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APPLICATION OF EDGE BIMAGIC MEAN LABELING IN DATA SECURITY

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Abstract. Cryptography plays an essential role in today's computerized environment for safely transferring information between at least two entities. Confidentiality is an essential issue to cope with in today's advanced conditions, and it is accomplished through numerous competencies. We investigate the use of graph theory in cryptography, which has been explored in numerous papers based on applications of graph theory. This paper proposes an edge bimagic mean labeling approach that is validated for latitude graph and slanting ladder graph. The same labeling technique is used to analyze both the encryption and decryption processes of a text using an affine cipher.

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Key words and phrases. latitude graph; slanting ladder graph; mean labeling; edge bimagic mean labeling; affine cipher; encryption; decryption.

1. Introduction

During the earlier period of 1960s graph labeling was developed. It is possible to observe the benefit of graph labeling in [1]. According to [2], standard terminologies and notations are used. The concept of bimagic labeling was first introduced by Babujee in [5]. Encrypting the communication is the first step of cryptography, which prevents the cipher text from revealing the original message to anyone. The receiver's side decryption is the other component, which reverses the encryption process and recovers the original message [11]. Security is achieved through encryption, which transforms the original communication into a scrambled configuration known as cipher content. Various security goals are provided by cryptanalysis to ensure that information is protected [6].

Cryptography is a legitimate technique for protecting information from unauthorized access. In cryptography, there are myriad encryption techniques for data security. Cryptology is a branch of

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mathematics that draws on algebra, graph theory, number theory, algebraic geometry, lattice theory, probability, and statistics. It deals with the issue of confidentiality, privacy, the authentication process, passwords, e-signature verification, digital currency, credibility, non-repudiation, etc. It is become an essential component of modern civilization.

Cryptography calculations are often classified into two major types: Public key (asymmetric) and private key (symmetric) cryptography [10]. Every sender and beneficiaries in symmetric-key cryptography shared a key that was used to garble and unscramble communications, but this had the drawback that executives needed to keep the key secure. In public key cryptography, a public key and a private key (a public key pair) are linked to an entity that wants to electronically authenticate its identification or sign or encrypt data. The associated private key is kept secret and each public key is made available. The affine cipher, a sort of monoalphabetic substitution cipher [4], each letter of the alphabet is converted into its numerical equivalent, encrypted using a fundamental mathematical formula, and then turned back to a letter. According to the procedure, each letter in the cipher text encrypts to one other letter and back again, making it entirely a conventional substitution cipher with a rule governing which letter goes where.

The concept of Face Magic Mean Labeling is introduced in [3]. The persuasion of Face Magic Mean Labeling has been established in [8] Mean graphs have been examined in [7] for composition of graphs, duplication of graphs, middle graphs etc. antimagic path, cycle, and felicitous Path with an even number of vertices, as well as a Complete Graph showing antimagic labeling in [9]. The conversion of the plaintext to cipher text by using Caesar and Bifid cipher method has been investigated in [13]. Affine Cipher has two degrees of security for encrypting content on social networking [12]. New Results on Face Magic Mean Labeling of Graphs [14] has been verified by S. Vani Shree and S. Dhanalakshmi. Face Bimagic Mean Labeling is introduced by S. Vani Shree and S. Dhanalakshmi [15] and it verified for Duplication of a Path Graph. The intent of this current study is to describe about an edge bimagic mean labeling approach, which is validated for latitude graph and slanting ladder graph. The same labeling technique is used to analyze both the encryption and decryption processes of a text using an affine cipher.

2. Proposed Work

Edge bimagic mean labeling is initially defined and analysed for latitude graph and slanting ladder graph. Then affine cipher technique is applied for encryption and decryption of the original text.

An *Edge Bimagic Mean labeling* of a graph $G(V,E)$ with V vertices and E edges is a bijection $f\colon V(G) \rightarrow \{1,2,3,\ldots|V|\}$ such that $f^*(uv) = \lfloor\frac{f(u)+f(v)}{2}\rfloor$ is either k_1 or k_2 a constant for any edge $uv \in E(G)$.

