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DERIVATIVE PRICING USING HYBRID STOCHASTIC VOLATILITY MODELS

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Abstract. Stochastic volatility models can generally be considered modifications of the popular Black-Scholes model. These particular models are more realistic to the obtaining in the financial markets. This paper examines a hybrid Heston-SABR model, which combines elements from both the Heston and Stochastic Alpha Beta Rho (SABR) models. This model captures the main features of both models, namely the mean-revertion for the stochastic volatility process and the skewness regarding the distribution of the underlying asset returns. We analyze stylized facts of the hybrid Heston-SABR model. Using Monte Carlo simulations, we plot the simulated paths for European option and examine them for the different correlation parameter under this model. Further, we conduct numerical simulations to compute option prices for the European style using Milstein and Euler-Maruyama methods.

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Key words and phrases. Stochastic volatility; Heston-SABR model; Monte Carlo; Heston's model; SABR model.

1. Introduction

Stochastic volatility based model are significant in financial market. Such models play a crucial role in finance and investment practice, as they enhance pricing accuracy, enable effective risk management, provide valuable insights for trading and investment decisions, and capture the complex dynamics of financial markets. So far, a good number of studies have been conducted to price derivatives using stochastic volatility models. Such as, an analytical solution for options with dynamic volatility, applied to bond and currency options, as provided by [10] and managing smile risk as given by [9] among the several valuable research work in this regard. These models, such as the Heston model, have

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lately served as foundation for the development of subsequent hybrid stochastic volatility models. As explained by [7], the Heston model ranks among the extensively utilized models for stochastic volatility for pricing derivative, as it considers the non-lognormal characteristics of the asset return distribution, incorporates the impact of leverage, and accommodates the notable mean-reverting behavior of volatility. As stated in [15], stochastic volatility models naturally extend the Black-Scholes model to address the skew and smile patterns observed in real data. Also, observed study shows that volatility is stochastic and leads to an incomplete market [2,13,17,18], in contradiction to the popular Black-Scholes model where market completeness is assumed [16].

In our study, we consider a hybrid stochastic volatility model based on [10] and [9]. First, we explore the model's properties that aim to address the constraints of the Black-Scholes model, additionally we observe that, in as much as these individual models can represent the volatility smile and skew observed in option prices, stylized facts are more pronounced in the hybrid model. Secondly, we analyze the pricing of the Vanilla options for the model. Finally, we employ Monte Carlo simulations, to provide numerical simulation for Milstein and Euler-Maruyama methods for the European options under this model.

The organization of this paper is outlined as follows: Section 2 gives the links to literature giving relevant background information to our workings. We introduce the hybrid model in section 3, along with properties required for model existence and scrutinize stylized facts for the model. In section 4, we get our simulations and give numerical results. Our conclusions are presented in section 5.

2. Literature Review

2.1. **Financial Derivatives.** Financial Derivatives are contracts or products that derive their value from underlying assets, which can be any kind of financially relevant quantity. According to [11], the term "asset" is employed to denote any financial entity with a known present value that is subject to change over time, with a stock serving as a prime illustration of such an asset.

Many definitions of financial derivatives have been given by different scholars but all point to one underlying concept that identifies them from other assets. For instance, [12] specified that derivative securities are financial contracts whose value is derived from underlying cash market instruments, including stocks, bonds, currencies, and commodities. It can also be described as a financial asset whose valuation is contingent upon, or derived from, the values of other fundamental underlying variables, [13]. It is observed from the definitions above that the value of a derivative heavily depends on other basic cash instruments in a market which is the underlying. A derivative can therefore be described as a financial instrument or contract whose price is hinged on the value of a basic underlying variable that can be exchanged for cash in the market. The value of the contracts traded as derivatives depend on the price of the underlying which can be stock, currency or other assets that have financial value as explained by [12]. In reality though, the underlying can be anything of financial value: from the price of sand to the amount of sunshine on a particular day.

In many cases, to value a derivative requires one to first know the current and establish the possible future price for the underlying. The value of the underlying and the duration within which the contract is valid are essential in valuing derivatives. Each trader of derivatives must know how much a contract can cost before buying it and the possible amount of profit or benefit that may be gained.The derivative markets have drawn a diverse range of traders and exhibit substantial liquidity. According to [13], he categorizes traders into three groups: hedgers, speculators, and arbitrageurs. Hedgers employ derivatives to mitigate potential market variable movements, reducing their risk. Speculators, on the other hand, speculate on the future direction of market variables. Arbitrageurs adopt offsetting positions to either limit or entirely eliminate liabilities. For farther information with regard to the valuation and pricing of options we can refer to [11–14, 17].

