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A PRODUCTION-INVENTORY MODEL WITH VARIABLE PRODUCTION COST AND PROBABILISTIC DETERIORATION

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ABSTRACT. This paper deals with an economic production quantity (EPQ) model for deteriorating items with price and advertisement dependent demand. We have considered three types of continuous probabilistic deterioration functions to determine the total inventory cost. Here, the rate of replenishment is considered to be a variable and the generalized unit production cost function is formulated by incorporating costs of several factors like raw material, labour, replenishment rate, advertisement and other factors of the manufacturing system. The selling price of a unit is determined by a mark-up over the production cost. This model aids in minimizing the total inventory cost of the manufacturer by finding the optimal cycle length and the optimal quantity. The optimal solution of the model is illustrated with the help of numerical examples. A numerical comparison between the three models is also given. Finally sensitivity analysis and graphical representations of the total cost functions are given to demonstrate the model.
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1. Introduction

In the production system, the output (i.e., product) of a firm depends upon the combination of production factors. These factors may be raw material, number of labours, production procedure, firm size, quality of the product etc. Because of these changes, production rate and cost are changed too. Moreover, due to increased equipment wear, overtime labor, greater defect rates, etc, the production rate should be a decision variable in determining the lot size or run length of a manufactured item. Also, in practice, the unit production cost varies with production rate, raw material cost, labour charge and advertisement cost of the production process. In this investigation, unit production cost is dependent on the cost of raw materials, labour charges, advertisement cost, produced units, etc.

Generally, high selling price of an item affects the demand, which in turn affects the decisions about production and inventory policies. The advertising by the sales team is one of the most important factors used to increase the retailer's profit in modern marketing system. The purpose of the advertisement is to enhance potential customer's responses to a business organization. In general, this strategy is only to sell more items in a short time. Increase in the advertising intensity not only increases the probability of successful marketing targets but also the demand from the customers. Therefore, the more investment in advertising gives more profits for the company. In this direction, our model encourage the retailers to consider the demand as an increasing function of advertising parameter with decreasing value of selling price.

Deterioration plays a significant role in many inventory systems. Deterioration is defined as decay, damage, dryness and spoilage. It is the process in which an item loses its utility and becomes useless. In the real life situation, it is too difficult to preserve highly volatile items like alcohol, liquid medicines, blood, etc., for all manufacturing sectors. These types of items may deteriorate over time. So decay or deterioration of physical goods in stock is a very realistic factor and there is a big need to consider this in inventory modeling. The next section reviews the relevant literature on inventory management issues from the perspective of the production industries.

2. Literature Review

In the production lot size models, both production rate and production cost are assumed to be constant and independent of each other. Several researchers developed inventory models for a single item or multiple items with a constant or variable production rate (as a function of demand and/or on-hand inventory). In this connection, one may refer the works of Misra [1], Mandal [2], Mandal [3] and Maiti [4]. In their models, the production cost is assumed to be constant. Khouja [5] provided an economic production lot size model under volume flexibility where unit production cost depends upon the raw material, labour force and tool wear out cost incurred. Here, unit production cost is a function of production rate. Bhandari [6] extended the work of Khouja [5] by including the marketing cost and taking a generalized unit cost function in to account.

In the present competitive market, the marketing policies and promotion of a product in the form of advertisement, display, etc. change the demand pattern of that item amongst the customers and have a motivational effect on the people to buy more. Also, the selling price of an item is one of the important factors in selecting an item for use. It is commonly seen that higher selling price causes decrease in demand whereas lower selling price has the reverse effect. Hence, it can be concluded that the demand of an item is a function of marketing cost and selling price of an item. Kotler [7] incorporated marketing policies into inventory decisions and studied the relationship between economic order quantity and decision. Ladany [8] discussed the effect of price variation on demand and consequently on EOQ. Subramanyan [9], Urban [10], Goyal [11], Luo [12] developed inventory models incorporating the effect of price variations and advertisement on demand. Mondal [13] developed an inventory models for defective items incorporating marketing decisions with variable production cost. Chang [14] studied an economic manufacturing quantity model for a two-stage assembly system with imperfect processes and variable production rate. Soni [15] investigated an optimal strategy for an integrated inventory system involving variable production and defective items under retailer partial trade credit policy. Deane [16] considered a model for scheduling online advertisements to maximize revenue under variable display frequency.

Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreased usefulness. Ghare and Schrader [17] were the first authors who considered the effect of deteriorating items in inventory model. They discussed the general EOQ (economic order quantity) model with direct spoilage and exponential deterioration. Covert and Philip [18] extended the work of Ghare and Schrader [17] with Weibull distribution and gamma distribution. Philip [19] deduced a three parameter Weibull distribution for the deteriorating time. Misra [20] developed optimal production lotsize model with finite production rate and different types of deterioration rates but without any backordering. Shah [21] discussed an order-level lot size model for both exponential and Weibull distributed deterioration with backordering. Optimal selling price and lot size, time varying deterioration and partial backlogging was developed by Sana [22]. Ghosh [23] discussed an optimal price and lot size determination for a perishable product under conditions of finite production, partial backordering and lost sale. Mahata [24] gave an optimal strategy for an EOQ model with non-instantaneous receipt and exponentially deteriorating items under permissible delay in payments. Maihami [25] considered an inventory control model for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand. Krishnamoorthi [26] developed an economic production lot size model for product life cycle (maturity stage) with defective items with shortages. Sarkar [27] found out an EOQ model with delay in payments and a variable deterioration rate. Sarkar [28] derived a probabilistic deterioration model to find the integer number of deliveries and lot size with the help of an algebraic procedure. Sarkar [29] investigated an economic manufacturing quantity model with probabilistic deterioration in a production system. Palanivel [30] developed an EPQ model for deteriorating items with variable production cost, time dependent holding cost and partial backlogging under inflation.

In the present work, a deterministic EPQ model for deteriorating items with price and advertisement dependent demand and variable production rate is considered. Moreover, the unit production cost is a function of raw materials, labour charges, advertisement cost, produced units, etc. and the selling price is a mark-up over the unit production cost. Also in this model, we have considered that the deterioration function follows probability distribution like (a) uniform distribution, (b) triangular distribution, and (c) beta distribution. We have used the suitable numerical examples to illustrate the model. Sensitivity analysis of the optimal solution with respect to major parameters of the system is carried out.

The rest of the paper is organized as follows: In Section 3, the assumptions and notations, which are used throughout this article, are described. In Section 4, the mathematical model to minimize the total annual inventory cost is established. Section 5 presents solution procedure to find the optimal cycle length and optimal production quantity. Numerical examples are provided in Section 6 to illustrate the theory and the solution procedure. This is followed by sensitivity analysis and conclusion.

3. Assumptions and Notations

To develop the mathematical model, the following assumptions are being made:

3.1 Assumptions

1. A single item is considered over an infinite planning horizon.
2. The demand rate D is a deterministic function of selling price, s and advertisement cost, A_c per unit item i.e. $D(A_c, s) = A_c^{\gamma}(x - ys), x.y, \gamma \geq 0$.
3. The production rate per unit time P is variable, which is more than the demand rate.
4. The unit production cost $v(P) = C_{rw} + A_c + L/P^g + KP^h$ where C_{rw}, L and K are non - negative real numbers to be provide the best fit for the estimated unit cost function. C_{rw}, L and K represent the raw material cost, labour charges and a positive constant. Also g, h are chosen to provide the feasible solution to the model. We shall use v and $v(P)$ interchangeable in the rest of the paper.
5. The selling price is determined by a mark - up over the unit production cost, v i.e. $s = n.v, n > 1$, where n is a mark - up .
6. Deterioration follows continuous probability distribution function as (a) uniform distribution (b) triangular distribution (c) beta distribution.
7. There is no replacement or repair of deteriorated items takes place in a given cycle.
8. The lead time is zero and shortages are not allowed.

3.2 Notations

In addition, the following notations are used throughout this paper:

t_1	Duration of the replenishment rate.
T	The length of the production inventory cycle.
$q_1(t)$	The inventory level at time $t, 0 \leq t \leq t_1$.
$q_2(t)$	The inventory level at time $t, t_1 \leq t \leq T$.
θ	The probabilistic deterioration rate of the item.

A	The setup cost per cycle.
C_1	The holding cost per unit per unit time.
Q	The production lot size per cycle.
TC	The total cost of the system.

4. Formulation and solution of the model

The inventory system is developed as follows: The inventory cycle starts at $t = 0$ with zero inventory and increases up to time t_1 at a rate P and also simultaneously decreases due to demand and deterioration. In the interval $[t_1, T]$, the inventory level is decreasing only due to demand rate and deterioration. Finally, the inventory reaches the zero level at time T . The related figure-1 of the model is as follows.

