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$(\lambda, \mu)-$ FUZZY INTERIOR IDEALS OF ORDERED SEMIRINGS

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Abstract. For all $\lambda, \mu \in [0, 1]$ such that $\lambda < \mu$, we introduce the notion of $(\lambda, \mu)-$fuzzy ideals and $(\lambda, \mu)-$fuzzy interior ideals of an ordered semiring. We characterize the regular ordered semiring and the simple ordered semiring in terms of $(\lambda, \mu)-$fuzzy interior ideals.

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1. Introduction

Historically semirings first appear implicitly in Dedekind and later in Macaulay, Neither and Lorenzen in connection with the study of a ring. However semirings first appear explicitly in Vandiver [35], also in connection with the axiomatization of arithmetic of natural numbers. Semirings have been studied by various researchers in an attempt to broaden techniques coming from semigroup theory, ring theory or in connection with applications. The developments of the theory in semirings have been taking place since 1950. Semirings abound in the mathematical world around us. A semiring is one of the fundamental structures in mathematics. Indeed the first mathematical structure we encounter the set of natural numbers is a semiring. Other semirings arise naturally in such diverse areas of mathematics as combinatorics, functional analysis, topology, graph theory, Euclidean geometry, probability theory, commutative ring theory, the mathematical modeling of quantum physics and parallel computation system. Semigroup, as the basic algebraic structure was used in the areas of theoretical computer science as well as in the solutions of graph theory, optimization theory and in particular for studying automata, coding theory and formal languages. The notion of a semiring was first introduced by Vandiver in 1934 but semirings had appeared in earlier studies on the theory of ideals of rings. A universal algebra $S = (S, +, \cdot)$ is called a semiring if and only if $(S, +), (S, \cdot)$ are semigroups which are connected by distributive laws, $i.e., a(b + c) = ab + ac, \ (a + b)c = ac + bc,$ for all $a, b, c \in S$. A natural example of semiring is the set of all natural numbers under usual addition and multiplication of numbers. In particular, if I is the unit interval on the real line, then $(I, \max, \min)$ is a semiring in which 0 is the additive identity and 1 is the mutiplicative identity. The theory of rings and the theory of semigroups have considerable impact on the development of the theory

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of semirings. In structure, semiring lies between semigroup and ring.Many semirings have order structure in addition to their algebraic structure. Additive and multiplicative structures of a semiring play an important role in determining the structure of a semiring. Semirings are useful in the areas of theoretical computer science as well as in the solutions of graph theory and optimization theory in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches of mathematics. In 1995, M.Murali Krishna Rao [14-22] introduced the notion of $\Gamma-$semiring as a generalization of $\Gamma-$ ring, ternary semiring and semiring. The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty was introduced by L.A. Zadeh [38] in 1965. There are many extensions of fuzzy sets, for example, intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, bipolar fuzzy sets, cubic sets etc. The fuzzification of algebraic structure was introduced by Rosenfeld [32] and he introduced the notion of fuzzy subgroups in 1971. Mandal [12] studied fuzzy ideals and fuzzy interior ideals in ordered semiring. Murali Krishna Rao [23-30] studied fuzzy ideals, fuzzy soft ideals and fuzzy interior ideals in $\Gamma-$semirings. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc. K.L. N. Swamy and U. M. Swamy [34] studied fuzzy prime ideals in rings in 1988. In 1982, Liu [11] defined and studied fuzzy subrings as well as fuzzy ideals in rings. Muhammed et al. [13] studied $(\alpha, \beta)-$fuzzy ideals in semirings Dutta et al. [3] studied fuzzy ideals of $\Gamma-$semirings. Kuroki [8] studied fuzzy interior ideals in semigroups.In this paper, based on the concepts of $(\lambda, \mu)-$fuzzy ideals introduced by Yao[36,37], we introduce the concepts of $(\lambda, \mu)-$fuzzy ideals and $(\lambda, \mu)-$fuzzy interior ideals of ordered semiring, which are generalization of fuzzy ideals and fuzzy interior ideals respectively. We characterize the regular ordered semiring and the simple ordered semiring in terms of $(\lambda, \mu)-$fuzzy interior ideals.

2. Preliminaries

In this section we will recall some of the fundamental concepts and definitions necessary for this paper.

Definition 2.1. A semiring $(M, +, \cdot)$ is an algebraiac structure with two binary operations " $+$ " and " $\cdot$ " such that $(M, +)$ and $(M, \cdot)$ are semigroups and the following distributive laws hold.
$x(y+z) = xy + xz$
$(x+y)z = xz + yz,$ for all $x, y, z \in M.$

Definition 2.2. A semiring $(M, +, \cdot)$ is said to be division semiring if $(M \setminus 0, \cdot)$ is a group.

Example 2.3. Let M be the set of all natural numbers. Then $(M,\ max,\ min)$ is a semiring.

Definition 2.4. An element a of a semiring S is called a regular element if there exists an element b of S such that $a = aba.$

Definition 2.5. A semiring S is called a regular semiring if every element of S is a regular element.