When valuing derivatives such as options, a critical step is to identify a model that accurately represents the underlying stock price. This model serves as the foundation for pricing options according to the underlying price. Typically, we employ stochastic processes to model the security price.

2.2. **Stochastic Volatility Models.** In 1973, Fisher Black and Myron Scholes released a seminal paper on option valuation, in which they presented the Black-Scholes model. This paper had major effect on the world of finance. Some of the most important assumptions of the model are that the underlying asset's price process is continuous and that of constant volatility [16]. Nevertheless, these models fail to account for enduring characteristics of the surface depicting implied volatility, such as the presence of a volatility smile and skew. These features signify that implied volatility indeed exhibits variations relative to both strike price and maturity. Stochastic volatility models offer a solution to address a limitation of the Black–Scholes model. Instead of assuming that the volatility of the underlying price is constant, these models consider it as a random process. This approach enables a more accurate modeling of derivatives. They are used in derivative pricing to account for the volatility of the underlying asset, which can change over time. In these models, the volatility itself is treated as a random variable, which is typically modeled using a stochastic process. Generally, these models, under a risk-neutral probability, when $X(t)$ represents the asset price, has an equation of the form:

$$dX\left(t\right) = rX\left(t\right) + f\left(\psi\right)X\left(t\right)dW\left(t\right)$$

where r is the interest rate, W is the Brownian motion, and $f\left(\psi\right)$ is a positive process that can be correlated with W. In these models generally $f\left(\psi\right)$ are considered to be volatilities, where f a positive function and ψ is a diffusion process and satisfies the stochastic differential equation:

$$d\psi\left(t\right) = A\left(t, \psi\left(t\right)\right)dt + B\left(t, \psi\left(t\right)\right)dW\left(t\right)$$

where A and B are deterministic functions. Popularly used function f are the square root, the exponential or absolute value and for the driving diffusion ψ we may have the Cox-Ingersol-Ross(CIR), mean-reverting Ornstein-Uhlenbeck (OU) or the log-normal. A mixture of the aforementioned f and ψ has resulted into some of the prominent stochastic volatility model, including the Heston and SABR models.

The Heston model is a mathematical framework for characterizing the behavior of asset prices that incorporates stochastic volatility. It was developed by [10] and has subsequently emerged as one of the most extensively employed models for pricing derivatives. The Heston model postulates that the volatility of the underlying asset is dynamic, governed by a stochastic process. This stochastic process is driven by two factors: a mean-reverting level of volatility and a stochastic volatility term that captures the random fluctuations of volatility around the mean level. The Heston model also posits that the underlying stock follows a geometric Brownian motion, a prevalent assumption in the field of finance. By merging stochastic volatility with geometric Brownian motion, the Heston model can effectively encapsulate the intricate dynamics of asset prices and the corresponding option values.

The SABR model is a mathematical framework employed in quantitative finance to represent the volatility of an underlying asset, be it a stock or a commodity. It accommodates variable volatility and is applicable for pricing options on a diverse spectrum of underlying assets, spanning interest rate derivatives, foreign exchange, and commodities. It uses a set of stochastic differential equations to describe how the volatility of the underlying asset experiences variations over time. This model derives its name from the trio of parameters employed to delineate the stochastic behavior of volatility: alpha, beta, and rho. Alpha represents the initial level of the volatility, beta represents the response of volatility to fluctuations in the spot price, and rho indicates the relationship between the spot price and the volatility. It was designed to incorporate the volatility smile, which is the observed phenomenon where implied volatilities change with the exercise price of the option. The SABR model assumes that the asset's volatility is governed by a stochastic process, influenced by the value of the underlying asset, and maintains a consistent correlation with it. It also postulates that the volatility process exhibits mean-reversion, signifying its tendency to return to a long-term average value as time progresses. This model is extensively employed in the financial sector for pricing options and other derivatives, especially those that are exposed to extreme market conditions or that have long maturities. Its flexibility in capturing the volatility smile and its ability to handle negative and low interest rates make it a popular choice among practitioners.

