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A UNIQUE COMMON COUPLED FIXED POINT THEOREM FOR JUNGCK TYPE MAPS IN COMPLEX VALUED b-METRIC SPACES

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Abstract. In this paper, we obtain a unique common coupled fixed point theorem for Jungck type maps satifying rational inequality in complex valued b-metric spaces. Also we give an example to illustrate our main theorem.
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1. Introduction and Preliminaries

It is a well-known fact that the mathematical results regarding fixed points of contraction type mappings are very useful for determining the existence and uniqeness of solutions to various mathematical models.

Azam et al.[1] introduced the notion of a complex valued metric space which is a generalization of the classical metric space and obtained sufficent conditions for the existence of common fixed points of a pair of mappings satisfying a rational contractive condition.Though complex valued metric spaces form a special class of cone metric spaces, yet this idea is intended to define rational expressions which are not meaningful in cone metric spaces and thus many results of analysis cannot be generalized to cone metric spaces. However, in complex valued metric spaces, one can study improvements of a host of results of analysis involving divisions.Later several authors proved fixed and common fixed point theorems in complex valued metric spaces ,for example, refer [2, 3, 6, 8, 10, 11, 12, 13, 14, 15, 16, 18, 20].

In this paper,we prove a unique common coupled fixed point theorem for two Gungck type of mappings satisfying a contractive condition of rational type in the frame work of complex valued metric spaces . The proved result generalizes and extends some of the results in the literature.

To begin with, we recall some basic definitions, notations and results.

Throughout this paper $\mathcal{R}$, $\mathcal{R}^+$, $\mathcal{N}$ and $\mathbb{C}$ denote the set of all real numbers, non-negative real numbers , positive integers and complex numbers respectively.First we refer the following preliminaries.

Let $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ follows:
$z_1 \precsim z_2$ if and only if $Re(z_1) \leq Re(z_2), Im(z_1) \leq Im(z_2)$.
Thus $z_1 \precsim z_2$ if one of the following holds:
(1)$Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(2)$Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$,
(3)$Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$,
(4)$Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.
Clearly $z_1 \precsim z_2 \Rightarrow |z_1| \leq |z_2|$.
We will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (2), (3) and (4) is satisfied. Also we will write $z_1 \prec z_2$ if only (4) is satisfied.

Remark 1.1. *One can easily check that the following statements :*
(i) if $0 \precsim z_1 \precnsim z_2$ *then* $|z_1| < |z_2|$*;*
(ii) if $z_1 \precsim z_2$ *and* $z_2 \prec z_3$*, then* $z_1 \prec z_3$.

Definition 1.2. *(*[1]*) Let* X *be a non empty set. A function* $d : X \times X \to \mathbb{C}$ *is called a complex valued metric on* X *if for all* $x, y, z, \in X$ *the following conditions are satisfied:*
(i) $0 \precsim d(x, y)$ *and* $d(x, y) = 0$ *if and only if* $x = y$;
(ii) $d(x, y) = d(y, x)$;
(iii) $d(x, y) \precsim d(x, z) + d(z, y)$.
The pair (X, d)*is called a complex valued metric space.*

Now, we briefly recall the notation about complex valued b-metric spaces introduced by Rao et al.[7].

Definition 1.3. *(*[7]*) Let* X *be a non empty set and* $s \geq 1$. *A function* $d : X \times X \to \mathbb{C}$ *is called a complex valued* b*- metric on* X *if for all* $x, y, z \in X$ *the following conditions are satisfied:*
(i) $0 \precsim d(x, y)$ *and* $d(x, y) = 0$ *if and only if* $x = y$;

(ii) $d(x,y) = d(y,x)$;
(iii) $d(x,y) \precsim s\,[d(x,z) + d(z,y)]$.
The pair (X,d)*is called a complex valued* b*- metric space.*

Remark 1.4. *Let* (X,d) *be a complex valued* b*-metric space. Then*
(i)$|d(x,y)|$ *or* $|d(u,v)| < |1 + d(x,y) + d(u,v)|$, $\forall x,y,u,v \in X$.
(ii) If $x \neq y$ *then* $|d(x,y)| > 0$.
(iii) For $0 \leq k < 1$ *and* $z,w \in \mathbb{C}$ *, if* $|z| \leq k\,|w|$ *and* $|w| \leq k\,|z|$ *then* $z = w = 0$.

