ON THE FOURIER-STIELTJES TRANSFORM OF COMPACT OPERATOR VALUED MEASURES ASSOCIATED TO DISCRETE SERIES REPRESENTATIONS

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ABSTRACT. The Fourier-Stieltjes transform of compact operators valued measures on a locally compact unimodular group is considered. The transformed measures are evaluated at a square integrable irreducible representation of the group. We obtain a decomposition of the transformed measures in terms of compact operators and an integral representation of some compact operators valued transformers on the completion of a certain tensor product space.

2020 Mathematics Subject Classification. 42A38, 48G10.

Key words and phrases. Fourier-Stieltjes transform; vector measure; locally compact group; square integrable representation; compact operator.

1. INTRODUCTION

The Fourier transform has shown its importance in Mathematics and its applications to Engineering, Signal processing, Physics, etc. This transform has been extended to other objects such as distributions, measures, operators...The extension to measures is known in the literature as the Fourier-Stieltjes transform.

In [3] the authors studied properties of the Fourier-Stieltjes transform of Banach space valued measures on compact groups. In [13], an interpretation of the Fourier-Stieltjes transform in terms of tensor product space was given. Other useful references are given along this paper. The aim of this paper is to pursue the study of the Fourier-Stieltjes transform of vector measures on a locally compact unimodular group. Here we are interested in C^*-algebra valued

measures with particular emphasis on the C^*-algebra of compact operators. Recent works in that direction are [11, 15].

This article is organized as follows. Section 2 is devoted to some preliminaries. Basic facts about vector measures and group representation theory are recalled. Section 3 contents the main results.

2. Preliminary notes

This section is devoted to collect the necessary ingredients which we may need in the sequel. More precisely we are concerned with C^*-algebras valued measures and group representations.

2.1. C^*-**algebra-valued measures.** We summarize some facts about vector measures. Many assertions here are borrowed from [13]. More details about vector measures can be found in [6]. Let G be a locally compact group. We denote by $\mathcal{B}(G)$ the σ-field of Borel subsets of G. Here we set some basic facts about a general C^*-algebra valued measure though in Section 3 we will deal with a particular C^*-algebra. Let $\mathcal{A}$ be a C^*-algebra [4, 12]. The bracket $\langle \psi, x \rangle$, $\psi \in \mathcal{A}^*$, $x \in \mathcal{A}$ denotes the duality between $\mathcal{A}$ and its topological dual $\mathcal{A}^*$. Following [6] we call a vector measure any countably additive set function $m : \mathcal{B}(G) \to \mathcal{A}$ such that for any sequence $(A_n)_{n \geq 1}$ of pairwise disjoint elements of $\mathcal{B}(G)$ one has

$$(2.1) \qquad\qquad m(\overset{\infty}{\underset{n=1}{\cup}} A_n) = \sum_{n=1}^{\infty} m(A_n).$$

For a vector measure m, the measure defined by

$$(2.2) \qquad\qquad \psi m(A) := \langle \psi, m(A) \rangle, A \in \mathcal{B}(G),$$

where $\psi \in \mathcal{A}^*$, is a complex measure. The m-integrability of a complex Borel-measurable function f defined on G is given by the following conditions:

1- $\forall \psi \in \mathcal{A}^*$, f is ψm-integrable

2- $\forall A \in \mathcal{B}(G), \exists y \in \mathcal{A}, \forall \psi \in \mathcal{A}^*, \langle \psi, y \rangle = \int_A f d\psi m.$

Then we set $y = \int_G f dm.$

An $\mathcal{A}$-valued function f defined on G is m-integrable if the function $x \mapsto \|f(x)\|$ is m-integrable.

A vector measure m is said to be bounded if there exists $M > 0$ such that

$$(2.3) \qquad\qquad \forall A \in \mathcal{B}(G), \|m(A)\| \leq M.$$

The variation of ψm is defined as

$$(2.4) \qquad |\psi m|(A) = \sup_{\pi} \sum_{n} |\psi m(A_n)|$$

where the supremum is taken over all the partitions π of A into finite number of pairwise disjoint measurable subsets of $\mathcal{B}(G)$ [6, page 2]. The *semivariation* of a vector measure m is the nonnegative set function $|m|$ defined on $\mathcal{B}(G)$ by

$$(2.5) \qquad |m|(A) = \sup\{|\psi m|(A) : \psi \in \mathcal{A}^*, \|\psi\| \leq 1\}, \forall A \in \mathcal{B}(G)$$

where the supremum is taken over all the partitions π of A into pairwise disjoint measurable subsets of A. If $|m|(G) < \infty$ then m is called a *vector measure of bounded semivariation*. Indeed, a vector measure is bounded if and only if it is of bounded semivariation [6, page 4].

