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FIXED POINTS OF ORDERED FUZZY CYCLIC CONTRACTIONS WITHOUT MONOTONE PROPERTY

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ABSTRACT. The purpose of this paper is to prove some fixed point theorems for fuzzy cyclic contractions without monotone property in O-complete fuzzy metric spaces. Our results extend the result of Gregori and Sapena [22] for fuzzy cyclic contractions without monotone property in O-complete fuzzy metric spaces. Illustrative examples in support of new results are provided.
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1. Introduction and preliminaries

The evolution of fuzzy mathematics started with an introduction of the notion of fuzzy sets by Zadeh [15], as a new way to represent the vagueness in every day life. There are many practical problems where the nature of uncertainty in the behavior of a given system possesses fuzzy rather than stochastic nature (see [9], [14], [19], [21], [25]).

The concept of a fuzzy metric space was introduced and generalized in many ways ([3],[10],[26]). George and Veeramani [2],[3] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [10]. They obtained a Hausdorff and first countable topology on modified fuzzy metric spaces, which has very important applications in quantum particle physics, particularly in connection with both string and ϵ^{∞} theory (see, [17] and references therein). In fuzzy metric spaces given by Kramosil and Michalek [10], Grabiec [22] gave the fuzzy version of Banach contraction principle. Subsequently, Many authors proved fixed point and common fixed point theorems in fuzzy metric spaces ([9], [19], [21]).

Kirk et al. [24] introduced the notion of a cyclic representation and characterized the Banach contraction principle in the context of a cyclic mapping. A fixed point result for cyclic contraction in fuzzy metric spaces can be seen in [8].

Existence of fixed points of a self map T on an ordered metric space was first investigated in 2004 by Ran and Reurings [1], and then by Nieto and Lopez [12],[13]. In these papers, the mapping T was assumed to be monotonic.

Recently, the fixed point results on partially ordered sets are investigated via a weaker property than the monotone property of T (see [6],[7],[11],[18]). In this article, it is shown that the monotone property in fixed point results for fuzzy cyclic contractions can be replaced by a weaker property, even in a more general setting of O-complete fuzzy metric spaces.

Let $(X, \preceq)$ be a partially ordered set and $x, y \in X$. If x, y are comparable, then we will write $x \asymp y$.

Let f be a self mapping on partially ordered set $(X, \preceq)$. The set $(UF)_f^X = \{x \in X : f(x) \preceq x\}$ ($(LF)_f^X = \{x \in X : x \preceq f(x)\}$) is called upper (resp. lower) fixed point set of f. The set of all fixed points of f is defined by $(F)_f^X = \{x \in X : f(x) = x\}$. Fixed point problem is to find some x in X such that $fx = x$ and we denote it by $FP(f, X)$.

Denote by $(X)_f$, the set of all those elements x of X which are comparable to the image $f(x)$, that is, $(X)_f = \{x \in X : f(x) \asymp x\}$. Then it is clear that $(X)_f = (UF)_f^X \cup (LF)_f^X$ and $(F)_f^X = (UF)_f^X \cap (LF)_f^X$.

Example 1. *Let $X = [0, \infty)$ be endowed with the usual ordering. Let $f : X \to X$ be defined by $fx = x^{\frac{1}{3}}$. Then $(LF)_f^X = \{x \in X : x \leq x^{\frac{1}{3}}\} = [0, 1] \subset X$, $(UF)_f^X = \{x \in X : x^{\frac{1}{3}} \leq x\} = \{0\} \cup [1, \infty)$, $(X)_f = X$ and $(F)_f^X = \{0, 1\}$.*

Example 2. *Let $X = [0, \infty)$ be endowed with the usual ordering. Define $f : X \to X$ by*

$$fx = \begin{cases} \sqrt{x}, & \textit{for} \quad x \in [0, 1); \\ x^2, & \textit{for} \quad x \in [1, 10); \\ 100, & \textit{for} \quad x \in [10, \infty). \end{cases}$$

Then $(LF)_f^X = \{x \in X : x \leq fx\} = [0, 100] \subset X$, $(UF)_f^X = \{x \in X : fx \leq x\} = \{0, 1\} \cup [100, \infty)$, $(X)_f = X$ and $(F)_f^X = \{0, 1, 100\}$.