Definition 2.6. An element a of a semiring S is called a multiplicatively idempotent (an additively idempotent) element if $aa = a(a + a = a).$

Definition 2.7. An element b of a semiring M is called an inverse element of a of M if $ab = ba = 1$.

Definition 2.8. A non-empty subset A of semiring M is called

(i) a subsemiring of M if A is an additive subsemigroup of M and $AA \subseteq A$.

(ii) a left(right) ideal of M if A is an additive subsemigroup of M and $MA \subseteq A(AM \subseteq A)$.

(iii) an ideal if A is an additive subsemigroup of $M,\ MA \subseteq A$ and $AM \subseteq A$.

(iv) a $k-$ideal if A is a subsemiring of $M, AM \subseteq A, MA \subseteq A$ and $x \in M,\ x + y \in A, y \in A$ then $x \in A$.

Definition 2.9. A semiring M is called a division semiring if for each non-zero element of M has multiplication inverse.

Definition 2.10. A semiring M is called an ordered semiring if it admits a compatible relation $\leq$. i.e. $\leq$ is a partial ordering on M satisfies the following conditions. If $a \leq b$ and $c \leq d$ then

(i) $a + c \leq b + d, c + a \leq d + b$

(ii) $ac \leq bd$

(iii) $ca \leq db$, for all $a, b, c, d \in M$

Example 2.11. Let $M = [0, 1]$, binary operations be defined as $x + y = max\{x, y\}, xy = min\{x, y\}$, for all $x, y \in M$. Then M is an ordered semiring with respect to usual ordering.

Definition 2.12. An ordered semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0x = x0 = 0,$ for all $x \in M$.

Definition 2.13. An ordered semiring M is said to be commutative semiring if $xy = yx$, for all $x, y \in M$

Definition 2.14. A non zero element a in an ordered semiring M is said to be a zero divisor if there exists non zero element $b \in M$, such that $ab = ba = 0$.

Definition 2.15. An ordered semiring M with unity 1 and zero element 0 is called an integral ordered semiring if it has no zero divisors.

Definition 2.16. An ordered semiring M is said to be totally ordered semiring M if any two elements of M are comparable.

Definition 2.17. In an ordered semiring M

(i) the semigroup $(M, +)$ is said to be positively ordered , if $a \leq a + b$ and $b \leq a + b,$ for all $a, b \in M$.

(ii) the semigroup $(M, +)$ is said to be negatively ordered, if $a + b \leq a$ and $a + b \leq b,$ for all $a, b \in M$.

(iii) the semigroup $(M, \cdot)$ is said to be positively ordered, if $a \leq ab$ and $b \leq a\alpha b,$ for all $\in \Gamma, a, b \in M$.

(iv) the semigroup $(M, \cdot)$ is said to be negatively ordered if $ab \leq a$ and $ab \leq b$ for all $a, b \in M$.

Definition 2.18. A non-empty subset A of an ordered semiring M is called a subsemiring M if $(A, +)$ is a subsemigroup of $(M, +)$ and $ab \in A$ for all $a, b \in A$.

Definition 2.19. Let M be an ordered semiring. A non-empty subset I of M is called a left (right) ideal of an ordered semiring M if I is closed under addition, $MI \subseteq I$ ($IM \subseteq I$) and if for any $a \in M,\ b \in I,\ a \leq b \Rightarrow a \in I$. I is called an ideal of M if it is both a left ideal and a right ideal of M.

Definition 2.20. A non-empty subset A of ordered $\Gamma-$semiring M is called a $k-$ideal if A is an ideal and $x \in M,\ x + y \in A, y \in A$ then $x \in A$.

Definition 2.21. Let M and N be ordered semirings. A mapping $f : M \to N$ is called a homomorphism if

(i) $f(a + b) = f(a) + f(b)$

(ii) $f(ab) = f(a)f(b)$, for all $a, b \in M$.

(iii). If $a \leq b$ then $f(a) \leq f(b)$, for all $a, b \in M$.

Definition 2.22. Let M be an ordered semiring. A mapping $f : M \to M$ is called an endomorphism if

(i) f is an onto ,

(ii) $f(a + b) = f(a) + f(b)$,

(iii) $f(ab) = f(a)f(b)$, for all $a, b \in M$.

(iii). If $a \leq b$ then $f(a) \leq f(b)$, for all $a, b \in M$.

Definition 2.23. Let M be a non-empty set. Then a mapping $f : M \to [0, 1]$ is called a fuzzy subset of M.

Definition 2.24. Let f be a fuzzy subset of a non-empty set M. For $t \in [0, 1]$, the set $f_t = \{x \in M \mid f(x) \geq t\}$ is called a level subset of M with respect to f.

Definition 2.25. Let M be a semiring. A fuzzy subset μ of M is said to be fuzzy subsemiring of M if it satisfies the following conditions

$(i)\ \mu(x + y) \geq min\ \{\mu(x), \mu(y)\}$

$(ii)\ \mu(xy) \geq min\ \{\mu(x), \mu(y)\}$, for all $x, y \in M$.

