Asia Pac. J. Math. 2023 **10**:36

MATHEMATICAL MODELLING OF THE DRINKING BEHAVIOUR EFFECT ON SOCIETY

K. CHINNADURAI[1], S. ATHITHAN[2,*]

[1]Department of Mathematics, Faculty of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chengalpattu - 603 203, India

[2]Department of Mathematics, Faculty of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chengalpattu - 603 203, India

[*]Corresponding author: athithas@srmist.edu.in

Received Sep. 10, 2023

Abstract. This paper presents an analysis of poverty dynamics in conjunction with populations affected by alcohol consumption using a deterministic perspective. A mathematical model is developed based on assumptions about the behaviour and challenges faced by individuals struggling with alcohol addiction and poverty. Our primary focus is examining how impoverished individuals can overcome alcohol addiction and reduce poverty within a specific area. We provide a comprehensive description of the model, emphasizing the existence and stability of different equilibria. Additionally, we conduct numerical simulations to assess the efficacy of our model. To further investigate the impact of critical parameters on the prevalence of illness and the basic reproduction number, we employ sensitivity analysis using PRCC. Our findings are illustrated through numerical simulations, effectively depicting the societal effects of alcohol and poverty through the mathematical model. We support our conclusions with suitable numerical explanations, utilizing visual representations of the respective compartments. We hope that our suggestions can contribute to effective poverty reduction strategies.

2020 Mathematics Subject Classification. 92B05, 34D20.

Key words and phrases. poverty and alcohol consumption; sensitivity analysis; numerical simulation; parameter estimation.

1. Introduction

The mathematical model establishes a direct relationship between poverty and alcohol addiction. Presently, there is a growing trend in endemic research towards adopting more practical approaches that focus on social issues. Consequently, we have embarked on developing a mathematical modeling framework that delves into the correlation between poverty, alcohol addiction, and related factors.

DOI: 10.28924/APJM/10-36

This latest toolkit offers comprehensive support to researchers in several key areas:

(i) It emphasizes the significance of considering gender, sex, and intersectionality in infectious disease and implementation research.

(ii) It guides researchers in designing, developing, and reporting studies using an intersectional gender lens through a step-by-step approach.

(iii) The toolkit helps researchers comprehend the barriers that hinder the effective implementation of health interventions to prevent and control infectious diseases.

(iv) It encourages researchers to explore solutions that promote equality in access to healthcare, ensuring that no individual is left behind.

While the toolkit prioritizes research on preventing and controlling poverty-related infectious diseases, it also acknowledges the equal importance of other health research and initiatives. This inclusive approach aims to address a broad spectrum of health challenges to create a healthier and more equitable world [1].

Increased rainfall can significantly affect African drylands, particularly in already impoverished regions. This rise in precipitation may lead to an increase in vector-borne water-related illnesses, putting additional pressure on the vulnerable populations residing in these areas [2].

Tobacco use can serve as a significant cause of heart disease and respiratory illnesses, as well as over 20 different types of cancer, among other serious health issues. This is primarily due to the high addictiveness of nicotine present in tobacco products [3,4].

Tobacco use can have deadly consequences not only for smokers but also for nonsmokers. Secondhand smoke has been linked to harmful health effects, leading to 1.2 million deaths annually. Shockingly, nearly 50% of children are exposed to tobacco-polluted air, resulting in the tragic loss of 65,000 young lives each year due to illnesses associated with secondhand smoking. Moreover, smoking during pregnancy can lead to long-term health problems for the baby.

Alcohol, on the other hand, is a toxic and euphoric substance with addictive properties. Alcoholic beverages have become an integral part of many cultures' social experiences. This is especially true for individuals with a high social status and cultural influence, both within their national communities and on the international stage, where the presence of liquor is common. [3,5].

Dangerous alcohol use accounts for 7.1% and 2.2% of the global disease burden in men and women, respectively. It stands as the primary cause of premature death and injury among individuals aged 15 to 49, contributing to 10% of all fatalities within this age group. Moreover, alcohol-related deaths and illnesses are more prevalent in impoverished and marginalized communities [6].