Definition 1. *Let $(X, \preceq)$ be a partially ordered set. A mapping $f : X \to X$ is said to be:*

(a) monotone on X if it is increasing or decreasing with respect to $\preceq$;

(b) weakly monotone on X if $x \asymp y$ implies that $fx \asymp fy$ for for all $x, y \in X$;
(c) partially increasing on X if $x \in (LF)_f^X$ implies that $fx \in (LF)_f^X$;
(d) partially decreasing on X if $x \in (LF)_f^X$ implies that $fx \in (UF)_f^X$;
(e) weakly partial monotone on X if $x \in (X)_f$ implies that $fx \in (X)_f$.

It is shown in [6] that $(a) \Rightarrow (b) \Rightarrow (e)$ and the reverse implications do not hold in general.

Note that any increasing mapping (decreasing mapping) f is partially increasing (resp. partially decreasing). The following example shows that the weakly partially monotone mappings need not be partially increasing. Moreover, partially increasing mappings need not be monotone and weakly partial monotone.

Example 3. *Let $X = \{a, b, c, d\}$. Define a partial order relation $\preceq$ on X by*

$$\preceq = \{(a,a), (b,b), (c,c), (d,d), (b,a), (c,d), (d,a), (c,a)\}.$$

Define $f, g\colon X \to X$ by

$$f := \begin{pmatrix} a & b & c & d \\ b & c & d & d \end{pmatrix} \text{ and } g := \begin{pmatrix} a & b & c & d \\ b & b & c & a \end{pmatrix}$$

Then, (i) f is not weakly partial monotone (ii) f is not monotone; (iii) f is partially increasing (iv) g is not partially increasing (v) g is weakly partial monotone.

Proof. (i) As $b \preceq a$, that is, $fa \preceq a$, and so $a \in (X)_f$, but $fa \not\asymp ffa$, that is, $fa \notin (X)_f$. Therefore f is not weakly partially monotone.
(ii) Note that, $d \preceq a$ but $fd \npreceq fa$. Therefore f is not nondecreasing. Also $b \preceq a$ but $fa \npreceq fb$. So f is not nonincreasing. Hence f is not monotone.
(iii) As, $c \preceq fc$ and $d \preceq fd$, so $c, d \in (LF)_f^X$. Note that $fc \preceq ffc$ and $fd \preceq ffd$ so fc, fd $\in (LF)_f^X$. Therefore f is partially increasing.
(iv) As $d \preceq a$, that is, $d \preceq gd$, so $d \in (LF)_g^X$. Note that $gd \npreceq ggd$, that is, $gd \notin (LF)_g^X$. Therefore, g is not partially increasing.
(v) As $a \asymp ga$, $b \asymp gb$, $c \asymp gc$ and $d \asymp gd$, so $a, b, c, d \in (X)_g$. Note that, $ga \asymp gga$, $gb \asymp ggb$, $gc \asymp ggc$ and $gd \asymp ggd$, that is, $ga, gb, gc, gd \in (X)_g$. Therefore g is weakly partial monotone. □ □

Similarly, partially decreasing mappings need not be monotone and weakly partially monotone. Therefore, it is significant to consider the partially increasing and decreasing mappings.

Following definitions and known results are needed in the sequel.

Definition 2. [15] *A fuzzy set A in a nonempty set X is characterized by a function with domain X and values in $[0,1]$.*

Definition 3. [5] *A binary operation $\star\colon [0,1]\times[0,1]\to[0,1]$ is a continuous t-norm if $\{[0,1],\star\}$ is an abelian topological monoid with unit 1 (that is, $\star$ is commutative and associative, $\star$ is continuos and $a\star 1=a$ for all a in $[0,1]$) and $a\star b\leq c\star d$ whenever $a\leq c$ and $b\leq d$, $a,b,c,d\in[0,1]$.*

Three typical examples of continuous $t-$ norms are $a\star b=\min\{a,b\}$ (minimum $t-$ norm), $a\star b=ab$ (product $t-$ norm), and $a\star b=\max\{a+b-1,0\}$ (Lukasiewicz $t-$ norm).

