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ON STARLIKE AND CONVEX FUNCTIONS OF COMPLEX ORDER WITH FIXED SECOND COEFFICIENT

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Abstract. Let $F_p(b,M)$ denote the class of functions $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ which are analytic in the open unit disc $U = \{z : |z| < 1\}$ and satisfy the inequality

$$\left| \frac{b - 1 + \frac{zf'(z)}{f(z)}}{b} - M \right| < M$$

for $b \neq 0$, complex, $M > \frac{1}{2}, |a_2| = 2p, 0 \leq p \leq \left(\frac{1+m}{2}\right)|b|$,

$m = 1 - \frac{1}{M}$ and for all $z \in U$. Further $f(z)$ is in the class $G_p(b,M)$ if $zf'(z)$ is in the class $F_p(b,M)$. In the present paper, we obtain lower bounds for the classes introduced above and apply them to determine γ-spiral radiu for functions of the class $F_p(b,M)$ and γ-convex radius for functions of the class $G_p(b,M)$.

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1. Introduction.

Let A denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \qquad (k \in N = \{1, 2, ...\}), \tag{1.1}$$

which are analytic in the open unit disc $U = \{z \in \boldsymbol{C} : |\, z \,| < 1\}$, and let S denote the subclass of A . In [17] Nasr and Aouf denote the class of bounded starlike functions of complex order $F(b,M)(b \neq 0$, complex, M fixed , $M > \frac{1}{2}$ and $\frac{f(z)}{z} \neq 0$) by

(1.2) $$\left|\frac{b-1+\frac{zf'(z)}{f(z)}}{b}-M\right|<M, \quad (z\in U)$$

Also in [18] Nasr and Aouf denote and introduced the class of bounded convex function of complex order $G(b, M)$ ($b\neq 0$, complex, M fixed and $M>\frac{1}{2}$) by

(1.3) $$\left|\frac{b+\frac{zf''(z)}{f'(z)}}{b}-M\right|<M, \qquad (z\in U)\,.$$

.From equations (1.2) and (1.3) , we get

(1.4) $$f(z)\in G(b,M) \text{ if and only if } zf'(z)\in F(b,M).$$

By specializing b and M, we obtain the following subclasses studied by earlier authors:

(1) $F(b,\infty)=S(1-b)$ (Nasr and Aouf [19]);

(2) $G(b,\,\infty)=C(b)$ (Wiatrowski [29], Nasr and Aouf [16], and Aouf [2]);.

(3) $F((1-\alpha)e^{-i\lambda}\cos\lambda,\infty)=S^{\lambda}(\alpha)$ $(|\lambda|<\frac{\pi}{2},\ \ 0\leq\alpha<1)$ (Libera [11] and Patil and Thakare [21]) ;

(4) $G((1-\alpha)e^{-i\lambda}\cos\lambda,\infty)=C^{\lambda}(\alpha)$ $\left(|\lambda|<\frac{\pi}{2},\ \ 0\leq\alpha<1\right)$ (Chichra [7] and Sizuk [28]);

(5) $F(1-\alpha,1)=\overline{S}_{\alpha}(0\leq\alpha<1)$ (Wright [30] and MaCarty [13]);

(6) $F(1,1)=S^{*}$ (Singh [26]);

(7) $F(1,M)$, $(M>\frac{1}{2})$ (Singh and Singh [27]);

(8) $F(e^{-i\lambda}\cos\lambda,(\cos\lambda)^{-1})=H(\lambda)(|\lambda|<\frac{\pi}{2})$ (Geol [9]) ;

(9) $F(e^{-i\lambda}\cos\lambda,M)=F_{\lambda,M}$ and ; $G(e^{-i\lambda}\cos\lambda,M)=G_{\lambda,M}$ $(|\lambda|<\frac{\pi}{2},\ \ M>\frac{1}{2})$ (Kulshrestha [10]);

(10) $F((1-\alpha)e^{-i\lambda}\cos\lambda,M)=F_M(\lambda,\alpha)$, and $G((1-\alpha)e^{-i\lambda}\cos\lambda,M)=G_M(\lambda,\alpha)$ $(|\lambda|<\frac{\pi}{2},\ \ M>\frac{1}{2},\ 0\leq\alpha<1)$ (Aouf [3]);

(11) $F(1-a-d,\ \dfrac{d}{a+d-1}) = S(a,d), \quad (a+d \geq 1,\ d \leq a < d+1)$ (Silverman [23]) ;

(12) $F((1-a-d)e^{-i\lambda}\cos\lambda,\ \ \dfrac{d}{a+d-1}) = S(\lambda,a,d) \quad (|\lambda| < \dfrac{\pi}{2},\ a+d \geq 1,\ \ d \leq a \leq d+1)$ (Silvia [25]) ;

Definition 1. *A functions* $f(z)$ *defined by* (1.1) *is said to be in the class* $F_p(b,M)(b \neq 0, \text{complex}, M > \dfrac{1}{2},\ |a_2| = 2p, 0 \leq p \leq (\frac{1+m}{2})\,|b|\,,\ \ m = 1 - \dfrac{1}{M})$ *if it satisfies* (1.2).