Definition 2.26. A fuzzy subset μ of a semiring M is called a fuzzy left (right) ideal of M if it satisfies the following conditions

$(i)\ \mu(x + y) \geq min\{\mu(x), \mu(y)\}$

$(ii)\ \mu(xy) \geq \mu(y)\ (\mu(x))$, for all $x, y \in M$.

Definition 2.27. A fuzzy subset μ of a semiring M is called a fuzzy ideal of M if it satisfies the following conditions

$(i)\ \mu(x + y) \geq min\{\mu(x), \mu(y)\}$

$(ii)\ \mu(xy) \geq max\ \{\mu(x), \mu(y)\}$, for all $x, y \in M$

Definition 2.28. An ideal I of a semiring M is called a $k-$ideal if for all $x, y \in M,\ x + y \in I, y \in I \Rightarrow x \in I$.

Definition 2.29. Let A be non-empty subset of a semiring M. The characteristic function of A is a fuzzy subset of M and it is defined by

$$\chi_{_A}(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A. \end{cases}$$

Definition 2.30. Let S and T be non empty sets and $\phi : S \to T$ be a any function. A fuzzy subset f of S is called a $\phi-$invariant if $\phi(x) = \phi(y) \Rightarrow f(x) = f(y)$.

3. $(\lambda, \mu)-$FUZZY INTERIOR IDEALS OF ORDERED SEMIRINGS

In this section.based on the concepts of $(\lambda, \mu)-$fuzzy ideals introduced by B. Yao, we introduce the concepts of $(\lambda, \mu)-$fuzzy ideals and $(\lambda, \mu)-$fuzzy interior ideals of an ordered semiring$M$., which are generalization Throughout this paper, we assume that $0 \leq \lambda < \mu \leq 1$, $a \vee b$ denotes max $\{a, b\}$, $a \wedge b$ denotes $\min\{a, b\}$

Definition 3.1. A fuzzy subset f of an ordered semiring M is called a $(\lambda, \mu)-$fuzzy right(left) ideal of M if

(i). $f(x + y) \vee \ \lambda \geq \min\{f(x), f(y)\} \wedge \mu$
(ii). $f(xy) \vee \lambda \geq f(x)(f(y)) \wedge \mu$
(iii). If $x \leq y$ then $f(x) \vee \lambda \geq f(y) \wedge \mu$, for all $x, y \in M$.

Definition 3.2. A fuzzy subset f of ordered semiring M is said to be $(\lambda, \mu)-$fuzzy ideal of M if it is a left $(\lambda, \mu)-$fuzzy ideal and a right $(\lambda, \mu)-$fuzzy ideal.

Definition 3.3. An additive subsemiring$A$ of M is called an interior ideal of an ordered semiring M if

(i). $MAM \subseteq A$.
(ii). If $a \in A, b \in M$ and $b \leq a$ then $b \in A$.

Definition 3.4. A fuzzy subset f of M is called a $(\lambda, \mu)-$fuzzy interior ideal of M if the following are satisfied

(i). $f(x + y) \vee \lambda \geq \min\{f(x), f(y)\} \wedge \mu$.
(ii). $f(xay) \vee \lambda \ \geq f(a) \wedge \mu$.
(iii). $if x \leq y$ then $f(x) \vee \lambda \geq f(y) \wedge \mu$, for all $x, y \in M$.

Remark 3.5. By talking $\lambda = 0, \mu = 1$, every fuzzy ideal, fuzzy interior ideal are $(\lambda, \mu)-$fuzzy ideal, $(\lambda, \mu)-$fuzzy interior ideals respectively of ordered semigroup.

Definition 3.6. An ordered semiring $M$is called a regular if for each $a \in M$ there exist $x \in M$, such that $a \leq axa$.

Definition 3.7. An ordered semiring M is called a simple if M has no proper ideals.

Definition 3.8. An ordered semiring M is called a $(\lambda, \mu)-$fuzzy simple if for any $(\lambda, \mu)-$fuzzy ideal f of M, we have $f(a) \vee \lambda \geq f(b) \wedge \mu$ for all $a, b \in M$ and $a < b$.

Theorem 3.9. *Let M be an ordered semiring. Then f is a $(\lambda, \mu)-$fuzzy interior ideal of M if and only if f_t is an interior ideal of M for all $t \in (\lambda, \mu]$.*

Proof. Let f be a $(\lambda,\mu]-$fuzzy interior ideal of the ordered semiring M and $x,y\in M$, and $a\in f_t.(t\in(\lambda,\mu])$

$$
\begin{aligned}
f(xay)\vee\lambda &\geq f(a)\wedge\mu\\
&\geq t\wedge\mu\\
&= t\\
\Rightarrow f(xay) &\geq t.
\end{aligned}
$$

Therefore $xay\in f_t$.
Suppose $b\in f_t,\ b\in M$ and $a\leq b$. Then $f(a)\vee\lambda\geq f(b)\wedge\mu$.

$$
\begin{aligned}
&\Rightarrow\ f(a)\vee\lambda\geq t\wedge\mu=t\\
&\Rightarrow\ f(a)\geq t\\
&\Rightarrow\ a\in f_t.
\end{aligned}
$$

Suppose $a,b\in f_t\Rightarrow\ f(a)\geq t, f(b)\geq t$

$$
\begin{aligned}
f(a+b)\vee\lambda &\geq \min\{f(a),f(b)\}\wedge\mu\\
&\geq t\wedge\mu=t.
\end{aligned}
$$

Therefore $a+b\in f_t$.