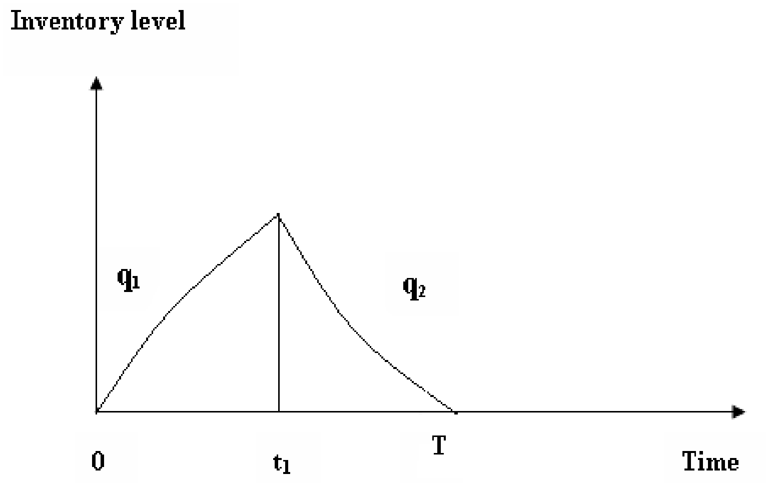

Figure 1. Graphical representation of the inventory system

Based on the above description, during the time interval $[0, t_1]$, the differential equation representing the inventory status is given by

$$\frac{dq_1(t)}{dt} + \theta q_1(t) = P - D, \ 0 \le t \le t_1 \tag{1}$$

With the condition $q_1(0) = 0$, the solution of equation (1) is

$$q_1(t) = \frac{P - D}{\theta}\left[1 - e^{-\theta t}\right], \ 0 \le t \le t_1 \tag{2}$$

In the second interval $[t_1, T]$, the differential equation below represents the inventory status:

$$\frac{dq_2(t)}{dt} + \theta q_2(t) = -D, \ t_1 \le t \le T \tag{3}$$

With the condition $q_2(T) = 0$, we get the solution of equation (3) is

$$q_2(t) = \frac{D}{\theta}\left[e^{\theta(T-t)} - 1\right], \ t_1 \le t \le T \tag{4}$$

Put $t = t_1$ in equations (2) and (4) we find the value of t_1 as

$$t_1 = \frac{1}{\theta} \ln \left\{ 1 + \frac{D}{P}[e^{\theta T} - 1] \right\}. \tag{5}$$

Since the production occurs in the continuous time-span $[0, t_1]$, then the production lot size in the problem is,

$$Q = Pt_1. \tag{6}$$

The maximum inventory level S in the problem is given by,

$$S = q(t_1) = \frac{P - D}{\theta} \left[1 - e^{-\theta t_1} \right]. \tag{7}$$

Now we want to find the different inventory costs as:
Setup cost =A.
The holding cost HC is given by

$$\begin{aligned} HC &= C_1 \left[\int_0^{t_1} q_1(t)dt + \int_{t_1}^{T} q_2(t)dt \right] \\ &= \frac{C_1}{\theta^2} \left\{ P \left[\theta t_1 + e^{-\theta t_1} - 1 \right] + D \left[e^{\theta(T-t_1)} - \theta T - e^{-\theta t_1} \right] \right\} \end{aligned} \tag{8}$$

Since $q_1(t_1) = q_2(t_1)$, which implies equation (8) can be rearranged as follows:

$$HC = \frac{C_1}{\theta} \left[Pt_1 - DT \right]. \tag{9}$$

The deteriorating cost DC is given by

$$DC = p \left[Pt_1 - DT \right]. \tag{10}$$

Total average cost per cycle = setup cost + inventory holding cost + deterioration cost. So, the total variable cost per unit time is

$$\begin{aligned} TC &= \frac{1}{T} \left[A + HC + DC \right] \\ &= \frac{A}{T} + \frac{C_1 + \theta p}{\theta T} \left[Pt_1 - DT \right] \\ &= \frac{A}{T} + \frac{C_1 + \theta p}{\theta T} \left[\frac{P}{\theta} \ln \left\{ 1 + \frac{D}{P}[e^{\theta T} - 1] \right\} - DT \right] \end{aligned} \tag{11}$$

Now, we consider the deterioration rate θ follows three different types of probability distribution function as $\theta = E[f(x)]$, where $f(x)$ follows (1) uniform distribution, (2) triangular distribution and (3) beta distribution.