2.3. **Monte Carlo Methods.** According to [6], the Monte Carlo method has shown beyond doubt to be adaptable and extremely useful computational tool in mathematical finance. They constitute a category of computational algorithms that employ random sampling to address problems. They find applications across various domains, encompassing physics, engineering, finance, and biology, for modeling intricate systems and studying their responses under diverse scenario. The fundamental concept underlying Monte Carlo methods involves generating a significant number of random samples according to the established or assumed probability distribution of the inputs for the system under investigation. These samples are then used to simulate the behavior of the system over time, and the results are analyzed statistically to estimate the probability distribution of the system's outputs. In [5], they introduce efficient ways to to apply Monte Carlo methods.

Monte Carlo Simulation plays a pivotal role in financial applications across various dimensions. Firstly, it's instrumental in options valuation, enabling the analysis of potential risks associated with pricing equity options. By simulating the fluctuations in underlying stock values across numerous price paths, it calculates option payoffs for different scenarios. The average of these payoffs provides the current option price. Secondly, it's utilized for portfolio valuation. This approach simulates the factors influencing the value of multiple portfolios to assess a wide array of potential outcomes. Ultimately, it derives the overall average value of these simulated portfolios, offering a highly accurate portfolio assessment. Lastly, Monte Carlo Simulation is integral to sensitivity analysis within financial modeling. In this context, our primary focus is on option valuation.

3. Methods and Materials

3.1. **Model Formulation.** Consider a complete stochastic foundation denoted as $(\Omega, \mathbb{F}, \{\mathcal{F}\}, \mathbb{P})$. This includes a right-continuous filtered probability space with the filtration $\{\mathcal{F}_t\}$, $0 \leq t \leq T$ and is $\mathbb{P}$-complete. All stochastic processes within the filtered complete probability space $(\Omega, \mathbb{F}, \{\mathcal{F}\}, \mathbb{P})$ are required to be well-defined and adapted. This holds within the context of a finite time horizon T. We examine a financial market scenario involving a single investor whose portfolio encompasses two key components: a risk-free security, which could be a bond or a money market account, represented as $\mathcal{M}(t)$, and a risky security, which could be a stock or a derivative, denoted as $X(t)$

Let the price dynamics of the risk-free security $\mathcal{M}(t)$ evolve as follows:

$$\begin{cases} d\mathcal{M}(t) = R\mathcal{M}(t)dt, \\ \mathcal{M}(0) = 1, \end{cases} \tag{1}$$

with a fixed interest rate R .

Let the price evolution of the risky security, a stock (or share) $X(t)$, be described by the stochastic differential equation:

$$\begin{cases} dX(t) = r(t)X(t)dt + \sqrt{\nu}X(t)dW^s(t), \\ X(0) = X > 0. \end{cases} \tag{2}$$

where $r(t)$ represents the risk-free interest rate, while ν denotes the instantaneous volatility, and dW^s is the standard Brownian motion and the dynamics of the Heston model for the instantaneous variance

ν is given by:

$$\begin{cases} d\nu\left(t\right)=\kappa(\theta-\nu\left(t\right))dt+\eta\sqrt{\nu\left(t\right)}dW^{v}\left(t\right), \\ \nu(0)=\nu_{0}>0, \end{cases} \tag{3}$$

where κ represents the mean reversion speed, θ stands for the long-term variance mean, η denotes the volatility of volatility, and $dW^{v}\left(t\right)$ corresponds to another Wiener process, independent of $dW^{s}\left(t\right)$.
The following stochastic differential equation, indigeneously defines the SABR model

$$\begin{cases} dF=\sigma F^{\beta}dW_{t} \\ d\sigma=\alpha\sigma dB_{t} \\ dW_{t}dB_{t}=\rho dt \end{cases} \tag{4}$$

