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SOLVING LFPP BY CONVERSION TECHNIQUE IN NEUTROSOPHIC ENVIRONMENT

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Abstract. In this paper, we present a new technique for resolving the Neutrosophic Linear Fractional Programming Problem (NLFPP) by converting it into a Neutrosophic Linear Programming Problem (NLPP). An algorithm is proposed for the conversion. We used simplex method in neutrosophic environment to generate the optimal solution. Numerical examples are illustrated.

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1. Introduction

Linear programming is a method for achieving the best outcome (such as maximum profit or minimum cost) in a mathematical model represented by linear relationships. Linear fractional programming (LFP) deals with that class of mathematical programming problems in which the relations among the variables are linear: the constraint relations must be in linear form and the objective function to be optimized must be a ratio of two linear functions. Nowadays linear fractional criterion is frequently encountered in business and economics such as: debt-to-equity ratio[Min], return on investment[Max], Risk asset to Capital[Min], Actual capital to required capital[Max] etc. So, the importance of LFP problems is evident.

Linear Fractional Programming Problem was first introduced by Charnes and Cooper [3] and subsequently developed by Bitrand and Navaes [2], Hasan and Acharjee [6]. Abdel-Baset, Hezam and Smarandache [1] introduced neutrosophy which is the study of neutralities as an extension of dialectics. Neutrosophic is the derivative of neutrosophy and it includes neutrosophic set, neutrosophic probability, neutrosophic statistics and neutrosophic logic.

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Neutrosophic theory applied in many branches of science, in order to solve problems related to indeterminacy. Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle both incomplete and indeterminate information. Neutrosophic sets are characterized by three independent degrees namely truth membership degree (T), indeterminacy-membership degree (I), and falsity membership degree (F), where T, I and F are standard or non-standard subsets of $]0^-, 1^+[$. The decision makers in neutrosophic set want to increase the degree of truth-membership and decrease the degree of indeterminacy and falsity membership.

In this paper, we develop a technique to solve Neutrosophic Linear Fractional Programming Problem by converting it into a Neutrosophic Linear Programming Problem. The rest of the paper is structured as follows; In section 2, preliminaries are given. In section 3, standard form of NLPP and NLFPP are given. In section 4, conversion procedure is explained in detail. In section 5, algorithm is given. Numerical examples are included in section 6. Finally conclusion is given in section 7.

2. Preliminaries

In this section we recall some definitions and results, which are necessary for our discussion.

Definition 2.1. *Single Valued Triangular Neutrosophic Number (SVTNN) In the following* a_i^l, b_i^m, c_i^u $(i = 1, 2, ..)$ *are real numbers. A SVTNN is defined by* $\tilde{a}^n = \left\{\left(a_1^l,\ b_1^m,\ c_1^u\right); t_a,\ i_a,\ \omega_a\right\}$ *(refer figure 1) whose three membership functions for the truth, indeterminacy, and a falsity of* x *are given by*

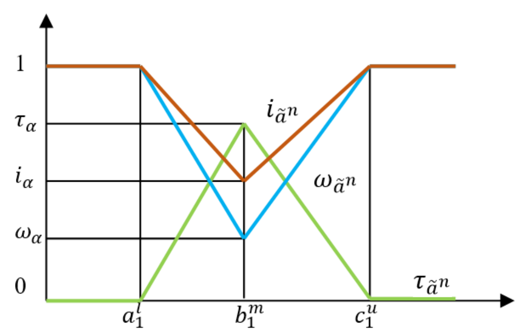

Figure 1. $\tilde{a}^n = \left\{\left(a_1^l,\ b_1^m,\ c_1^u\right); t_a,\ i_a,\ \omega_a\right\}$ SVTNN

$$t_{\tilde{a}^n}(x) = \begin{cases} \dfrac{\left(x - a_1^l\right) t_a}{b_1^m - a_1^l} & (a_1^l \leq x < b_1^m) \\ t_a & (x = b_1^m) \\ \dfrac{\left(c_1^u - x\right) t_a}{c_1^u - b_1^m} & (b_1^m \leq x < c_1^u) \\ 0 & \text{Otherwise,} \end{cases}$$

$$i_{\tilde{a}^n}(x) = \begin{cases} \dfrac{(b_1^m - x)\, i_a}{b_1^m - a_1^l} & (a_1^l \le x < b_1^m) \\ i_a & (x = b_1^m) \\ \dfrac{(x - c_1^u)\, i_a}{c_1^u - b_1^m} & (b_1^m \le x < c_1^u) \\ 1 & \text{Otherwise,} \end{cases}$$

$$\omega_{\tilde{a}^n}(x) = \begin{cases} \dfrac{(b_1^m - x)\, \omega_a}{b_1^m - a_1^l} & (a_1^l \le x < b_1^m) \\ \omega_a & (x = b_1^m) \\ \dfrac{(x - c_1^u)\, \omega_a}{c_1^u - b_1^m} & (b_1^m \le x < c_1^u) \\ 1 & \text{Otherwise,} \end{cases}$$

where $0 \le t_{\tilde{a}^n}(x) + i_{\tilde{a}^n}(x) + \omega_{\tilde{a}^n}(x) \le 3$, $x \in \tilde{a}^n$. Additionally, when $a_1^l > 0$, $\tilde{a}^n$ is called a positive SVTNN. Similarly, when $a_1^l < 0$, $\tilde{a}^n$ becomes a negative SVTNN.