Case 1: θ follows uniform distribution

We consider that θ follows uniform distribution as $\theta = E[f(x)] = (a+b)/2, a > 0, b > 0, a < b$. Now from equation (11), we have

$$TC_1 = \frac{A}{T} + \frac{2C_1 + (a+b)p}{(a+b)T} \left[\frac{2P}{(a+b)} \ln \left\{ 1 + \frac{D}{P} \left[e^{(a+b)T/2} - 1 \right] \right\} - DT \right] \tag{12}$$

Case 2: θ follows triangular distribution

Now, we assume that θ follows triangular distribution as $\theta = E[f(x)] = (a+b+c)/3$, where $f(x)$ is the probability density function of triangular distribution with lower limit a, upper limit b and mode c as well as $a < b$ and $a \leq c \leq b$.

Therefore, the equation of TC can be written as

$$TC_2 = \frac{A}{T} + \frac{3C_1 + (a+b+c)p}{(a+b+c)T}\left[\frac{3P}{(a+b+c)}\ln\left\{1 + \frac{D}{P}\left[e^{(a+b+c)T/3} - 1\right]\right\} - DT\right] \tag{13}$$

Case 3: θ follows beta distribution

Now, we consider that θ follows beta distribution as $\theta = E[f(x)] = \alpha/(\alpha+\beta)$, where $f(x)$ follows beta distribution which is a continuous probability distributions defined on the interval (0, 1) parameterized by two positive parameters, denoted by α and β.

From equation (11), we have

$$TC_3 = \frac{A}{T} + \left[C_1 + \frac{\alpha p}{\alpha+\beta}\right]\left[\frac{\alpha+\beta}{\alpha T}\right]\left[\frac{P(\alpha+\beta)}{\alpha}\ln\left\{1 + \frac{D}{P}\left[e^{\alpha T/(\alpha+\beta)} - 1\right]\right\} - DT\right] \tag{14}$$

5. Solution procedure

To find the optimal solution, the following procedures are considered: Now the production rate that minimizes the unit production cost is given by $v'(P) = 0$. Therefore $v'(P) = 0$ implies

$$\frac{-gL}{P^{g+1}} + KhP^{h-1} = 0$$

$$\Rightarrow P = \left(\frac{Lg}{Kh}\right)^{\frac{1}{g+h}} \tag{15}$$

5.1 Determination of the optimal cycle length T

Here the objective is to minimize the total inventory cost by finding the optimal cycle length. First we need the following theorem and lemma which are used to find the optimal cycle length.

Theorem 1: (Intermediate Value Theorem)

Let g be a continuous function on the closed interval $[a, b]$ and let $g(a).g(b) < 0$. Then there exists $c \in (a, b)$ such that $g(c) = 0$.

Lemma 1:

If $f(t)$ is a continuous function on (a, b) and if $\frac{df}{dt}$ is non-decreasing, then $f(t)$ is convex.

Case 1: θ follows uniform distribution

We have $\frac{dTC_1(T)}{dT} = \frac{f_1(t)}{T^2}$, where

$$\begin{aligned} f_1(t) = -A - \frac{2C_1 + (a+b)p}{(a+b)} \Bigg\{ \left[\frac{2P}{(a+b)} \ln \left(1 + \frac{D}{P} \left[e^{(a+b)T/2} - 1 \right] \right) \right] \\ - \left[\frac{PTDe^{(a+b)T/2}}{P + D\left[e^{(a+b)T/2} - 1 \right]} \right] \Bigg\} \end{aligned} \tag{16}$$

Then both $f_1(T)$ and $\frac{dTC_1(T)}{dT}$ have the same sign. The optimal value of T, say T_1^*, is obtained by solving the equation $f_1(T) = 0$.

We also have $\frac{df_1(t)}{dT} = PDT(P-D)e^{(a+b)T/2}\left(C_1 + p(a+b)/2\right) > 0$ if $T > 0$, since $P > D$. Hence $f_1(t)$ is increasing on $(0, \infty)$, and so $\frac{dTC_1(T)}{dT}$ is increasing on $(0, \infty)$.