Let $\mathcal{M}(G, \mathcal{A})$ denote the space of bounded $\mathcal{A}$-valued measures on G. We define the norm of a measure $m \in \mathcal{M}(G, \mathcal{A})$ by:

$$(2.6) \qquad \|m\| := \int_G \chi_G d|m| = |m|(G).$$

2.2. **Basic group representations theory and Haar measure.** Here we are interested in the theory of group representations. Interested readers are refered to [2,5,8–10]. Its applications to wavelets analysis can be found in [1,9].

A u*nitary representation* of a locally compact group G is a group homomorphism $\sigma : G \mapsto \mathcal{U}(H_\sigma)$ where $\mathcal{U}(H_\sigma)$ is the group of unitary operators on the complex Hilbert space H_σ. The space H_σ is called *the representation space* of σ and its dimension is called the *dimension* of σ. A unitary representation is said to be *continuous* if for any $\alpha \in H_\sigma$, the function $x \mapsto \sigma(x)\alpha$ from G into H_σ is continuous. However, since the weak and the strong operator topologies coincide on $\mathcal{U}(H_\sigma)$ then the representation σ is continuous if for $\alpha, \beta \in H_\sigma$, the mapping $x \mapsto \langle \sigma(x)\alpha, \beta \rangle$ from G into $\mathbb{C}$ is continuous. An intertwining operator for unitary representations $\sigma_1 : G \to \mathcal{U}(H_{\sigma_1})$ and $\sigma_2 : G \to \mathcal{U}(H_{\sigma_2})$ is a bounded linear map $T : H_{\sigma_1} \to H_{\sigma_2}$ such that

$$(2.7) \qquad T\sigma_1(x) = \sigma_2(x)T, \ \forall x \in G.$$

Let us denote by $\mathcal{C}(\sigma_1, \sigma_2)$ the set of all intertwining operators of σ_1 and σ_2. If $\mathcal{C}(\sigma_1, \sigma_2)$ is non empty then the representations σ_1 and σ_2 are said to be *unitary equivalent*.

A subspace M of H_σ is said to be *invariant* if $\forall x \in G$, $\sigma(x)M \subset M$. If σ has no invariant subspace except $\{0\}$ and H_σ then σ is said to be *irreducible*. The set of all equivalent classes of

unitary irreducible representations of G is called the *unitary dual* of G and it is denoted by $\widehat{G}$. A locally compact group G admits a left Haar measure, that is a nonzero Radon measure μ on G such that $\mu(xE) = \mu(E)$ for every Borel set E in G and every x in G and a right Haar measure, that is a nonzero Radon measure μ on G such that $\mu(Ex) = \mu(E)$ for every Borel set E in G and every x in G [8, page 37]. One may decide to consider one or other of these measures. A left Haar measure μ is unique up to a strictly positive multiplicative constant, that is, if ν is another left Haar measure then $\nu = a\mu$ where a is a positive constant. Let μ be a left Haar measure. For $s \in G$, set $\nu(f) = \mu(f_{s^{-1}}), \forall f \in \mathcal{K}(G)$ where $\mathcal{K}(G)$ designates the space of all compact support functions on G and $f_{s^{-1}}$ the right translated of f by s. One can easily verify that ν is a left Haar measure. Then there exists a unique positive number $\Delta_G(s)$ such that $\nu(f) = \Delta_G(s)\mu(f)$. This means that

$$(2.8) \qquad \int_G f(xs^{-1})d\mu(x) = \Delta_G(s) \int_G f(x)d\mu(x).$$

Let us denote by $\mathbb{R}_+^*$ the multiplicative group of positive real numbers. The function Δ_G called the *modular function* of G, is a continuous homomorphism from G into $\mathbb{R}_+^*$, that is Δ_G is continuous and

$$(2.9) \qquad \forall s_1, s_2 \in G, \Delta_G(s_1 s_2) = \Delta_G(s_1)\Delta_G(s_2).$$

The group G is said to be *unimodular* if $\Delta_G \equiv 1$. Therefore in a unimodular group every left Haar measure is also a right Haar measure. Abelian groups, compact groups and the Heisenberg group are some examples of unimodular groups.

A continuous unitary representation σ of a locally compact group G is said to be *square-integrable* if for all $\alpha, \beta \in H_\sigma$ the following functions $x \mapsto \langle \sigma(x)\alpha, \beta \rangle$, called the *coefficients* of σ, are square integrable with respect the left (or right) Haar measure of G [10, page 424]. The set of equivalent classes of square integrable representations is called the *discrete series* of G. The proposition below will be useful in the sequel (see [10]).

Proposition 2.1. *Let G be a locally compact unimodular group with left Haar measure dx. Let σ be a square integrable irreducible representation of G. Then there exists a positive constant d_σ such that*

$$(2.10) \qquad \int_G \langle \sigma(x)\alpha, \beta \rangle \overline{\langle \sigma(x)\gamma, \delta \rangle} dx = \frac{\langle \alpha, \gamma \rangle \overline{\langle \beta, \delta \rangle}}{d_\sigma}.$$

The constant d_σ is called the *formal degree* of the representation σ. Proposition 2.1 extends to non-unimodular locally compact groups by the Duflo-Moore theorem [7].