The analytical formula we have developed enables us to determine the equilibrium and tipping points between high and low alcohol levels in the model. We have identified various factors, such as ignorance, an unfavorable public environment, incapability to cope with life and stress, lack of psychological and social support, family and societal damages, family problems, forced relationships, and academic underachievement, all of which are directly or indirectly linked to poverty. Consequently, due to their impoverished circumstances, many individuals resort to alcohol abuse. In order to gain a deeper understanding of the dynamic behavior within this system and explore ways to minimize the impact of poverty and alcohol use, we employ a mathematical model. This model includes a dedicated section for individuals struggling with addiction, allowing us to explore a combination of government and non-government treatments to address these issues effectively [7].

This paper is organized as followings: Section 2 is described the initial assumption of model and analysis which comprises the subsections as basic reproduction number, equilibria, bifurcation analysis and their stability analysis. Section 3 analyze the parameter sensitivity of the model, section 4 as we display the numerical results of model and PRCC's of the numerical simulation are calculte in program to show the figures. Finally, we have given the results discussion and conclusion of the section 5.

2. The Model and Analysis

We have proposed and analyzed a non-linear model for the relationship between Poverty and Alcohol Consumption, dividing the entire population into four distinct categories. Specifically, we have focused on the age group of adults, as they are primarily involved in alcohol addiction, and used this age group to represent the population affected by poverty.

Let $N(t)$, $M(t)$, $A^R(t)$, $A^P(t)$, and $R(t)$ denote the fractions of Non-alcohol users, Moderate drinkers, Alcohol-addicted individuals from the rich group, Alcohol-addicted individuals from the poor group, and the Recovered population, respectively, at time t. We make the assumption that the total populations are varying and homogeneously mixed throughout the analysis [7].

$$
\begin{aligned}
\frac{dN}{dt} &= \Lambda - \mu N - \beta NM + \eta R, \\
\frac{dM}{dt} &= \beta NM - (\gamma_1 + \mu + \alpha) M, \\
\frac{dA^R}{dt} &= a\alpha M - (\gamma_2 + \mu + \mu_1) A^R, \\
\frac{dA^P}{dt} &= (1-a)\alpha M - (\gamma_3 + \mu + \mu_2) A^P, \\
\frac{dR}{dt} &= \gamma_1 M + \gamma_2 A^R + \gamma_3 A^P - \eta R.
\end{aligned} \tag{1}
$$

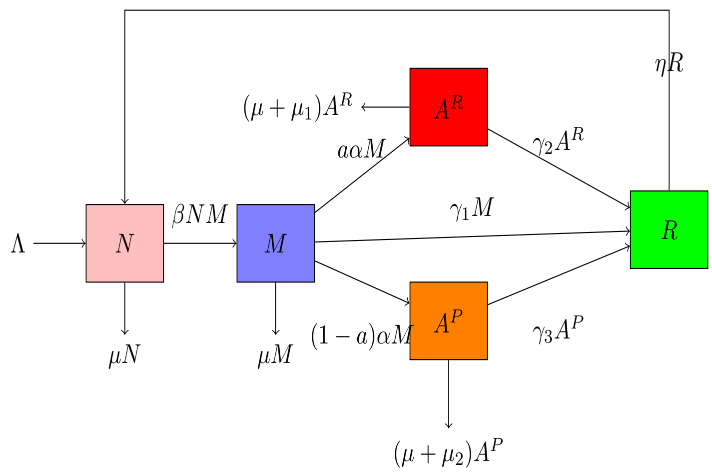

FIGURE 1. Model flow diagram

TABLE 1. Description of parameters

Parameter	Description
Λ	Recruitment rate
μ	Natural death rate
μ_1	Due to alcohol related disease affected rich population of death rate
μ_2	Due to alcohol related disease affected poor population of death rate
α	Progression rate of alcohol drinking behaviour with/without awareness
β	Population of progression rate from non-alcohol user to the moderate drinker
γ_1	Rates of moderate drinking people may leave the habit self aware
γ_2	Rates of alcohol addicted people take the treatment
γ_3	Rates of alcohol addicted people undertaken the NGO's to take the treatment
η	Rate of popoulation moves from recovery to Non-alcohol user

The model (1) can be simplified as following:

$$\begin{aligned}
\frac{dN}{dt} &= \Lambda - \mu N - \beta NM + \eta R, \\
\frac{dM}{dt} &= \beta NM - k_1 M, \\
\frac{dA^R}{dt} &= a\alpha M - k_2 A^R, \\
\frac{dA^P}{dt} &= (1-a)\alpha M - k_3 A^P, \\
\frac{dA^P}{dt} &= \gamma_1 M + \gamma_2 A^R + \gamma_3 A^P - \eta R.
\end{aligned} \tag{2}$$