From lemma 1, $TC_1(T)$ is convex function on $(0, \infty)$. Also $f_1(0) = -A < 0$ and $\lim_{T\to\infty} f_1(T) = \infty > 0$ implies that

(17)

$$\frac{dTC_1(T)}{dT} = \begin{cases} < 0 & if\ T \in (0, T_1^*) \\ = 0 & if\ T = T_1^* \\ > 0 & if\ T \in (T_1^*, \infty) \end{cases}$$

Based upon the above arguments, the intermediate value theorem shows that the optimal solution, T_1^*, exists and unique.

Case 2: θ follows triangular distribution

Now We have $\frac{dTC_2(T)}{dT} = \frac{f_2(t)}{T^2}$, where

$$\begin{aligned} f_2(t) = -A - \frac{3C_1 + (a+b+c)p}{(a+b+c)} \Bigg\{ \left[\frac{3P}{(a+b+c)} \ln \left(1 + \frac{D}{P} \left[e^{(a+b+c)T/3} - 1 \right] \right) \right] \\ - \left[\frac{PTDe^{(a+b+c)T/3}}{P + D\left[e^{(a+b+c)T/3} - 1 \right]} \right] \Bigg\} \end{aligned} \tag{18}$$

Then both $f_2(T)$ and $\frac{dTC_2(T)}{dT}$ have the same sign. The optimal value of T, say T_2^*, is obtained by solving the equation $f_2(T) = 0$.

We also have $\frac{df_2(t)}{dT} = PDT(P-D)e^{(a+b+c)T/3}\left(C_1 + p(a+b+c)/3\right) > 0$ if $T > 0$, since $P > D$. Hence $f_2(t)$ is increasing on $(0, \infty)$, and so $\frac{dTC_2(T)}{dT}$ is increasing on $(0, \infty)$.

From lemma 1, $TC_2(T)$ is convex function on $(0, \infty)$. Also $f_2(0) = -A < 0$ and $\lim_{T\to\infty} f_2(T) = \infty > 0$ implies that

(19)

$$\frac{dTC_2(T)}{dT} = \begin{cases} < 0 & if\ T \in (0, T_2^*) \\ = 0 & if\ T = T_2^* \\ > 0 & if\ T \in (T_2^*, \infty) \end{cases}$$

Based upon the above arguments, the intermediate value theorem shows that the optimal solution, T_2^*, exists and unique.

Case 3: θ follows beta distribution

Now $\frac{dTC_3(T)}{dT} = \frac{f_3(t)}{T^2}$, where

$$f_3(t) = -A - \left[C_1 + \frac{\alpha p}{\alpha+\beta}\right]\left[\frac{\alpha+\beta}{\alpha}\right]\left\{\left[\frac{P(\alpha+\beta)}{\alpha}\ln\left\{1 + \frac{D}{P}\left[e^{\alpha T/(\alpha+\beta)} - 1\right]\right\}\right] - \left[\frac{PTDe^{\alpha T/(\alpha+\beta)}}{P + D\left[e^{\alpha T/(\alpha+\beta)} - 1\right]}\right]\right\} \tag{20}$$

Then both $f_3(T)$ and $\frac{dTC_3(T)}{dT}$ have the same sign. The optimal value of T, say T_3^*, is obtained by solving the equation $f_3(T) = 0$.

We also have $\frac{df_3(t)}{dT} = PDT(P-D)e^{\alpha T/(\alpha+\beta)}\left(C_1 + p\alpha/(\alpha+\beta)\right) > 0$ if $T > 0$, since $P > D$. Hence $f_3(t)$ is increasing on $(0, \infty)$, and so $\frac{dTC_3(T)}{dT}$ is increasing on $(0, \infty)$.

From lemma 1, $TC_3(T)$ is convex function on $(0, \infty)$. Also $f_3(0) = -A < 0$ and $\lim_{T\to\infty} f_3(T) = \infty > 0$ implies that

(21)

$$\frac{dTC_3(T)}{dT} = \begin{cases} < 0 & if\ T \in (0, T_3^*) \\ = 0 & if\ T = T_3^* \\ > 0 & if\ T \in (T_3^*, \infty) \end{cases}$$

Based upon the above arguments, the intermediate value theorem shows that the optimal solution, T_3^*, exists and unique.

