Asia Pac. J. Math. 2023 **10**:25

# SEMIDETACHED UP (BCC)-SUBALGEBRAS OF UP (BCC)-ALGEBRAS

AIYARED IAMPAN[1,*], N. RAJESH[2]

[1]Fuzzy Algebras and Decision-Making Problems Research Unit, Department of Mathematics, School of Science, University of Phayao, Mae Ka, Mueang, Phayao 56000, Thailand

[2]Department of Mathematics, Rajah Serfoji Government College (affiliated to Bharathidasan University), Thanjavur-613005, Tamilnadu, India

*Corresponding author: aiyared.ia@up.ac.th

Received Aug. 6, 2023

Abstract. The concept of semidetached UP (BCC)-subalgebras is introduced, and their properties are investigated. Several conditions for a semidetached structure in UP (BCC)-algebras to be a semidetached UP (BCC)-subalgebra are provided. The concepts of $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy UP (BCC)-subalgebras, $k$-left ($k$-right) $(q_k, \overline{\in} \vee \overline{q_k})$-fuzzy UP (BCC)-subalgebras, $(q_k, \overline{\in} \vee \overline{q_k})$-fuzzy UP (BCC)-subalgebras, and $(\overline{\in} \vee \overline{q_k}, \overline{\in} \vee \overline{q_k})$-fuzzy UP (BCC)-subalgebras are introduced, and relative relations and properties are discussed.

2020 Mathematics Subject Classification. 03G25, 03E72, 08A72.

Key words and phrases. UP (BCC)-algebra; semidetached structure; semidetached UP (BCC)-subalgebra; $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy UP (BCC)-subalgebra.

## 1. Introduction

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [8], played a vital role in generating some different types of fuzzy subgroups, called fuzzy subgroups, introduced by Bhakat and Das [1]. In particular, the idea of an $(\in, \in \vee q)$-fuzzy subgroup is an important and useful generalization of the Rosenfeld's fuzzy subgroup [9]. In UP-algebras, the concept of fuzzy UP-subalgebras, which is studied in [10], is also important and useful generalization of the well-known concepts. The notion of UP-algebras (see [4]) and the concept of BCC-algebras (see [7]) are the same concept, as shown by Jun et al. [6] in 2022. We shall refer to it as BCC rather than UP in this article out of respect for Komori, who initially described it in 1984. In Bordbar et. al. [2] introduced the notion of semidetached subalgebras, and investigated their properties.

DOI: 10.28924/APJM/10-25

In this paper, we introduce the concepts of $(\in, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebras, $k$-left ($k$-right) $(q_k, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebras, $(q_k, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebras, and $(\overline{\in} \vee \overline{q_k}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebras, and investigate relative relations and properties. We provide several conditions for a semidetached structure in BCC-algebras to be a semidetached BCC-subalgebra.

## 2. Preliminaries

The concept of BCC-algebras (see [7]) can be redefined without the condition (2.6) as follows:

An algebra $X = (X, \cdot, 0)$ of type $(2, 0)$ is called a *BCC-algebra* (see [3]) if it satisfies the following conditions:

$$(\forall x, y, z \in X)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0) \tag{2.1}$$

$$(\forall x \in X)(0 \cdot x = x) \tag{2.2}$$

$$(\forall x \in X)(x \cdot 0 = 0) \tag{2.3}$$

$$(\forall x, y \in X)(x \cdot y = 0 = y \cdot x \Rightarrow x = y) \tag{2.4}$$

After this, we assign $X$ instead of a BCC-algebra $(X, \cdot, 0)$ until otherwise specified.

We define a binary relation $\leq$ on $X$ as follows:

$$(\forall x, y \in X)(x \leq y \Leftrightarrow x \cdot y = 0) \tag{2.5}$$

In $X$, the following assertions are valid (see [4]).

$$(\forall x \in X)(x \leq x) \tag{2.6}$$

$$(\forall x, y, z \in X)(x \leq y, y \leq z \Rightarrow x \leq z) \tag{2.7}$$

$$(\forall x, y, z \in X)(x \leq y \Rightarrow z \cdot x \leq z \cdot y) \tag{2.8}$$

$$(\forall x, y, z \in X)(x \leq y \Rightarrow y \cdot z \leq x \cdot z) \tag{2.9}$$

$$(\forall x, y, z \in X)(x \leq y \cdot x, \text{ in particular, } y \cdot z \leq x \cdot (y \cdot z)) \tag{2.10}$$

$$(\forall x, y \in X)(y \cdot x \leq x \Leftrightarrow x = y \cdot x) \tag{2.11}$$

$$(\forall x, y \in X)(x \leq y \cdot y) \tag{2.12}$$

$$(\forall a, x, y, z \in X)(x \cdot (y \cdot z) \leq x \cdot ((a \cdot y) \cdot (a \cdot z))) \tag{2.13}$$

$$(\forall a, x, y, z \in X)(((a \cdot x) \cdot (a \cdot y)) \cdot z \leq (x \cdot y) \cdot z) \tag{2.14}$$

$$(\forall x, y, z \in X)((x \cdot y) \cdot z \leq y \cdot z) \tag{2.15}$$

$$(\forall x, y, z \in X)(x \leq y \Rightarrow x \leq z \cdot y) \tag{2.16}$$

$$(\forall x, y, z \in X)((x \cdot y) \cdot z \leq x \cdot (y \cdot z)) \tag{2.17}$$

$$(\forall a, x, y, z \in X)((x \cdot y) \cdot z \leq y \cdot (a \cdot z)) \tag{2.18}$$

A *fuzzy set* [11] in a nonempty set $X$ is defined to be a function $\lambda : X \to [0, 1]$, where $[0, 1]$ is the unit closed interval of real numbers.

**Definition 2.1.** [5] A fuzzy set $\lambda$ in a nonempty set $X$ of the form

$$\lambda(y) = \begin{cases} t \in (0, 1] & \text{if} \quad y = x, \\ 0 & \text{if} \quad y \neq x, \end{cases}$$

is said to be a *fuzzy point with support $x$ and value $t$* and is denoted by $x_t$.