Definition 2.2. *Let $\tilde{a}^n$and $\tilde{b}^n$ be two SVTNN's and $\gamma \neq 0$. Here $\tilde{a}^n = \{(a_1^l,\ b_1^m,\ c_1^u);t_a,\ i_a,\ \omega_a\}$ and $\tilde{b}^n = \{(a_2^l,\ b_2^m,\ c_2^u);t_b,\ i_b,\ \omega_b\}$. Then*

(a) *Addition*

$$\tilde{a}^n + \tilde{b}^n = \{\left(a_1^l + a_2^l,\ b_1^m + b_2^m,\ c_1^u + c_2^u\right); t_a \wedge t_b,\ i_a \vee i_b,\ \omega_a \vee \omega_b\}$$

(b) *Subtraction*

$$\tilde{a}^n - \tilde{b}^n = \{\left(a_1^l - c_2^u,\ b_1^m - b_2^m,\ c_1^u - a_2^l\right); t_a \wedge t_b,\ i_a \vee i_b,\ \omega_a \vee \omega_b\}$$

(c) *Multiplication*

$$\tilde{a}^n \cdot \tilde{b}^n$$
$$= \{\left(\text{Min}\left(a_1^l a_2^l, a_1^l c_2^u, c_1^u a_2^l, c_1^u c_2^u\right), b_1^m b_2^m,\ \text{Max}\left(a_1^l a_2^l, a_1^l c_2^u, c_1^u a_2^l, c_1^u c_2^u\right)\right); t_a \wedge t_b,\ i_a \vee i_b,\ \omega_a \vee \omega_b\}$$

(d) *Division*

$$\frac{\tilde{a}^n}{\tilde{b}^n} = \left\{\left(\text{Min}\left(\frac{a_1^l}{a_2^l}, \frac{a_1^l}{c_2^u}, \frac{c_1^u}{a_2^l}, \frac{c_1^u}{c_2^u}\right), \frac{b_1^m}{b_2^m}, \text{Max}\left(\frac{a_1^l}{a_2^l}, \frac{a_1^l}{c_2^u}, \frac{c_1^u}{a_2^l}, \frac{c_1^u}{c_2^u}\right)\right); t_a \wedge t_b,\ i_a \vee i_b,\ \omega_a \vee \omega_b\right\}$$

(e) *Scalar Multiplication*

$$\gamma \tilde{a}^n = \begin{cases} \{(\gamma a_1^l,\ \gamma b_1^m,\ \gamma c_1^u); t_a,\ i_a,\ \omega_a\},\ (\gamma > 0) \\ \{(\gamma c_1^u,\ \gamma b_1^m,\ \gamma a_1^l); t_a,\ i_a,\ \omega_a\},\ (\gamma < 0). \end{cases}$$

3. Mathematical Form of NLPP and NLFPP

The standard form of the NLPP is

$$\text{Max(or)Min } \tilde{z}(\tilde{x}) = \tilde{c}_i\tilde{x}_i + \tilde{\alpha} \quad \forall\, i = 1, 2, \ldots, n$$

subject to

$$\tilde{A}_{ji}\tilde{x}_i \leq \tilde{b}_j \quad \forall\, j = 1, 2, \ldots, m$$

and $$\tilde{x}_i \geq \{(0, 0, 0)\,; 1, 0, 0\}$$

and that of an NLFPP is

$$\text{Max(or)Min } \tilde{z}(\tilde{x}) = \frac{\tilde{c}_i\tilde{x}_i + \tilde{\alpha}}{\tilde{d}_i\tilde{x}_i + \tilde{\beta}} \qquad \forall\, i = 1, 2, \ldots, n \tag{1}$$

subject to

$$\tilde{A}_{ji}\tilde{x}_i \leq \tilde{b}_j \qquad \forall\, j = 1, 2, \ldots, m \tag{2}$$

and $$\tilde{x}_i \geq \{(0, 0, 0)\,; 1, 0, 0\}$$

where $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$; $\tilde{x}_i$, $\tilde{c}_i$ and $\tilde{d}_i \in R^n$; $\tilde{b}_j \in R^m$; $\tilde{A}_{ji}$ is a $m \times n$-SVTNN coefficient matrix; $\tilde{\alpha}$ and $\tilde{\beta}$ are neutrosophic constants.

4. Conversion From NLFPP to NLPP

In this conversion we are going to deal with two cases, (i) is in objective function (1) we going to consider $\tilde{\alpha}$ is positive and $\tilde{\beta}$ is negative and in case (ii) we going to consider both $\tilde{\alpha}$ and $\tilde{\beta}$ are negative.

4.1. **Case - (i).**

4.1.1. *Conversion of objective function.* Now (1) will be changed into

$$\text{Max (or) Min } \tilde{z}(\tilde{x}) = \frac{\tilde{c}_i\tilde{x}_i + \tilde{\alpha}}{\tilde{d}_i\tilde{x}_i + (-\tilde{\beta})}, \tag{3}$$

where $\tilde{\alpha}$ is positive and $\tilde{\beta}$ is negative.

$$\tilde{Z}(\tilde{x}) = \frac{\tilde{z}(\tilde{x}) + 1}{\tilde{z}(\tilde{x}) - 1}$$

$$\tilde{Z}(\tilde{x}) = \frac{(\tilde{c}_i + \tilde{d}_i)\tilde{x}_i + (\tilde{\alpha} - \tilde{\beta})}{(\tilde{c}_i - \tilde{d}_i)\tilde{x}_i + (\tilde{\alpha} + \tilde{\beta})}$$

$$= \frac{\tilde{C}_i\tilde{x}_i + \tilde{\lambda}}{\tilde{D}_i\tilde{x}_i + \tilde{\mu}} \tag{4}$$

where $\tilde{C}_i = \tilde{c}_i + \tilde{d}_i$; $\quad \tilde{D}_i = \tilde{c}_i - \tilde{d}_i \quad \tilde{\lambda} = \tilde{\alpha} - \tilde{\beta} \quad \tilde{\mu} = \tilde{\alpha} + \tilde{\beta}$

In Equation (4), Multiply & divide by $\tilde{\mu}$, and simplify it.