Definition 4. [2] *The 3-tuple $(X,M,\star)$ is a fuzzy metric space if X is an arbitrary set, $\star$ is a continuous t-norm, M is a fuzzy set in $X^2\times(0,\infty)$ satisfying the following conditions for all $x,y,z\in X$ and $s,t>0$:*

(1) $M(x,y,t)>0$;
(2) $M(x,y,t)=1$ *if and only if* $x=y$;
(3) $M(x,y,t)=M(y,x,t)$;
(4) $M(x,y,t)\star M(y,z,s)\leq M(x,z,t+s)$;
(5) $M(x,y,\cdot)\colon(0,\infty)\to[0,1]$ *is continuous*

Here M with $\star$ is called a fuzzy metric on X. Note that, $M(x,y,t)$ can be thought of as the definition of nearness between x and y with respect to t. It is known that $M(x,y,.)$ is nondecreasing for all $x,y\in X$ (see [3]). For examples of fuzzy metric spaces we refer to [4] and [23].

Let $(X,M,\star)$ be a fuzzy metric space. For $t>0$, the open ball $B(x,r,t)$ with center $x\in X$ and radius $0<r<1$ is defined by

$$B(x,r,t)=\{y\in X\colon M(x,y,t)>1-r\}.$$

The collection $\{B(x,y,t)\colon x\in X, 0<r<1, t>0\}$ is a neighborhood system for a topology τ on X induced by the fuzzy metric M. This topology is Hausdorff and first countable.

Lemma 1. [3] *Let $(X,M,\star)$ be a fuzzy metric space. Then M is a continuous function on $X^2\times(0,\infty)$.*

Definition 5. [2],[16] *A sequence $\{x_n\}$ in a fuzzy metric space $(X,M,\star)$ is said to be:*

(1) convergent to $x\in X$ if for each $\varepsilon\in(0,1)$ and $t>0$ there exists $n_0\in\mathbb{N}$ such that $M(x_n,x,t)>1-\varepsilon$ for all $n>n_0$;
(2) Cauchy sequence if for each $\varepsilon\in(0,1)$ and each $t>0$ there exists $n_0\in\mathbb{N}$ such that $M(x_n,x_m,t)>1-\varepsilon$ for all $n,m>n_0$;

(3) G-Cauchy if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ *for all* $t > 0, p \geq 1$.

The above definitions of Cauchy sequences are different (see [20]).

Definition 6. *A fuzzy metric space in which every Cauchy sequence is convergent is called a complete fuzzy metric space. A fuzzy metric space in which every G-Cauchy sequence is convergent is called a G-complete fuzzy metric space.*

Theorem 2. [2] *A sequence* $\{x_n\}$ *in a fuzzy metric space* $(X, M, \star)$ *converges to* $x \in X$ *if and only if* $M(x_n, x, t) \to 1$ *as* $n \to \infty$.

It is well known that every closed subset of a complete fuzzy metric space is complete. In what follows, by a partially ordered fuzzy metric space we mean a 4-tuple $(X, M, \star, \preceq)$, where X is a nonempty set, $\preceq$ a partial order on X, $\star$ a continuous t-norm and M a fuzzy metric on X.

Denote by $\mathcal{CM}_u$, the class of all sequences $\{x_n\}$ in X such that $\{x_n\}$ is G-Cauchy and nondecreasing. Denote by $\mathcal{CM}_l$, the class of all sequences $\{x_n\}$ in X such that $\{x_n\}$ is G-Cauchy and nonincreasing. Denote by $\mathcal{CM}$, the class of all sequences $\{x_n\}$ in X such that $\{x_n\}$ is G-Cauchy and monotonic. It is obvious that $\mathcal{CM} = \mathcal{CM}_l \bigcup \mathcal{CM}_u$.

Definition 7. *A partially ordered fuzzy metric space* $(X, M, \star, \preceq)$ *is called:*

(1) u-O-complete, if every sequence in $\mathcal{CM}_u$ *converges in* X;

(2) l-O-complete, if every sequence in $\mathcal{CM}_l$ *converges in* X;

(3) O-complete, if every sequence in $\mathcal{CM}$ *converges in* X.

Note that, if $X = (0, 1]$ and $Y = [0, 1)$ are equipped with usual order $\leq$ and M is the standard fuzzy metric, that is,

$$M(x, y, t) = \frac{t}{t + |x - y|}$$

for all $x, y \in [0, 1]$, $t > 0$ and $\star$ is the usual product. Then $(X, M, \star, \leq)$ is u-O-complete but not l-O-complete, while $(Y, M, \star, \leq)$ is l-O-complete but not u-O-complete. Also, an O-complete fuzzy metric space is u-O-complete as well as l-O-complete. It can be seen easily that a closed subset of an O-complete, u-O-complete, l-O-complete fuzzy metric space is O-complete, u-O-complete, l-O-complete, respectively.