Conversely suppose that f_t is an interior ideal of $M$for all $t\in(\lambda,\mu]$.
Suppose there exist $x,a,y\in M$, such that $f(xay)\vee\lambda<f(a)\wedge\mu$ and $f(a)\wedge\mu=t\in(\lambda,\mu]$.
Then $f(a)\geq t$ and $f(xay)<t$
$\Rightarrow\ a\in f_t$ and $xay\notin f_t$, which is a contradiction.
Hence $f(xay)\vee\lambda\geq f(a)\wedge\mu$ for all $x,a,y\in M$.
Suppose $x,y\in M$ such that $x\leq y$ and $f(x)\vee\lambda<f(y)\wedge\mu=t$.
Then $t\in(\lambda,\mu],\ f(y)\geq t$ and $f(x)<t$
$\Rightarrow y\in f_t$ and $x\notin f_t$, which is a contradiction.
Hence $f$is a $(\lambda,\mu)-$fuzzy interior ideal of the ordered semiring M. □

Theorem 3.10. *Let A be an interior ideal of an ordered semiring M. Then for every $t\in(01]$, there exists a $(\lambda,\mu)-$fuzzy interior ideal f of M such that $f_t=A$.*

Proof. Let A be an interior ideal of the ordered semiring M and f be a fuzzy subset is defined by

$$
f(x)\ =\ \begin{cases} t, & \text{if } x\in A\\ 0, & \text{if } x\notin A,\ \text{where } t\in(0,\ 1].\end{cases}
$$

Choose λ,μ such that $t>\lambda>\mu$.
Suppose $x,y\in A$, then $x+y, xy\in A$.
Therefore $f(x+y)\vee\lambda=t\vee\lambda\geq\min\{f(x),f(y)\}\wedge\mu$

and $f(xy) \vee \lambda = t \geq \min\{f(x), f(y)\} \wedge \mu$.
Let $x, y \in M$ and $a \in A$. Then $xay \in A$.

$$f(xay) \vee \lambda = t \vee \lambda = t$$
$$= f(a) \wedge \mu.$$

If $a \notin A$ then $f(a) = 0 \Rightarrow f(xay) \vee \lambda = \lambda \geq f(a) \wedge \mu$.
Suppose $x \leq y$ and $x, y \in A$. Then $f(x) = f(y) = t$.

$$\text{Therefore } f(x) \vee \lambda = t$$
$$f(y) \wedge \mu = t \wedge \mu = \mu$$
$$f(x) \vee \lambda \geq f(y) \wedge \mu.$$

Similarly we can prove for all other cases. Hence f is a $(\lambda, \mu)-$interior fuzzy ideal and $f_t = A$. □

Corollary 3.11. *Let χ_A be a characteristic function of a non-empty subset A of an ordered semiring M. Then χ_A is a $(\lambda, \mu)-$fuzzy interior ideal of an ordered semiring M if and only if A is an interior ideal of M.*

Theorem 3.12. *Let A and B be any two $(\lambda, \mu)-$fuzzy interior ideals of an ordered semiring M. Then $A \cap B$ is a $(\lambda, \mu)-$fuzzy interior ideal of M.*

Proof. Let $x, y, z \in M$,

$$(i)\ A \cap B(x+y) \vee \lambda = \min\{A(x+y) \vee \lambda, B(x+y) \vee \lambda\}$$
$$\geq \min\ \{\min\{A(x), A(y)\} \wedge \mu, \min\{B(x), B(y)\} \wedge \mu\}$$
$$= \min\{\min\{A(x), B(x)\}, \min\{A(y), B(y)\}, \mu\}$$
$$= \{A \cap B(x), A \cap B(y)\} \wedge \mu$$
$$(ii)\ A \cap B(xyz) \vee \lambda = \min\{A(xyz) \vee \lambda, B(x\alpha y\beta z) \vee \lambda\}$$
$$\geq \min\{A(y) \wedge \mu, B(y) \wedge \mu\}$$
$$= A \cap B(y) \wedge \mu.$$

Suppose $x \leq y$ then $A(x) \vee \lambda \geq A(y) \wedge \mu, B(x) \vee \lambda \geq B(y) \wedge \mu$.
Then $A(x) \wedge B(x) \vee \lambda \geq A(y) \wedge B(y) \wedge \mu \Rightarrow A \cap B(x) \vee \lambda \geq (A \cap B)y \wedge \mu$. Therefore $A \cap B$ is a $(\lambda, \mu)-$fuzzy interior ideal of the ordered semiring M. □