The *latitude graph* is a graph that consist of a cycle of even length n having vertex set $u_k : 1 \leq k \leq n$ in which the edge set is formed by the edges $u_1\ u_2$,$u_2\ u_3 \ldots u_{n-1}\ u_n$,$u_n\ u_1$ and by joining u_k and u_{n+2-k} for$2 \leq k \leq \frac{n}{2}$.

A *slanting ladder graph* SL_n,$n \geq 2$ is a ladder graph with 2n vertices and is obtained from two paths of length $n-1$ with $V(G) = u_k, v_k : 1 \leq k \leq n$ and $E(G) = u_k u_{k+1}, v_k v_{k+1} : 1 \leq k \leq n-1 \cup u_k v_{k+1} : 1 \leq k \leq n-1$.

3. Encryption and Decryption Procedure

Edge bimagic mean labeling is analyzed for Latitude graph Ln_n, $n \geq 2$ and slanting ladder graph SL_n, $n \geq 2$ with significant illustration.

3.1. **Functioning of Encryption process.**

- Examine the graph G, by computing $C \equiv aP + k(mod 26)$, with the requirement that $gcd(a, 26) = 1$. Here a and k are the number of edges of a graph G and the bimagic constant k_1 which is computed.
- Convert the letters in the original message M to their corresponding ordinal numbers with the help of the normal chart and name it as P which is a sequence of ordinal number.
- To generate the sequence of encrypted numbers each text is assigned to a numerical value (cipher text) which is obtained by step 1.
- Encrypted message was created as a resultant of transforming the respective encrypted numbers.

3.2. **Functioning of Decryption process.**

- Obtained encrypted numbers are transformed into ordinal numbers using the normal chart.
- By Solving, $P \equiv a^{-1}(C-k)(mod 26)$, a unique solution P for each C. The numeric string is obtained by Converting the other words is same way.
- The original message is attained by transforming the resulting sequence of numbers into their corresponding letters from normal chart.

Note: The Affine Cipher perception is utilised, and Core Java is also implemented for the verification process. The Images of some Java programmes are shown in section 4.2.4 for better understanding.

4. Results and Discussions

4.1. **Main Result: Encryption and decryption - Latitude graph.**

Theorem 4.1. *Latitude graph $Ln_n, n \geq 2$ admits Edge Bimagic labeling*

Proof. Let $G = Ln_n$ be a latitude graph,$n \geq 2$ has V vertices and E edges where $V = \{u_l, v_l; 1 \leq l \leq n\}$ and $E = \{u_l u_{l+1}, v_l v_{l+1}; 1 \leq l \leq n-1\} \cup \{u_l v_l; 1 \leq l \leq n\}$.
Define a mapping $\phi : V(G) \to \{1, 2, 3, ...2n\}$ as follows,