Now, we get the new objective function,

$$\tilde{Z}\left(\tilde{y}\right) = \tilde{E}_i \tilde{y}_i + \tilde{\rho}. \tag{5}$$

where $\tilde{E}_i = \dfrac{\tilde{C}_i \tilde{\mu} - \tilde{D}_i \tilde{\lambda}}{\tilde{\mu}}$; $\tilde{y}_i = \dfrac{\tilde{x}_i}{\tilde{D}_i \tilde{x}_i + \tilde{\mu}}$ and $\tilde{\rho} = \dfrac{\tilde{\lambda}}{\tilde{\mu}}$.

4.1.2. *Conversion of the constraints* (2).

$$\tilde{A}_{ji} \tilde{x}_i \leq \tilde{b}_j.$$

We shall now determine the value of $\tilde{x}_i$.

Here

$$\begin{aligned}
\tilde{y}_i &= \frac{\tilde{x}_i}{\tilde{D}_i \tilde{x}_i + \tilde{\mu}} \\
\tilde{x}_i &= \tilde{y}_i \left(\tilde{D}_i \tilde{x}_i + \tilde{\mu}\right) \\
\tilde{x}_i &= \tilde{D}_i \tilde{x}_i \tilde{y}_i + \tilde{y}_i \tilde{\mu} \\
\tilde{x}_i - \tilde{D}_i \tilde{x}_i \tilde{y}_i &= \tilde{y}_i \tilde{\mu} \\
\tilde{x}_i \left(1 - \tilde{y}_i \tilde{D}_i\right) &= \tilde{y}_i \tilde{\mu} \\
\tilde{x}_i &= \frac{\tilde{y}_i \tilde{\mu}}{1 - \tilde{y}_i \tilde{D}_i}.
\end{aligned} \tag{6}$$

Further substituting $\tilde{x}_i$ in

$$\begin{aligned}
\tilde{A}_{ji} \tilde{x}_i &\leq \tilde{b}_j \\
\tilde{A}_{ji} \left(\frac{y_i \mu}{1 - y_i D_i}\right) &\leq \tilde{b}_j, \qquad \text{where } \tilde{x}_i = \frac{\tilde{y}_i \tilde{\mu}}{1 - \tilde{y}_i \tilde{D}_i} \\
\tilde{A}_{ji} \tilde{y}_i \tilde{\mu} &\leq \tilde{b}_j \left(1 - \tilde{y}_i \tilde{D}_i\right) \\
\tilde{A}_{ji} \tilde{y}_i \tilde{\mu} &\leq \tilde{b}_j - \tilde{y}_i \tilde{D}_i \tilde{b}_j \\
\tilde{A}_{ji} \tilde{y}_i \tilde{\mu} + \tilde{y}_i \tilde{D}_i \tilde{b}_j &\leq \tilde{b}_j \\
\left(\tilde{A}_{ji} \tilde{\mu} + \tilde{b}_j \tilde{D}_i\right) \tilde{y}_i &\leq \tilde{b}_j \\
\tilde{B}_{ji} \tilde{y}_i &\leq \tilde{b}_j,
\end{aligned}$$

where $\tilde{B}_{ji} = \tilde{A}_{ji} \tilde{\mu} + \tilde{b}_j \tilde{D}_i$ and $\tilde{y}_i = \dfrac{\tilde{x}_i}{\tilde{D}_i \tilde{x}_i + \tilde{\mu}}$.
We get the new constraint

$$\tilde{B}_{ji} \tilde{y}_i \leq \tilde{b}_j. \tag{7}$$

4.1.3. *Calculation of unknown variables.* After solving the converted NLPP, we obtain the values of $\tilde{y}_i$. Then find $\tilde{x}_i$ using

$$\tilde{x}_i = \frac{\tilde{y}_i \tilde{\mu}}{1 - \tilde{y}_i \tilde{D}_i}.$$

On substituting the values of $\tilde{x}_i$ in the objective function, in

$$\text{Max (or) Min } \tilde{z}(\tilde{x}) = \frac{\tilde{c}_i\tilde{x}_i + \tilde{\alpha}}{\tilde{d}_i\tilde{x}_i - \tilde{\beta}}$$

We get the optimal solution.

4.2. **Case - (ii).**

4.2.1. *Objective function for this case is.*

$$\text{Max (or) Min } \tilde{z}(\tilde{x}) = \frac{\tilde{c}_i\tilde{x}_i - \tilde{\alpha}}{\tilde{d}_i\tilde{x}_i - \tilde{\beta}}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are negative.

We can also consider,

$$\text{Max (or) Min } \tilde{z}(\tilde{x}) = \frac{-\tilde{c}_i\tilde{x}_i + \tilde{\alpha}}{-\tilde{d}_i\tilde{x}_i + \tilde{\beta}} \tag{8}$$

Substituting $\tilde{C}_i = -\tilde{c}_i; \quad \tilde{D}_i = -\tilde{d}_i \quad \tilde{\lambda} = \tilde{\alpha} \quad \tilde{\mu} = \tilde{\beta}$

$$\tilde{Z}(\tilde{x}) = \frac{\tilde{C}_i\tilde{x}_i + \tilde{\lambda}}{\tilde{D}_i\tilde{x}_i + \tilde{\mu}}$$

In Equation (8), Multiply & divide by $\tilde{\mu}$, and simplify it.

Now, we get the new objective function,

$$\tilde{Z}\left(\tilde{y}\right) = \tilde{E}_i\tilde{y}_i + \tilde{\rho}. \tag{9}$$

where $\tilde{E}_i = \dfrac{\tilde{C}_i\tilde{\mu} - \tilde{D}_i\tilde{\lambda}}{\tilde{\mu}}$; $\tilde{y}_i = \dfrac{\tilde{x}_i}{-\tilde{D}_i\tilde{x}_i + \tilde{\mu}}$ and $\tilde{\rho} = \dfrac{\tilde{\lambda}}{\tilde{\mu}}$.