Where $k_1 = \gamma_1 + \mu + \alpha,\ k_2 = \gamma_2 + \mu + \mu_1,\ k_3 = \gamma_3 + \mu + \mu_2.$

2.1. **The Basic Reproduction Number R_0.** The basic reproduction number R_0 is defined the number of secondary alcohol addiction is generated by the typical poverty individual in otherwise the disease free population. For our model, the reproduction number (R_0) is calculated by using the method to describe in [8] and using the same notation as in [8] the matrices $\mathcal{F}$ and $\mathcal{V}$ are followed by

$$\mathcal{F} = \begin{pmatrix} \beta NM \\ a\alpha M \\ (1-a)\alpha M \end{pmatrix}, \ \mathcal{V} = \begin{pmatrix} k_1 M \\ k_2 A^R \\ k_3 A^p \end{pmatrix}.$$

Now, the matrix F and V are evaluated to both alcohol and poverty free equilibrium point are given by

$$F = \begin{pmatrix} \beta N & 0 & 0 \\ a\alpha & 0 & 0 \\ (1-a)\alpha & 0 & 0 \end{pmatrix}, \ V = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k_3 \end{pmatrix}.$$

The basic reproduction number of model (2) is given by the eigenvalue of FV^{-1} is called the R_0 and is attained the following:

$$R_0 = \frac{\beta\Lambda}{k_1\mu}$$

Here, the R_0 denotes the number of secondary Alcohol addiction is generated to the single poverty people his/her whole alcohol usage of life. When, the whole population is considered under the poverty.

2.2. **Existence of Equilibria.** To determine the equilibria of the model used to the right hand side of the system (2) is equal to zero [9,10]. The following equilibria are namely,

(i) Alcohol free equilibrium(AFE) $E^0(N^0, 0, 0, 0, 0) = (\frac{\Lambda}{\mu}, 0, 0, 0, 0)$

(ii) Endemic equilibrium (EE) $E^* \left(N^*, M^*, A^{R*}, A^{P*}, R^*\right)$, where

$$
\begin{aligned}
N^* &= \frac{k_1}{\beta}, \\
M^* &= \frac{\mu k_1 - \Lambda\beta}{\beta(\gamma_1 - k_1 + \frac{\gamma_2 \alpha a}{k_2} + \frac{\gamma_3 \alpha (1-a)}{k_3})}, \\
A^{R*} &= \frac{\alpha a M}{k_2}, \\
A^{P*} &= \frac{(1-a)\alpha M}{k_3}, \\
R^* &= \frac{\mu k_1 - \beta k_1 M - \beta\Lambda}{\eta\beta}.
\end{aligned}
$$

2.3. **Stability Analysis.**

2.3.1. *Stability analysis of alcohol free equilibrium.*

Theorem 2.1. *The alcohol free equilibrium E^0 of the model (1) is locally asymptotically stable [11].*

Proof. The study of stability to the alcohol free equilibria of the matrix $\mathbf{M}_1$ corresponding to the system E^0 is obtained as

$$
\mathbf{M}_1 = \begin{pmatrix} -\mu & -\beta N & 0 & 0 & \eta \\ \beta M & \beta N - k_1 & 0 & 0 & 0 \\ 0 & a\alpha & -k_2 & 0 & 0 \\ 0 & (1-a)\alpha & 02 & -k_3 & 0 \\ 0 & \gamma_1 & \gamma_2 & \gamma_3 & -\eta \end{pmatrix}
$$

The eigenvalues of the variational matrix $\mathbf{M}_1$are given by the following roots are,

$$
\lambda_1 = -\mu, \qquad \lambda_2 = -k_1, \qquad \lambda_3 = -k_2,
$$
$$
\lambda_4 = -k_3, \qquad \lambda_5 = -\eta.
$$

We get all the eigenvalues of $\mathbf{M_1}$ are negative or negative real parts. Hence the equilibrium point E^0 is locally asymptotically stable . □

2.3.2. *Stability analysis of endemic equilibrium.*

Theorem 2.2. *The endemic equilibrium E^* of the model (1) is locally asymptotically stable by using Routh–Hurwitz criteria [12] [13].*