In this context, F represents the forward value of the underlying security, while σ denotes the spot volatility, which is characterized as a stochastic process. The parameters α and β are subsequently introduced to complete the description, with $\alpha>0$ and $\beta\in[0,1]$.
The implied volatility corresponding to an option featuring a strike price of K and a maturity of T expressed as follows

$$\sigma(K,T)=\alpha(K,T)(F(K,T))^{(\beta(K,T))} \tag{5}$$

where $F(K,T)$ is as already defined, and $\alpha(K,T)$ and $\beta(K,T)$ are parameters. We combine these two models by replacing the volatility parameter in the Heston model with the SABR volatility formula:

$$\nu=f(\sigma(K,T),F(K,T),K,T) \tag{6}$$

where f is a function that maps the SABR parameters to the Heston volatility parameter. The function f that maps the SABR parameters to the Heston volatility parameter is expressed as:

$$f(\sigma(K,T),F(K,T),K,T)=\nu_{0}+\rho\sigma(K,T)(\sigma(K,T)-\sigma_{0}) \tag{7}$$

where ν_0 is the initial variance, σ_0 is the initial volatility.

3.1.1. *SABR Implied Volatility.* The distinctive characteristic of the SABR model enables one to calculate the implied volatility corresponding to a specified strike price. By using the Black's formula for European option prices provided in [4] as:

$$\begin{aligned} &C_{B}=e^{-rT}\left[FN\left(d_{1}\right)-KN\left(d_{2}\right)\right], \qquad P_{B}=V_{0}^{call}+e^{-rT}\left[K-F\right] \\ &\text{for} \quad d_{1}=\frac{\ln\left(\frac{F}{K}\right)+\frac{1}{2}\sigma_{B}^{2}T}{\sigma_{B}\sqrt{T}} \qquad \text{and} \quad d_{2}=d_{1}-\sigma_{B}\sqrt{T} \end{aligned} \tag{8}$$

where σ_B is assumed to be constant volatility in the Blacks model.
Thus, equipped with the formula in 8, and using singular perturbation methods, [9] derived an

analytical expression from the SABR-model for the implied volatility, also referred to as the Hagan formula:

$$\sigma_B(F,K) = \left[\frac{\alpha}{(FK)^{\frac{1-\beta}{2}}\left\{1+\frac{(1-\beta)^2}{24}\left(\ln\frac{F}{K}\right)^2+\frac{(1-\beta)^4}{1920}\left(\ln\frac{F}{K}\right)^4+\ldots\right\}}\frac{z}{\chi(z)}\right]$$
$$\left[1+\left(\frac{(1-\beta)^2}{24}\frac{\alpha^2}{(FK)^{(1-\beta)}}+\frac{1}{4}\frac{\alpha\beta\rho\nu}{(FK)^{\frac{(1-\beta)}{2}}}+\frac{2-3\rho^2}{24}\nu^2\right)T\right]+\ldots \tag{9}$$

Here

$$z = \frac{\nu}{\alpha}(FK)^{\frac{(1-\beta)}{2}}\ln\frac{F}{K}$$
$$\chi(x) = \ln\left(\frac{\sqrt{1-2\rho z+z^2}+z-\rho}{1-\rho}\right)$$

Where; $\sigma_B(F,K)$ is the implied volatility at strike K , α is the initial volatility level, F is the current forward price of the underlying asset, β is the parameter that controls the skewness of the volatility surface, ρ is as previously specified, ν is the volatility of volatility.

At-the-money, where $F = K$, the formula in 9 is further simplified to:

$$\sigma_B(F,K) = \frac{\alpha}{(F)^{(1-\beta)}}\left(\frac{z}{\chi(z)}\right)\times$$
$$\left[1+\left(\frac{(1-\beta)^2}{24}\frac{\alpha^2}{(F)^{\frac{(1-\beta)}{2}}}+\frac{1}{4}\frac{\alpha\beta\rho\nu}{(F)^{(1-\beta)}}+\frac{2-3\rho^2}{24}\nu^2\right)T\right]+\ldots \tag{10}$$

As mentioned in [9], the terms indicated by "..." , can be left out since they are too small.