Theorem 2:

a) $TC_1(T)$ has the unique optimal solution T_1^* on the non-negative interval $(0, \infty)$.
b) $TC_2(T)$ has the unique optimal solution T_2^* on the non-negative interval $(0, \infty)$.
c) $TC_3(T)$ has the unique optimal solution T_3^* on the non-negative interval $(0, \infty)$.

Proof: The above arguments imply that Theorem 2 holds.

From equations (5), (12), (13), (14) and (15), the optimal value of $t_1^*, TC_1^*, TC_2^*, TC_3^*$ and P^* respectively can be obtained.

6. Numerical Examples

Example 1

Consider an inventory system with the following data: Setup cost A = \$500/year; Holding cost C_1 = \$10/unit/year; Advertisement cost A_c = \$50/Advertisement; Labour Charge L = \$1500/year; Raw material Cost C_{rw} = \$45/unit/year; γ= 0.01; x = 200; y = 0.6; g = 0.76; h = 1.5; K= 0.01; n= 1.18; a = 0.15; b= 0.25.

Then we get the optimal values as t_1 = 0.6332 years, T = 0.8904 years, P = 144 units, D = 100 units and TC = \$1087.2. That is, the manufacturer should produce the product upto t_1 = 0.6332 years and maintain the inventory level upto T = 0.8904 years.

Figure - 2 shows that, TC is convex with respect to T when θ follows uniform distribution. If the deterioration follows uniform distribution, then the total cost decreases with the cycle length and it attains the minimum value \$1087.2 at T = 0.8904 years. If the production inventory cycle length is longer than 0.8904, then the total cost increases.

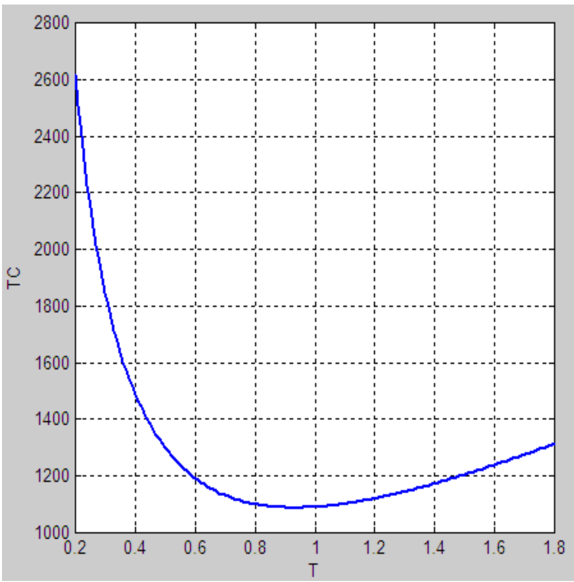

FIGURE 2. The total cost function with respect to T when θ follows uniform distribution.

Example 2

Consider an inventory system with the following data: Setup cost A = \$500/year; Holding cost C_1 = \$10/unit/year; Advertisement cost A_c = \$50/Advertisement; Labour Charge L = \$1500/year; Raw material Cost C_{rw} = \$45/unit/year; γ= 0.01; x = 200; y = 0.6; g = 0.76; h = 1.5; K= 0.01; n= 1.18; a = 0.15; b= 0.35; c = 0.25.

Then we get the optimal values as t_1 = 0.5818 years, T = 0.8151 years, P = 144 units, D = 100 units and TC = \$1182.2. That is, the manufacturer should produce the product upto t_1 = 0.5818 years and maintain the inventory level upto T = 0.8151 years.

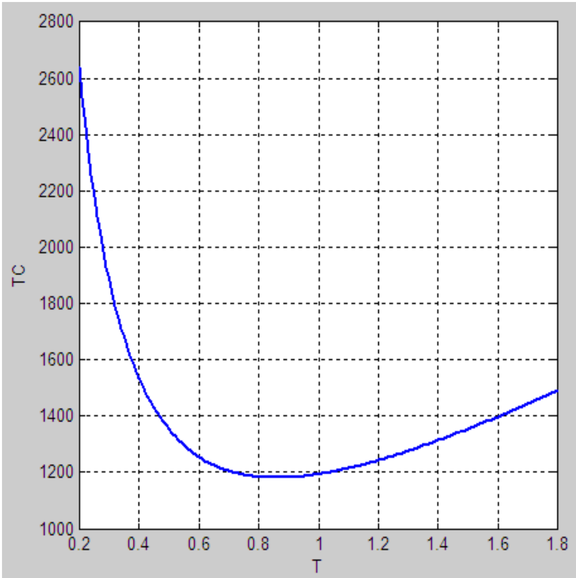

FIGURE 3. The total cost function with respect to T when θ follows triangular distribution.