4.2.2. *Conversion of the constraints* (2).

$$\tilde{A}_{ji}\tilde{x}_i \leq \tilde{b}_j.$$

We shall now determine the value of $\tilde{x}_i$. Here

$$\begin{aligned}
\tilde{y}_i &= \frac{\tilde{x}_i}{-\tilde{D}_i\tilde{x}_i + \tilde{\mu}} \\
\tilde{x}_i &= \tilde{y}_i\left(-\tilde{D}_i\tilde{x}_i + \tilde{\mu}\right) \\
\tilde{x}_i &= -\tilde{D}_i\tilde{x}_i\tilde{y}_i + \tilde{y}_i\tilde{\mu} \\
\tilde{x}_i + \tilde{D}_i\tilde{x}_i\tilde{y}_i &= \tilde{y}_i\tilde{\mu} \\
\tilde{x}_i\left(\tilde{1} + \tilde{y}_i\tilde{D}_i\right) &= \tilde{y}_i\tilde{\mu} \\
\tilde{x}_i &= \frac{\tilde{y}_i\tilde{\mu}}{\tilde{1} + \tilde{y}_i\tilde{D}_i}.
\end{aligned}$$

Further substituting $\tilde{x}_i$ in

$$\begin{aligned}\tilde{A}_{ji}\tilde{x}_i &\leq \tilde{b}_j\\ \tilde{A}_{ji}\left(\frac{y_i\mu}{1+y_iD_i}\right) &\leq \tilde{b}_j, \qquad \text{where } \tilde{x}_i=\frac{\tilde{y}_i\tilde{\mu}}{\tilde{1}+\tilde{y}_i\tilde{D}_i}\\ \tilde{A}_{ji}\tilde{y}_i\tilde{\mu} &\leq \tilde{b}_j\left(\tilde{1}+\tilde{y}_i\tilde{D}_i\right)\\ \tilde{A}_{ji}\tilde{y}_i\tilde{\mu} &\leq \tilde{b}_j+\tilde{y}_i\tilde{D}_i\tilde{b}_j\\ \tilde{A}_{ji}\tilde{y}_i\tilde{\mu}-\tilde{y}_i\tilde{D}_i\tilde{b}_j &\leq \tilde{b}_j\\ \left(\tilde{A}_{ji}\tilde{\mu}+\tilde{b}_j\tilde{D}_i\right)\tilde{y}_i &\leq \tilde{b}_j\\ \tilde{B}_{ji}\tilde{y}_i &\leq \tilde{b}_j,\end{aligned}$$

where $\tilde{B}_{ji}=\tilde{A}_{ji}\tilde{\mu}-\tilde{b}_j\tilde{D}_i$ and $\tilde{y}_i=\dfrac{\tilde{x}_i}{-\tilde{D}_i\tilde{x}_i+\tilde{\mu}}$.
We get the new constraint

$$\tilde{B}_{ji}\tilde{y}_i \leq \tilde{b}_j. \tag{10}$$

4.2.3. *Calculation of unknown variables.* After solving the converted NLPP, we obtain the values of $\tilde{y}_i$. Then find $\tilde{x}_i$ using

$$\tilde{x}_i=\frac{\tilde{y}_i\tilde{\mu}}{\tilde{1}+\tilde{y}_i\tilde{D}_i}.$$

On substituting the values of $\tilde{x}_i$ in the objective function, in

$$\text{Max (or) Min } \tilde{z}(\tilde{x})=\frac{\tilde{c}_i\tilde{x}_i-\tilde{\alpha}}{\tilde{d}_i\tilde{x}_i-\tilde{\beta}}$$

We get the optimal solution.

5. Transformation Procedure

We now present the algorithm for finding the solution of a NLFPP. Consider the NLFPP

$$\text{Max (or) Min } \tilde{z}\left(\tilde{x}\right)=\frac{\tilde{c}_i\tilde{x}_i+\tilde{a}}{\tilde{d}_i\tilde{x}_i+\tilde{\beta}} \quad \forall\, i=1,2,\ldots,n$$

subject to

$$\tilde{A}_{ji}\tilde{x}_i \leq \tilde{b}_j$$

and $$\tilde{x}_i \geq \{(0,0,0)\,;1,0,0\}\,.$$

Step 1. Calculate $\tilde{E}_i$, $\tilde{\rho}$ from the values of $\tilde{C}_i$, $\tilde{D}_i$, $\tilde{\lambda}$, $\tilde{\mu}$.

Step 2. Form the new objective function as $\tilde{Z}\left(\tilde{y}\right)=\tilde{E}_i\tilde{y}_i+\tilde{\rho}$.

Step 3. Calculate $\tilde{B}_{ji}$ from the values of $\tilde{A}_{ji}$, $\tilde{\mu}$, $\tilde{b}_j$, $\tilde{D}_i$. This yields $\tilde{B}_{ji}\tilde{y}_i \leq \tilde{b}_j$; is the new constraints for the above objective function obtained at step 2.

Step 4. Solve the above NLP problem obtained from steps 2 and 3 by Neutrosophic simplex algorithm.

Step 5. The $\tilde{y}_i$ values are obtained from the simplex table.

Step 6. Using the values of $\tilde{y}_i$, $\tilde{\mu}$, $\tilde{D}_i$, calculate $\tilde{x}_i$ and Max (or) Min $\tilde{z}(\tilde{x})$, which is the final Optimal Neutrosophic Solution.