Further in [1], an approximate formula has been provided when $\beta = 1$ for formula 10 becomes:

$$\sigma_B(F,K) \approx \alpha\left[1+\left(\frac{\rho\alpha\nu}{4}+\frac{2-3\rho^2}{24}\nu^2\right)T\right]. \tag{11}$$

3.2. **Heston-SABR Model.** The hybrid stochastic volatility models belong to a category of mathematical models that are utilized to describe the behavior of financial markets. They are used to model the volatility of asset prices, which is the degree of variation of prices over time. Numerous varieties of hybrid stochastic volatility models exist, and they can be used for a wide range across various cases, including but not limited to option valuation, risk assessment, and portfolio enhancement. In our model formulation we have adopted the Hybrid Model which is a combination of the two stochastic models; the Heston and the SABR Models. Following [19], this model is mathematically define by:

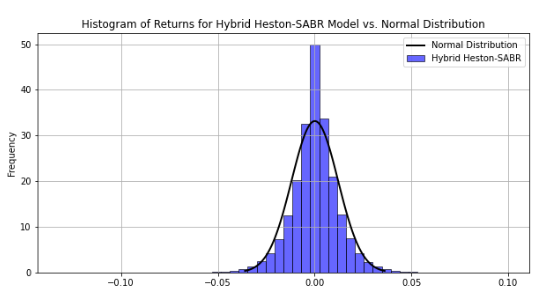
$$\begin{cases} dX(t) = rX(t)\,dt + \sqrt{\nu(t)}X^{\beta}(t)\,dW^s(t) \\ d\nu(t) = \kappa(\theta-\nu(t))dt + \eta\sqrt{\nu(t)}dW^{\nu}(t), \\ \text{where} \quad r\in\mathbb{R}, \quad \beta\in[0,1] \quad \text{and} \quad \kappa,\theta,\eta,\nu(0)>0 \end{cases} \tag{12}$$

with the notations as previously described, $dW^s(t)$ and $dW^v(t)$ are standard brownian motion.

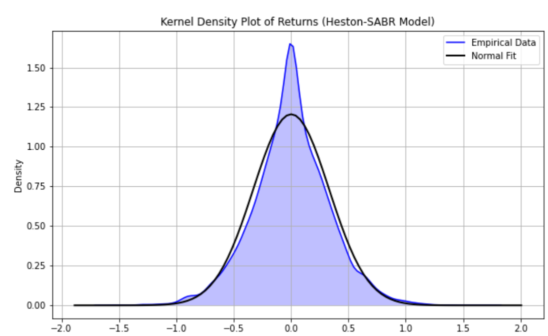
3.2.1. *Some Properties of the model Parameters.* To ensure the validity of the model, our parameters will be estimated whilst abiding by the following conditions: The parameters κ, θ, η, ν_0, should all be positive to indicate respectively; the volatility mean reverts towards its long-term average value, a non-zero long-term average value for volatility, volatility itself has volatility, the initial volatility level is positive. Furthermore, the correlation denoted as ρ between the asset price and its volatility must fall within the range of -1 to 1. This ensures the validity of the relation, signifying that the stock price and volatility exhibit corresponding or inverse movements, contingent on the sign of ρ. Accordingly, as in [3,7], $2\kappa\theta > \eta^2$, the Feller condition.

3.2.2. *Stylized Facts of the Model.* In this part, we analyze these empirical observations, for the discription and understanding of the financial market under this model. There are several well known stylized facts, however, in our case we simply look at the fat-tails, volatility clustering and leverage effects properties. In figure 1, by using the histogram and the kernel density, we have shown that the simulated returns are not normally distributed due to high peaks and not clearly symmetrical. Additionally, in support of the above assertions, a quantile-quantile plot is given in figure 2, hence, these returns for the hybrid Heston-SABR model typically exhibits characteristics that are more in line with empirical financial data, such as fat tails, which are not accounted for by a simple normal distribution. This suggests that the hybrid Heston-SABR model provides a better representation of market behavior and is suitable for capturing the stylized facts of financial returns, hence, accounts for higher levels of market volatility and unexpected events.

The plotted rolling volatility under the hybrid model in figure 3a reveals the presence of volatility clustering and this reaffirms the validity of the model in capturing real-world financial dynamics. Further, in figure 3b, we demonstrate that the hybrid Heston-SABR model incorporates the leverage characteristics of both models.