Definition 1.5. *([7]) Let* (X,d) *be a complex valued* b*-metric space.*

(i) *A point* $x \in X$ *is called interior point of a set* $A \subseteq X$ *whenever there exists* $0 \prec r \in \mathbb{C}$ *such that* $B(x,r) = \{y \in X : d(x,y) \prec r\} \subseteq A$.

(ii) *A point* $x \in X$ *is called a limit point of a set* $A \subseteq X$ *whenever there exists* $0 \prec r \in \mathbb{C}$ *such that* $B(x,r) \cap (X - A) \neq \phi$.

(iii) *A subset* $B \subseteq X$ *is called open whenever each point of* B *is an interior point of* B.

(iv) *A subset* $B \subseteq X$ *is called closed whenever each limit point of* B *is in* B.

(v) *The family* $F = \{B(x,r) : x \in X \text{ and } 0 \prec r\}$ *is a sub basis for a topology on* X. *We denote this complex topology by* τ_c. *Indeed, the topology*τ_c *is Hausdorff.*

Let $\{x_n\}$ be a sequence in X and $x \in X$.If for every $c \in \mathbb{C}$ with $0 \preceq c$ there is $n_0 \in \mathcal{N}$ such that for all $n > n_0, d(x_n,x) \prec c$,then $\{x_n\}$ is said to be convergent to x and x is the limit point of $\{x_n\}$.We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$. If for every $c \in \mathbb{C}$ with $0 \prec c$ there is $n_0 \in \mathcal{N}$ such that for all $n > n_0, d(x_n, x_{n+m}) \prec c$, where $m \in \mathcal{N}$,then $\{x_n\}$ is called a Cauchy sequence in (X,d). If every Cauchy sequence is convergent in (X,d) then (X,d) is called a complete complex valued b-metric space. We require the follwing lemmas.

Lemma 1.6. *([7]) Let* (X,d) *be a complex valued* b*-metric space and let* $\{x_n\}$ *be a sequence in X.Then* $\{x_n\}$ *converges to x if and only if* $|d(x_n,x)| \to 0$ *as* $n \to \infty$.

Lemma 1.7. *([7]) Let* (X,d) *be a complex valued* b*-metric space and let* $\{x_n\}$ *be a sequence in X.Then* $\{x_n\}$ *is a Cauchy sequence if and only if*
$|d(x_n, x_{n+m})| \to 0$ *as* $n,m \to \infty$.

One can easily prove the following lemma

Lemma 1.8. *Let* (X,d) *be a complex valued* b*-metric space and let* $\{x_n\}$ *and* $\{y_n\}$ *be sequences in* X *converging to* x *and* y *respectively. Then*

(i) $\frac{1}{s}\,|d(x,z)| \leq \lim_{n\to\infty} |d(x_n,z)| \leq s\ \ |d(x,z)|$ *for all* $z \in X$,
(ii) $\frac{1}{s^2}\,|d(x,y)| \leq \lim_{n\to\infty} |d(x_n,y_n)| \leq s^2\,|d(x,y)|$.

Bhaskar and Lakshmikantham [17] introduced the concept of coupled fixed points and Lakshmikantham and Ciric [19] defined the common coupled fixed points.Later several authors proved coupled and common coupled fixed point theorems in various spaces. Now, we mention the following definitions which are needed for further discussions.

Definition 1.9. *Let* $F : X \times X \to X$ *and* $f : X \to X$ *be mappings.*
(i)*(*[19]*) An element* $(x,y) \in X \times X$ *is called a coupled coincidence point of* F *and* f *if* $fx = F(x,y)$ *and* $fy = F(y,x)$
(ii)*(*[19]*) An element* $(x,y) \in X \times X$ *is called a common coupled fixed point of* F *and* f *if* $x = fx = F(x,y)$ *and* $y = fy = F(y,x)$.
(iii)*(*[9]*) The pair* (F,f) *is called* w*-compatible if* $f(F(x,y)) = F(fx,fy)$ *whenever* $x,y \in X$ *such that* $fx = F(x,y)$ *and* $fy = F(y,x)$.

Definition 1.10. *(*[5]*) Let* $F, G : X \times X \to X$ *be mappings. An element* $(x,y) \in X \times X$ *is called*
(i) *coupled coincidence point of* F *and* G *if* $F(x,y) = G(x,y)$ *and* $F(y,x) = G(y,x)$.
(ii) *common coupled fixed point of* F *and* G *if* $x = F(x,y) = G(x,y)$ *and* $y = F(y,x) = G(y,x)$.
(iii) *The pair* (F,G) *is called* $\widetilde{w}$*-compatible if* $x,y \in X$ *such that* $F(x,y) = G(x,y)$ *and* $F(y,x) = G(y,x) \Rightarrow F(G(x,y),G(y,x)) = G(F(x,y),F(y,x))$.

