P}{m^3P}\right]$$

As$h_b > h_v$ and $\delta < \frac{a}{b}$.the second derivative is positive. Consequently, TP is strictly convex in m. Solving $\frac{\partial TP}{\partial m}$, we get

(6) $$m = \sqrt{\frac{A_v\left[2h_v(a-b\delta) + (h_b+h_v)P\right]}{A_b\left[h_v(P-a+b\delta) + Pg(a-b\delta)\theta\right]}}$$

Recalling the integrity constraint on m one gets,

$$m^*(m-1) \le \frac{A_v\left[2h_v(a-b\delta) + (h_b+h_v)P\right]}{A_b\left[h_v(P-a+b\delta) + Pg(a-b\delta)\theta\right]} \le m^*(m-1)$$

6. Algorithm

Step 1.
Set $\theta = \theta_0$ and Repeat (i) - (iii) until no change occurs in the values of Q, m, θ. Denote these solution by Q^*, m^*, θ^* respectively.
(i) Substitute θ into equation (6) to find m.
(ii) Use θ and m to compute Q from equation (3).
(iii) Use Q and m to determine θ from equation (4).
Step 2.
If $\theta \leq \theta_0$, then the solution found in step 1 is optimal solutions and go to step (4). Otherwise go to step (3).
Step 3.
Set $\theta^* = \theta_0$, then substitute θ^* into equation (6) to compute m^* and determine Q^* from equation (3) using the values of θ^*, m^* . Then go to step (4).
Step 4.
Determine $TP\left(Q^*, m^*, \theta^*\right)$ from equation (2) using the values of Q^*, m^*, θ^*. $TP\left(Q^*, m^*, \theta^*\right)$ is a set of optimal solutions.

7. Numerical Examples

Consider an inventory system with the following characteristics $P = 3200$/year, $A_v = 400$/unit, $A_b = 25$/unit, $h_b = 5$/unit, $h_v = 4$/unit, $i = 0.2$, $g = 15$/unit, $q(\theta) = 400log\left(\frac{\theta_0}{\theta}\right)$, where $\theta_0 = 0.0002$. Besides, we take $q_1 = 400$, $a = 1500$, $b = 10$, $\delta = 15.50$/unit/year. Applying the solution procedure we have number of shipments $m^* = 5$, optimal order quantity $Q^* = 128$ units process quality $\theta^* = 0.000012392$ units , Total profit $TP = 18420$. In order to gain insight into this effect, different levels of a, b, δ have been considered in the table (1), (2) and (3) and figures (1)-(6).

Table 1. Decision variable under joint optimization (a)

a	m	Q	θ^*	TP
500	3	93	0.000011082	3936
1000	4	114	0.000027683	11084
1500	5	128	0.000012392	18420

8. Conclusion

In this paper, we developed an integrated production inventory marketing model for joint optimization model. In this paper demand function is more price-sensitive

Table 2. Decision variable under joint optimization (b)

a	m	Q	θ^*	TP
10	5	128	0.000012392	18420
20	5	118	0.000015193	16124
30	4	129	0.000019973	13846
40	4	117	0.000025900	11591
50	4	104	0.000035367	9358
60	3	115	0.000054242	7160

Table 3. Decision variable under joint optimization (δ)

a	m	Q	θ^*	TP
5.50	6	117	0.000010515	5460
10.50	5	131	0.0000116743	12189
15.50	5	128	0.000012392	18420