For a fuzzy point $x_t$ and a fuzzy set $\lambda$ in a nonempty set $X$, Pu and Liu [8] introduced the symbol $x_t \alpha \lambda$, where $\alpha \in \{\in, q, \in \vee q, \in \wedge q\}$. To say that $x_t \in \lambda$ (resp., $x_t q \lambda$), we mean $\lambda(x) \geq t$ (resp., $\lambda(x) + t > 1$), and in this case, $x_t$ is said to *belong to* (resp., *be quasi-coincident with*) a fuzzy set $\lambda$. To say that $x_t \in \vee q \lambda$ (resp., $x_t \in \wedge q \lambda$), we mean $x_t \in \lambda$ or $x_t q \lambda$ (resp., $x_t \in \lambda$ and $x_t q \lambda$). We assign the symbol $x_t \overline{\alpha} \lambda$ to the negation of $x_t \alpha \lambda$.

Jun [5] considered the general form of the symbol $x_t q \lambda$ as follows: for an arbitrary element $k \in [0, 1)$, we say that

$$x_t q_k \lambda \Leftrightarrow \lambda(x) + t + k > 1, \tag{2.19}$$

$$x_t \in \vee q_k \lambda \Leftrightarrow x_t \in \lambda \text{ or } x_t q_k \lambda. \tag{2.20}$$

**Definition 2.2.** [10] A fuzzy set $\lambda$ in $X$ is called a *fuzzy BCC-subalgebra* of $X$ if it satisfies:

$$(\forall x, y \in X)(\lambda(x \cdot y) \geq \min\{\lambda(x), \lambda(y)\}). \tag{2.21}$$

**Definition 2.3.** [10] For any fuzzy set $\lambda$ in a nonempty set $X$ and any $t \in [0, 1]$, the set $U(\lambda, t) = \{x \in X \mid \lambda(x) \geq t\}$ is called a *level subset* of $\lambda$.

**Definition 2.4.** [5] A fuzzy set $\lambda$ in $X$ is called an $(\in, \in \vee q_k)$*-fuzzy BCC-subalgebra* of $X$ if it satisfies:

$$(\forall x, y \in X, \forall t, r \in (0, 1])(x_t \in \lambda, y_r \in \lambda \Rightarrow (x \cdot y)_{\min\{t, r\}} \in \vee q_k \lambda). \tag{2.22}$$

## 3. Semidetached BCC-Subalgebras

Given a set $X$ and a subinterval $\Omega$ of $[0, 1]$, a *semidetached structure* over $\Omega$ is defined to be a pair $(X, f)$, where $f : \Omega \to P(X)$ is a mapping when $P(X)$ is represented as the power set of $X$.

**Definition 3.1.** A semidetached structure $(X, f)$ over $\Omega$ is called a *semidetached BCC-subalgebra* over $\Omega$ with respect to $t \in \Omega$ (briefly, $t$-semidetached BCC-subalgebra over $\Omega$) if $f(t)$ is a BCC-subalgebra of $X$.

We say that $(X, f)$ is a *semidetached BCC-subalgebra* over $\Omega$ if it is a $t$-semidetached BCC-subalgebra over $\Omega$ with respect to all $t \in \Omega$.

Given a fuzzy set $\lambda$ in a set $X$, consider the following mappings:

$$\ell_U^\lambda : \Omega \to P(X), t \mapsto U(\lambda, t), \tag{3.1}$$

$$\ell_{Q_k}^\lambda : \Omega \to P(X), t \mapsto Q_k(\lambda, t), \tag{3.2}$$

$$\ell_{\mathscr{E}_k}^\lambda : \Omega \to P(X), t \mapsto \mathscr{E}_k(\lambda, t), \tag{3.3}$$

where $Q_k(\lambda, t) = \{x \in X \mid x_t q_k \lambda\}$ and $\mathscr{E}_k(\lambda, t) = \{x \in X \mid x_t \in \vee q_k \lambda\}$, which are called the *$q_k$-set* and *$\in \vee q_k$-set* with respect to $t$ (briefly, $t$-$q_k$-set and $t$-$\in \vee q_k$-set), respectively, of $\lambda$. A $t$-$q_k$-set with $k = 0$ is called a *$t$-$q$-set* and is denoted by $Q(\lambda, t)$. A $t$-$\in \vee q_k$-set with $k = 0$ is called a *$t$-$\in \vee q$-set* and is denoted by $\mathscr{E}(\lambda, t)$. Note that, for any $t, r \in (0, 1]$, if $t \geq r$, then every $r$-$q_k$-set is contained in the $t$-$q_k$-set, that is, $Q_k(\lambda, r) \subseteq Q_k(\lambda, t)$. Obviously, $\mathscr{E}_k(\lambda, t) = U(\lambda, t) \cup Q_k(\lambda, t)$.

**Lemma 3.2.** [10] *A fuzzy set $\lambda$ is a fuzzy BCC-subalgebra of $X$ if and only if $U(\lambda, t)$ is a BCC-subalgebra of $X$ for all $t \in [0, 1]$ if it is nonempty.*

**Theorem 3.3.** *A semidetached structure $(X, \ell_U^\lambda)$ is a semidetached BCC-subalgebra over $\Omega = (0, 1]$ if and only if $\lambda$ is a fuzzy BCC-subalgebra of $X$.*

*Proof.* Straightforward from Lemma 3.2. □

**Theorem 3.4.** *If $\lambda$ is an $(\in, \in)$-fuzzy BCC-subalgebra (or equivalently, $\lambda$ is a fuzzy BCC-subalgebra) of $X$, then a semidetached structure $(X, \ell_{Q_k}^\lambda)$ is a semidetached BCC-subalgebra over $\Omega = (0, 1]$.*

*Proof.* Let $x, y \in \ell_{Q_k}^\lambda(t)$ for $t \in \Omega = (0, 1]$. Then $x_t q_k \lambda$ and $y_t q_k \lambda$, that is, $\lambda(x) + t + k > 1$ and $\lambda(y) + t + k > 1$. It follows from (2.21) that $\lambda(x \cdot y) + t + k \geq \min\{\lambda(x), \lambda(y)\} + t + k = \min\{\lambda(x) + t + k, \lambda(y) + t + k\} > 1$. Hence, $(x \cdot y)_t \in \vee q_k \lambda$, and so $x \cdot y \in \ell_{Q_k}^\lambda(t)$. Therefore, $\ell_{Q_k}^\lambda(t)$ is a BCC-subalgebra of $X$. Consequently, $(X, \ell_{Q_k}^\lambda)$ is a semidetached BCC-subalgebra over $\Omega = (0, 1]$. □