Definition 2. *A functions* $f(z)$ *defined by* (1.1) *is said to be in the class* $G_p(b,M)(b \neq 0, \text{complex}, M > \dfrac{1}{2},\ |a_2| = p,\ 0 \leq p \leq (\dfrac{1+m}{2})\,|b|\,,\ \ m = 1 - \dfrac{1}{M})$ *if it satisfies* (1.3) .

In [11] Libera (see also [14]) introduced the concept of " γ-spiral radius" for the classes of univalent functions as follows

Definition 3. *If* $f \in S$ *and* $|\gamma| < \dfrac{\pi}{2}$*, then* γ*-spiral radius of* f *given by*

$$\gamma - s.r.\{f\} = \sup\{r : \text{Re}(e^{i\gamma}\frac{zf'(z)}{f(z)}) > 0, \qquad |z| < r\}, \tag{1.4}$$

and if $F \subset S$, then γ-spiral radius of F is

$$\gamma - s.r.F = \inf_{f \in F}[\gamma - s.r.\{f\}]. \tag{1.5}$$

In this paper, we obtain convolution conditions for the classes $K(A,B,p,\alpha),\ S^*(A,B,p,\alpha),\ K_\lambda(A,B,p,\alpha)$ and $S^*_\lambda(A,B,p,\alpha)$ as described below:

Definition 4. *If* $f \in A$ *and* $|\gamma| < \dfrac{\pi}{2}$*,then the* $\gamma-$*convex radius of* f

$$\gamma - c.r.\{f\} = \sup\{r : \text{Re}\{e^{i\gamma}(1 + \frac{zf''(z)}{f'(z)})\} > 0, z \in U\}. \tag{1.6}$$

Definition 5. *If* $E \subset A$*, and* $|\gamma| < \dfrac{\pi}{2}$*, then the* γ*-convex radius of* E *is*

$$\gamma - c.r.E = \inf_{f \in E}[\gamma - c.r.\{f\}]. \tag{1.7}$$

Results in terms of a fixed second coefficient have been obtained for varoius subclasses of S. Finkelstein [8] investigated the classes $F_p(1,\infty)$ and $G_p(1,\infty)$ the starlike and convex functions with pre-assigned second coefficient. Extensions of these results can be found in ([1], [4], [5], [6], [14], [23] and [34]).

In the present paper, we obtain lower bounds for the classes introduced above and apply them to determine γ-spiral radiu for functions of the class $F_p(b, M)$ and γ-convex radius for functions of the class $G_p(b, M)$.

2. Growth estimates

To prove our results, we need the following lemma.

Lemma 1. *Let* $w(z) = d_1 z + d_2 z^2 + ...$, *be an analytic map of the unit disc into itself. Then*

$$|d_1| \leq 1$$

and

$$|w(z)| \leq \frac{r(r + |d_1|)}{(1 + |d_1| r)} \quad (|z| = r).$$

Equality holds at some point $z(z \neq 0)$ if and only if

$$w(z) = \frac{e^{-it} z(z + d_1 e^{it})}{1 + \overline{d}_1 e^{-it} z} \quad (t \geq 0).$$

This lemma is an iterated form of Schwarz's Lemma [20] and is due to Lowner [12].

To obtain growth estimates for the classes $F_p(b, M)$ and $G_p(b, M)$ it is useful to consider the following class.

Definition 6. *A function* $h(z) = 1 + 2a_2 z + ...$, *analytic in the unit disc U, is in the class* $H_p(b, M)$ $(b \neq 0$, *complex,* $M > \frac{1}{2}, |a_2| = p$, $0 \leq p \leq (\frac{1+m}{2}) |b|, m = 1 - \frac{1}{M})$ *if the inequality*

$$\left| \frac{b - 1 + h(z)}{b} - M \right| < M \tag{2.1}$$

holds for $b \neq 0$, complex, $M > \frac{1}{2}$, and for all $z \in U$.