Case (i): n is odd $\phi(u_{2l-1}) = 2l-1; 1 \leq l \leq \frac{n+1}{2};$

$\phi(u_{l+1}) = 2n-l; 1 \leq l \leq \frac{n+1}{2}; l \equiv 1(mod\ 2)$

$\phi(v_{2l-1}) = 2n-2l+2; 1 \leq l \leq \frac{n+1}{2}$

$\phi(v_{2l}) = 2l; 1 \leq l \leq \frac{n-1}{2}; l \equiv 1(mod\ 2)$

Hence, the Edge Bimagic Mean constants in G is obtained as,

$\phi^*(u_l u_{l+1}) = \lfloor n \rfloor \ \ or \ \ \lfloor n+1 \rfloor$

$\phi^*(v_l v_{l+1}) = \lfloor n \rfloor \ \ or \ \ \lfloor n+1 \rfloor$

$\phi^*(u_l v_l) = \lfloor n \rfloor$

Case (i): n is even

$\phi(u_{2l-1}) = 2l-1; 1 \leq l \leq \frac{n}{2};$

$\phi(u_{l+1}) = 2n-l; 1 \leq l \leq n; l \equiv 1(mod\ 2)$

$\phi(v_{2l-1}) = 2n-2l+2; 1 \leq l \leq \frac{n}{2}$

$\phi(v_{2l}) = 2l; 1 \leq l \leq \frac{n}{2};$

Hence, the Edge Bimagic Mean constants in G is obtained as,

$\phi^*(u_l u_{l+1}) = \lfloor n \rfloor \ \ or \ \ \lfloor n+1 \rfloor$

$\phi^*(v_l v_{l+1}) = \lfloor n \rfloor \ \ or \ \ \lfloor n+1 \rfloor$

$\phi^*(u_l v_l) = \lfloor n \rfloor$

□

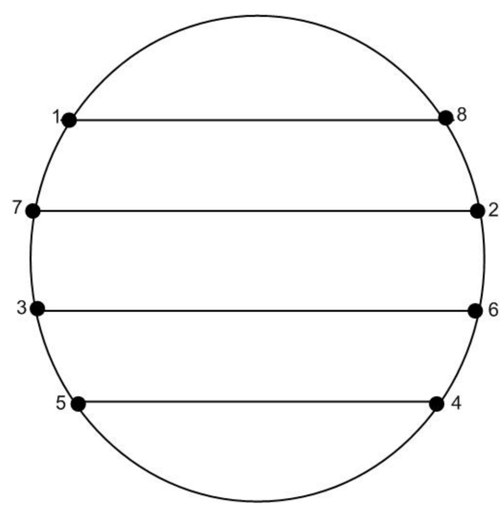

Figure 1. Edge Bimagic Mean Labeling of Ln_4

4.1.1. *The procedure of encrypting a latitude graph.*

Input: The original text EVERY CLOUD HAS A SILVER LINING and the latitude graph.

Output: The encrypted message NINAB JOHTY GFP F PVOINA OVSVSR.

(1) From the graph Ln_5, $a = 3n = 15$ where a is the number of edges, $k = n = 5$ is the bimagic constant k_1 is calculated.

(2) The plain text EVERY CLOUD HAS A SILVER LINING is replaced with its corresponding ordinal number – 0421041724 0211142003 070018 00 180811210417 110813081306. Let the ordinal numbers in the resulting sequence to be P in a text is convert each text into a numeric value using a linear system. $C \equiv 15p + 5(mod\, 26)$

For example P=04, $C \equiv 15(04) + 5(mod\, 26)$

By solving the aforementioned equations,

The solution $C \equiv 13$ is obtained.

(3) Convert the other text in the similar form, yields the numeric string-1308130001 0914071924 060515 05 152114081300 142118211817.

(4) As a result, the cipher text is NINAB JOHTY GFP F PVOINA OVSVSR, with the relevant letters translated based on the ordinal numbers.

4.1.2. *The procedure of decrypting a latitude graph.*

Input: The encrypted message NINAB JOHTY GFP F PVOINA OVSVSR

Output: The decrypted message EVERY CLOUD HAS A SILVER LINING

(1) The encrypted message NINAB JOHTY GFP F PVOINA OVSVSR is replaced by 1308130001 0914071924 060515 05 152114081300 142118211817 using the normal chart.

(2) On solving the linear system, $P \equiv a^{-1}(c - k)(mod\, 26)$, computation of a^{-1} to be done initially. The condition required to find a^{-1} is $ax(mod\, 26) = 1$.

(3) Based on the illustration taken, it is obtained as $(15 * 7(mod\, 26) = 1)$, where $a^{-1} = 7$. A unique solution for P is obtained, for each cipher text C.

For example $C = 13 = N$, $P \equiv 7(13 - 5)(mod\, 26) = 56(mod 26)$ $P = 4 = E$

By solving the obtained solution is $P = 04$

(4) The sequence P was derived in a similar manner for the sequence with regard to C. By converting the sequence of number - 0421041724 0211142003 070018 00 180811210417 110813081306 is obtained by using normal chart based on its corresponding letters.

(5) As a result the corresponding plain text is – EVERY CLOUD HAS A SILVER LINING.

4.2. **Main Result: Encryption and decryption - Slanting ladder graph.**

Theorem 4.2. *Slanting ladder graph $SL_n, n \geq 2$ admits Edge Bimagic labeling*

Proof. Let $G = SL_n$ be a Slanting ladder graph,$n \geq 2$ has V vertices and E edges where $V = \{u_l, v_l; 1 \leq l \leq n\}$ and $E = \{u_l u_{l+1}, v_l v_{l+1}, u_l v_{l+1}; 1 \leq l \leq n-1\}$.