Note that every G-complete partially ordered fuzzy metric space is O-complete but the converse may not be true in general. For example, If $X = \mathbb{R}^+ \setminus \mathbb{N}$ then $(X, M, \star, \preceq)$ is not a G-complete fuzzy metric space, where $\mathbb{R}^+$ is the set of all nonnegative reals and M is the standard fuzzy metric on X, $\star$ is the usual product and partial order $\preceq$

is defined by

$$\preceq = \bigcup_{n=0}^{\infty} \{(x,y)\colon x,y \in A_n \text{ with } x \leq y\} \bigcup \{(x,x)\colon x \in X\},$$

where

$$A_n = \left[n + \frac{1}{2(n+2)}, n + \frac{1}{n+2}\right] \text{ for all } n \geq 0.$$

Note that $(X, M, \star, \preceq)$ is an O-complete ordered fuzzy metric space.

In [24], the following concept of cyclic representation of a set was defined. Let X be a nonempty set and $T\colon X \to X$ a mapping. A collection $\{A_1, A_2, \ldots, A_m\}$ of nonempty subsets of X is a cyclic representation of X with respect to T if

(1) $X = \bigcup_{i=1}^{m} A_i$

(2) $T(A_1) \subset A_2, \ldots, T(A_{m-1}) \subset T(A_m), T(A_m) \subset T(A_1)$.

Definition 8. *Let $(X, M, \star, \preceq)$ be ordered fuzzy metric space, $Y \subset X$ and $T\colon Y \to Y$. Suppose that $\{A_1, A_2, \ldots, A_m\}$ is a cyclic representation of Y with respect to T. Then the operator T is called a fuzzy ordered cyclic contraction if there exists $k \in [0,1)$ such that*

(1)
$$\frac{1}{M(Tx, Ty, t)} - 1 \leq k \left[\frac{1}{M(x,y,t)} - 1\right]$$

for any $x \in A_i, y \in A_{i+1}$ with $x \preceq y$ ($i = 1, 2, \ldots, m$ where $A_{m+1} = A_1$) and each $t > 0$.

2. Main results

In this section, we prove some fixed point results for partially increasing and partially decreasing fuzzy ordered cyclic contraction on u-O-complete and l-O-complete fuzzy metric spaces.

Theorem 3. *Let $(X, M, \star, \preceq)$ be an u-O-complete fuzzy metric space, $Y \subset X$, $T\colon Y \to Y$ and the collection $\{A_1, A_2, \ldots, A_m\}$ of closed subsets of X is a cyclic representation of Y with respect to T. Suppose T is a fuzzy ordered cyclic contraction and partially increasing on Y such that $(LF)_T^Y \neq \phi$. Then T has a fixed point $u \in \bigcap_{i=1}^{m} A_i$ provided that for any nondecreasing sequence $\{x_n\}$ in X converging to some $z \in X$, we have $x_n \preceq z$ for all $n \in \mathbb{N}$. Furthermore, $(F)_T^Y$ is well ordered if and only if T has a unique fixed point.*

Proof. As $(LF)_T^Y \neq \phi$, suppose that $x_0 \in Y$ such that $x_0 \preceq Tx_0$. Define a sequence $\{x_n\}$ by $x_n = Tx_{n-1}$ for all $n \in \mathbb{N}$. Since T is partially increasing, $x_1 = Tx_0 \in (LF)_T^Y$. Similarly, we obtain $x_n = Tx_{n-1} \in (LF)_T^Y$ for all $n \in \mathbb{N}$, that is, $x_n \preceq Tx_n = x_{n+1}$, so $\{x_n\}$ is a nondecreasing sequence in Y.

We shall show that $\{x_n\}$ is G-Cauchy in Y. As, $x_0 \in Y = \bigcup_{i=1}^{m} A_i$, so for all $n \in \mathbb{N}$ there exists i such that $x_n \in A_i$, $1 \leq i \leq m$, and $x_{n+1} = Tx_n \in A_{i+1}$. Since for any $n \in \mathbb{N}$ we have $x_{n-1} \preceq x_n$, and T is fuzzy ordered cyclic contraction therefore we obtain that

$$\frac{1}{M(x_n, x_{n+1}, t)} - 1 = \frac{1}{M(Tx_{n-1}, Tx_n, t)} - 1 \leq k\left[\frac{1}{M(x_{n-1}, x_n, t)} - 1\right].$$