Proof. E^* corresponding to the variational matrix $\mathbf{M_2}$ is given by

$$
\mathbf{M_2} = \begin{pmatrix} -\mu - \beta M & -\beta N & 0 & 0 & \eta \\ \beta N & \beta N - k_1 & 0 & 0 & 0 \\ 0 & a\alpha & -k_2 & 0 & 0 \\ 0 & (1-a)\alpha & 0 & -k_3 & 0 \\ 0 & \gamma_1 & \gamma_2 & \gamma_3 & -\eta \end{pmatrix}
$$

The eigenvalues of the $\mathbf{M_2}$ is given by the roots of the following characteristic equation in λ.

$$\lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_4 = 0$$

Where,

$$\begin{aligned}
a_4 &= -(\beta N - k_1 - k_2 - k_3 - \eta - \mu - \beta M),\\
a_3 &= -[(\mu + \beta N)(\beta N - k_1 - k_2 - k_3 - \eta)\\
&\quad + (\beta N - k_1)(k_2 + k_3 + \eta)\\
&\quad - \eta k_2 - \eta k_3 - k_2 k_3 - \beta^2 MN],\\
a_2 &= -[\beta\eta\gamma_1 M - \beta^2 MN(k_2 + k_3 + \eta)\\
&\quad + (-\mu\beta N - \beta^2 MN + k_1\mu + k_1\beta M)(\eta - k_2 - k_3)\\
&\quad - k_2(k_3 + \eta)[(\mu + \beta M) - (\beta N - k_1)]\\
&\quad + \eta k_3(\beta(N - M) - \mu - k_1 - K_2)],\\
a_1 &= -[k_2(\mu + \beta M)(\beta N - k_1)(-k_3 + \eta)\\
&\quad + k_3\eta(\mu + \beta M)(\beta N - k_1)\\
&\quad + k_2 k_3\eta(\beta N - k_1 - \mu - \beta M) - \beta^2 MN k_2(k_3 + \eta)\\
&\quad + \beta M\eta(-\beta N k_3 + \gamma_2 a\alpha + \gamma_1 k_2 + \gamma_3(1 - a)\alpha + \gamma_1 k_3)],\\
a_0 &= -[\eta k_2 k_3(\beta^2 MN + \mu\beta N - \mu k_1 + \beta^2 MN - \beta M k_1)\\
&\quad + k_3\eta\beta M(\gamma_1 k_2 + \gamma_2 a\alpha)\\
&\quad + \beta M(1 - a)\alpha k_2\eta\gamma_3].
\end{aligned}$$

By using Routh–Hurwitz criteria, E^* will be locally asymptotically stable if the following conditions are satisfied: $a_i > 0, i = 1, 2, 3, 4, 5., a_1a_2a_3 > a_3^2 + a_1^2 a4$ and $(a_1a_4 - a_5)(a_1a_2a_3 - a_3^2 - a_1^2 a_4) > a_5(a_1a_2 - a_3)^2 + a_1a_5^2$. Hence, the E^* is locally asymptotically stable by Routh–Hurwitz criteria([6]-[29]). □

3. Sensitivity Analysis

We carry out sensitivity analysis for the parameter elaborate in reproduction number (R_0). It's utilised to find parameters with a lot of influence on R_0 that should be addressed by intervention tactics, when a parameter is altered, sensitivity indices can be used to assess the relative alter in a variable. We do this by looking at a variable's forward sensitivity index in relation to a specific parameter, which is outlined because the magnitude relation of the relative amendment within the variable to the relative amendment within the parameter. If such variable is differentiable with reference to the parameter, then the sensitivity index is outlined exploitation partial derivatives, as follows [14].

$$\frac{\partial R_0}{\partial \beta}\frac{\beta}{R_0} = 1 > 0$$

$$\frac{\partial R_0}{\partial \Lambda}\frac{\Lambda}{R_0} = 1 > 0$$

$$\frac{\partial R_0}{\partial \gamma_1}\frac{\gamma_1}{R_0} = -\frac{\gamma_1}{k_1} < 0$$

$$\frac{\partial R_0}{\partial \alpha}\frac{\alpha}{R_0} = -\frac{\alpha}{k_1} < 0$$

$$\frac{\partial R_0}{\partial \mu}\frac{\mu}{R_0} = -\frac{(2\mu + \alpha + \gamma_1)}{k_1} < 0$$