Define a mapping $\phi : V(G) \to \{1, 2, 3, ...3n-3\}$ as follows,

$\phi(u_{l-1}) = l; 1 \leq l \leq n+1; l \equiv 0(mod\, 2)$

$\phi(u_{2l}) = 2n - l; 1 \leq l \leq n-1; l \equiv 1(mod\, 2)$

$\phi(v_l) = l; 1 \leq l \leq n; l \equiv 1(mod\, 2)$

$\phi(v_{l+1}) = 2n - l + 1; 1 \leq l \leq n-1; l \equiv 1(mod\, 2)$

Hence, the Edge Bimagic Mean constants in G is obtained as,

$\phi^*(u_l u_{l+1}) = \lfloor n \rfloor \ or \ \lfloor n+1 \rfloor$

$\phi^*(v_l v_{l+1}) = \lfloor n \rfloor \ or \ \lfloor n+1 \rfloor$

$\phi^*(u_l v_l) = \lfloor n \rfloor, 2 \leq l \leq n;$

□

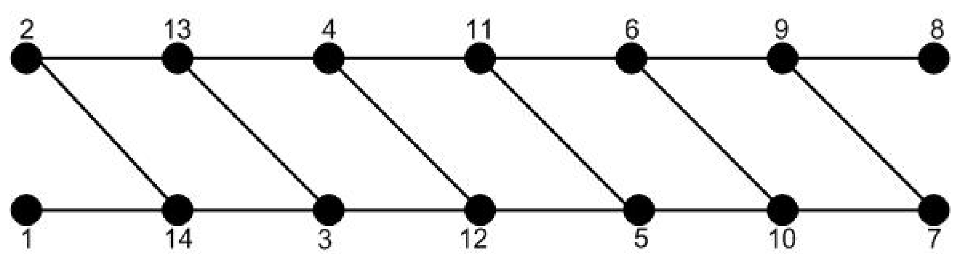

FIGURE 2. Edge Bimagic Mean Labeling of SL_7

4.2.1. *The procedure of encrypting a slanting ladder graph.*

Input: The original text EVERY CLOUD HAS A SILVER LINING and the slanting ladder graph.

Output: The encrypted message OHOBS YFQMT ZIW I WUFHOB FUVUVE.

(1) From the graph SL_8, $a = 3n - 3 = 21$ where a is the number of edges, $k = n = 8$ is the bimagic constant k_1 is calculated.

(2) The plain text EVERY CLOUD HAS A SILVER LINING is replaced with its corresponding ordinal number – 0421041724 0211142003 070018 00 180811210417 110813081306. Let the ordinal numbers in the resulting sequence to be P in a text is convert each text into a numeric value using a linear system. $C \equiv 21p + 8(mod\, 26)$

For example P=04, $C \equiv 21(04) + 8(mod\, 26)$

By solving the aforementioned equations,

The solution $C \equiv 14$ is obtained.

(3) Convert the other text in the similar form, yields the numeric string – 1407140118 2405161219 250822 08 222005071401 052021202104.

(4) As a result, the cipher text is OHOBS YFQMT ZIW I WUFHOB FUVUVE, with the relevant letters translated based on the ordinal numbers.

4.2.2. *The procedure of decrypting a slanting ladder graph.*

Input: The encrypted message OHOBS YFQMT ZIW I WUFHOB FUVUVE

Output: The decrypted message EVERY CLOUD HAS A SILVER LINING

(1) The encrypted message OHOBS YFQMT ZIW I WUFHOB FUVUVE is replaced by 1407140118 2405161219 250822 08 222005071401 052021202104 using the normal chart.

(2) On solving the linear system, $P \equiv a^{-1}(c-k)(mod\, 26)$, computation of a^{-1} to be done initially. The condition required to find a^{-1} is $ax(mod\, 26) = 1$.

(3) Based on the illustration taken, it is obtained as $(21 * 5)(mod\, 26) = 1)$, where $a^{-1} = 5$. A unique solution for P is obtained, for each cipher text C.

For example $C = 14 = N$, $P \equiv 5(14-8)(mod\, 26) = 30(mod\, 26)$, $P = 4 = E$

By solving the obtained solution is $P = 04$

(4) The sequence P was derived in a similar manner for the sequence with regard to C. By converting the sequence of number - 0421041724 0211142003 070018 00 180811210417 110813081306 is obtained by using normal chart based on its corresponding letters.