(a) Histogram



(b) Kennel Density

FIGURE 1. Histogram and Kennel Density of Returns

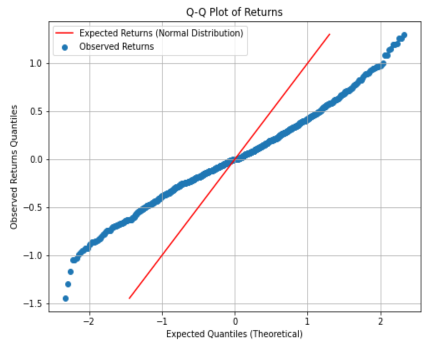
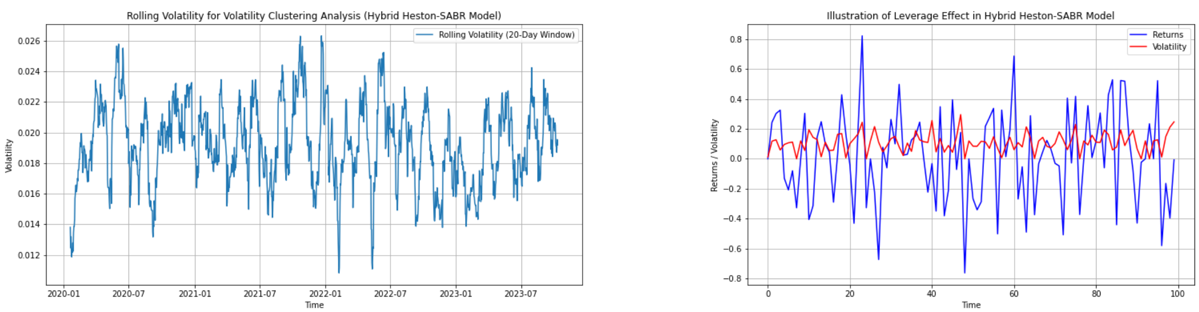


FIGURE 2. Quantile-Quantile of returns



(a) Rolling Volatility

(b) Leverage Effect

FIGURE 3. Volatility Clustering and Leverage Effect for the Model

4. Discussion and Results

It this section, we delve into the Monte Carlo estimation of the model. Firstly, we discretize the stochastic differential equations and simulation of the underlying asset.

4.1. **Simulation Of the Hybrid Heston-SABR Model.** The Monte Carlo simulation method is a commonly employed approach for modeling the prices of financial assets and derivatives. Accordingly, we apply this method to simulate our hybrid Heston-SABR model. In our implementation of these simulation in Matlab R2021a, we firstly choose the model parameters of the hybrid Heston-SABR model. Next, we create the Brownian motion, which is subsequently employed for simulating both the underlying asset's value and its volatility. To achieve this, we generated two sequences of independent normal random variables Z_s and Z_v, each of length n. Afterwards, employ these sequences to model the Brownian motion of the underlying asset's price and volatility by utilizing the following equations:

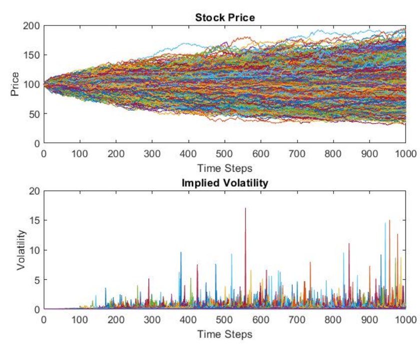
$$X_t = X_{t-1}e^{(r-\frac{1}{2}\nu_{t-1})\Delta t} + \sqrt{\nu_{t-1}}\sqrt{\Delta t}Z_{s,t} \tag{13}$$

$$\nu_t = |\nu_{t-1} + \kappa(\theta - \nu_{t-1})\Delta t + \sigma\sqrt{\nu_{t-1}}\sqrt{\Delta t} Z_{\nu,t}| \tag{14}$$

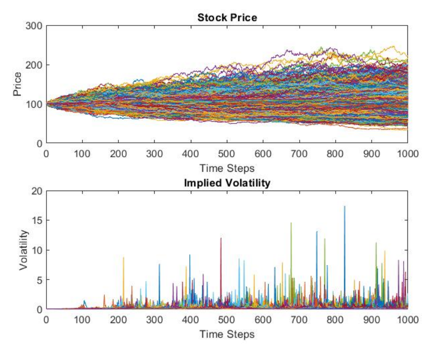
where X_t and ν_t are the simulated values of the underlying stocks's price and volatility at time t, r is the riskless rate, Δt is the time step, and $\kappa, \theta, \sigma, \rho$ are the parameters of the Heston model. By using the simulated values of the underlying asset's price and volatility to calculate the log-returns, we finally simulate the implied volatility and then use the following equation

$$\sigma_{imp,t} = \alpha \frac{S_t^{\beta}}{(\alpha S_t^{\beta} + (1-\alpha)K^{\beta})^{\frac{1}{\beta}}} \frac{\nu}{\nu_t} \tag{15}$$