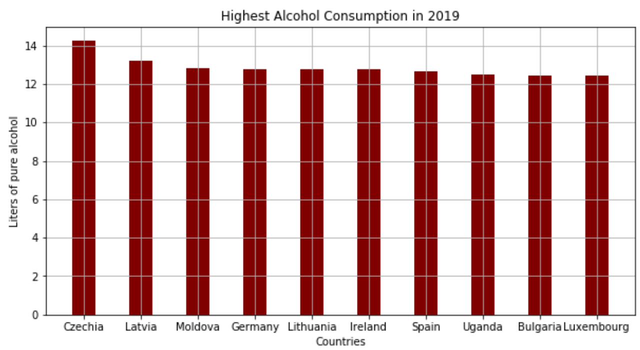

Figure 2. In 2019, the 10 most popular nations with the highest alcohol consumption are as follows: (in litres of pure alcohol per capita)

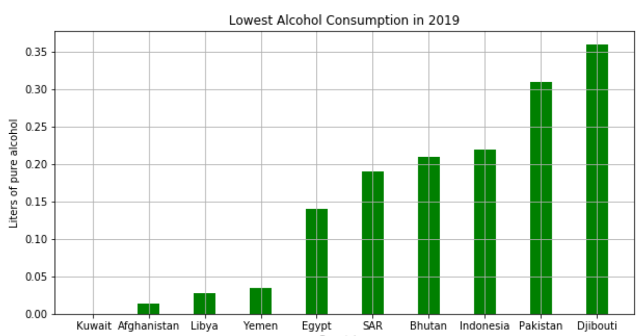

Figure 3. In 2019, the 10 most popular Nations with the Minimum Alcohol Consumption (in litres of pure alcohol per capita)

One will simply calculate an analytical expression for the sensitivity of R_0, using the above formula, to every parameter that it includes. The effect of the parameters is also visible on the R_0 in the Fig. 4. From the figures we can concludes that β , Λ, γ_1 , μ and α have same reaction on the R_0, which means if we have to rise the value of these parameters, poor and alcohol consumption rate will rise in the population. we have to control the poverty population rates due to binge drinking behaviuor between and poor population and non-alcohol drinker populations. Also, it is monitor that the parameter α, γ_1 and β are very sensistive and control in this parameters can lead to a considerable reduction in value of R_0.

Each country's alcohol consumption varies widely and is influenced by its laws, culture, and other factors. For all beer, wine, spirits, and other alcoholic drinks, the WHO reported each country's alcohol consumption in litres of pure alcohol.

In Fig.(2) is shows the details of Czechia nationals used the most alcohol per capita in 2019, according to the WHO, with 14.26 litres of pure alcohol drunk per person. Latvia and Moldova come in second and third, with 13.19 and 12.85 points, respectively. There are 38 nations on that peoples consume more than 10 litres of pure alcohol per year. The vast majority of these nations are found in Europe. https://worldpopulationreview.com/country-rankings/alcohol-consumption-by-country.

In Fig.(3) is shows the details of Alcohol drinking is frowned upon by the Middle East's leading religions, as seen by the region's extremely low consumption statistics.

Again, we do the sensitivity analysis to determine the sensitivity of the numerical simulation outputs to changes in each model parameter. As described in ([13], [15]), we use Latin Hypercube Sampling (LHS) and partial rank correlation coefficients (PRCC). We assume that all model parameters have a uniform distribution. We generate partial rank correlation coefficients (PRCCs) to identify the model parameters that have the greatest impact on the human drinking behaviours (A^R). To acquire the PRCC values, we use Latin Hypercube Sampling (LHS) using stratified sampling without replacement for the input parameters.

The PRCC values are shown in Figs.(4 & 5) for alcohol drinking humans at the 1000-day time point. Here, it is seen that PRCC values are always between -1 and 1. Positive numbers indicate a correlation between the parameter and the model output. Likewise, a negative PRCC score shows a negative correlation between the parameter and model output. Figure (5) demonstrates that, β,η,γ_2, and Λ are the most influential factors on the alcohol drinking behaviour infected human population.