**Corollary 3.5.** *If $\lambda$ is an $(\in, \in)$-fuzzy BCC-subalgebra (or equivalently, $\lambda$ is a fuzzy BCC-subalgebra) of $X$, then a semidetached structure $(X, \ell_U^\lambda)$ is a semidetached BCC-subalgebra over $\Omega = (0, 1]$.*

Given a fuzzy set $\lambda$ in $X$, $x, y \in X$, $t, r \in [0, 1]$, and $k \in [0, 1)$, we consider the following condition:

$$x_t q_k \lambda, y_r q_k \lambda \Rightarrow (x \cdot y)_{\min\{t, r\}} \in \vee q_k \lambda. \tag{3.4}$$

**Definition 3.6.** A fuzzy set $\lambda$ in $X$ is called a *$k$-left* (resp., $k$-right) *$(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra* of $X$ if it satisfies the condition (3.4) for all $x, y \in X$ and $t, r \in (0, \frac{1-k}{2}]$ (resp., $t, r \in (\frac{1-k}{2}, 1]$).

**Theorem 3.7.** *Every $k$-right $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$ is an $(\in, \in \vee q_k)$-fuzzy BCC-subalgebra.*

*Proof.* Let $\lambda$ be a $k$-right $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$. Let $x, y \in X$ and $t, r \in (0, 1]$ be such that $x_t \in \lambda$ and $y_r \in \lambda$. Then $\lambda(x) \geq t$ and $\lambda(y) \geq r$. Suppose that $(x \cdot y)_{\min\{t,r\}} \overline{\in q_k} \lambda$. Then $\lambda(x \cdot y) < \min\{t, r\}$ and $\lambda(x \cdot y) + \min\{t, r\} + k \leq 1$. It follows that $\lambda(x \cdot y) < \frac{1-k}{2}$ and so that $\lambda(x \cdot y) < \min\{t, r, \frac{1-k}{2}\}$. Hence, $1 - k - \lambda(x \cdot y) > 1 - k - \min\{t, r, \frac{1-k}{2}\} = \max\{1 - k - t, 1 - k - r, 1 - k - \frac{1-k}{2}\} \geq \max\{1 - k - \lambda(x), 1 - k - \lambda(y), \frac{1-k}{2}\}$, and so there exists $\delta \in (0, 1]$ such that $1 - k - \lambda(x \cdot y) \geq \delta > \max\{1 - k - \lambda(x), 1 - k - \lambda(y), \frac{1-k}{2}\}$. Then $\delta \in (\frac{1-k}{2}, 1], \lambda(x) + \delta + k > 1$ and $\lambda(y) + \delta + k > 1$, that is, $(x, \delta) q_k \lambda$ and $(y, \delta) q_k \lambda$. Since $\lambda$ is a $k$-right $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$, it follows that $(x \cdot y, \delta) \in \vee q_k \lambda$. On the other hand, $1 - k - \lambda(x \cdot y) \geq \delta$ implies that $\lambda(x \cdot y) + \delta + k \leq 1$, that is, $(x \cdot y, \delta) \overline{q_k} \lambda$ and $\lambda(x \cdot y) \leq 1 - \delta - k < 1 - k - \frac{1-k}{2} = \frac{1-k}{2} < \delta$, that is, $(x \cdot y, \delta) \overline{\in} \lambda$. Hence, $(x \cdot y, \delta) \overline{\in \vee q_k} \lambda$, which is a contradiction. Therefore, $(x \cdot y)_{\min\{t,r\}} \in \vee q_k \lambda$, and thus $\lambda$ is an $(\in, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$. $\square$

**Corollary 3.8.** *Every 0-right $(q, \in \vee q)$-fuzzy BCC-subalgebra of $X$ is an $(\in, \in \vee q)$-fuzzy BCC-subalgebra.*

We consider the converse of Theorem 3.7.

**Theorem 3.9.** *If every fuzzy point has the value $t$ in $(0, \frac{1-k}{2}]$, then every $(\in, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$ is a $k$-left $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra.*

*Proof.* Let $\lambda$ be an $(\in, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$. Let $x, y \in X$ and $t, r \in (0, \frac{1-k}{2}]$ be such that $x_t q_k \lambda$ and $y_r q_k \lambda$. Then $\lambda(x) + t + k > 1$ and $\lambda(y) + r + k > 1$. Since $t, r \in (0, \frac{1-k}{2}]$, it follows that $\lambda(x) > 1 - t - k \geq \frac{1-k}{2} \geq t$ and $\lambda(y) > 1 - r - k \geq \frac{1-k}{2} \geq r$, that is, $x_t \in \lambda$ and $y_r \in \lambda$. It follows from (3.4) that $(x \cdot y)_{\min\{t,r\}} \in \vee q_k \lambda$. Hence, $\lambda$ is a $k$-left $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$. $\square$

**Corollary 3.10.** *If every fuzzy point has the value $t$ in $(0, 0.5]$, then every $(\in, \in \vee q)$-fuzzy BCC-subalgebra of $X$ is a 0-left $(q, \in \vee q)$-fuzzy BCC-subalgebra.*

**Proposition 3.11.** *If $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$, then*

$$(\forall x, y \in X, \forall t, r \in \Omega)(x_t \in \lambda, y_r \in \lambda \Rightarrow (x \cdot y)_{\max\{t,r\}} q_k \lambda). \tag{3.5}$$