From figure 3, it is observed that the total cost TC decreases with T and it attains the minimum value \$1182.2 at T = 0.8151 years, when the deterioration follows triangular distribution. If T crosses 0.8151 years, the total cost then increases. The graph (Fig. 3) shows that the function TC is convex with respect to T when θ follows triangular distribution.

Example 3

Consider an inventory system with the following data: Setup cost A = \$500/year; Holding cost C_1 = \$10/unit/year; Advertisement cost A_c = \$50/Advertisement; Labour Charge L = \$1500/year; Raw material Cost C_{rw} = \$45/unit/year; γ= 0.01; x = 200; y = 0.6; g = 0.76; h = 1.5; K= 0.01; n= 1.18; α = 0.15; β= 0.35.

Then we get the optimal values as t_1 = 0.5413 years, T = 0.7559 years, P = 144 units, D = 100 units and TC = \$1269.9. That is, the manufacturer should produce the product upto t_1 = 0.5413 years and maintain the inventory level upto T = 0.7559 years.

Figure 4 illustrates that, TC is convex with respect to T when θ follows beta distribution. Suppose deterioration follows beta distribution, the total cost decreases with the cycle length and it attains the minimum value \$1269.9 at T = 0.7559 years. Then the total cost increases, if the inventory cycle length is longer than 0.7559.

6.1. Comparison between the three models by the graphical representations

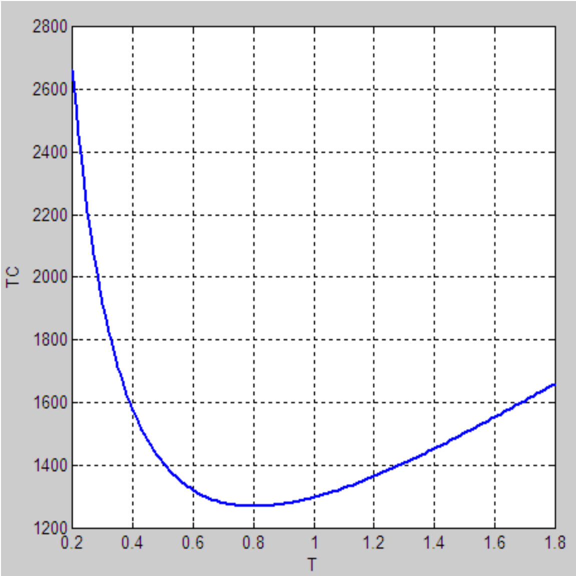

FIGURE 4. The total cost function with respect to T when θ follows beta distribution.

The comparison between the three probabilistic deteriorated models is done with the help of graphical representations. The following plots are due to the change of the three probabilistic deterioration functions.

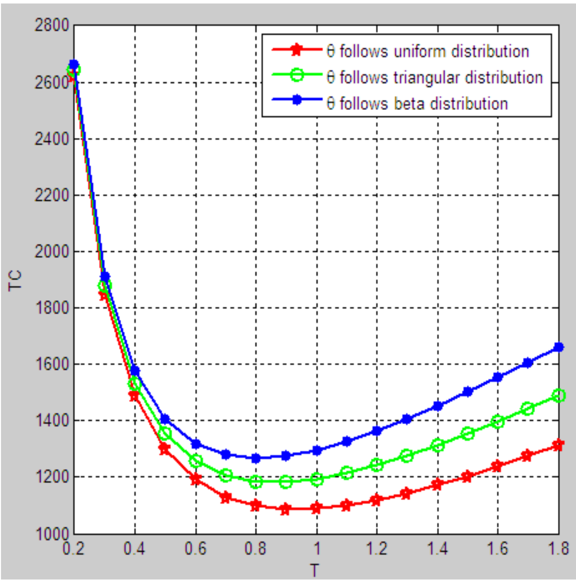

FIGURE 5. Total cost with respect to production cycle length for different distributions of θ

There are two figures (figure 5 and figure 6) and each of the two figures contains a combination of three plots in which the above plot is for beta distribution, middle-plot is for triangular distribution and last (downside) plot is for uniform distribution. From figures 5 and 6, it is