Theorem 3.13. *Let M be an ordered semiring. If f is a $(\lambda, \mu)-$fuzzy ideal of M then f is a $(\lambda, \mu)-$fuzzy interior ideal of M.*

Proof. let $x, a, y \in M$, and f be a $(\lambda, \mu)-$fuzzy ideal of the ordered semiring M.
Then by Definition of $(\lambda, \mu)-$fuzzy ideal of M, we have

$f(x+y) \vee \lambda \geq \min\{f(x), f(y)\} \wedge \mu.$

Since f is a $(\lambda, \mu)-$fuzzy left ideal of the ordered semiring$M$, we have

$$\begin{aligned} f(xay) \vee \lambda &= f(xay)) \vee \lambda \\ &\geq f(ay) \wedge \mu \\ f(ay) \vee \lambda &\geq f(a) \wedge \mu. \end{aligned}$$

Since f is a $(\lambda, \mu)-$fuzzy right ideal,we have

$$\begin{aligned} f(xay) \vee \lambda &= \{f(xay) \vee \lambda\} \vee \lambda \\ &\geq \{f(ay) \wedge \mu\} \vee \lambda \\ &= f(ay) \vee \lambda \wedge \mu \vee \lambda \\ &\geq f(a) \wedge \mu. \end{aligned}$$

Hence f is a $(\lambda, \mu)-$fuzzy interior ideal of the ordered semiring M. □

Theorem 3.14. *Let M be a regular ordered semiring. If f is a $(\lambda, \mu)-$fuzzy interioridealof M then f is a $(\lambda, \mu)-$fuzzyidealof M.*

Proof. Let M be a regular ordered semiring, $x \in M$. Then there exist $y \in M$ such that $x \leq xyx$.

$$\begin{aligned} xy &\leq xyxy \\ \Rightarrow f(x\,y) \vee \lambda &\geq f(xyxy) \wedge \mu \\ f(xy) \vee \lambda &= f(xy) \vee \lambda) \vee \lambda \\ &\geq (f(xyxy) \wedge \mu) \vee \lambda \\ &= (f(xyxy) \vee \lambda) \wedge (\mu \vee \lambda) \\ &\geq (f(x) \vee \lambda) \wedge \mu \\ &\geq f(x) \wedge \mu. \end{aligned}$$

Therefore f is a $(\lambda, \mu)-$fuzzy rightidealof M. Similarly we can prove that f is a $(\lambda, \mu)-$fuzzy leftidealof M. Thus f is a $(\lambda, \mu)-$fuzzyidealof M. □

Theorem 3.15. *Let M be an ordered semiring. If f is a $(\lambda, \mu)-$fuzzy rightidealof M and $a \in M$ then $I_a = \{b \in M \mid f(b) \vee \lambda \geq f(a) \wedge \mu\}$ is a rightidealof M.*

Proof. Let f be a $(\lambda, \mu)-$fuzzy right ideal of the ordered semiring$M$, and $a \in M$. Suppose $x, y \in I_a$, then

$$f(x) \vee \lambda \geq f(a) \wedge \mu \text{ and } f(y) \vee \lambda \geq f(a) \wedge \mu$$

$$\begin{aligned} f(x+y) \vee \lambda &\geq \min\{f(x), f(y)\} \vee \lambda \\ &= \min\{f(x) \vee \lambda,\ f(y) \vee \lambda\} \\ &\geq f(a) \wedge \mu. \end{aligned}$$

Therefore $x + y \in I_a$.

Let $b \in I_a, x \in M$. Since $b \in I_a$, $f(b) \vee \lambda \geq f(a) \wedge \mu$

$$f(bx) \vee \lambda \geq f(b) \wedge \mu$$

$$\begin{aligned} f(bx) \vee \lambda &= (f(bx) \vee \lambda) \vee \lambda \\ &\geq (f(b) \wedge \mu) \vee \lambda \\ &= (f(b) \vee \lambda) \wedge (\mu \vee \lambda) \\ &\geq f(a) \wedge \mu. \end{aligned}$$

Therefore $bx \in I_a$.