Definition 1.11. *(*[4]*) Let* $f,g : X \to X$ *be mappings. The pair* (f,g) *is said to be weakly compatible if* $fgx = gfx$ *whenever* $x \in X$ *such that* $fx = gx$.

If (X,d) is a complex valued b-metric space, we endow the product set $X \times X$ by the complex valued b-metric p defined by $p((x,y),(u,v)) = d(x,u) + d(y,v), \forall (x,y),(u,v) \in X \times X$.
Now we prove our main result.

2. Main Result

Theorem 2.1. . *Let* (X,d) *be a complex valued* b*-metric space and* $F, G : X \times X \to X$ *be satisfying*

(2.1.1)

$$\begin{aligned} d(F(x,y),F(u,v)) \precsim\ & a_1 d(G(x,y),G(u,v)) + a_2 d(G(y,x),G(v,u)) \\ & + a_3 d(F(x,y),G(x,y)) + a_4 d(F(y,x),G(y,x)) \\ & + a_5 d(F(u,v),G(u,v)) + a_6 d(F(v,u),G(v,u)) \\ & + a_7 \tfrac{d(F(x,y),G(x,y))\ d(F(u,v),G(u,v))}{1+d(G(x,y),G(u,v))+d(G(y,x),G(v,u))} \\ & + a_8 \tfrac{d(F(x,y),G(u,v))\ d(F(u,v),G(x,y))}{1+d(G(x,y),G(u,v))+d(G(y,x),G(v,u))} \end{aligned}$$

for all $x, y, u, v \in X$*, where* $a_i \geq 0$*,* $i = 1, 2, 3, ..., 8$ *with* $\sum_{i=1}^{8} a_i < \frac{1}{s}$*,*

(2.1.2) $F(X \times X) \subseteq G(X \times X)$*,*

(2.1.3) $\{(G(x,y), G(y,x)) : x, y \in X\}$ *is a complete subspace of* $(X \times X, p)$*,*

(2.1.4) *the pair* (G, T) *is* $\widetilde{w}$*-compatible.*

Then F *and* G *have a unique common coupled fixed point in* $X \times X$ *.*

Proof. Let $(x_0, y_0) \in X \times X$ be an arbitrary point.
From (2.1.2) there exist sequences $\{x_n\}, \{y_n\}, \{z_n\}$ and $\{w_n\}$ in X such that
$z_n = F(x_n, y_n) = G(x_{n+1}, y_{n+1})$,
$w_n = F(y_n, x_n) = G(y_{n+1}, x_{n+1})$, $n = 0, 1, 2, ...$
Let $R_n = \max\{d(z_n, z_{n+1}), d(w_n, w_{n+1})\}$.
Case(i): Suppose $R_n = 0$ for some $n \in \mathcal{N} \cup \{0\}$.
Then $z_n = z_{n+1}$ and $w_n = w_{n+1}$. From (2.1.1), we have

$$\begin{aligned} & d(z_{n+1}, z_{n+2}) = d(F(x_{n+1}, y_{n+1}), F(x_{n+2}, y_{n+2})) \\ & \precsim a_1 d(z_n, z_{n+1}) + a_2 d(w_n, w_{n+1}) + a_3 d(z_{n+1}, z_n) \\ & + a_4 d(w_{n+1}, w_n) + a_5 d(z_{n+2}, z_{n+1}) + a_6 d(w_{n+2}, w_{n+1}) \\ & + a_7 \tfrac{d(z_{n+1}, z_n)\ d(z_{n+2}, z_{n+1})}{1+d(z_n, z_{n+1})+d(w_n, w_{n+1})} + a_8 \tfrac{d(z_{n+1}, z_{n+1})\ d(z_{n+2}, z_n)}{1+d(z_n, z_{n+1})+d(w_n, w_{n+1})} \\ & = a_5 d(z_{n+1}, z_{n+2}) + a_6 d(w_{n+1}, w_{n+2}). \end{aligned}$$