6. Numerical Example

Example 1. Consider the NLFPP.

$$\text{Max } \tilde{z} = \frac{\{(3,4,5);.3,.4,.6\}\,\tilde{x}_1 + \{(5,6,7);.4,.6,.5\}\,\tilde{x}_2 + \{(0,\ 0,\ 0);1,0,0\}}{\{(1,1,2);.4,.5,.3\}\,\tilde{x}_1 + \{(1,2,3);.5,.3,.2\}\,\tilde{x}_2 - \{(2,\ 3,\ 4);.5,.4,.3\}}$$

subject to

$$\begin{aligned}
\{(1,2,3);.2,.4,.3\}\,\tilde{x}_1 + \{(2,4,6);.3,.5,.7\}\,\tilde{x}_2 &\leq \{(14,\ 15,\ 16);.7,.5,.3\} \\
\{(1,2,4);.4,.6,.2\}\,\tilde{x}_1 + \{(1,4,8);.6,.5,.4\}\,\tilde{x}_2 &\leq \{(18,\ 19,\ 20);.6,.4,.2\} \\
\text{and } \ \tilde{x}_1,\ \tilde{x}_2 &\geq \{(0,0,0);1,\ 0,\ 0\}.
\end{aligned}$$

Solution. Here

$$\begin{aligned}
\tilde{c}_1 &= \{(3,4,5);.3,.4,.6\}, & \tilde{c}_2 &= \{(5,6,7);.4,.6,.5\}, \\
\tilde{\alpha} &= \{(0,\ 0,\ 0);1,0,0\}, & \tilde{d}_1 &= \{(1,1,2);.4,.5,.3\}, \\
\tilde{d}_2 &= \{(1,2,3);.5,.3,.2\}, & \tilde{\beta} &= \{(-4,\ -3,\ -2);.5,.4,.3\}, \\
\tilde{A}_{11} &= \{(1,2,3);.2,.4,.8\}, & \tilde{A}_{12} &= \{(2,4,6);.3,.5,.7\}, \\
\tilde{b}_1 &= \{(14,\ 15,\ 16);.7,.5,.3\}, & \tilde{A}_{21} &= \{(1,2,4);.4,.6,.2\}, \\
\tilde{A}_{22} &= \{(1,4,8);.6,.5,.4\}, & \tilde{b}_2 &= \{(18,\ 19,\ 20);.6,.4,.2\},
\end{aligned}$$

where $\tilde{A}_{11},\ \tilde{A}_{12}$ and $\tilde{b}_1$ are the coefficients of the first constraint and $\tilde{A}_{21},\ \tilde{A}_{22}$ and $\tilde{b}_2$ are the coefficients of the second constraint.

We now calculate

$$\begin{aligned}
\tilde{C}_1 = \tilde{c}_1 + \tilde{d}_1 &= \{(3,4,5);.3,.4,.6\} + \{(1,1,2);.4,.5,.3\} \\
&= \{(4,5,7);.3,.5,.6\} \\
\tilde{C}_2 = \tilde{c}_2 + \tilde{d}_2 &= \{(5,6,7);.4,.6,.5\} + \{(1,2,3);.5,.3,.2\} \\
&= \{(6,8,10);.4,.6,.5\} \\
\tilde{D}_1 = \tilde{c}_1 - \tilde{d}_1 &= \{(3,4,5);.3,.4,.6\} - \{(1,1,2);.4,.5,.3\} \\
&= \{(1,3,4);.3,.5,.6\} \\
\tilde{D}_2 = \tilde{c}_2 - \tilde{d}_2 &= \{(5,6,7);.4,.6,.5\} - \{(1,2,3);.5,.3,.2\} \\
&= \{(2,4,6);.4,.6,.5\}
\end{aligned}$$

$$\tilde{\lambda} = \tilde{\alpha} - \tilde{\beta} = \{(0,\ 0,\ 0)\,;1,0,0\} - \{(-4,\ -3,\ -2)\,;.5,.4,.3\}$$
$$= \{(2,3,4)\,;.5,.4,.3\}$$
$$\tilde{\mu} = \tilde{\alpha} + \tilde{\beta} = \{(0,\ 0,\ 0)\,;1,0,0\} + \{(-4,\ -3,\ -2)\,;.5,.4,.3\}$$
$$= \{(-4,-3,-2)\,;.5,.4,.3\}$$
$$\tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}} = \frac{\{(2,3,4)\,;.5,.4,.3\}}{\{(-4,-3,-2)\,;.5,.4,.3\}} = \{(-2,-1,-0.5)\,;.5,.4,.3\}$$

$$\tilde{E}_1 = \frac{C_1\tilde{\mu} - \tilde{\lambda}\tilde{D}_1}{\tilde{\mu}} = \tilde{C}_1 - \left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)\tilde{D}_1 = \tilde{C}_1 - \tilde{\rho}\tilde{D}_1 = \{(4.5,8,15)\,;.3,.5,.6\}$$
$$\tilde{E}_2 = \frac{C_2\tilde{\mu} - \tilde{\lambda}\tilde{D}_2}{\tilde{\mu}} = \tilde{C}_2 - \left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)\tilde{D}_2 = \tilde{C}_2 - \tilde{\rho}\tilde{D}_2 = \{(7,12,22)\,;.4,.6,.5\}$$

The new objective function is $\tilde{Z}(\tilde{y}) = \tilde{E}_i\tilde{y}_i + \tilde{\rho}$

$$\text{Max } \tilde{Z}(\tilde{y}) = \tilde{E}_1\tilde{y}_1 + \tilde{E}_2\tilde{y}_2 + \tilde{\rho}$$
$$= \{(4.5,8,15)\,;.3,.5,.6\}\,\tilde{y}_1 + \{(7,12,22)\,;.4,.6,.5\}\,\tilde{y}_2$$
$$+ \{(-2,-1,-0.5)\,;.5,.4,.3\}\,.$$