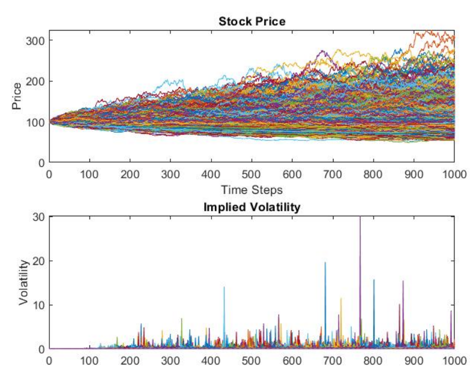
where $\sigma_{imp,t}$ represents the simulated implied volatility at time t, K stands for the strike price, while α, β, ρ, ν denote the parameters of the SABR model.



(a) fig



(b) fig



(c) fig

Figure 4. Simulations of the Hybrid Model

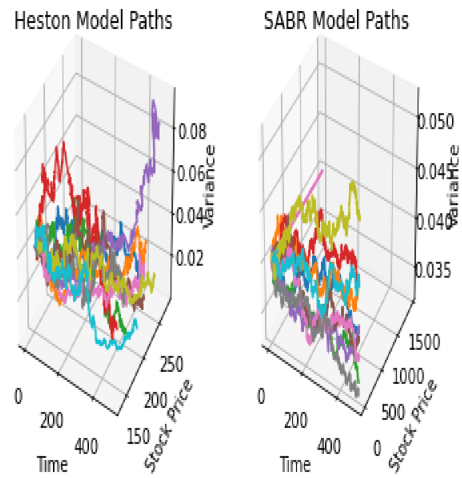


Figure 5. Heston and SABR Models Paths

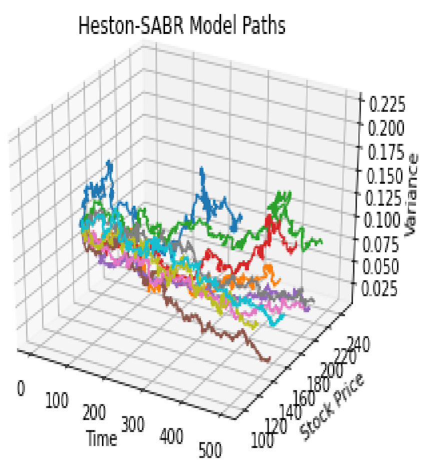


Figure 6. Hybrid Heston-SABR Model Paths

In figure 4, we have produced two charts: the upper chart displays the simulated stock price as a time-dependent function, and the lower chart illustrates the implied volatility over time. The stock price plot shows the simulated stock price for each simulation over time. It shows how the stock price changes over time and how it varies for different simulations. We can use it to analyze the behavior of the stock price under different market conditions and to evaluate the risk associated with a particular investment. The implied volatility plot shows the simulated implied volatility for each time step and simulation. The plot shows how the implied volatility changes over time and how it varies between different simulations. Given that implied volatility reflects market anticipations of future volatility based on option prices at each time step, it can serve as a tool for studying how implied volatility behaves in varying market conditions and for assessing the risk associated with a specific investment. In other words, these simulations and plots provide insights into the behaviour of the stock price and implied volatility over time based on the specified Heston and SABR model parameters. Thus, can be used for pricing of financial derivatives, such as options. In the figure 4: a, b, c above are reproduced for $\rho = -0.5, \quad \rho = 0 \quad$ and $\quad \rho = 0.5$ respectively, (that is, negative correlation, no correlation and positive correlation).

In figure 5 and figure 6, we employ python to compare the behaviour of simulated paths for the underlying asset price(stock price) and variance for individual models and the hybrid model , respectively.