*Proof.* Let $x, y \in X$ and $t, r \in \Omega = (\frac{1-k}{2}, 1]$ be such that $x_t \in \lambda$ and $y_r \in \lambda$. Then $\lambda(x) \geq t > \frac{1-k}{2}$ and $\lambda(y) \geq r > \frac{1-k}{2}$, which imply that $\lambda(x) + t + k > 1$ and $\lambda(y) + r + k > 1$, that is, $x_t q_k \lambda$ and $y_r q_k \lambda$. It follows that $x, y \in \ell^{\lambda}_{Q_k}(\max\{t, r\})$ and $\max\{t, r\} \in (\frac{1-k}{2}, 1]$. Since $\ell^{\lambda}_{Q_k}(\max\{t, r\})$ is a BCC-subalgebra of $X$ by assumption, we have $x \cdot y \in \ell^{\lambda}_{Q_k}(\max\{t, r\})$ and so $(x \cdot y)_{\max\{t,r\}} q_k \lambda$. $\square$

**Corollary 3.12.** *If $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (0.5, 1]$, then*

$$(\forall x, y \in X, \forall t, r \in \Omega)(x_t \in \lambda, y_r \in \lambda \Rightarrow (x \cdot y)_{\max\{t,r\}} q \lambda). \tag{3.6}$$

**Proposition 3.13.** *If $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (0, \frac{1-k}{2}]$, then*

$$(\forall x, y \in X, \forall t, r \in \Omega)(x_t q_k \lambda, y_r q_k \lambda \Rightarrow (x \cdot y)_{\max\{t,r\}} \in \lambda). \tag{3.7}$$

*Proof.* Let $x, y \in X$ and $t, r \in \Omega = (0, \frac{1-k}{2}]$ be such that $x_t q_k \lambda$ and $y_r q_k \lambda$. Then $x \in \ell^{\lambda}_{Q_k}(t)$ and $y \in \ell^{\lambda}_{Q_k}(r)$. It follows that $x, y \in \ell^{\lambda}_{Q_k}(\max\{t, r\})$ and $\max\{t, r\} \in \Omega = (0, \frac{1-k}{2}]$. Thus $x \cdot y \in \ell^{\lambda}_{Q_k}(\max\{t, r\})$ since $\ell^{\lambda}_{Q_k}(\max\{t, r\})$ is a BCC-subalgebra of $X$ by the assumption. Hence, $\lambda(x \cdot y) + k + \max\{t, r\} > 1$ and so $\lambda(x \cdot y) > 1 - k - \max\{t, r\} \geq \frac{1-k}{2} \geq \max\{t, r\}$. Thus $(x \cdot y)_{\max\{t,r\}} \in \lambda$. □

**Corollary 3.14.** *If $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (0, 0.5]$, then*

$$(\forall x, y \in X, \forall t, r \in)(x_t q \lambda, y_r q \lambda \Rightarrow (x \cdot y)_{\max\{t,r\}} \in \lambda). \tag{3.8}$$

**Theorem 3.15.** *If $\lambda$ is a $k$-right $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$, then $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$.*

*Proof.* Let $x, y \in \ell^{\lambda}_{Q_k}(t)$ for $t \in (\frac{1-k}{2}, 1]$. Then $x_t q_k \lambda$ and $y_t q_k \lambda$. Since $\lambda$ is a $k$-right $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$, we have $(x \cdot y)_t \in \vee q_k \lambda$, that is, $(x \cdot y)_t \in \lambda$ or $(x \cdot y)_t q_k \lambda$. If $(x \cdot y)_t \in \lambda$, then $\lambda(x \cdot y) \geq t > \frac{1-k}{2} > 1 - t - k$ and so $\lambda(x \cdot y) + t + k > 1$, that is, $(x \cdot y)_t q_k \lambda$. Hence, $x \cdot y \in \ell^{\lambda}_{Q_k}$. If $(x \cdot y)_t q_k \lambda$, then $x \cdot y \in \ell^{\lambda}_{Q_k}(t)$. Therefore, $\ell^{\lambda}_{Q_k}(t)$ is a BCC-subalgebra of $X$, and consequently $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$. □

**Corollary 3.16.** *If $\lambda$ is a 0-right $(q, \in \vee q)$-fuzzy BCC-subalgebra of $X$, then $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (0.5, 1]$.*

**Theorem 3.17.** *For a BCC-subalgebra $A$ of $X$, let $\lambda$ be a fuzzy set in $X$ such that*

(1) $\lambda(x) \geq \frac{1-k}{2}$ *for all* $x \in A$,
(2) $\lambda(x) = 0$ *for all* $x \in X \backslash A$.

*Then $\lambda$ is a $k$-left $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$.*

*Proof.* Let $x, y \in X$ and $t, r \in (0, \frac{1-k}{2}]$ be such that $x_t q_k \lambda$ and $y_r q_k \lambda$. Then $\lambda(x) + t + k > 1$ and $\lambda(y) + r + k > 1$, which imply that $\lambda(x) > 1 - t - k \geq \frac{1-k}{2}$ and $\lambda(y) > 1 - r - k \geq \frac{1-k}{2}$. Hence, $x \in A$ and $y \in A$. Since $A$ is a BCC-subalgebra of $X$, we get $x \cdot y \in A$ and so $\lambda(x \cdot y) \geq \frac{1-k}{2} \geq \max\{t, r\}$. Thus $(x \cdot y)_{\max\{t,r\}} \in \lambda$, and so $(x \cdot y)_{\max\{t,r\}} \in \vee q_k \lambda$. Therefore, $\lambda$ is a $k$-left $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$. □

**Corollary 3.18.** *For a BCC-subalgebra $A$ of $X$, let $\lambda$ be a fuzzy set in $X$ such that*

(1) $\lambda(x) \geq 0.5$ *for all* $x \in A$,
(2) $\lambda(x) = 0$ *for all* $x \in X \backslash A$.