Let $y \in I_a$ and $x \in M$ such that $x \leq y$. Since f is a $(\lambda, \mu)-$fuzzy rightidealof M,we have $f(x) \vee \lambda \geq f(y) \wedge \mu$. Since $y \in I_a$, we have

$$f(y) \vee \lambda \geq f(a) \wedge \mu$$

$$\begin{aligned} f(x) \vee \lambda &= (f(x) \vee \lambda) \vee \lambda \\ &\geq (f(y) \wedge \mu) \vee \lambda \\ &= (f(y) \vee \lambda) \wedge (\mu \vee \lambda \\ &\geq f(a) \wedge \mu. \end{aligned}$$

Therefore $x \in I_a$. Hence I_a is a right ideal of the ordered semiring M. □

Corollary 3.16. *Let M be an ordered semiring. If f is a $(\lambda, \mu)-$fuzzy leftidealof M and $a \in M$ then $I_a = \{b \in M \mid f(b) \vee \lambda \geq f(a) \wedge \mu\}$ is a leftidealof M.*

Corollary 3.17. *Let M be an ordered semiring. If f is a $(\lambda, \mu)-$fuzzy idealof M and $a \in M$ then $I_a = \{b \in M \mid f(b) \vee \lambda \geq f(a) \wedge \mu\}$ is anidealof M.*

Theorem 3.18. *Let M be an ordered semiring. Then I is anidealof M if and only if characteristic function χ_I is a (λ, μ) is a fuzzyidealof M.*

Proof. Suppose I is anidealof an ordered semiring M and $x \in M$.

Case(i). If $x \in I, y \in M$, then $xy \in I$.

Therefore $\chi_I(xy) = \chi_I(x) = 1$.

Hence $\chi_I(xy) \vee \lambda \geq \chi_I(x) \wedge \mu$ similarly $\chi_I(yx) \vee \lambda \geq \chi_I(x) \wedge \mu$.

Case(ii). $x \notin I$. Then $\chi_I(x) = 0$ so $\chi_I(xry) \vee \lambda \geq \chi_I(x) \wedge \mu$

and $\chi_I(yrx) \vee \lambda \geq \chi_I \wedge \mu$.

If $x, y \in M$ then we can prove $\chi_I(xry) \vee \lambda \geq \min\{\chi_I(x), \chi_I(y)\} \wedge \mu$.

Thus χ_I is a $(\lambda, \mu)-$fuzzyidealof M. Conversely suppose that χ_I is a $(\lambda, \mu)-$fuzzy idealof M.

Let $x \in I$, Then $\chi_I(x) = 1$. Therefore $\chi_I(xy) \vee \lambda \geq \chi_I(x) \wedge \mu = \mu$

and $\chi_I(yx) \vee \lambda \geq \chi_I(x) \wedge \mu = \mu$.

Thus $\chi_I(xy) = 1$ and $\chi_I(yx) = 1$.

Therefore $xy \in I$ for all $x, y \in I$ We have $\chi_I(x + y) \vee \lambda \geq \min\{\chi_I(x), \chi_I(y)\} \wedge \mu$.

Suppose $x \leq y$ $x, y \in M$ and $y \in I$.

$\Rightarrow \chi_I(x) \geq \chi_I(y), \Rightarrow \chi_I(x) \geq 1, \Rightarrow x \in I$. Hence I is anidealof M. □

Theorem 3.19. *Let M be an ordered semiring. M is a simple ordered semiring if and only if M is a $(\lambda, \mu)-$fuzzy simple.*

Proof. Suppose M is a simple ordered semiring. Let f be a $(\lambda, \mu)-$fuzzy ideal of M , $aandb \in M$. By Corollary [3.17], I_a is an idealof M. Since M is simple, we have $I_a = M$. Then $b \in I_a$. Therefore $f(b) \vee \lambda \geq f(a) \wedge \mu$. Hence M is $(\lambda, \mu)-$fuzzy simple.

Conversely suppose that M is a $(\lambda, \mu)-$fuzzy simple. Let I be a proper ideal of M, $a \in I$ and $x \in M$. By Theorem [3.18] χ_I is a $(\lambda, \mu)-$fuzzy ideal of M. Since M is a $(\lambda, \mu)-$fuzzy simple, we have

$$\chi_I(x) \vee \lambda \geq \chi_I(a) \wedge \mu$$
$$= \mu.$$

$$\Rightarrow \chi_I(x) \geq \mu$$
$$\Rightarrow \chi_I(x) = 1$$
$$\Rightarrow x \in I.$$

Therefore $M \subseteq I$ and $M = I$, which is a contradiction. Hence M is a simple ordered semiring. □

Theorem 3.20. *Let M be an ordered semiring. Then M is a simple if and only if $a \in M$ and $M = (MaM] = \{x \in M \mid x \leq y$ and $y \in MaM\}$.*

Proof. Let M be a simple ordered semiringand $a \in M$. Then $(MaM]$ is anidealof M. Therefore $M = (MaM]$. Suppose $M = (MaM]$ for all $a \in M$.

Let I be a properidealof ordered semiring M. Let $x \in M$ and $a \in I$. We have $M = (MaM]$. There $x \leq xay,\ x, y \in M$. Since $xay \in I$ and I is anideall. Therefore $x \in I$.