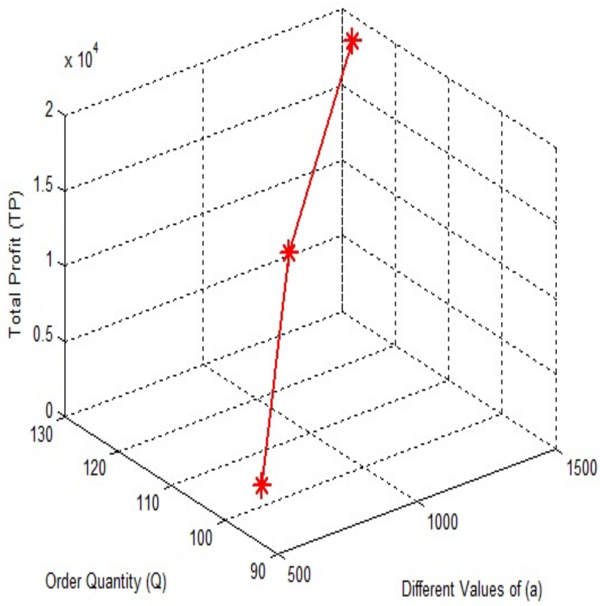

Figure 1. Graphical representation for optimal solution in Different values a

demand. In the present work we have developed an integrated production inventory model in which the objective is to maximize the total profit of the buyer and the vendor by optimizing the optimal order quantity, number of shipments and process quality. Moreover, a capital investment, which is necessary to improve the quality of the product, is also considered in the total profit function. This paper attempts to determine the optimal order quantity and capital investments in process quality improvement for production system such that the total profit is maximized. The cost of

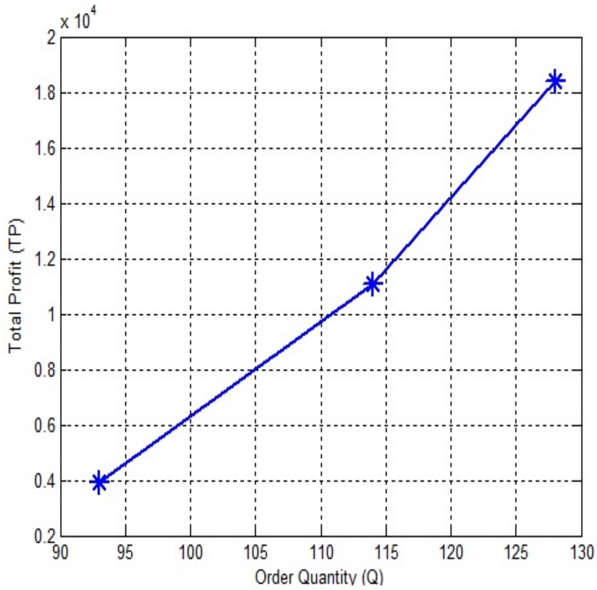

FIGURE 2. Graphical representation optimal solution

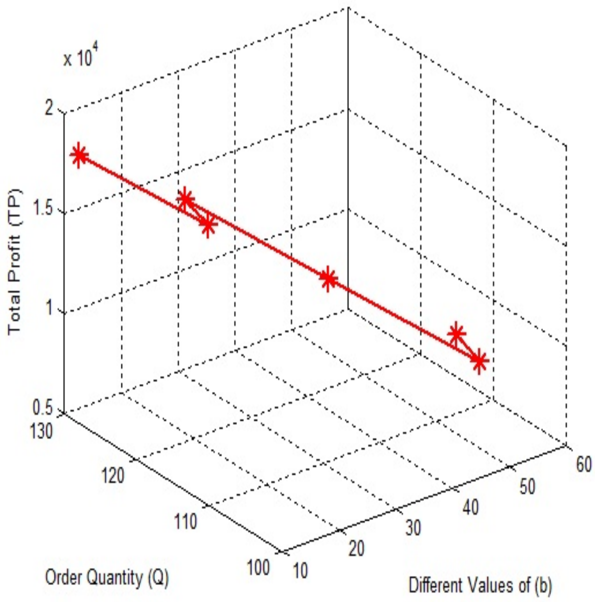

FIGURE 3. Graphical representation for optimal solution in Different values b

capital (i.e., opportunity cost) is one of the key factors in making the inventory and investment decisions. The main purpose of this paper is to present the vendor buyer integrated production inventory model with investment for quality improvement. In