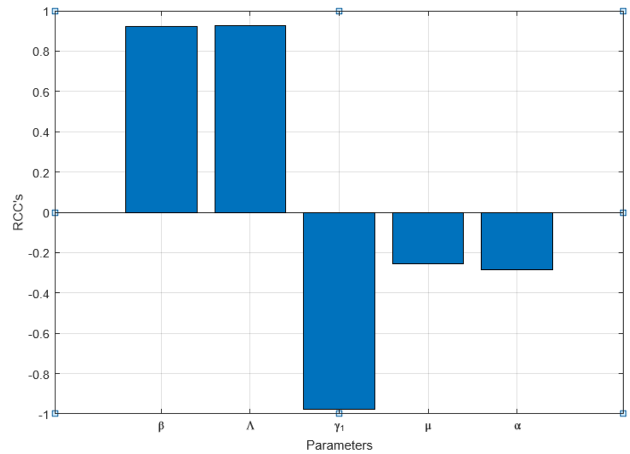

FIGURE 4. PRCC findings with R_0 displaying model parameter sensitivity indices

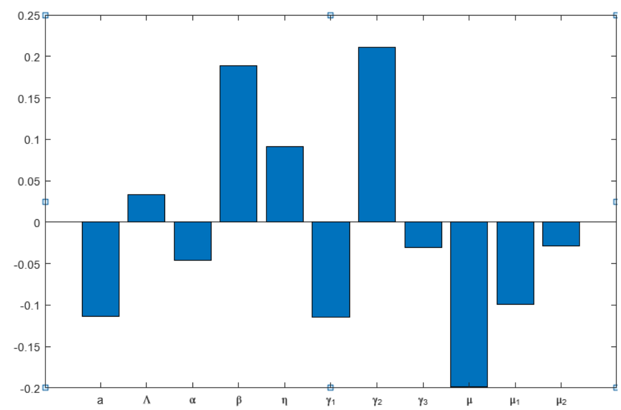

FIGURE 5. PRCC results showing sensitivity indices of the model parameters with A^R

4. Numerical Simulation

In this section, to validate our analytic findings, we need to apply numerical simulation. We verified the analysis results using numerical simulation. Experimentally validated results are justified and presented here.

The Numerical- simulations will be helped us to discuss and determined the effective of our analytical approach in this section.

System (1) is simulated for various values set of parameters which is satisfied locally asymptotically stable (see Fig. 9) the EE condition E^*. We are considered the parameters set $S_1 = \{\Lambda = 100, a = 0.08, \alpha = 0.06, \beta = 0.0001, \mu = 0.014, \mu_1 = 0.03, \mu_2 = 0.023, \gamma_1 = 0.0004, \gamma_2 = 0.00023, \gamma_3 = 0.00019, \eta = 0.003\}$, and some of the parameters are assum with feasible values.

The set parameters are E^0 the system (1) has only the AFE is locally asymptotically stable (see Fig. 8). The parameters values are $S_3 = \{\Lambda = 100, a = 0.2, \alpha = 0.01, \beta = 0.0001, \mu = 0.014, \mu_1 = 0.03, \mu_2 = 0.023, \gamma_1 = 0.51, \gamma_2 = 0.13, \gamma_3 = 0.19, \eta = 0.03\}$

The system (1) has four feasible equilibria; There are EE and AFE are locally asymptotically stable [9] (see Fig. 9& 8) respectively.

The effects of the parameter of the system (1) has to attain the stability of the equilibrium point in different position in the ($\alpha, \beta, \gamma_1, \gamma_2, \gamma_3$) parameter are how its work and its shown in the variation of pointwise effects like (see Fig. 10, 11, 12, 13 & 14) respectively.