*Then $\lambda$ is a 0-left $(q, \in \vee q)$-fuzzy BCC-subalgebra of $X$.*

**Proposition 3.19.** *If $(X, \ell^{\lambda}_{\mathscr{E}_k})$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$, then*

$$(\forall x, y \in X, \forall t, r \in \Omega)(x_t q_k \lambda, y_r q_k \lambda \Rightarrow (x \cdot y)_{\max\{t,r\}} \in \vee q_k \lambda). \tag{3.9}$$

*Proof.* Let $x, y \in X$ and $t, r \in \Omega = (\frac{1-k}{2}, 1]$ be such that $x_t q_k \lambda$ and $y_r q_k \lambda$. Then $x \in \ell^{\lambda}_{Q_k}(t) \subseteq \ell^{\lambda}_{\mathscr{E}_k}(t)$ and $y \in \ell^{\lambda}_{Q_k}(r) \subseteq \ell^{\lambda}_{\mathscr{E}_k}(r)$. It follows that $x, y \in \ell^{\lambda}_{\mathscr{E}_k}(\max\{t, r\})$ and so from the hypothesis that $x \cdot y \in \ell^{\lambda}_{\mathscr{E}_k}(\max\{t, r\})$. Hence, $(x \cdot y)_{\max\{t,r\}} \in \vee q_k \lambda$. □

**Corollary 3.20.** *If $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$, then*

$$(\forall x, y \in X, \forall t, r \in \Omega)(x_t q \lambda, y_r q \lambda \Rightarrow (x \cdot y)_{\max\{t,r\}} \in \vee q \lambda). \tag{3.10}$$

**Lemma 3.21.** *A fuzzy set $\lambda$ in $X$ is an $(\in, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$ if and only if it satisfies:*

$$(\forall x, y \in X)(\lambda(x \cdot y) \geq \min\{\lambda(x), \lambda(y), \frac{1-k}{2}\}). \tag{3.11}$$

*Proof.* The proof is straightforward by Definition 2.4. □

**Theorem 3.22.** *If $\lambda$ is an $(\in, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$, then $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$.*

*Proof.* Let $x, y \in \ell^{\lambda}_{Q_k}(t)$ for $t \in \Omega = (\frac{1-k}{2}, 1]$. Then $x_t q_k \lambda$ and $y_t q_k \lambda$, that is, $\lambda(x) + t + k > 1$ and $\lambda(y) + t + k > 1$. It follows from Lemma 3.21 that $\lambda(x \cdot y) + t + k \geq \min\{\lambda(x), \lambda(y), \frac{1-k}{2}\} + t + k = \min\{\lambda(x) + t + k, \lambda(y) + t + k, \frac{1-k}{2} + t + k\} > 1$. Hence, $(x \cdot y)_t q_k \lambda$, and so $x \cdot y \in \ell^{\lambda}_{Q_k}(t)$. Therefore, $\ell^{\lambda}_{Q_k}(t)$ is a BCC-subalgebra of $X$ for all $t \in (\frac{1-k}{2}, 1]$, and consequently $X$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$. □

**Corollary 3.23.** *If $\lambda$ is an $(\in, \in \vee q)$-fuzzy BCC-subalgebra of $X$, then $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (0.5, 1]$.*

**Theorem 3.24.** *If $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$, then $\lambda$ is an $(\in, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$.*

*Proof.* Assume that there exist $a, b \in X$ such that $\lambda(a \cdot b) < \min\{\lambda(a), \lambda(b), \frac{1-k}{2}\} = t_0$. Then $t_0 \in (0, \frac{1-k}{2}]$, $a, b \in U(\lambda, t_0) \subseteq \ell^{\lambda}_{\mathscr{E}_k}(t_0)$, which implies that $a \cdot b \in \ell^{\lambda}_{\mathscr{E}_k}(t_0)$. Hence $\lambda(a \cdot b) \geq t_0$ or $\lambda(a \cdot b) + t_0 + k > 1$. This is a contradiction. Thus $\lambda(x \cdot y) \geq \min\{\lambda(x), \lambda(y), \frac{1-k}{2}\}$ for all $x, y \in X$. It follows from Lemma 3.21 that $\lambda$ is an $(\in, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$. □

**Corollary 3.25.** *If $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (0.5, 1]$, then $\lambda$ is an $(\in, \in \vee q)$-fuzzy BCC-subalgebra of $X$.*

**Theorem 3.26.** *If $\lambda$ is an $(\in, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$, then $(X, \ell^{\lambda}_{\mathscr{E}_k})$ is a semidetached BCC-subalgebra over $\Omega = (0, \frac{1-k}{2}]$.*

*Proof.* Let $x, y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$ for $t \in \Omega = (0, \frac{1-k}{2}]$. Then $x_t \in \vee q_k \lambda$ and $y_t \in \vee q_k \lambda$. Hence, we have the following four cases:

(1) $x_t \in \lambda$ and $y_t \in \lambda$,

(2) $x_t \in \lambda$ and $y_t q_k \lambda$,

(3) $x_t q_k \lambda$ and $y_t \in \lambda$,

(4) $x_t q_k \lambda$ and $y_t q_k \lambda$.

The first case implies that $(x \cdot y)_t \in \vee q_k \lambda$ and so $x \cdot y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$. For the second case, $y_t q_k \lambda$ induces $\lambda(y) > 1 - t - k \geq t$, that is, $y_t \in \lambda$. Hence, $(x \cdot y)_t \in \vee \overline{q_k} \lambda$ and so $x \cdot y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$. Similarly, the third case implies $x \cdot y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$. The last case induces $\lambda(x) > 1 - t - k \geq t$ and $\lambda(y) > 1 - t - k \geq t$, that is, $x_t \in \lambda$ and $y_t \in \lambda$. It follows that $(x \cdot y)_t \in \vee q_k \lambda$ and so that $x \cdot y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$. Therefore, $\ell_{\mathscr{E}_k}^{\lambda}(t)$ is a BCC-subalgebra of $X$ for all $t \in (0, \frac{1-k}{2}]$. Hence, $(X, \ell_{\mathscr{E}_k}^{\lambda})$ is a semidetached BCC-subalgebra over $\Omega = (0, \frac{1-k}{2}]$. □

**Corollary 3.27.** *If $\lambda$ is an $(\in, \in \vee q)$-fuzzy BCC-subalgebra of $X$, then $(X, \ell_{\mathscr{E}_k}^{\lambda})$ is a semidetached BCC-subalgebra over $\Omega = (0, 0.5]$.*

**Theorem 3.28.** *If $\lambda$ is a $(q_k, \in \vee q_k)$-fuzzy BCC-subalgebra of $X$, then $(X, \ell_{\mathscr{E}_k}^{\lambda})$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$.*

*Proof.* Let $x, y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$ for $t \in \Omega = (\frac{1-k}{2}, 1]$. Then $x_t \in \vee q_k \lambda$ and $y_t \in \vee q_k \lambda$. Hence, we have the following four cases:

(1) $x_t \in \lambda$ and $y_t \in \lambda$,

(2) $x_t \in \lambda$ and $y_t q_k \lambda$,

(3) $x_t q_k \lambda$ and $y_t \in \lambda$,

(4) $x_t q_k \lambda$ and $y_t q_k \lambda$.