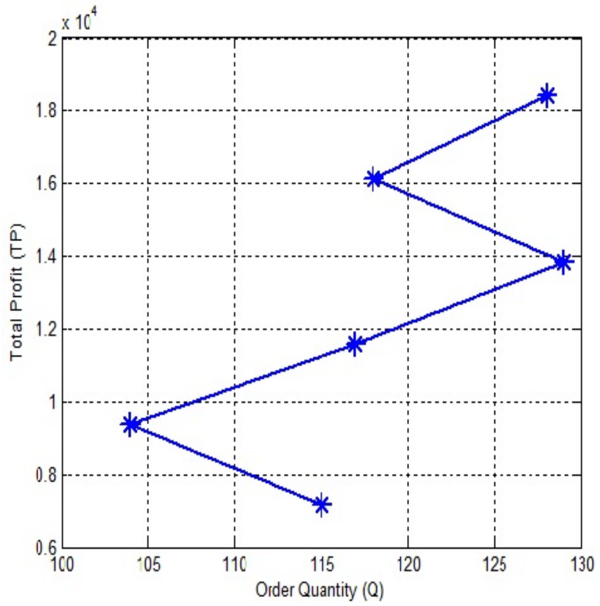

FIGURE 4. Graphical representation optimal solution

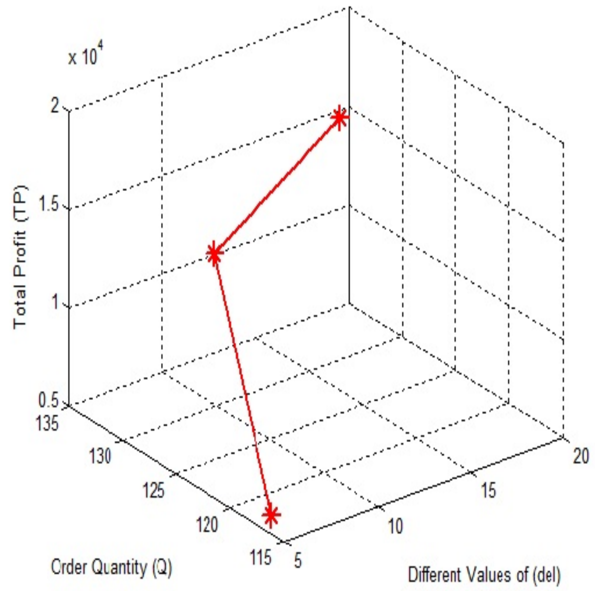

FIGURE 5. Graphical representation for optimal solution in Different values δ

our model, the capital investment for quality improvement is assumed to be a logarithmic function. Also the solution procedure is developed in order to find the total profit of the vendor and the buyer which is to be maximized. Then, an algorithm procedure is developed in order to find that the optimal order quantity, number of shipments, process quality and total profit is maximized, and our approach is illustrated through

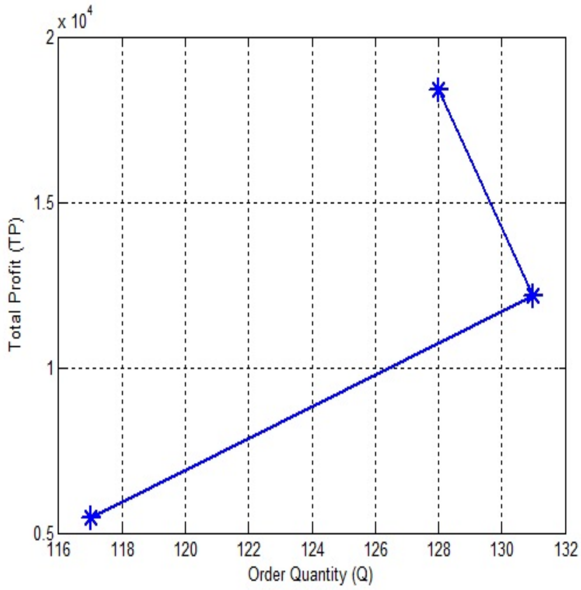

FIGURE 6. Graphical representation optimal solution

a numerical example is given to illustrate the solution procedure.Graphical representation is also presented to illustrate the proposed model. Developing the model to the multi-supplier case is also proposed for the future research.