Again $\tilde{B}_{ji}$

$$\tilde{B}_{11} = \tilde{A}_{11}\tilde{\mu} + \tilde{b}_1\tilde{D}_1 = \{(2,\ 39,\ 62)\,;.2,.5,.6\}$$
$$\tilde{B}_{12} = \tilde{A}_{12}\tilde{\mu} + \tilde{b}_1\tilde{D}_2 = \{(4,\ 48,\ 92)\,;.3,.6,.7\}$$
$$\tilde{B}_{21} = \tilde{A}_{21}\tilde{\mu} + \tilde{b}_2\tilde{D}_1 = \{(2,\ 57,\ 78)\,;.3,.6,.6\}$$
$$\tilde{B}_{22} = \tilde{A}_{22}\tilde{\mu} + \tilde{b}_2\tilde{D}_2 = \{(4,\ 64,\ 118)\,;.4,.6,.5\}\,.$$

We now find the transformed constraints $\tilde{B}_{ji}\tilde{y}_i \leq \tilde{b}_j$

$$\{(2,\ 39,\ 62)\,;.2,.5,.6\}\,\tilde{y}_1 + \{(4,\ 48,\ 92)\,;.3,.6,.7\}\,\tilde{y}_2 \leq \{(14,\ 15,\ 16)\,;.7,.5,.3\}$$
$$\{(2,\ 57,\ 78)\,;.3,.6,.6\}\,\tilde{y}_1 + \{(4,\ 64,\ 118)\,;.4,.6,.5\}\,\tilde{y}_2 \leq \{(18,\ 19,\ 20)\,;.6,.4,.2\}$$
$$\text{and} \quad \tilde{y}_1,\ \tilde{y}_2 \geq \{(0,\ 0,\ 0)\,;1,0,0\}$$

The standard form of NLPP

$$\text{Max } \tilde{Z}(\tilde{y}) = \{(4.5,8,15)\,;.3,.5,.6\}\,\tilde{y}_1 + \{(7,12,22)\,;.4,.6,.5\}\,\tilde{y}_2$$
$$+ \{(-2,-1,-0.5)\,;.5,.4,.3\}$$

subject to

$$\{(2,\ 39,\ 62)\,;.2,.5,.6\}\,\tilde{y}_1+\{(4,\ 48,\ 92)\,;.3,.6,.7\}\,\tilde{y}_2\leq\{(14,\ 15,\ 16)\,;.7,.5,.3\}$$

$$\{(2,\ 57,\ 78)\,;.3,.6,.6\}\,\tilde{y}_1+\{(4,\ 64,\ 118)\,;.4,.6,.5\}\,\tilde{y}_2\leq\{(18,\ 19,\ 20)\,;.6,.4,.2\}$$

$$\text{and}\quad \tilde{y}_1,\ \tilde{y}_2\geq\{(0,\ 0,\ 0)\,;1,0,0\}$$

Using Neutrosophic Simplex algorithm, we solve this problem and obtain the values of $\tilde{y}_i$ as

	$\tilde{c}_j'$	{(4.5,8,15); .3, .5, .6}	{(7,12,22); .4, .6, .5}	{(0,0,0); 1,0,0}	{(0,0,0); 1,0,0}		
$\tilde{c}_B'$	Basic Variables	$\tilde{y}_1$	$\tilde{y}_2\downarrow$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{b}_j$	Ratio
{(0,0,0); 1,0,0}	$\tilde{y}_3$	{(2,39,62); .2, .5, .6}	{(4,48,92); .3, .6, .7}	{(1,1,1); 1,0,0}	{(0,0,0); 1,0,0}	{(14,15,16); .7, .5, .3}	{(0.15,0.31,4); .3, .6, .7}
{(0,0,0); 1,0,0}	$\leftarrow\tilde{y}_4$	{(2,57,78); .3, .6, .6}	{(4,64,118); .4, .6, .5}	{(0,0,0); 1,0,0}	{(1,1,1); 1,0,0}	{(18,19,20); .6, .4, .2}	{(0.15,0.29,5); .4, .6, .5}
$\check{C}_j^*=\tilde{Z}_j-\tilde{c}_j'$		{(−15, −8, −4.5); .3, .5, .6}	{(−22, −12, −7); .4, .6, .5}	{(0,0,0); 1,0,0}	{(0,0,0); 1,0,0}		
{(7,12,22); .4, .6, .5}	$\tilde{y}_2$	{(0.04,0.81,15.5); .2, .6, .7}	{(0.04,1,23); .3, .6, .7}	{(0.01,0.02,0.25); .3, .6, .7}	{(0,0,0); .3, .6, .7}	{(0.15,0.31,4); .3, .6, .7}	–
{(0,0,0); 1,0,0}	$\tilde{y}_4$	{(−990,5,76.61); .2, .6, .7}	{(−1468,0,115.22); .3, .6, .7}	{(0,0,0); .3, .6, .7}	{(1,1,1); .3, .6, .7}	{(0.15,0.29,5); .4, .6, .5}	–
$\check{C}_j^*=\tilde{Z}_j-\tilde{c}_j'$		{(−14.84,1.75,336.5); .2, .6, .7}	{(−21.69,0,499); .3, .6, .7}	{(0.07,0.24,5.5); .3, .6, .7}	{(0,0,0); .3, .6, .7}		

FIGURE 2. Neutrosophic Simplex Table

$$\tilde{y}_1=\{(0,\ 0,\ 0)\,;1,0,0\}\,,$$

$$\tilde{y}_2=\{(0.15,\ 0.31,\ 4)\,;.3,.6,.7\}\,.$$

From the values of $\tilde{y}_i$, we find the values of $\tilde{x}_i$

$$\tilde{x}_1=\frac{\tilde{y}_1\tilde{\beta}}{\tilde{1}-\tilde{y}_1\tilde{d}_1}=\{(0,\ 0,\ 0)\,;.3,.6,.7\}$$

$$\tilde{x}_2=\frac{\tilde{y}_2\tilde{\beta}}{\tilde{1}-\tilde{y}_2\tilde{d}_2}=\{(-18.82,\ -1.02,\ 0.02)\,;.3,.6,.7\}\,.$$