Observe that $f(z) \in F_p(b, M)$ if and only if $\frac{zf'(z)}{f(z)} \in H_p(b, M)$ and $f(z) \in G_p(b, M)$ if and only if $1 + \frac{zf''(z)}{f'(z)} \in H_p(b, M)$.

Theorem 2. *Suppose* $h(z) \in H_p(b, M)$, $|\gamma| < \frac{\pi}{2}$ *and*

$$u = pr + (\frac{1+m}{2})\,|b|\,, \qquad v = (\frac{1+m}{2})\,|b|\,r + p. \tag{2.2}$$

Then, for $|z| = r < 1$ and for all $b, M, p (b \neq 0,$ complex, $M > \frac{1}{2},\ 0 \leq p \leq (\frac{1+m}{2})\,|b|\,,\ m = 1 - \frac{1}{M})$,

$$\begin{aligned} \operatorname{Re}\{e^{i\gamma} h(z)\} \ \geq\ & \frac{1}{u^2 - m^2 v^2 r^2}\{u^2 \cos\gamma - (1+m)\,|b|\,uvr \\ & + mv^2[(m+1)(\operatorname{Re}\{b\}\cos\gamma - \operatorname{Im}\{b\}\sin\gamma) - m\cos\gamma]r^2\}. \end{aligned} \tag{2.3}$$

The result is sharp.

Proof. Since $h(z) \in H_p(b, M)$, an application of Schwarz's Lemma [20] gives

$$h(z) = \frac{1 + [(1+m)b - m]w(z)}{1 - mw(z)}, \qquad m = 1 - \frac{1}{M}, \tag{2.4}$$

where $w(z) = d_1 z + ...(|d_1| = \frac{2p}{(1+m)\,|b|})$ satisfies the hypothesies of the lemma. Thus

$$|w(z)| \leq \frac{v}{u} r, \tag{2.5}$$

where u and v are given by (2.2) and $|z| = r$. Let $B(z) = e^{-i\gamma} h(z)$ and $|\gamma| < \frac{\pi}{2}$. Then (2.4) may be written as

$$w(z) = -\frac{e^{i\gamma} - B(z)}{mB(z) + e^{i\gamma}[(1+m)b - m]}. \tag{2.6}$$

From (2.5) and (2.6), we have

$$\left|\frac{e^{i\gamma} - B(z)}{mB(z) + e^{i\gamma}[(1+m)b - m]}\right| \leq \frac{v}{u} r. \tag{2.7}$$

Setting $B(z) = \zeta + i\eta$ and simplifying, (2.7) gives

$$\left|\zeta + i\eta - e^{i\gamma}[1 + \frac{m(1+m)bv^2r^2}{u^2 - m^2v^2r^2}]\right| \leq \frac{(1+m)\,|b|\,uvr}{u^2 - m^2v^2r^2}. \tag{2.8}$$

From (2.8) it follows that

$$\operatorname{Re}\{B(z)\} \geq \operatorname{Re}\{e^{i\gamma}[1 + \frac{m(1+m)bv^2r^2}{u^2 - m^2v^2r^2}]\} - \frac{(1+m)\,|b|\,uvr}{u^2 - m^2v^2r^2}. \tag{2.9}$$

Now the result follows immediately from (2.9).

The bound in (2.3) is sharp for the function

$$h(z)=\begin{cases}\dfrac{u+[m-(1+m)b]vz}{u+mvz}, & m\neq 0\,,\\ 1-\dfrac{bv}{u}z, & m=0\end{cases} \tag{2.10}$$

and

$$\zeta=\begin{cases}\dfrac{r[mvr-\sqrt{\frac{\bar{b}}{b}}\,e^{-i\gamma}u]}{u-m\sqrt{\frac{\bar{b}}{b}}\,e^{-i\gamma}vr}, & m\neq 0,\\ r\sqrt{\dfrac{\bar{b}}{b}}\,e^{-i\gamma}\,, & m=0.\end{cases} \tag{2.11}$$

3. The γ-spiral and γ-convex radius.

Theorem 3. $\gamma-s.r.F_p(b,M)$ *is the smallest positive root* r_0 *of the equation*

$$\begin{aligned}&u^2\cos\gamma-(1+m)\,|b|\,uvr+mv^2[(1+m)(\mathrm{Re}\{b\}\cos\gamma-\\&\mathrm{Im}\{b\}\sin\gamma)-m\cos\gamma]r^2=0,\end{aligned} \tag{3.1}$$

where u and v are given by (2.2). The result is sharp for all admissible values of b, M and p.