(5) As a result the corresponding plain text is – EVERY CLOUD HAS A SILVER LINING.

4.2.3. *Flow chart of encryption and decryption process for latitude graph and slanting ladder graph.*

The following figure represents the structural outline of encryption and decryption process for slanting ladder graph.

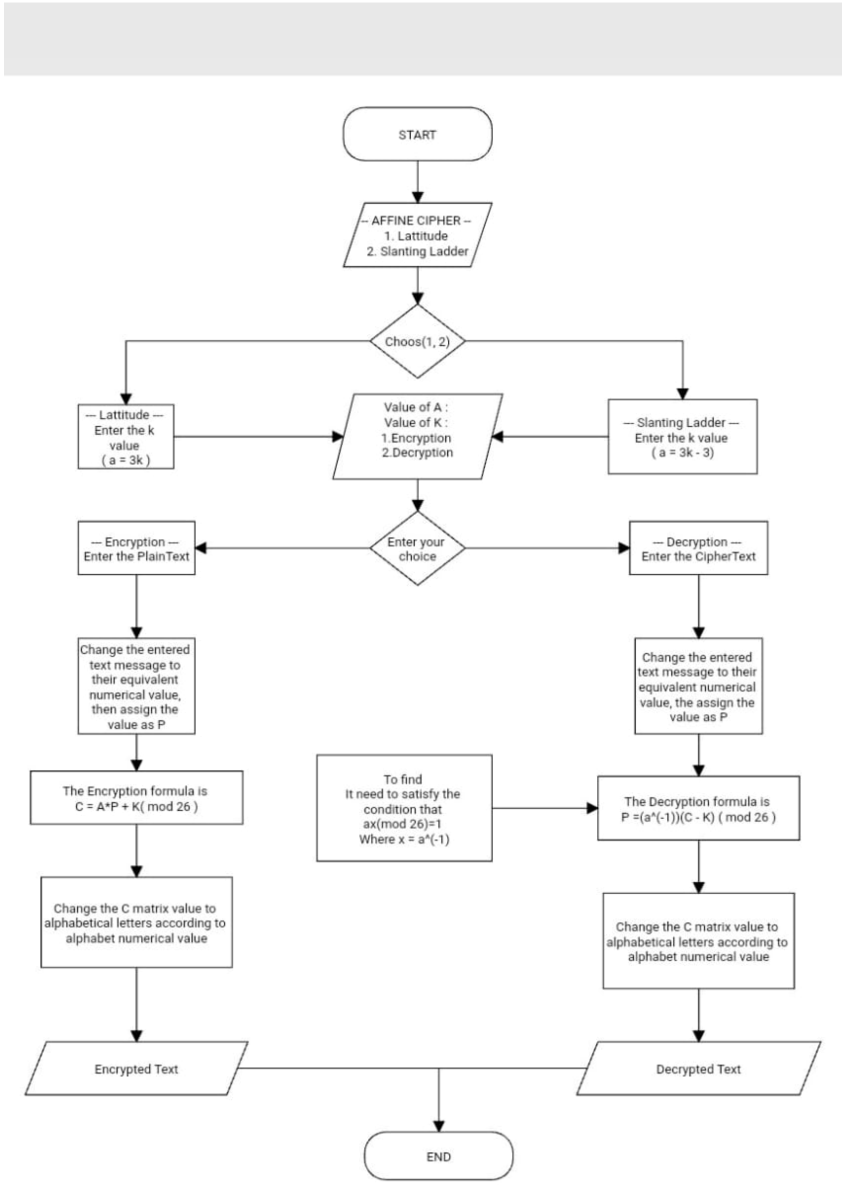

FIGURE 3. Schema Chart - encryption and decryption process for Ln_n and SL_n.