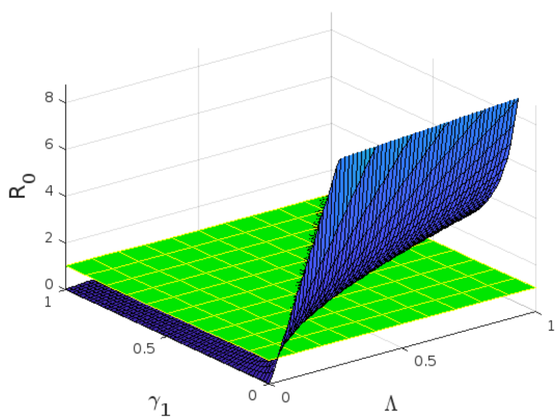

FIGURE 6. In terms of the parameters Λ and γ_1 are showing the effect of R_0

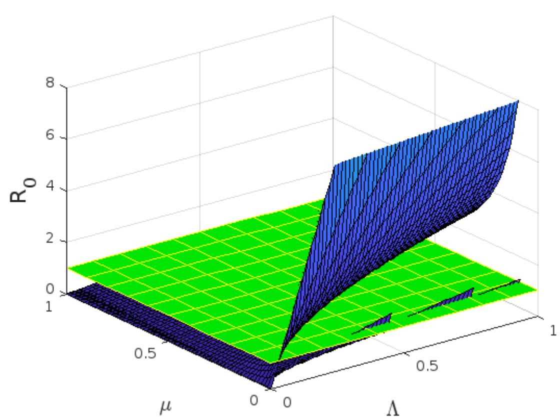

FIGURE 7. In terms of the parameters μ and Λ are showing the effect of R_0

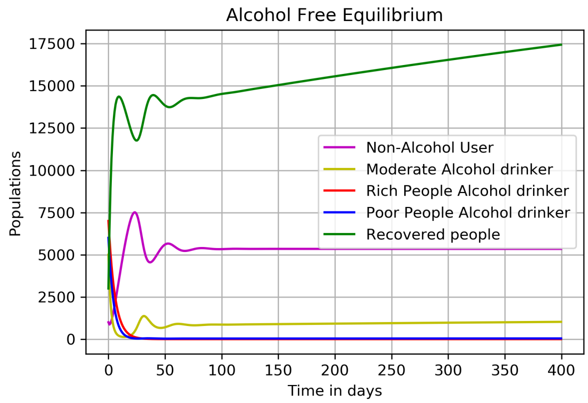

Figure 8. Variation of populations with time showing the stability of alcohol free equilibrium E^0

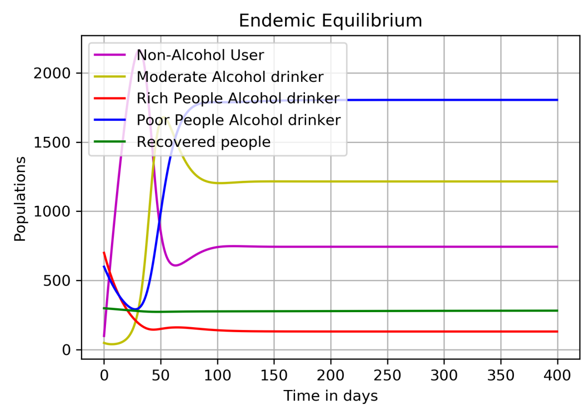

Figure 9. Time evolution depicting the dynamical behavior of the system (1) stable point of E^*

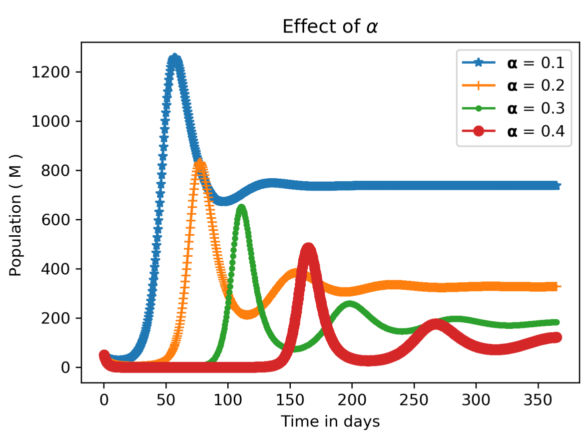

Figure 10. Variation of M population with time for different values of the parameter effects at each point α.

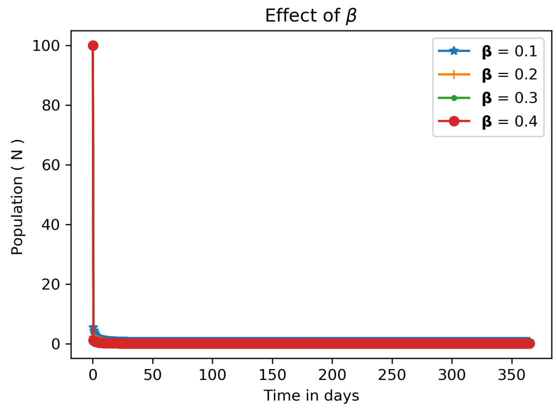

FIGURE 11. Variation of N population with time for different values of the parameter effects at each point β