If we take $M_n(t) = M(x_n, x_{n+1}, t)$ for all $t > 0$, then we obtain

$$\frac{1}{M_n(t)} - 1 \leq k\left[\frac{1}{M_{n-1}(t)} - 1\right].$$

Successive application of above inequality gives

$$\frac{1}{M_n(t)} - 1 \leq k^n\left[\frac{1}{M_0(t)} - 1\right],$$

that is,

(2) $$\frac{1}{\frac{k^n}{M_0(t)} + 1 - k^n} \leq M_n(t).$$

For any $p \geq 1$, we have

$$\begin{aligned}
M(x_n, x_{n+p}, t) &\geq M(x_n, x_{n+1}, t/2) \star M(x_{n+1}, x_{n+p}, t/2) \\
&\geq M(x_n, x_{n+1}, t/2) \star M(x_{n+1}, x_{n+2}, t/2^2) \star M(x_{n+2}, x_{n+p}, t/2^2) \\
&\geq M(x_n, x_{n+1}, t/2) \star M(x_{n+1}, x_{n+2}, t/2^2) \star M(x_{n+2}, x_{n+3}, t/2^3) \\
&\quad \star \cdots \star M(x_{n+p-1}, x_{n+p}, t/2^{p-1}) \\
&= M_n(t/2) \star M_{n+1}(t/2^2) \star M_{n+2}(t/2^3) \star \cdots \star M_{n+p-1}(t/2^{p-1}).
\end{aligned}$$

Using (2) in above inequality we obtain

$$\begin{aligned}
M(x_n, x_{n+p}, t) &\geq \frac{1}{\frac{k^n}{M_0(t/2)} + 1 - k^n} \star \frac{1}{\frac{k^{n+1}}{M_0(t/2^2)} + 1 - k^{n+1}} \star \frac{1}{\frac{k^{n+2}}{M_0(t/2^3)} + 1 - k^{n+2}} \\
&\quad \star \cdots \star \frac{1}{\frac{k^{n+p-1}}{M_0(t/2^{p-1})} + 1 - k^{n+p-1}} \\
&\geq \frac{1}{\frac{k^n}{M_0(t/2)} + 1} \star \frac{1}{\frac{k^n}{M_0(t/2^2)} + 1} \star \frac{1}{\frac{k^n}{M_0(t/2^3)} + 1} \star \cdots \star \frac{1}{\frac{k^n}{M_0(t/2^{p-1})} + 1}.
\end{aligned}$$

Taking limit $n \to \infty$ in above inequality we obtain

$$\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1 \text{ for all } t > 0 \text{ and } p \geq 1.$$

Therefore, $\{x_n\}$ is a nondecreasing G-Cauchy sequence in Y. As, Y is closed and X is u-O-complete, there exists $u \in Y$ such that

(3) $$\lim_{n\to\infty} M(x_n, u, t) = 1 \quad \text{for all} \quad t > 0.$$

We shall show that u is a fixed point point of T.
As Y has a cyclic representation with respect to T, the sequence $\{x_n\}$ has infinitely many terms in each A_i for $i \in \{1, 2, \dots, m\}$. Suppose that $u \in A_i$ then we have $Tu \in A_{i+1}$. Take a subsequence $\{x_{n_k}\}$ such that $x_{n_k} \in A_{i+1}$. Then from given assumption, we have $x_{n_k} \preceq u$ for all $k \in \mathbb{N}$. So for any $t > 0$ we obtain from (1) that

$$\begin{aligned} \frac{1}{M(x_{n_k+1}, Tu, t)} - 1 &= \frac{1}{M(Tx_{n_k}, Tu, t)} - 1 \\ &\leq k\left[\frac{1}{M(x_{n_k}, u, t)} - 1\right]. \end{aligned}$$

Taking limit as $k \to \infty$ in above inequality and using (3) we obtain

$$\frac{1}{M(u, Tu, t)} - 1 \leq k\,[1 - 1] = 0.$$

That is, $M(u, Tu, t) = 1$ for all $t > 0$ which implies that $Tu = u$. Suppose, $(F)_T^Y$ is well ordered. We shall show that the fixed point of T is unique. Assume the contrary, that $u, v \in (F)_T^Y \subset Y = \bigcup_{i=1}^{m} A_i$ and $u \neq v$. As $(F)_T^Y$ is well ordered, so we may assume that $u \preceq v$. Note that, $u \in A_i$ for some $1 \leq i \leq m$, so $u = Tu \in A_{i+1}$ and so on. Hence, $u \in \bigcap_{i=1}^{m} A_i$. Similarly, $v \in \bigcap_{i=1}^{m} A_i$. Thus from (1), we have

$$\frac{1}{M(u, v, t)} - 1 = \frac{1}{M(Tu, Tv, t)} - 1 \leq k\left[\frac{1}{M(u, v, t)} - 1\right] < \frac{1}{M(u, v, t)} - 1,$$

a contradiction. Therefore $u = v$. Converse is straightforward. □ □

The following example shows the case when the known results are not applicable but the above result is applicable.