Thus $I = M$. Hence M is a simple ordered semiring. □

Theorem 3.21. *Every $(0, 1)-$ fuzzy interioridealof an ordered semiring M is a constant function then M is a simple semiring.*

Proof. Suppose every $(0,1)-$ fuzzy interior ideal$f$ of the ordered semiring $M.$ is a constant function and I is a properidealof an ordered semiring M. Then by Theorem [3.18], χ_I is a $(0,1)$fuzzy ideal. By Theorem [3.13],χ_I is a $(0,1)-$fuzzy interiorideal. Therefore $\chi_I$is a constant function. Then $I = M$. Hence M is a simple ordered semiring M. □

Theorem 3.22. *If f is a $(0,1)-$fuzzy interior ideal of a simple ordered semiring then f is a constant function.*

Proof. Let f be a $(0,1)-$ fuzzy interior ideal of a simple ordered semiring and $a, x \in M$. By Theorem [3.20],$M = (MaM]$

Therefore $x \leq cay,\ c, y \in M$,

$$\Rightarrow f(x) \vee 0 \geq f(cay) \wedge 1$$

$$\Rightarrow f(x) \geq f(a) \wedge 1$$

$$\Rightarrow f(x) \geq f(a).$$

Similarly we can prove $f(a) \leq f(x)$. Therefore $f(x) = f(a)$. Hence f is a constant function. □

Theorem 3.23. *Let M be an ordered semiring. Then M is a simple if and only if $f(a) \vee \lambda \geq f(b) \wedge \mu$ for all $(\lambda, \mu)-$fuzzy interior ideals f of $M$and $a, b \in M$.*

Proof. Suppose M is a simple and $a \in M$. By Theorem [3.20],$M = (MaM]$. Suppose $b \in M = (MaM]$. Then $b \leq xay$ $, x, y \in M$. Since f is a$(\lambda, \mu)-$fuzzy interior ideal of M,

$$\text{we have } f(b) \vee \lambda \geq f(xay) \wedge \mu$$

$$\begin{aligned} f(b) \vee \lambda &= (f(b) \vee \lambda) \vee \lambda \\ &\geq [f(xay) \wedge \mu] \vee \lambda \\ &= (f(xay) \vee \lambda) \wedge (\mu \vee \lambda) \\ &\geq (f(a) \wedge \mu) \wedge (\mu) \\ &= f(a) \wedge \mu. \end{aligned}$$

Conversely, suppose that $f(a) \vee \lambda \geq f(b) \wedge \mu$for any $(\lambda, \mu)-$fuzzy interior ideal$f$ of $M$and $a, b \in M$. Let f be a $(\lambda, \mu)-$fuzzyidealof M. By Theorem[3.13], f is a $(\lambda, \mu)-$fuzzy interiorîdealof M. Therefore $f(a) \vee \lambda \geq f(b) \wedge \mu$ for all $a, b \in M$. By Theorem[3.19], M is a simple ordered semiring. □

The following theorem follows from Theorems [3.19] to [3.23]

Theorem 3.24. *Let M be an ordered semiring. Then the following are equivalent.*

(i). *M is a simple.*

(ii). *$M = (MaM]$ for all $a \in M$.*

(iii). *M is a $(\lambda, \mu)-$fuzzy simple.*

(iv). *$f(a) \vee \lambda \geq f(b) \wedge \mu$ for all $(\lambda, \mu)-$fuzzy interior ideals $f$and $a, b \in M$.*

(v). *If f is a $(0,\ 1)-$fuzzy interior ideal then f is a constant function.*

4. Conclusion

We introduced the notions of $(\lambda, \mu)-$fuzzy ideals and $(\lambda, \mu)-$fuzzy interior ideals of an ordered semiring and characterized the regular ordered semirings and simple ordered semirings in terms of $(\lambda, \mu)-$fuzzy interior ideals. In continuation of this paper,we study $(\lambda, \mu)-$fuzzy soft ideals and $(\lambda, \mu)-$fuzzy soft interior ideals over ordered $\Gamma-$ semirings.

References

[1] P. J. Allen, A fundamental theorem of homomorphism for semirings, Proc. Amer. Math. Sco., 21 (1969), 412–416.

[2] K. T. Attensov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1) (1986), 87–96.

[3] T. K. Dutta, S. K. Sardar and S. Goswami, Operations on fuzzy ideals of $\Gamma-$semirings, Fuzzy Inf. Engg. arXiv:1101.4791v1 [math.GM], (2011), DOI: 10.1007/s12543-012-0115-z.

[4] J. S. Golan, Semirings and their Applications, Kluwer Acad. Publ. (1999).

[5] M. Henriksen, Ideals in semirings with commutative addition, Amer. Math. Soc. Notices, 5 (1958), 321.

[6] N. Kahayopula and M. Tsingels, Fuzzyidealls in ordered semirings, Quasigroups and related sustems, 15(2017), 279-289.

[7] Kyung Ho Kim and Young Bae Jun, Intuitionistic fuzzy interior ideals of semirings, Int. J. of Math. and Math. Sci. 27 (5) (2001) 261–267.

[8] N. Kuroki, On fuzzy semirings, Information Sciences, 53(3)(1991), 203–236.

[9] H. Lehmer, A ternary analogue of abelian groups, Amer. J. of Math., 59 (1932), 329–338.