Proof. Setting $h(z)=\dfrac{zf'(z)}{f(z)}$, in Theorem 1, we get

$$\begin{aligned}\mathrm{Re}\{e^{i\gamma}\frac{zf'(z)}{f(z)}\} \quad > \quad & \frac{1}{u^2-m^2v^2r^2}\{u^2\cos\gamma-(1+m)\,|b|\,uvr\\ &+mv^2[(1+m)(\mathrm{Re}\{b\}\cos\gamma-\\ &\mathrm{Im}\{b\}\sin\gamma)-m\cos\gamma]r^2\}.\end{aligned} \tag{3.2}$$

Thus, from (1.5) and the inequality (3.2), $f(z)$ is γ-spiral in $|z|<r_0$, where r_0 is the smallest positive root of the equation (3.1). Hence the theorem.

The result is sharp for the function $f(z)$ ginen by

$$f(z)=\begin{cases} z(u+mvz)^{-(\frac{1+m}{m})b}, & m\neq 0,\\ z\exp(-b\frac{v}{u}z), & m=0,\end{cases} \tag{3.3}$$

where ζ is given by (2.11).

Corollary 4. $\gamma-s.r.S_p(1-b)$*is the smallest positive root* r_0 *of the equation*

(3.4) $$u^2\cos\gamma - 2\,|b|\,uvr + v^2[2(\mathrm{Re}\{b\}\cos\gamma - \mathrm{Im}\{b\}\sin\gamma) - \cos\gamma]r^2 = 0,$$

where $u = pr + |b|\, and\, v = |b|\, r + p.$ The result is sharp.

The above result is obtained by fixing $m = 1$ in Theorem2. Further, taking $p = |b|$ in Corollary 1, we get the following result.

Corollary 5. $\gamma - s.r.S(1-b)$ *is the smallest positive root* r_0 *of the equation*

(3.5) $$\cos\gamma - 2\,|b|\,r + [2\,(\mathrm{Re}\{b\}\cos\gamma - \mathrm{Im}\{b\}\sin\gamma) - \cos\gamma]\,r^2 = 0.$$

The result is sharp.

Corollary 6. $\gamma - s.r.S_p^{\lambda}(\alpha)$ *is the smallest positive root* r_0 *of the equation*

$$u^2\cos\gamma - 2(1-\alpha)ruv\ \cos\,\lambda +$$

(3.6) $$v^2\,[2(1-\alpha)\cos\lambda\ \cos(\gamma-\lambda) - \cos\gamma]\,r^2 = 0,$$

where $u = pr + (1-\alpha)\ \cos\lambda$ and $v = (1-\alpha)r\ \cos\ \lambda + p$.The result is sharp.

The above result is obtained by fixing $b = (1-\alpha)e^{-i\lambda}\cos\lambda(|\lambda| < \frac{\pi}{2},$ $0 \le \alpha < 1)$ and $m = 1$in Theorem 3.1. This result was also obtained by Mogra [14, Corollary1]. Further, taking $p = (1-\alpha)\cos\lambda$ in Corollary 3.4, we get the reult obtained by Mogra and Ahuja [15] and Mogra [14].

Corollary 7. $\gamma - s.r.F_{M,p}(\lambda,\alpha)$ *is the smallest positive root* r_0 *of the equation*

$$u^2\cos\gamma - (1+m)\,uvr\,(1-\alpha)\ \cos\ \lambda\ +$$

(3.7) $$mv^2[(1+m)\ (1-\alpha)\ \ \cos\ \lambda\,\cos\,(\gamma-\lambda) - m\ \ \cos\ \gamma\,]\,r^2 = 0,$$

where $u = pr + \left(\dfrac{1+m}{2}\right)\,(1-\alpha)\cos\lambda$ and $v = \left(\dfrac{1+m}{2}\right)r\,(1-\alpha)\cos\lambda + p$.The result is sharp.

The above result is obtained by choosing $b = (1-\alpha)\,e^{-i\lambda}\cos\,\lambda\,, |\lambda| < \frac{\pi}{2}$ and $0 \le \alpha < 1,$ in Theorem 3.1.