Example 4. *Let* $X = \{1, 2, 3, 4, 5, 6\}$ *and* $\star$ *be the usual product. Consider the fuzzy set* $M\colon X^2 \times (0, \infty) \to [0, 1]$ *defined by*

$$M(x, y, t) = \begin{cases} \dfrac{x}{y}, & \textit{if } x \leq y; \\ \dfrac{y}{x}, & \textit{if } y \leq x. \end{cases} \quad \textit{for all} \quad x, y \in X \textit{ and } t > 0.$$

Let, $A_1 = \{1, 2, 4, 5, 6\}, A_2 = \{1, 3, 5\}$ *and* $Y = A_1 \bigcup A_2$. *Define,* $T\colon Y \to Y$ *by*

$$T := \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 4 & 5 & 5 & 1 \end{pmatrix}.$$

Define a partial order relation $\preceq$ *on* X *by*

$$\preceq\; = \{(x, x)\colon x \in X\} \bigcup \{(2, 3), (3, 4), (2, 4), (2, 5), (4, 5), (3, 5), (2, 6)\}$$

Then $(X, M, \star)$ *is an O-complete fuzzy metric space and* $\{A_1, A_2\}$ *is a cyclic representation of* Y *with respect to* T. *Note that* T *is not monotonic. Indeed* $(2,6) \in \preceq$ *but neither* $(T2, T6) \in \preceq$ *nor* $(T6, T2) \in \preceq$. *It is straightforward to check that* T *is fuzzy ordered cyclic contraction with* $k \in [\frac{3}{4}, 1)$. *All conditions of Theorem 3 are satisfied. Moreover* $(F)_T^Y = \{1, 5\}$. *Note that the set of fixed points of* T *is not well ordered as* $(1,5), (5,1) \notin \preceq$.

On the other hand, for $x = 1, y = 3$, *there exists no* $k \in [0,1)$ *such that*

$$\frac{1}{M(Tx,Ty,t)} - 1 \leq k \left[\frac{1}{M(x,y,t)} - 1\right].$$

Therefore, T *is not a fuzzy contraction (see* [22]*).*
Again, the results of [8] *are not applicable. To check this, if* $x = 3$, *and* $y = 6$, *then there exists no continuous, non-decreasing function* $\phi\colon [0,\infty) \to [0,\infty)$ *with* $\phi(r) > 0$ *for* $r > 0$ *and* $\phi(0) = 0$, *such that*

$$\frac{1}{M(Tx,Ty,t)} - 1 \leq \left(\frac{1}{M(x,y,t)} - 1\right) - \phi\left(\frac{1}{M(x,y,t)} - 1\right). \square$$

Following similar arguments to those given in Theorem 3 one can obtain the proof of following theorem.

Theorem 4. *Let* $(X, M, \star, \preceq)$ *be an l-O-complete fuzzy metric space,* $Y \subset X$, $T\colon Y \to Y$ *and the collection* $\{A_1, A_2, \ldots, A_m\}$ *of closed subsets of* X *is a cyclic representation of* Y *with respect to* T. *Suppose* T *is a fuzzy cyclic contraction and partially decreasing on* Y *such that* $(UF)_T^Y \neq \phi$. *Then* T *has a fixed point* $u \in \bigcap_{i=1}^{m} A_i$ *provided that for any nonincreasing sequence* $\{x_n\}$ *in* X *converging to some* $z \in X$, *we have* $z \preceq x_n$ *for all* $n \in \mathbb{N}$. *Furthermore, the set* $(F)_T^Y$ *is well ordered if and only if* T *has a unique fixed point.*

Remark 1. *Since an O-complete fuzzy metric space is u-O-complete as well as l-O-complete therefore all above results are also true in O-complete fuzzy metric spaces.*

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