Thus $|d(z_{n+1}, z_{n+2})| \leq \frac{a_6}{1-a_5} |d(w_{n+1}, w_{n+2})|$.
Similarly $|d(w_{n+1}, w_{n+2})| \leq \frac{a_6}{1-a_5} |d(z_{n+1}, z_{n+2})|$.
Hence from Remark 1.4(iii),we get $z_{n+1} = z_{n+2}$ and $w_{n+1} = w_{n+2}$.
Thus $z_n = z_{n+1} = z_{n+2} = ...$ and $w_n = w_{n+1} = w_{n+2} = ...$
Thus $\{z_n\}$ and $\{w_n\}$ are Cauchy sequences in X.
Case(ii): Assume that $R_n \neq 0$ for all $n \in \mathcal{N} \cup \{0\}$.
As in case(i),using Remark 1.4(i) ,we have
$|d(z_{n+1}, z_{n+2})| \leq (a_1 + a_2 + a_3 + a_4) |R_n| + (a_5 + a_6 + a_7) |R_{n+1}|$ and
$|d(w_{n+1}, w_{n+2})| \leq (a_1 + a_2 + a_3 + a_4) |R_n| + (a_5 + a_6 + a_7) |R_{n+1}|$
Thus $|R_{n+1}| \leq k |R_n|$, where $k = \frac{a_1+a_2+a_3+a_4}{1-a_5-a_6-a_7}$
Now consider

$d(z_{n+2}, z_{n+3}) = d(F(x_{n+2}, y_{n+2}), F(x_{n+3}, y_{n+3})$
$\precsim a_1 d(z_{n+1}, z_{n+2}) + a_2 d(w_{n+1}, w_{n+2}) + a_3 d(z_{n+2}, z_{n+1})$
$+ a_4 d(w_{n+2}, w_{n+1}) + a_5 d(z_{n+3}, z_{n+2}) + a_6 d(w_{n+3}, w_{n+2})$
$+ a_7 \frac{d(z_{n+2}, z_{n+1})\ d(z_{n+3}, z_{n+2})}{1+d(z_{n+1}, z_{n+2})+d(w_{n+1}, w_{n+2})} + a_8 \frac{d(z_{n+2}, z_{n+2})\ d(z_{n+3}, z_{n+1})}{1+d(z_{n+1}, z_{n+2})+d(w_{n+1}, w_{n+2})}$.
Thus $|d(z_{n+2}, z_{n+3})| \leq (a_1 + a_2 + a_3 + a_4)|R_{n+1}| + (a_5 + a_6 + a_7)|R_{n+2}|$
Similarly $|d(w_{n+2}, w_{n+3})| \leq (a_1 + a_2 + a_3 + a_4)|R_{n+1}| + (a_5 + a_6 + a_7)|R_{n+2}|$
Hence $|R_{n+2}| \leq k|R_{n+1}|$
Thus $|R_{n+1}| \leq k|R_n|$, $n = 0, 1, 2, 3, ...$
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.
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$\leq k^{n+1}|R_0|$ ....(1)
for $m > n$, using (1) we have

$$\begin{aligned}
|d(z_n, z_m)| &\leq s|d(z_n, z_{n+1})| + s^2|d(z_{n+1}, z_{n+2})| + ... + s^{m-n-1}|d(z_{m-1}, z_m)| \\
&\leq [sk^n + s^2k^{n+1} + ... + s^{m-n-1}k^{m-1}]|R_0|, from(1) \\
&\leq [(sk)^n + (sk)^{n+1} + ... + (sk)^{m-1}]|R_0| \\
&\leq \frac{(sk)^n}{1-sk}|R_0| \\
&\to 0 \ as \ n, m \to \infty
\end{aligned}$$

Thus $\{z_n\}$ is a Cauchy sequence in X. Similarly, we can show that $\{w_n\}$ is also a Cauchy sequence in X.
Now, $|p((z_n, w_n), (z_m, w_m))| \leq |d(z_n, z_m)| + |d(w_n, w_m)| \to 0$ as $n, m \to \infty$.
This implies that $\{(z_n, w_n)\}$ is a Cauchy sequence in $(X \times X, p)$.
Since $(z_n, w_n) = (G(x_{n+1}, y_{n+1}), G(y_{n+1}, x_{n+1})) \in \{(G(x, y), G(y, x)) : x, y \in X\}$, by (2.1.3),there exist $(z, w) \in \{(G(x, y), G(y, x)) : x, y \in X\}$ such that $p((z_n, w_n), (z, w)) \to 0$ as $n \to \infty$.
This implies that there exist $a, b \in X$ such that $z_n \to z = G(a, b)$ and $w_n \to w = G(b, a)$.
Now from(2.1.1), we have