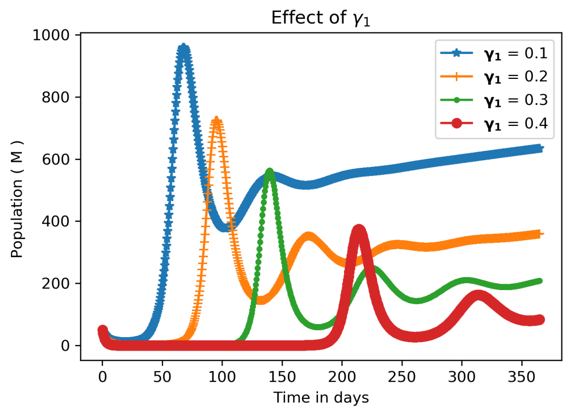

FIGURE 12. Variation of M population with time for different values of the parameter effects at each point γ_1

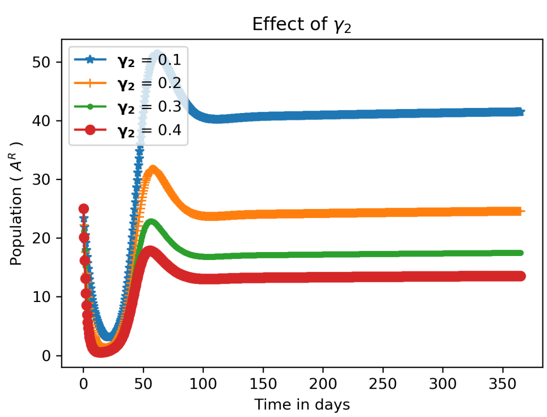

FIGURE 13. Variation of A^R population with time for different values of the parameter effects at each point γ_2

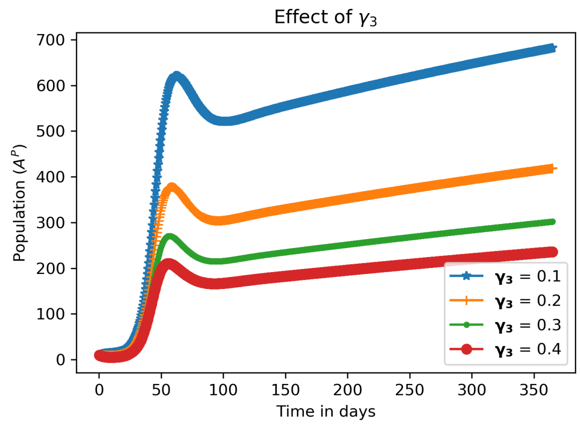

FIGURE 14. Variation of A^P population with time for different values of the parameter effects at each point of γ_3

5. CONCLUSION

In this section, we are discussing the effect of our model compartements and parameters on reducing the poverty and alcohol inttervention. It is a well known fact that there is dealing with a difficult situation (a solution) for poverty reduction for socially and economically, we expand the various aspects to reduce the poverty growth and alcoholism with the effects of generalised methods or specific methods and spread through poverty and its impact on the population.

Therefore, to minimize poverty and its causes, identification/detection is needed. All these are probably if we mobilize ourselves to eliminate suffering for a prosperous world. Each of us the socially responsible for reducing the persistence of poverty reduction in society, and we are taking steps to forecast poverty reduction using potential methods by designing mathematical models.

This model is suggested and evaluated to minimize poverty with this factor in mind. Through this model, through computational simulation, we found the feasibility of the demographic progression from alcohol addiction to recovery. In figure 10, shows that the effect of α value increased so automatically the moderate drinking behaviour changed into binge drinking. In this way they are affected in alcohol related diseases. In figure 11, demonstrate the effect of parameter β value is increased at the time the alcohol users are increased. The figure 12, exhibits the effect of parameter γ_1 the moderate drinking peoples to take the decision to avoid the drinking habit by the social media. In figure 13, display that the effect of parameter γ_2 the alcohol addicted rich people to avoid the drinking behaviour day by day because they are getting the psychological treatment and counselling methods. In figure 14, shows the effect of parameter γ_3 the alcohol addicted poor people to decrease the drinking behaviour day today by helping the government and non- government (NGO) organizations.

Simulations of the systems are showed the variation of the population at different times in a particular period we are going to find out the maximum rates of intervention that will be reduced the poverty and alcohol addiction. Result of this paper, we will reduce the maximum number of members in both poverty and alcohol consumption.

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