Further, taking $p = \left(\dfrac{1+m}{2}\right)(1-\alpha)\ cos\lambda$ in Corollary 3.5 , we get the following result:

Corollary 8. $\gamma - s.r.F_M\,(\lambda,\alpha)$ *is the smalllest positive root* r_0 *of the equation*

(3.8) $cos\gamma\;-\;(1+m)\,r(1-\alpha)\;\cos\lambda+m[(1+m)(1-\alpha)\cos\lambda\cos\,(\gamma-\lambda)-m\cos\gamma\,]\,r^2=0$

The result is sharp

Remark 1. *(1) Putting $\alpha=0$ in Corollary 3.5, we get the corresponding result for the class $F_{\lambda,M,p}(\lambda,0)=F_{\lambda,M,p}$.*

(2) Putting $\alpha=0$ in Corollary 3.6, we get the result obtained by Mogra and Ahuja [15, Corollary 6] for the class $F_M(\lambda,0)=F_{\lambda,M}$

Corollary 9. $\gamma-s.r.H_p(\lambda)$ *is the smallest positive root* r_0 *of the equation*

$$u^2\cos\gamma-(2-\cos\;\lambda\,)\,ruv\;\cos\lambda+(1-\cos\lambda)\,v^2[(2-\cos\lambda)\;\cos\lambda\;\cos(\gamma-\lambda)$$

$$-(1-\cos\lambda)\;\cos\gamma]\,r^2=0, \qquad (3.9)$$

where $u=pr+(\dfrac{2-\cos\lambda}{2})\cos\lambda$ and $v=(\dfrac{2-\cos\lambda}{2})r\cos\lambda+p$. The result is sharp.

The above result is obtained by choosing $b=e^{i\lambda}\cos\lambda$ and $M=(\cos\lambda)^{-1}\left(|\lambda|<\frac{\pi}{2}\right)$in Theorem3.1. Further,taking $p=(\dfrac{2-\cos\lambda}{2})\cos\lambda$ in corollary 7, we get the result obtaine by Mogra and Ahuja [15,Corollary7].

Corollary 10. $\gamma-s.r.S_p(\lambda,a,d)$ *is the smallest positive root* r_0*of the equation*

$$u^2\cos\gamma-(\frac{\rho}{d}uv\cos\lambda)r-(\frac{1-a}{d})v^2\;\left[\frac{\rho}{d}\cos\lambda\cos(\gamma-\lambda)\right.$$

$$\left.+(\frac{1-a}{d})\cos\gamma\right]\,r^2=0, \qquad (3.10)$$

where $\rho=d^2-(1-a)^2, u=pr+(a+d-1)\cos\lambda$ and $v=(a+d-1)r\cos\lambda+p$.The result is sharp.

The above result obtained by choosing

$b=(1-a-d)e^{-i\lambda}\cos\lambda$ and $M=\dfrac{d}{a+d-1}\,(|\lambda|<\dfrac{\pi}{2},a+d\geq1,d\leq a<d+1)$ in Theorem 3.1.Further, taking $p=(a+d-1)\cos\lambda$ in Corollary 3.7 get the following result:

Corollary 11. $\gamma-s.r.S(\lambda,a,d)$ *is the smallest positive root* r_0, *of the equation*

$$\cos\gamma-(\frac{\rho}{d}\cos\lambda)r-(\frac{1-a}{d})\left[\frac{\rho}{d}\cos\lambda\cos(\gamma-\lambda)+(\frac{1-a}{d})\cos\gamma\right]r^2=0, \qquad (3.11.)$$

The result is sharp.

Remark 2. *(1) Putting $\gamma = 0$ in Corollary 3.9, we get the result obtained by Silvia [25, Theorem 3].*

(2) Putting $\lambda = 0$ in Corollary 3.8 and Corollary 3.9, respectively, we get the corresponding results for the classes $S_p(a,d)$ and $S(a,d)$, respectively.

Setting $h(z) = 1 + \frac{zf''(z)}{f'(z)}$ in Theorem 2.3 and using Definition 1.4, we get the following result for the class $G_p(b, M)$.

Theorem 12. $\gamma - c.r.G_p(b,M)$. *is the smallest positive root* r_0 *of the equation* (3.1)*, where* u *and* v *are given by* (2.2)*. The result is sharp for the functions* $f(z)$ *given by*

$$f'(z) = \begin{cases} (u+mvz)^{-(\frac{1+m}{m})b} & m \neq 0\,, \\ \exp(-b\frac{v}{u}z), & m = 0 \end{cases} \tag{3.12}$$

where ζ is given by (2.11).

Remark 3. *On taking the appropriate values of* b *and* M *the above theorem can give* $\gamma - c.r.$ *for the functions in the different classes.*

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