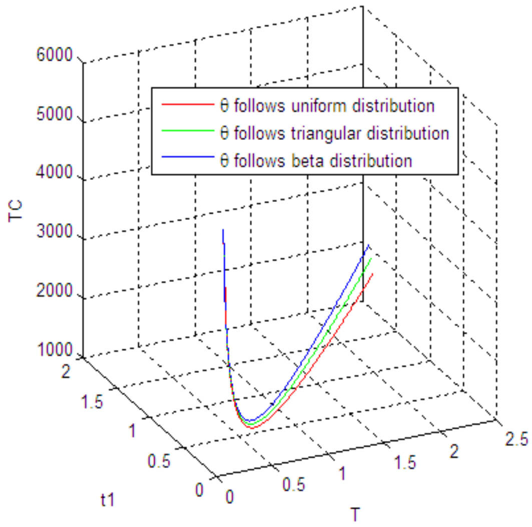

FIGURE 6. Total cost with respect to t_1 and T for different distributions of θ

observed that the total cost will be minimized when the deterioration follows uniform distribution.

7. Sensitivity Analysis

We now study the effects of changes in the values of the system parameters $A, C_1, A_c, L, C_{rw}, n, x$ and y on the optimal replenishment policy. We change one parameter at a time keeping the other parameters unchanged. The results are summarized in Table 1.

Based on our numerical results, we obtain the following managerial phenomena:

1. When the setup cost A and the holding cost C_1 are increasing, total cost TC is also increasing. That is, minimum setup cost and minimum cost for holding the items will minimize the total cost of the manufacturer.
2. When the advertisement cost A_c is increasing, the total cost TC is increasing. That is, the minimum advertisement cost will minimize the total cost of the manufacturer but more advertisement cost implies more demand of the product.
3. When the labour charge L and raw material cost C_{rw} are increasing, the total cost TC is also increasing. That is, the increasing of labour charge and raw material cost will increase the total cost of the manufacturer. In order to minimize the cost, the manufacturer should decrease the labour charge and raw material cost.
4. When the mark-up value n and the parameter y are increasing, the total cost TC will increase. That is, the increasing of n and y will increase the total cost of the manufacturer. But the increase in the value of the parameter x will give the variable changes in the total cost.

Parameter	Parameter value	TC(uniform)	TC(triangular)	TC(beta)
A	250	771.2625	839.1004	901.6759
	500	1087.2000	1182.2000	1269.9000
	750	1328.2000	1443.8000	1550.4000
C_1	5	1014.9000	1116.3000	1208.8000
	10	1087.2000	1182.2000	1269.9000
	15	1155.0000	1244.8000	1328.2000
A_c	25	847.4981	916.9883	981.2353
	50	1087.2000	1182.2000	1269.9000
	75	1257.5000	1374.3000	1482.1000
L	1000	604.4070	654.9356	701.6338
	1500	1087.2000	1182.2000	1269.9000
	2000	1298.2000	1406.3000	1525.7000
C_{rw}	35	998.7199	1084.6000	1162.7000
	45	1087.2000	1182.2000	1269.9000
	55	1161.9000	1265.7000	1361.3000
n	1.0	923.2228	1003.1000	1076.7000
	1.2	1100.0000	1196.3000	1285.1000
	1.5	1191.3000	1297.5000	1395.5000
x	150	1134.2000	1237.3000	1332.8000
	175	1189.7000	1295.4000	1393.1000
	200	1087.2000	1182.2000	1269.9000
y	0.4	537.8091	583.8414	626.3811
	0.6	1087.2000	1182.2000	1269.9000
	0.9	1123.2000	1225.5000	1320.3000

TABLE 1. Sensitivity analysis for various inventory parameters.

8. CONCLUSION

In this paper, the economic production lot size model for determining the optimal production length and the optimal total cost for deteriorating items are developed. We also developed the model incorporating both marketing decision and variable unit price depending on the rate of production. Here the deterioration function follows probability distribution like (a) uniform distribution, (b) triangular distribution, and (c) beta distribution. In each case, we find the minimum total cost associated with the system. Furthermore, numerical examples are provided to illustrate the model and the solution procedure. Also a numerical comparison between the three models is shown graphically. Finally, sensitivity analysis is carried out with respect to the key parameters.

The proposed model can be adopted in inventory control of production system such as food industries, fish, fruits, domestic goods etc.,

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