On substituting these values in the given objective function, we obtain

$$\text{Max }\tilde{z}=\frac{\{(3,4,5)\,;.3,.4,.6\}\,\tilde{x}_1+\{(5,6,7)\,;.4,.6,.5\}\,\tilde{x}_2+\{(0,\ 0,\ 0)\,;1,0,0\}}{\{(1,1,2)\,;.4,.5,.3\}\,\tilde{x}_1+\{(1,2,3)\,;.5,.3,.2\}\,\tilde{x}_2-\{(2,\ 3,\ 4)\,;.5,.4,.3\}}$$
$$=\{(-0.0023,\ 1.2142,\ 67.9072)\,;.3,.6,.7\}$$

The optimal solution is

$$\text{Max }\tilde{z}=\{(-0.0023,\ 1.2142,\ 67.9072)\,;.3,.6,.7\}$$

$$\tilde{x}_1=\{(0,\ 0,\ 0)\,;.3,.6,.7\}$$

$$\tilde{x}_2=\{(-18.82,\ -1.02,\ 0.02)\,;.3,.6,.7\}\,.$$

Example 2. Consider the NLFPP

$$\text{Max } \tilde{z} = \frac{\{(1,2,3)\,;.3,.4,.6\}\,\tilde{x}_1 + \{(1,2,3)\,;.4,.6,.5\}\,\tilde{x}_2 - \{(6,\ 7,\ 8)\,;.2,.5,.3\}}{\{(5,6,7)\,;.4,.5,.3\}\,\tilde{x}_1 + \{(4,5,6)\,;.5,.3,.2\}\,\tilde{x}_2 - \{(3,\ 4,\ 5)\,;.5,.4,.3\}}$$

subject to

$$\begin{aligned}
\{(6,7,8)\,;.2,.4,.3\}\,\tilde{x}_1 + \{(2,3,4)\,;.3,.5,.7\}\,\tilde{x}_2 &\leq \{(11,\ 12,\ 13)\,;.7,.5,.3\} \\
\{(2,4,5)\,;.4,.6,.2\}\,\tilde{x}_1 + \{(1,2,3)\,;.6,.5,.4\}\,\tilde{x}_2 &\leq \{(6,\ 8,\ 10)\,;.6,.4,.2\} \\
\text{and } \ \tilde{x}_1,\ \tilde{x}_2 &\geq \{(0,0,0)\,;1,\ 0,\ 0\}\,.
\end{aligned}$$

Solution. Here

$$\begin{aligned}
\tilde{c}_1 &= \{(1,2,3)\,;.3,.4,.6\}\,, & \tilde{c}_2 &= \{(1,2,3)\,;.4,.6,.5\}\,, \\
\tilde{\alpha} &= \{(-8,\ -7,\ -6)\,;.2,.5,.3\}\,, & \tilde{d}_1 &= \{(5,6,7)\,;.4,.5,.3\}\,, \\
\tilde{d}_2 &= \{(4,5,6)\,;.5,.3,.2\}\,, & \tilde{\beta} &= \{(-5,-4,-3)\,;.5,.4,.3\}\,, \\
\tilde{A}_{11} &= \{(6,7,8)\,;.2,.4,.3\}\,, & \tilde{A}_{12} &= \{(2,3,4)\,;.3,.5,.7\}\,, \\
\tilde{b}_1 &= \{(11,\ 12,\ 13)\,;.7,.5,.3\}\,, & \tilde{A}_{21} &= \{(2,4,5)\,;.4,.6,.2\}\,, \\
\tilde{A}_{22} &= \{(1,2,3)\,;.6,.5,.4\}\,, & \tilde{b}_2 &= \{(6,\ 8,\ 10)\,;.6,.4,.2\}\,,
\end{aligned}$$

where $\tilde{A}_{11},\ \tilde{A}_{12}$ and $\tilde{b}_1$ are the coefficients of the first constraint and $\tilde{A}_{21},\ \tilde{A}_{22}$ and $\tilde{b}_2$ are the coefficients of the second constraint.

We now calculate

$$\begin{aligned}
\tilde{C}_1 &= \{(-3,-2,-1)\,;.3,.4,.6\}\,, & \tilde{C}_2 &= \{(-3,-2,-1)\,;.4,.6,.5\}\,, \\
\tilde{\lambda} &= \{(6,\ 7,\ 8)\,;.2,.5,.3\}\,, & \tilde{D}_1 &= \{(-7,-6,-5)\,;.4,.5,.3\}\,, \\
\tilde{D}_2 &= \{(-6,-5,-4)\,;.5,.3,.2\}\,, & \tilde{\mu} &= \{(3,4,5)\,;.5,.4,.3\}\,,
\end{aligned}$$

$$\tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}} = \{(1.2, 1.75, 2.67)\,;.2,.5,.3\}\,.$$

$$\begin{aligned}
\tilde{E}_1 &= \frac{C_1\tilde{\mu} - \tilde{\lambda}\tilde{D}_1}{\tilde{\mu}} = \tilde{C}_1 - \left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)\tilde{D}_1 = \tilde{C}_1 - \tilde{\rho}\tilde{D}_1 \\
&= \{(3, 8.5, 17.67)\,;.2,.6,.7\} \\
\tilde{E}_2 &= \frac{C_2\tilde{\mu} - \tilde{\lambda}\tilde{D}_2}{\tilde{\mu}} = \tilde{C}_2 - \left(\frac{\tilde{\lambda}}{\tilde{\mu}}\right)\tilde{D}_2 = \tilde{C}_2 - \tilde{\rho}\tilde{D}_2 \\
&= \{(1.8, 6.75, 15)\,;.2,.6,.5\}
\end{aligned}$$