For the first case, we have $\lambda(x) + t + k \geq 2t + k > 1$ and $\lambda(y) + t + k \geq 2t + k > 1$, that is, $x_t q_k \lambda$ and $y_t q_k \lambda$. Hence, $(x \cdot y)_t \in \vee q_k \lambda$, and so $x \cdot y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$. For the second case, $x_t \in \lambda$ implies $\lambda(x) + t + k \geq 2t + k > 1$, that is, $x_t q_k \lambda$. Hence, $(x \cdot y)_t \in \vee q_k \lambda$, and so $x \cdot y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$. Similarly, the third case implies $x \cdot y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$. For the last case, we have $(x \cdot y)_t \in \vee q_k \lambda$, and so $x \cdot y \in \ell_{\mathscr{E}_k}^{\lambda}(t)$. Consequently, $\ell_{\mathscr{E}_k}^{\lambda}(t)$ is a BCC-subalgebra of $X$ for all $t \in \Omega = (\frac{1-k}{2}, 1]$. Therefore, $(X, \ell_{\mathscr{E}_k}^{\lambda})$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$. □

**Corollary 3.29.** *If $\lambda$ is a $(q, \in \vee q)$-fuzzy BCC-subalgebra of $X$, then $(X, \ell_{\mathscr{E}_k}^{\lambda})$ is a semidetached BCC-subalgebra over $\Omega = (0.5, 1]$.*

For $\alpha \in \{\in, q_k\}$ and $t \in (0, 1]$, we say that $x_t \overline{\alpha} \lambda$ if $x_t \alpha \lambda$ does not hold.

**Definition 3.30.** A fuzzy set $\lambda$ in $X$ is called an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-*fuzzy BCC-subalgebra* of $X$ if it satisfies:

$$(\forall x, y \in X, \forall t, r \in (0, 1])((x \cdot y)_{\min\{t,r\}} \overline{\in} \lambda \Rightarrow x_t \overline{\in} \vee \overline{q_k} \lambda \text{ or } y_r \overline{\in} \vee \overline{q_k} \lambda). \tag{3.12}$$

An $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$ with $k = 0$ is called an $(\overline{\in}, \overline{\in} \vee \overline{q})$-*fuzzy BCC-subalgebra*.

We provide a characterization of an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra.

**Theorem 3.31.** *A fuzzy set $\lambda$ in $X$ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$ if and only if the following inequality is valid:*

$$(\forall x, y \in X)(\max\{\lambda(x \cdot y), \frac{1-k}{2}\} \geq \min\{\lambda(x), \lambda(y)\}). \tag{3.13}$$

*Proof.* Let $\lambda$ be an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. Assume that (3.13) is not valid. Then there exist $a, b \in X$ such that $\max\{\lambda(a \cdot b), \frac{1-k}{2}\} < \min\{\lambda(a), \lambda(b)\} = t$. Then $\frac{1-k}{2} < t \leq 1, a_t \in \lambda, b_t \in \lambda$, and $(a \cdot b)_t \overline{\in} \lambda$. It follows from (3.12) that $a_t \overline{q_k} \lambda$ or $b_t \overline{q_k} \lambda$. Hence, $\lambda(a) \geq t$ and $\lambda(a) + t + k \leq 1$ or $\lambda(b) \geq t$ and $\lambda(b) + t + k \leq 1$. In either case, we have $t \leq \frac{1-k}{2}$, which is a contradiction. Therefore, $\max\{\lambda(x \cdot y), \frac{1-k}{2}\} \geq \min\{\lambda(x), \lambda(y)\}$ for all $x, y \in X$.

Conversely, suppose that (3.13) is valid. Let $x, y \in X$ and $t, r \in (0, 1]$ be such that $(x \cdot y)_{\min\{t,r\}} \overline{\in} \lambda$. Then $\lambda(x \cdot y) < \min\{t, r\}$. If $\max\{\lambda(x \cdot y), \frac{1-k}{2}\} = \lambda(x \cdot y)$, then $\min\{t, r\} > \lambda(x \cdot y) \geq \min\{\lambda(x), \lambda(y)\}$ and so $\lambda(x) < t$ or $\lambda(y) < r$. Thus $x_t \overline{\in} \lambda$ or $y_r \overline{\in} \lambda$, which implies that $x_t \overline{\in} \vee \overline{q_k} \lambda$ or $y_r \overline{\in} \vee \overline{q_k} \lambda$. If $\max\{\lambda(x \cdot y), \frac{1-k}{2}\} = \frac{1-k}{2}$, then $\min\{\lambda(x), \lambda(y)\} \leq \frac{1-k}{2}$. Suppose $x_t \in \lambda$ or $y_r \in \lambda$. Then $t \leq \lambda(x) \leq \frac{1-k}{2}$ or $r \leq \lambda(y) \leq \frac{1-k}{2}$, and so $\lambda(x) + t + k \leq \frac{1-k}{2} + \frac{1-k}{2} + k = 1$ or $\lambda(y) + r + k \leq \frac{1-k}{2} + \frac{1-k}{2} + k = 1$. Therefore, $x_t \overline{q_k} \lambda$ or $y_r \overline{q_k} \lambda$ and so $x_t \overline{\in} \vee \overline{q_k} \lambda$ or $y_r \overline{\in} \vee \overline{q_k} \lambda$. Hence, $\lambda$ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. □

**Corollary 3.32.** *A fuzzy set $\lambda$ in $X$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy BCC-subalgebra of $X$ if and only if the following inequality is valid:*

$$(\forall x, y \in X)(\max\{\lambda(x \cdot y), 0.5\} \geq \min\{\lambda(x), \lambda(y)\}). \tag{3.14}$$

**Theorem 3.33.** *A fuzzy set $\lambda$ in $X$ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$ if and only if $(X, \ell_U^\lambda)$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$.*

*Proof.* Assume that $\lambda$ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. Let $x, y \in \ell_U^\lambda(t)$ for $t \in \Omega = (\frac{1-k}{2}, 1]$. Then $\lambda(x) \geq t$ and $\lambda(y) \geq t$. It follows from (3.13) that $\max\{\lambda(x \cdot y), \frac{1-k}{2}\} \geq \min\{\lambda(x), \lambda(y)\} \geq t$. Since $t > \frac{1-k}{2}$, it follows that $\lambda(x \cdot y) \geq t$ and so $x \cdot y \in \ell_U^\lambda(t)$. Thus $\ell_U^\lambda(t)$ is a BCC-subalgebra of $X$. Hence, $(X, \ell_U^\lambda)$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$.