4.2.4. *Images of some Java programmes.*

```java
import java.util.*;
public class affine {
   public static void main(String args[]) {
      Scanner sc = new Scanner(System.in);
      int key11 = 0, ch, ct, pt, l;
      int n2;
      int n1= 0 ;
      char t;
      int fact26[] = {1, 2, 13};
      int co = 0;
      char c[] = {'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H', 'I', 'J', 'K', 'L', 'M', 'N', 'O', 'P', 'Q', 'R', 'S', 'T', 'U', 'V', 'W', 'X', 'Y', 'Z', ' '};
      System.out.println("---AIFFINE CIPHER---");
      System.out.print("1. Lattitude\n2. Ladder\nChoose(1,2): ");
      int choice = sc.nextInt();
      System.out.print("Enter K value :");
      n2 = sc.nextInt();
      if (choice == 1) {
         System.out.println("---Lattitude---");
        n1 = 3 * n2;
      } else if (choice == 2) {
         System.out.println("---Ladder---");
         n1 = (3 * n2) - 3;
      } else {
         System.out.println("Invalid Choice");
      }

      System.out.println("Value of A :"+n1);
      System.out.println("Value of K :"+n2);

      System.out.println("1. Encryption");
      System.out.println("2. Decryption");
      System.out.print("Enter your choice :");
      ch = sc.nextInt();
      //   switch(ch)
```

```java
         System.out.println("---Encryption---");
         Scanner dc = new Scanner(System.in);
         System.out.print("Enter PlainText : ");
         String num = dc.nextLine();
         num = num.toUpperCase();
         l = num.length();
         int k = 0;
         int a[] = new int[l];
         char cte[] = new char[l];
         char[] num1 = num.toCharArray();
         for (int j = 0; j < num1.length; j++) {
            for (int i = 0; i < c.length; i++) {
               if (num1[j] == c[i]) {
                  if (num1[j] != ' ') {
                     a[k] = i;
                  } else {
                     a[k] = 32;
                  }
                  k = k + 1;
               }
            }
         }
         //   System.out.print("Enter Key 1 :");
         //   key1=sc.nextInt();
         int key1 = n1;

         for (int i = 0; i < 3; i++) {
            if (key1 % fact26[i] == 0)
               co++;
         }
         if (co > 1) {
            System.out.println("Key is inappropriate i.e. " + key1 + " and 26 is not coprime");

         }
```

```java
        System.out.print("Encrypted message :");
        for (int i = 0; i < a.length; i++) {
          if (a[i] != 32) {
            ct = (a[i] * key1 + key2) % 26;
            cte[i] = c[ct];
          } else {
            cte[i] = (char)a[i];
          }
          System.out.print(cte[i]);
        }


      }
      //case 2:
      else if (ch == 2) {
        System.out.println("---Decryption---");
        Scanner ac = new Scanner(System.in);
        System.out.print("Enter CipherText : ");
        String num = ac.nextLine();
        num = num.toUpperCase();
        l = num.length();
        int k1 = 0;
        int a1[] = new int[l];
        char cte1[] = new char[l];
        char[] num2 = num.toCharArray();
        for (int j = 0; j < num2.length; j++) {
          for (int i = 0; i < c.length; i++) {
            if (num2[j] == c[i]) {
              if (num2[j] != ' ') {
                a1[k1] = i;
              } else {
                a1[k1] = 32;
              }
              k1 = k1 + 1;
            }
          }
```

```java
        int key1 = n1;
        for (int i = 0; i < 3; i++) {
          if (key1 % fact26[i] == 0)
            co++;
        }
        if (co > 1) {
          System.out.println("Key is inappropriate i.e. " + key1 + " and 26 is not coprime");

        }
        //System.out.print("Enter Key 2 :");
        int key2 = n2;
        for (int i = 0; i < 26; i++) {
          key11 = i;
          if ((key1 * i) % 26 == 1)
            break;
        }
        System.out.print("Decrypted message :");
        for (int i = 0; i < a1.length; i++) {
          if (a1[i] != 32) {
            ct = ((a1[i] - key2) * key11) % 26;
            if (ct < 0) {
              ct = ct + 26;
            }
            cte1[i] = c[ct];
          } else {
            cte1[i] = (char)a1[i];
          }
          System.out.print(cte1[i]);
        }

        //default:   System.out.println("Wrong Choice");
      }
    }
```

```
---AIFFINE CIPHER---
1. Lattitude
2. Ladder
Choose(1,2): 1
Enter K value :5
---Lattitude---
Value of A :15
Value of K :5
1. Encryption
2. Decryption
Enter your choice :1
---Encryption---
Enter PlainText : EVERY CLOUD HAS A SILVER LINING
Encrypted message :NINAB JOHTY GFP F PVOINA OVSVSR
[Program finished]
```

```
---AIFFINE CIPHER---
1. Lattitude
2. Ladder
Choose(1,2): 1
Enter K value :5
---Lattitude---
Value of A :15
Value of K :5
1. Encryption
2. Decryption
Enter your choice :2
---Decryption---
Enter CipherText : NINAB JOHTY GFP F PVOINA OVSVSR
Decrypted message :EVERY CLOUD HAS A SILVER LINING
[Program finished]
```

```
---AIFFINE CIPHER---
1. Latitude
2. Slanting ladder
Choose(1,2): 2
Enter K value :8
---Slanting ladder---
Value of A :21
Value of K :8
1. Encryption
2. Decryption
Enter your choice :1
---Encryption---
Enter PlainText : EVERY CLOUD HAS A SILVER LINING
Encrypted message :OHOBS YFQMT ZIW I WUFHOB FUVUVE
[Program finished]
```

```
---AIFFINE CIPHER---
1. Latitude
2. Slanting ladder
Choose(1,2): 2
Enter K value :8
---Slanting ladder---
Value of A :21
Value of K :8
1. Encryption
2. Decryption
Enter your choice :2
---Decryption---
Enter CipherText : OHOBS YFQMT ZIW I WUFHOB FUVUVE
Decrypted message :EVERY CLOUD HAS A SILVER LINING
[Program finished]
```