Acknowledgements

The research work is supported by DST INSPIRE Fellowship, Ministry of Science and Technology, Government of India under the grant no. DST/INSPIRE Fellowship/2011/413 dated 13.03.2012, and UGC- SAP, Department of Mathematics, Gandhigram Rural Institute-Deemed University, Gandhigram - 624302, Tamilnadu, India.

References

[1] A. Banerjee, A joint economic-lot-size model for purchaser and vendor, Decision Sciences. 17 (1986), 292-311.

[2] A. H. Lau, H. S. Lau, Effects of a demand-curve's shape on the optimal solutions of a multi-echelon inventory/pricing model, European Journal of Operational Research. 147 (2003), 530-548.

[3] B. R. Sarker, E. R. Coates, Manufacturing setup cost reduction under variable lead times and finite opportunities for investment, International Journal of Production Economics. 49 (1997), 237-247.

[4] C. Hofmann, Investments in modern production technology and the cash flow-oriented EPQ-model, International Journal of Production Economics. 54 (1988), 193-206.

[5] E. L. Porteus, Optimal lot sizing, process quality improvement and setup cost reduction, Operations Research. 34 (1986), 137-144.

[6] F. Nasri, J. F. Affisco, M. J. Paknejad, Setup cost reduction in an inventory model with finite-range stochastic lead times, International Journal of Production Research. 28 (1990), 199-212.

[7] G. Keller, H. Noori, Impact of investing in quality improvement on the lot size model, International Journal of Management Science. 15 (1988), 595-601.

[8] H. Hwang, D. B. Kim, Y. D Kim, Multiproduct economic lot size models with investments costs for setup reduction and quality improvement, International Journal of Production Research. 31 (1993), 691-703.

[9] I. Moon, Multiproduct economic lot size models with investments costs for setup reduction and quality improvement: Review and extensions, International Journal of Production Research. 32 (1994), 2795-2801.

[10] I. S. Bakal, J. Geunes, H. E. Romeijn, Market selection decisions for inventory models with price-sensitive demand, Journal of Global Optimization. 41 (1988), 633-657.

[11] J. D. Hong, J. C. Hayya, Joint investment in quality improvement and setup reduction, Computers & Operations Research. 22 (1995), 567-574.

[12] L. R. Khan, R. A. Sarker, An optimal batch size for a JIT manufacturing system, Computers & Industrial Engineering. 42 (2002), 127-136.

[13] L. Y. Ouyang, H. C. Change, Impact of investing in quality improvement on (Q, r, L) model involving imperfect production process, Production Planning & Control. 11 (2000), 598-607.

[14] M. Ben-Daya, M. Darwish, K. Ertogral, The joint economic lot sizing problem: review and extensions, European Journal of Operational Research. 185 (2008), 726-742.

[15] M. J. Paknejad, J. F. Affisco, The effect of investment in new technology on optimal batch quantity, Proceedings of the Northeast Decision Sciences Institute, DSI, RI, USA. (1987), 118-120.

[16] M. J. Rosenblatt, H. L. Lee, Economic production cycles with imperfect production processes, IIE Transactions. 18 (1986), 48-55.

[17] M. S. Sajadieh, M. R. A. Jokar, Optimizing shipment, ordering and pricing policies in a two-stage supply chain with price-sensitive demand, Transportation Research Part. 45 (2009), 564-571.

[18] R. Hall, Zero Inventories, Homewood, Illinois: Dow Jones-Irwin, (1983).

[19] S. K. Goyal, A joint economic-lot-size model for purchaser and vendor: a comment, Decision Sciences. 19 (1988), 236-241.

[20] S. K. Goyal, An integrated inventory model for a single product system, Operations Research Quarterly. 28 (1977), 539-545.

[21] S. K. Goyal, An integrated model for a single supplier-single customer problem, International Journal of Production Research. 15 (1976), 107-111.

[22] S. Ray, Y. Gerchak E. M. Jewkes, Joint pricing and inventory policies for make-to-stock products with deterministic price-sensitive demand, International Journal of Production Economics. 97 (2005), 143-158.

[23] S. Viswanathan, Q. Wang, Discount pricing decisions in distribution channels with price-sensitive demand, European Journal of Operational Research. 149 (2003), 571-587.