Again $\tilde{B}_{ji}$

$$\tilde{B}_{11} = \tilde{A}_{11}\tilde{\mu} + \tilde{b}_1\tilde{D}_1 = \{(-73, -44, -15); .2, .5, .3\}$$

$$\tilde{B}_{12} = \tilde{A}_{12}\tilde{\mu} + \tilde{b}_1\tilde{D}_2 = \{(-72, -48, -24); .3, .5, .7\}$$

$$\tilde{B}_{21} = \tilde{A}_{21}\tilde{\mu} + \tilde{b}_2\tilde{D}_1 = \{(-64, -32, -5); .4, .6, .3\}$$

$$\tilde{B}_{22} = \tilde{A}_{22}\tilde{\mu} + \tilde{b}_2\tilde{D}_2 = \{(-57, -32, -9); .5, .5, .4\} .$$

So, we have the new objective function $\tilde{Z}\left(\tilde{y}\right) = \tilde{E}_i\tilde{y}_i + \tilde{\rho}$

$$\begin{aligned} \text{Max } \tilde{Z}\left(\tilde{y}\right) &= \tilde{E}_1\tilde{y}_1 + \tilde{E}_2\tilde{y}_2 + \tilde{\rho} \\ &= \{(3, 8.5, 17.67); .2, .6, .7\}\, y_1 + \{(1.8, 6.75, 15); .2, .6, .5\}\, y_2 \\ &\quad + \{(1.2, 1.75, 2.67); .2, .5, .3\} \end{aligned}$$

Now we get the new constraint $\widetilde{B}_{ji}\widetilde{y}_i \le \widetilde{b}_j$

$$\{(-73, -44, -15); .2, .5, .3\}\, \widetilde{y}_1 + \{(-72, -48, -24); .3, .5, .7\}\, \widetilde{y}_2 \le \{(11, 12, 13); .7, .5, .3\}$$

$$\{(-64, -32, -5); .4, .6, .3\}\, \widetilde{y}_1 + \{(-57, -32, -9); .5, .5, .4\}\, \widetilde{y}_2 \le \{(6, 8, 10); .6, .4, .2\}$$

$$\widetilde{y}_1,\ \widetilde{y}_2 \ge \{(0, 0, 0); 1, 0, 0\}$$

Using Neutrosophic Simplex algorithm,

	$\tilde{c}_j'$	{(3,8.5,17.67); .2,.6,.7}	{(1.8,6.75,15); .2,.6,.5}	{(0,0,0); 1,0,0}	{(0,0,0); 1,0,0}	
$\tilde{c}_B'$	Basic Variables	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{b}_j$
{(0,0,0); 1,0,0}	$\tilde{y}_3$	{(−73, −44, −15); .2, .5, .3}	{(−72, −48, −24); .3, .5, .7}	{(1,1,1); 1,0,0}	{(0,0,0); 1,0,0}	{(11,12,13); .7, .5, .3}
{(0,0,0); 1,0,0}	$\tilde{y}_4$	{(−64, −32, −5); .4, .6, .3}	{(−57, −32, −9); .5, .5, .4}	{(0,0,0); 1,0,0}	{(1,1,1); 1,0,0}	{(6,8,10); .6, .4, .2}
$\tilde{C}_j^* = \tilde{Z}_j - \tilde{c}_j'$		{(−17.67, −8.5, −3); .2, .6, .7}	{(−15, −6.75, −1.8); .2, .6, .7}	{(0,0,0); 1,0,0}	{(0,0,0); 1,0,0}	

Figure 3. Neutrosophic Simplex Table

We cannot solve this problem. Because of the constraints are negative. Therefore the problem is unbounded solution.

7. Conclusions

In this paper, a new technique for solving the NLFPP has been proposed. We have suggested the methodology that efficiently addresses the problem. In the proposed technique, we transform NLFPP's into NLPP by a processing technique and then resolve them using the simplex method in Neutrosophic environment. We illustrate with a numerical example to demonstrate our method.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] M. Abdel-Baset, I.M. Hezam, F. Smarandache, Neutrosophic goal programming, Neutrosophic Sets Syst. 11 (2016), 112–118.

[2] G.R. Bitran, A.G. Novaes, Linear programming with a fractional objective function, Oper. Res. 21 (1973), 22–29. https://doi.org/10.1287/opre.21.1.22.

[3] A. Charnes, W.W. Cooper, Programming with linear fractional functionals, Naval Res. Logist. Quart. 9 (1962), 181–186. https://doi.org/10.1002/nav.3800090303.

[4] A. Charnes, W.W. Cooper, An explicit general solution in linear fractional programming, Naval Res. Logist. Quart. 20 (1973), 449–467. https://doi.org/10.1002/nav.3800200308.

[5] N. El-Hefenawy, M.A. Metwally, Z.M. Ahmed, I.M. El-Henawy, A review on the applications of neutrosophic sets, J. Comput. Theor. Nanosci. 13 (2016), 936–944. https://doi.org/10.1166/jctn.2016.4896.

[6] M.B. Hasan, S. Acharjee, Solving LFP by converting it into a single LP, Int. J. Oper. Res. 8 (2011), 1–14.

[7] K. Swarup, Letter to the editor–linear fractional functionals programming, Oper. Res. 13 (1965), 1029–1036. https://doi.org/10.1287/opre.13.6.1029.

[8] S.F. Tantawy, A new method for solving linear fractional programming problems, Aust. J. Basic Appl. Sci. 1 (2007), 105–108.