Conversely, suppose that $(X, \ell_U^\lambda)$ is a semidetached BCC-subalgebra over $\Omega = (\frac{1-k}{2}, 1]$. If (3.13) is not valid, then there exist $a, b \in X$ such that $\max\{\lambda(a \cdot b), \frac{1-k}{2}\} < \min\{\lambda(a), \lambda(b)\} = t$. Then $t \in (\frac{1-k}{2}, 1], a, b \in \ell_U^\lambda(t)$, and $a \cdot b \notin \ell_U^\lambda(t)$. This is a contradiction, and so (3.13) is valid. Using Theorem 3.31, we know that $\lambda$ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. □

**Corollary 3.34.** *A fuzzy set $\lambda$ in $X$ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$-fuzzy BCC-subalgebra of $X$ if and only if $(X, \ell_U^\lambda)$ is a semidetached BCC-subalgebra over $\Omega = (0.5, 1]$.*

**Theorem 3.35.** *A fuzzy set* $\lambda$ *in* $X$ *is an* $(\overline{\in}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra of* $X$ *if and only if* $(X, \ell^{\lambda}_{Q_k})$ *is a semidetached BCC-subalgebra over* $\Omega = (0, \frac{1-k}{2}]$.

*Proof.* Assume that $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (0, \frac{1-k}{2}]$. If (3.13) is not valid, then there exist $a, b \in X, t \in (0, 1]$, and $k \in [0, 1)$ such that $\max\{\lambda(a \cdot b), \frac{1-k}{2}\} + t + k \leq 1 < \min\{\lambda(a), \lambda(b)\} + t + k$. It follows that $a_t q_k \lambda$ and $b_t q_k \lambda$, that is, $a, b \in \ell^{\lambda}_{Q_k}(t)$. But $(a \cdot b)_t \overline{q_k} \lambda$, that is, $a \cdot b \notin \ell^{\lambda}_{Q_k}$. This is a contradiction, and so (3.13) is valid. Using Theorem 3.31, we have $\lambda$ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$.

Conversely, suppose that $\lambda$ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. Let $x, y \in \ell^{\lambda}_{Q_k}(t)$ for $t \in \Omega = (0, \frac{1-k}{2}]$. Then $x_t q_k \lambda$ and $y_t q_k \lambda$, that is, $\lambda(x) + t + k > 1$ and $\lambda(y) + t + k > 1$. It follows from (3.13) that $\max\{\lambda(x \cdot y), \frac{1-k}{2}\} \geq \min\{\lambda(x), \lambda(y)\} > 1 - t - k \geq \frac{1-k}{2}$ and so $\lambda(x \cdot y) + t + k > 1$, that is, $x \cdot y \in \lambda$. Therefore, $\ell^{\lambda}_{Q_k}(t)$ is a BCC-subalgebra of $X$. Hence, $(X, \ell^{\lambda}_{Q_k})$ is a semidetached BCC-subalgebra over $\Omega = (0, \frac{1-k}{2}]$. □

Using Theorems 3.33 and 3.35, we have the following corollary.

**Corollary 3.36.** *For a fuzzy set* $\lambda$ *in* $X$*, the following are equivalent:*

(1) $(X, \ell^{\lambda}_{U})$ *is a semidetached BCC-subalgebra over* $\Omega = (\frac{1-k}{2}, 1]$*,*
(2) $(X, \ell^{\lambda}_{Q_k})$ *is a semidetached BCC-subalgebra over* $\Omega = (0, \frac{1-k}{2}]$.

**Definition 3.37.** A fuzzy set $\lambda$ in $X$ is called an $(\overline{\in} \vee \overline{q_k}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra* of $X$ if it satisfies:

$$(\forall x, y \in X, \forall t, r \in (0, 1])((x \cdot y)_{\min\{t,r\}} \overline{\in} \vee \overline{q_k} \lambda \Rightarrow x_t \overline{\in} \vee \overline{q_k} \lambda \text{ or } y_r \overline{\in} \vee \overline{q_k} \lambda). \tag{3.15}$$

**Theorem 3.38.** *Every* $(\overline{\in} \vee \overline{q_k}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra of* $X$ *is an* $(\overline{\in}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra.*

*Proof.* Let $x, y \in X$ and $t, r \in (0, 1]$ be such that $(x \cdot y)_{\min\{t,r\}} \overline{\in} \lambda$. Then $(x \cdot y)_{\min\{t,r\}} \overline{\in} \vee \overline{q_k} \lambda$, and so $x_t \overline{\in} \vee \overline{q_k} \lambda$ or $y_r \overline{\in} \vee \overline{q_k} \lambda$ by (3.15). Hence, $\lambda$ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. □

**Corollary 3.39.** *If* $\lambda$ *is an* $(\overline{\in} \vee \overline{q_k}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra of* $X$*, then*

(1) $\lambda$ *is an* $(\overline{\in}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra of* $X$*,*
(2) $\lambda$ *satisfies the condition* (3.13)*,*
(3) $(X, \ell^{\lambda}_{U})$ *is a semidetached BCC-subalgebra over* $\Omega = (\frac{1-k}{2}, 1]$*,*
(4) $(X, \ell^{\lambda}_{Q_k})$ *is a semidetached BCC-subalgebra over* $\Omega = (0, \frac{1-k}{2}]$.