4.2. **Numerical Pricing of the European option.** Since the probability distributions of the random variable is not known for our stated hybrid model, the price of the plain Vanilla option cannot be expressed using an analytical solution, thus, we use Monte Carlo simulations. In our work the Milstein and Euler-Maruyama methods are applied to update the stock price, Heston variance, and the SABR volatility paths at each time step. We have done $N = 10000$ simulations with $n = 252$ steps, with terminal time $T = 1.0$ for 9 different strike prices. We evaluate the mean option prices for the two schemes. A Corei5(Gen11), 1135G7@2.4GHz2.42GHz with 8GB RAM computer with Windows 10(x64) is employed to do computations in Python (Anaconda3) and the results given in table 1, for negative and positive correlations of the respective schemes. Table 2, show results when there is no correlation. To visualise our European Option prices under the hybrid Heston-SABR model, we plot figures 7, respectively for Milstein and Euler-Maruyama methods. Both approaches provide higher accuracy as observed in figure 8, which compares the the two schemes and differ slightly by the first decimal depending on the strike price, under the negative correlation.

Table 1. Mean-European Option Prices for the Model

n	Strike Price	Milstein*	Euler-Maruyama*	Milstein**	Euler-Maruyama**
0	80.0	32.210938	32.127348	32.203537	31.889112
1	85.0	28.498816	28.806606	28.711648	28.413035
2	90.0	25.699813	25.068208	25.327530	25.208082
3	95.0	22.357290	22.411554	21.980335	22.402860
4	100.0	19.707789	19.545275	19.587241	19.697529
5	105.0	17.358629	17.294781	17.330058	17.083570
6	110.0	14.763253	15.185878	15.003641	14.791138
7	115.0	13.165819	13.046696	13.206516	13.189936
8	120.0	11.450745	11.180395	11.393477	11.180545

Notes: $^*\rho = -0.5$, $^{**}\rho = 0.5$, *respectively , with* $S_0 = 100, r = 0.05, \beta = 0.5$.

Table 2. Mean-European Option Prices for the Model.

n	Strike Price	Milstein	Euler-Maruyama
0	80	32.062549	32.095253
1	85	28.436043	28.820786
2	90	25.434541	25.522703
3	95	22.078632	22.268216
4	100	19.809803	19.929522
5	105	17.164363	17.306409
6	110	14.788211	14.960135
7	115	12.934181	13.154319
8	120	11.275480	11.19896

$\rho = 0$, *with* $S_0 = 100, r = 0.05, \beta = 0.5$

(a) Milsteim method

(b) Euler-Maruyama method

FIGURE 7. Mean-European Option Prices(Hybrid Heston-SABR Model)

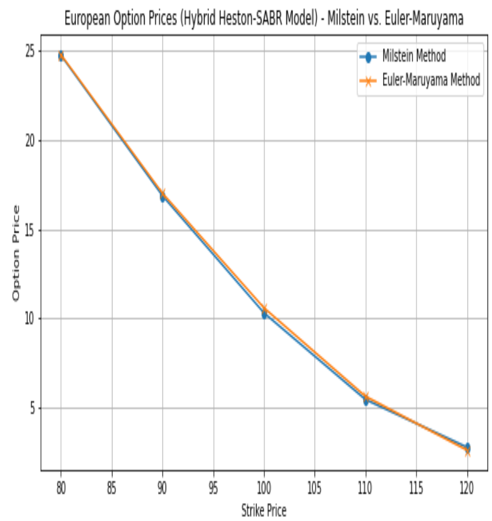


FIGURE 8. European Option Prices (Hybrid Heston-SABR Model)

5. Conclusion

Hybrid Stochastic volatility models play a crucial role in financial and investment practice. In this study, we have examined the hybrid Heston-SABR model. We have seen how the implied volatilities change over time for each simulation path and how they tend to vary with the underlying stock price. We have confirmed the better performance of ρ, when it is negative, as compared to when either positive or zero for this model. We have observed how the stock prices evolve over time, and how they can vary significantly across different simulation paths. These simulations are essential for understanding the dynamics of stock prices and implied volatilities, which are crucial for options pricing and risk management in quantitative finance. Further, we have performed numerical simulations to demonstrate that the hybrid Heston-SABR model encompasses a broader range of stylized facts than singly Heston or SABR models. Lastly, we used Monte Carlo simulations to compute the European Option prices for the model under different strike prices using Milstein and Euler-Maruyama methods.

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Authors' Contributions

Every author has reviewed and endorsed the final draft of the manuscript. The contributions to this work were equal among all authors.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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