$$\begin{aligned}
&\tfrac{1}{s}|d(F(a,b), G(a,b))| \\
&\leq \lim_{n\to\infty} |d(F(a,b), F(x_n, y_n))|, from\ Lemma\ 1.8 \\
&\leq \lim_{n\to\infty} \{a_1|d(G(a,b), G(x_n, y_n))| + a_2|d(G(b,a), G(y_n, x_n))| + a_3|d(F(a,b), G(a,b))| \\
&\ +a_4|d(F(b,a), G(b,a))| + a_5|d(F(x_n, y_n), G(x_n, y_n))| + a_6|d(F(y_n, x_n), G(y_n, x_n))| \\
&\ +a_7 \tfrac{|d(F(a,b),G(a,b))|\ |d(F(x_n,y_n),G(x_n,y_n))|}{|1+d(G(a,b),G(x_n,y_n))+d(G(b,a),G(y_n,x_n))|} + a_8 \tfrac{|d(F(a,b),G(x_n,y_n))|\ |d(F(x_n,y_n),G(a,b))|}{|1+d(G(a,b),G(x_n,y_n))+d(G(b,a),G(y_n,x_n))|}\} \\
&\leq a_1 s(0) + a_2 s(0) + a_3|d(F(a,b), G(a,b))| + a_4|d(F(b,a), G(b,a))| \\
&\ +a_5 s^2(0) + a_6 s^2(0) + a_7 s^2(0) + a_8 s^2(0), from\ Lemma\ 1.8.
\end{aligned}$$

Thus $|d(F(a,b),G(a,b))| \leq \frac{sa_4}{1-sa_3}|d(F(b,a),G(b,a))|$.
Similarly, we have $|d(F(b,a),G(b,a))| \leq \frac{sa_4}{1-sa_3}|d(F(a,b),G(a,b))|$.
Hence from Remark 1.4(iii),we obtain $z = F(a,b) = G(a,b)$ and $F(b,a) = G(b,a) = w$.
Since (F,G) is $\widetilde{w}$-compatible, we have
$F(z,w) = F(G(a,b),G(b,a)) = G(F(a,b),F(b,a)) = G(z,w)$ and $F(w,z) = F(G(b,a),G(a,b)) = G(F(b,a),F(a,b)) = G(w,z)$. Now consider

$$\begin{aligned}
&|d(z,F(z,w)|\\
&= |d(F(a,b),F(z,w)|\\
&\leq a_1|d(z,F(z,w))| + a_2|d(w,F(w,z))| + a_3|d(z,z)| + a_4|d(w,w)|\\
&\qquad +a_5|d(F(z,w),F(z,w))| + a_6|d(F(w,z),F(w,z))|\\
&\qquad +a_7\frac{|d(z,z)|\ |d(F(z,w),F(z,w))|}{|1+d(z,F(z,w))+d(w,F(w,z))|} + a_8\frac{|d(z,F(w,z))|\ |d(F(z,w),z)|}{|1+d(z,F(z,w))+d(w,F(w,z))|}\\
&\leq a_1|d(z,F(z,w))| + a_2|d(w,F(w,z))| + a_3(0) + a_4(0) + a_5(0)\\
&\ +a_6(0) + a_7(0) + a_8|d(F(z,w),z)|, from\ Remark\ 1.4(i)
\end{aligned}$$

which gives that $|d(z,F(z,w))| \leq \frac{a_2}{1-a_1-a_8}|d(w,F(w,z))|$.
Similarly, we can show that $|d(w,F(w,z))| \leq \frac{a_2}{1-a_1-a_8}|d(z,F(z,w))|$.
Hence, from Remark 1.4(iii), we obtain $z = F(z,w)$, $w = F(w,z)$.
Thus $z = F(z,w) = G(z,w)$ and $w = F(w,z) = G(w,z)$.
Hence (z,w) is a common coupled fixed point of F and G.
Suppose (z^1,w^1) be another common coupled fixed point.Then from (2.1.1),
$d(z,z^1) = d(F(z,w),F(z^1,w^1))$
$\leq a_1d(z,z^1) + a_2d(w,w^1) + a_3d(z,z) + a_4d(w,w) + a_5d(z^1,z^1) + a_6d(w^1,w^1)$
$+ a_7\frac{d(z,z)\ d(z^1,z^1)}{1+d(z,z^1)+d(w,w^1)} + a_8\frac{d(z,z^1)\ d(z^1,z)}{1+d(z,z^1)+d(w,w^1)}$.
Thus $|d(z,z^1)| \leq \frac{a_2}{1-a_1-a_8}|d(w,w^1)|$.
Similarly, we have $|d(w,w^1)| \leq \frac{a_2}{1-a_1-a_8}|d(z,z^1)|$. Hence from Remark 1.4(iii), we have $z = z^1$ and $w = w^1$.Thus (z,w) is the unique common coupled fixed point of F and G.
Now we give an example to support Theorem 2.1.