5. Conclusion

Edge Bimagic Mean Labeling is analysed for latitude graph and slanting ladder graph in this work. To strengthen the secrecy of data, the encryption process and the decryption process of a text using affine cipher is implemented for the above mentioned labeling technique. In future different cipher technologies can be employed for the various classes of graphs to compute the encryption process and the decryption process.

References

[1] J.A. Gallian. A dynamic survey of graph labeling, Electron. J. Comb. 2020 (2020), #DS6.

[2] F. Harary, Graph theory, Addison Wesley, New Delhi, 1969.

[3] J.B. Babujee, Bimagic Labeling in Path Graphs, Math. Educ. 38 (2004), 12–16.

[4] D.A. Angel Sherin, V. Maheswari, Encryption and decryption process using edge magic labeling, J. Phys.: Conf. Ser. 1362 (2019), 012024. https://doi.org/10.1088/1742-6596/1362/1/012024.

[5] J.B. Babujee, On edge bimagic labeling, J. Comb. Inf. Syst. Sci. 28 (2004), 239–244.

[6] T. Koshy, Elementary number theory with applications, 2nd ed, Academic Press, Amsterdam, 2007.

[7] S.K. Vaidya, L. Bijukumar, Some new families of mean graphs, J. Math. Res. 2 (2010), 169–176.

[8] S. Arockiaraj, A. Meena Kumari, On F-face magic mean labeling of some planar graphs, Int. J. Pure Appl. Math. 117 (2017), 1–8.

[9] D.K. Gurjar, A. Krishnaa, Lexicographic labeled graphs in cryptography, Adv. Appl. Discr. Math. 27 (2021), 209–232. https://doi.org/10.17654/dm027020209.

[10] D.R. Stinson, M.B. Paterson, Cryptography: theory and practice, Fourth edition, CRC Press, Boca Raton, 2018.

[11] W. Stallings, Cryptography and network security: principles and practice, 4th ed, Pearson/Prentice Hall, Upper Saddle River, 2006.

[12] I.W. Sudarsana, S.A. Suryanto, D. Lusianti, N.P.A.P.S. Putri, An application of super mean and magic graphs labeling on cryptography system, J. Phys.: Conf. Ser. 1763 (2021), 012052. https://doi.org/10.1088/1742-6596/1763/1/012052.

[13] B. Deepa, V. Maheswari, V. Balaji, Creating ciphertext and decipher using graph labeling techniques, Int. J. Eng. Adv. Technol. 9 (2019), 206–211.

[14] S.V. Shree, S. Dhanalakshmi, On face bimagic mean labeling of duplication of a path graph, TELEMATIQUE, 21 (2022), 3262–3267.

[15] S.V. Shree, S. Dhanalakshmi, New results on face magic mean labeling of graphs, Math. Stat. 11 (2023), 199–205. https://doi.org/10.13189/ms.2023.110123.