**Definition 3.40.** A fuzzy set $\lambda$ in $X$ is called a $(\overline{q_k}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra* of $X$ if it satisfies:

$$(\forall x, y \in X, \forall t, r \in (0, 1])((x \cdot y)_{\min\{t,r\}} \overline{q_k} \lambda \Rightarrow x_t \overline{\in} \vee \overline{q_k} \lambda \text{ or } y_r \overline{\in} \vee \overline{q_k} \lambda). \tag{3.16}$$

**Theorem 3.41.** *Assume that* $\min\{t, r\} \leq \frac{1-k}{2}$ *for any* $t, r \in (0, 1]$*. Then every* $(\overline{q_k}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra of* $X$ *is an* $(\overline{\in}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra.*

*Proof.* Let $\lambda$ be an $(\overline{q_k}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. Assume that $(x \cdot y)_{\min\{t,r\}} \overline{\in} \lambda$ for $x, y \in X$ and $t, r \in (0,1]$ with $\min\{t,r\} \leq \frac{1-k}{2}$. Then $\lambda(x \cdot y) < \min\{t,r\} \leq \frac{1-k}{2}$, and so $\lambda(x \cdot y) + k + \min\{t,r\} < \frac{1-k}{2} + \frac{1-k}{2} + k = 1$, that is, $(x \cdot y)_{\min\{t,r\}} q_k \lambda$. It follows from (3.16) that $x_t \overline{\in} \vee \overline{q_k} \lambda$ or $y_r \overline{\in} \vee \overline{q_k} \lambda$. Therefore, $\lambda$ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. □

**Corollary 3.42.** *Assume that* $\min\{t,r\} \leq 0.5$ *for any* $t, r \in (0,1]$. *Then every* $(\overline{q}, \overline{\in} \vee \overline{q})$*-fuzzy BCC-subalgebra of* $X$ *is an* $(\overline{\in}, \overline{\in} \vee \overline{q})$*-fuzzy BCC-subalgebra.*

**Theorem 3.43.** *Assume that* $\min\{t,r\} > \frac{1-k}{2}$ *for any* $t, r \in (0,1]$. *Then every* $(\overline{\in}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra of* $X$ *is a* $(\overline{q_k}, \overline{\in} \vee \overline{q_k})$*-fuzzy BCC-subalgebra.*

*Proof.* Let $\lambda$ be an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. Assume that $(x \cdot y)_{\min\{t,r\}} \overline{q_k} \lambda$ for $x, y \in X$ and $t, r \in (0,1]$ with $\min\{t,r\} > \frac{1-k}{2}$. If $(x \cdot y)_{\min\{t,r\}} \in \lambda$, then $\lambda(x \cdot y) \geq \min\{t,r\}$, and so $\lambda(x \cdot y) + k + \min\{t,r\} > \frac{1-k}{2} + \frac{1-k}{2} + k = 1$. Hence, $(x \cdot y)_{\min\{t,r\}} q_k \lambda$, which is a contradiction. Thus $(x \cdot y)_{\min\{t,r\}} \overline{\in}$, which implies from (3.12) that $x_t \overline{\in} \vee \overline{q_k} \lambda$ or $y_r \overline{\in} \vee \overline{q_k} \lambda$. Therefore, $\lambda$ is a $(\overline{q_k}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebra of $X$. □

**Corollary 3.44.** *Assume that* $\min\{t,r\} > 0.5$ *for any* $t, r \in (0,1]$. *Then every* $(\overline{\in}, \overline{\in} \vee \overline{q})$*-fuzzy BCC-subalgebra of* $X$ *is a* $(\overline{q}, \overline{\in} \vee \overline{q})$*-fuzzy BCC-subalgebra.*

## 4. Conclusion

In the present paper, we have introduced the notion of semidetached BCC-subalgebras of BCC-algebras. Several conditions for a semidetached structure in BCC-algebras to be a semidetached BCC-subalgebra are provided. The concepts of $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebras, $k$-left ($k$-right) $(q_k, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebras, $(q_k, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebras, and $(\overline{\in} \vee \overline{q_k}, \overline{\in} \vee \overline{q_k})$-fuzzy BCC-subalgebras are presented, and relative relations and properties are described.

## Acknowledgment

This work was supported by the revenue budget in 2023, School of Science, University of Phayao (Grant No. PBTSC66019).

## References

[1] S.K. Bhakat, P. Das, $(\in, \in \vee q)$-fuzzy subgroup, Fuzzy Sets Syst. 80 (1996), 359–368. https://doi.org/10.1016/0165-0114(95)00157-3.

[2] H. Bordbar, M. Rahim Bordbar, Y.B. Jun, A generalization of semidetached subalgebras in BCK/BCI-algebras, New Math. Nat. Comput. 15 (2019), 489–501. https://doi.org/10.1142/s1793005719500285.

[3] Y. Huang, BCI-algebra, Science Press, Beijing, China, 2006.

[4] A. Iampan, A new branch of the logical algebra: UP-algebras, J. Algebra Related Topics. 5 (2017), 35–54. https://doi.org/10.22124/jart.2017.2403.

[5] Y.B. Jun, Generalizations of $(\in, \in \vee q)$-fuzzy subalgebras in BCK/BCI-algebras, Comput. Math. Appl. 58 (2009), 1383–1390. https://doi.org/10.1016/j.camwa.2009.07.043.

[6] Y.B. Jun, B. Brundha, N. Rajesh, R.K. Bandaru, $(3, 2)$-fuzzy UP-subalgebras and $(3, 2)$-fuzzy UP-filters, J. Mahani Math. Res. 1 (2022), 1–14. https://doi.org/10.22103/JMMRC.2022.18786.1191.

[7] Y. Komori, The class of BCC-algebras is not a variety, Math. Japon. 29 (1984), 391–394.

[8] P. Pao-Ming, L. Ying-Ming, Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence, J. Math. Anal. Appl. 76 (1980), 571–599. https://doi.org/10.1016/0022-247x(80)90048-7.

[9] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971), 512–517. https://doi.org/10.1016/0022-247x(71)90199-5.

[10] J. Somjanta, N. Thuekaew, P. Kumpeangkeaw, A. Iampan, Fuzzy sets in UP-algebras, Ann. Fuzzy Math. Inf. 12 (2016), 739–756.

[11] L.A. Zadeh, Fuzzy sets, Inf. Control. 8 (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X.