[10] W. G. Lister, Ternary rings, Trans. Amer. Math. Soc., 154 (1971), 37–55.

[11] W. J. Liu, Fuzzy invariant subgroups and fuzzy ideals, Fuzzy Sets Syst. 8(2) (1982), 133–139.

[12] D. Mandal, Fuzzyidealls and fuzzy interior ideals in ordered semirings, Fuzzy info. Engg. 6 (2014), 101–114.

[13] S. Muhammed, B. Young and N. Yasir, Characterizations of regular semirings by $(\alpha, \beta)-$fuzzyidealls, Comp. Math. Appl. 59 (2010), 161-175.

[14] M. Murali Krishna Rao, Γ-semirings-I, Southeast Asian Bull. Math. 19(1)(1995), 49-54.

[15] M. Murali Krishna Rao, Γ-semirings-II, Southeast Asian Bull. Math. 21(3)(1997), 281-287

[16] : M.Murali Krishna Rao, The Jacobson radical of Γ-semiring, Southeast Asian Bull. Math. 23(1999)127-134

[17] M.Murali Krishna Rao,Ideals in ordered $\Gamma-$semirings, Discuss. Math. Gen. Algebra Appl. 38 (2018), 47-68.

[18] M. Murali Krishna Rao, $\Gamma-$ semiring with identity, Discuss. Math. Gen. Algebra Appl. 37 (2017) 189âĂŞ207.

[19] Marapureddy Murali Krishna Rao, A study of bi-quasi-interior ideals a new generalization ofidealof generalization of semiring, Bull. Int. Math. Virtual Inst. 8(2018), 519-535.

[20] M. Murali Krishna Rao and B. Venkateswarlu, On generalized right derivations of $\Gamma-$ incline,*Journal Of The International Mathematical Virtual Institute*, **6** (2016), 31-47.

[21] M. Murali Krishna Rao and B. Venkateswarlu,Right derivation of ordered $\Gamma-$semirings, Discuss. Math. Gen. Algebra Appl. 36 (2016), 209âĂŞ221 doi:10.7151/dmgaa.1258

.

[22] M. Murali Krishna Rao and B. Venkateswarlu, Regular $\Gamma-$incline and field $\Gamma-$semiring, Novi Sad J. Math. 45 (2) (2015), 155-171.

[23] M. Murali Krishna Rao, Fuzzy soft $\Gamma-$semiring and fuzzy soft $k$ideal over $\Gamma-$semiring, Ann. Fuzzy Math. Inform. 9 (2) (2015) 12–25.

[24] M. Murali Krishna Rao, Fuzzy soft Γ- semiring homomorphism, Ann. Fuzzy Math. Inform. 12 (4) (2016), 479–489.

[25] M. Murali Krishna Rao, T-fuzzyideals in ordered Γ - semirings, Ann. Fuzzy Math. Inform. 13 (2) (2017), 253–276.

[26] M. Murali Krishna Rao, Bi-quasiideals and fuzzy bi-quasi ideals of Γ - semirings. Bull. Int. Math. Virtual Inst. 7 (2) (2017), 231–242.

[27] M. Murali Krishna Rao and B. Venkateswarlu, $L-$ fuzzy ideals in $\Gamma-$semirings, Ann. Fuzzy Math. Inform. 10 (1) (2015), 1–16.

[28] M. Murali Krishna Rao and B. Venkateswarlu, Fuzzy soft $k$idealls over semiring and fuzzy soft semiring homomorphism, J. Hyperstructures, 4 (2) (2015), 93–116.

[29] M. Murali Krishna Rao, An intuitionistic normal fuzzy soft k -ideal over a Γ -semiring, Ann. Fuzzy Math. Inform. 11 (3) (2016), 361–376.

[30] M. Murali Krishna Rao, Fuzzy prime ideals in ordered Γ -semirings, J. Int. Math. Virtual Inst. 7 (2017) 85–99.

[31] N. Nobusawa, On a generalization of the ring theory, Osaka. J. Math., 1 (1964), 81 – 89.

[32] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35(1971), 512 – 517.

[33] M. K. Sen, On $\Gamma-$semiring, Proc. of Inter. Con. of Alg. and its Appl., Decker Publicaiton, New York, (1981), 301–308.

[34] U. M. Swamy and K. L. N. Swamy, Fuzzy primeidealls of rings, J. Math. Anal. Appl. 134 (1988), 94– 103.

[35] H. S. Vandiver, Note on a simple type of algebra in which cancellation law of addition does not hold, Bull. Amer. Math. Soc., (N.S.), 40 (1934), 914–920.

[36] B. Yao, $(\lambda, \mu)-$fuzzy subrings and $(\lambda, \mu)-$fuzzyidealls, J. fuzzy Math. 15(4) (2007), 981-987.

[37] B. Yao, Characterizations of regular semirings, Appl. Math. Inf. Sci., 8 (2) (2014), 715-719.

[38] L. A. Zadeh, Fuzzy sets, Inf. control, 8 (1965), 338–353.