Example 2.2. *Let* $X = [0,1]$ *and* $d(x,y) = i\ |x-y|^2, \forall\ x,y \in X$. *Define* $F,G : X \times X \to X$ *by* $F(x,y = Sin(\frac{x+y}{8})$ *and* $G(x,y) = \frac{x+y}{2}$ *for all* $x,y \in X$.

$$\begin{aligned}
d(F(x,y),F(u,v)) &= i\ \left|Sin(\tfrac{x+y}{8}) - Sin(\tfrac{u+v}{8})\right|^2\\
&= i\left|2cos(\tfrac{x+y+u+v}{16})\ Sin(\tfrac{x+y-u-v}{16})\right|^2\\
&\precsim 4i\ \left|\tfrac{x+y-u-v}{16}\right|^2\\
&= \tfrac{i}{16}\ \left|\tfrac{x+y}{2} - \tfrac{u+v}{2}\right|^2\\
&= \tfrac{1}{16}\ d(G(x,y),G(u,v))\\
&= \tfrac{1}{32}\ d(G(x,y),G(u,v)) + \tfrac{1}{32}\ d(G(y,x),G(v,u)).
\end{aligned}$$

Thus the condition $(2.1.1)$ *is satisfied with* $a_1 = a_2 = \frac{1}{32}, a_3 = a_4 = a_5 = a_6 = a_7 = a_8 = 0$. *One can easily verify the remaining conditions of Theorem* 2.1. *Clearly* $(0, 0)$ *is the common coupled fixed point of* F *and* G.

Corollary 2.3. *Let* (X, d) *be a complex valued* b*-metric space and* $F : X \times X \to X$ *and* $g : X \to X$ *be satisfying*

(2.3.1)

$$
\begin{aligned}
d(F(x,y), F(u,v)) \precsim\ & a_1 d(gx, gu) + a_2 d(gy, gv)) \\
& + a_3 d(F(x,y), gx) + a_4 d(F(y,x), gy) \\
& + a_5 d(F(u,v), gu) + a_6 d(F(v,u), gv) \\
& + a_7 \tfrac{d(F(x,y),gx)\ d(F(u,v),gu)}{1+d(gx,gu)+d(gy,gv)} \\
& + a_8 \tfrac{d(F(x,y),gu)\ d(F(u,v),gx)}{1+d(gx,gu)+d(gy,gv)}
\end{aligned}
$$

for all $x, y, u, v \in X$, *where* $a_i \geq 0$, $i = 1, 2, 3, ..., 8$ *with* $\sum_{i=1}^{8} a_i < \frac{1}{s}$,

(2.3.2) $F(X \times X) \subseteq g(X)$,

(2.3.3) $g(X)$ *is a complete subspace of* X,

(2.3.4) *the pair* (F, g) *is* w*-compatible.*

Then F *and* g *have a unique common coupled fixed point in* $X \times X$.

Corollary 2.4. *Let* (X, d) *be a complex valued* b*-metric space and* $f, g : X \to X$ *be satisfying*

(2.4.1)

$$
\begin{aligned}
d(fx, fy) \precsim\ & a_1 d(gx, gy) + a_2 d(fx, gx) + a_3 d(fy, gy) \\
& + a_4 \tfrac{d(fx,gx)\ d(fy,gy)}{1+d(gx,gy)} + a_5 \tfrac{d(fx,gy)\ d(fy,gx)}{1+d(gx,gy)}
\end{aligned}
$$

for all $x, y \in X$, *where* $a_i \geq 0$, $i = 1, 2, 3, 4, 5$ *with* $\sum_{i=1}^{5} a_i < \frac{1}{s}$,

(2.4.2) $f(X) \subseteq g(X)$,

(2.4.3) $g(X)$ *is a complete subspace of* X,

(2.4.4) *the pair* (f, g) *is weakly compatible.*

Then f *and* g *have a unique common fixed point in* X.

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