3. Fourier-Stieltjes transform

In this section, we deal with the Fourier-Stieltjes transform of vector measures on locally compact groups. We fix a square integrable irreducible representation σ of a locally compact unimodular group G. We denote by H_σ its representation space and by d_σ its formal degree. Finally we fix a basis $(\xi_\lambda^\sigma)_{\lambda \in \Lambda}$ of H_σ. Then the orthogonality relations in Proposition 2.1 become

$$(3.1) \qquad \int_G \langle \sigma(x)\xi_i^\sigma, \xi_j^\sigma \rangle \overline{\langle \sigma(x)\xi_k^\sigma, \xi_l^\sigma \rangle} dx = \frac{\langle \xi_i^\sigma, \xi_k^\sigma \rangle \overline{\langle \xi_j^\sigma, \xi_l^\sigma \rangle}}{d_\sigma} = \frac{1}{d_\sigma}\delta_{ik}\delta_{jl}$$

where $i, j, k, l \in \Lambda$.

We also consider a complex Hilbert space $\mathfrak{H}$ and denote the space of compact operators on $\mathfrak{H}$ by $\mathcal{B}_0(\mathfrak{H})$. It is well-known that $\mathcal{B}_0(\mathfrak{H})$ is a C*-algebra. In the sequel one can deal with a general C^*-algebra $\mathcal{A}$ but from our desire to be concret, we fix $\mathcal{A} = \mathcal{B}_0(\mathfrak{H})$.

Proposition 3.1. *Let G be a locally compact unimodular group. Let m be a bounded $\mathcal{B}_0(\mathfrak{H})$-valued measure on G. Then the mapping*

$$(3.2) \qquad \varphi : (\xi, \eta) \mapsto \int_G \langle \sigma(x^{-1})\xi, \eta \rangle dm(x)$$

is a bounded sesquilinear mapping from $H_\sigma \times H_\sigma$ into $\mathcal{B}_0(\mathfrak{H})$.

Proof. The sesquilinearity is trivial. Let us prove the boundedness.

$$\|\varphi(\xi, \eta)\| = \left\| \int_G \langle \sigma(x^{-1})\xi, \eta \rangle dm(x) \right\|$$

$$\leq \int_G |\langle \sigma(x^{-1})\xi, \eta \rangle| d|m|(x)$$

$$\leq \int_G \|\sigma(x^{-1})\xi\| \|\eta\| d|m|(x)$$

$$\leq \|\xi\| \|\eta\| \|m\|.$$

Thus φ is bounded and $\|\varphi\| \leq \|m\|$. $\qquad\square$

Following [3] where authors dealt with the Fourier-Stieltjes transform on compact groups we set the following definition.

Definition 3.1. *The σ-Fourier-Stieltjes transform of a bounded $\mathcal{B}_0(\mathfrak{H})$-valued measure m on a locally compact unimodular group G is the $\mathcal{B}_0(\mathfrak{H})$-valued bounded sesquilinear map $\mathcal{F}^\sigma m$ on $H_\sigma \times H_\sigma$ defined by*

$$(3.3) \qquad \mathcal{F}^\sigma m(\xi, \eta) = \int_G \langle \sigma(x^{-1})\xi, \eta \rangle dm(x), \quad \xi, \eta \in H_\sigma.$$

The σ-Fourier transform of a strong Bochner integrable $\mathcal{B}_0(\mathfrak{H})$-valued function f on G (with respect to the Haar measure of G) is defined by

$$(3.4) \qquad \mathcal{F}^\sigma f(\xi, \eta) = \int_G \langle \sigma(x^{-1})\xi, \eta \rangle f(x) dx, \; \xi, \eta \in H_\sigma.$$

Now we want to move from sesquilinear mappings to linear operators by the means of tensor product spaces [14]. Let us denote by $\mathcal{S}(H_\sigma \times H_\sigma, \mathcal{B}_0(\mathfrak{H}))$ the set of bounded sesquilinear maps from $H_\sigma \times H_\sigma$ into $\mathfrak{B}_0(\mathfrak{H})$. We denote by $\overline{H}_\sigma$ the conjugate Hilbert space of H_σ and by $H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma$ the completion of $H_\sigma \otimes \overline{H}_\sigma$ under the projective tensor product norm. Let us denote by $\mathcal{B}(H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma, \mathcal{B}_0(\mathfrak{H}))$ the space of bounded linear operators from $H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma$ into $\mathcal{B}_0(\mathfrak{H})$. Then we have the identification $\mathcal{S}(H_\sigma \times H_\sigma, \mathcal{B}_0(\mathfrak{H})) \simeq \mathcal{B}(H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma, \mathcal{B}_0(\mathfrak{H}))$ [13]. Therefore one can linearize the transformed measure $\mathcal{F}^\sigma m$ by viewing it as the element of the space $\mathcal{B}(H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma, \mathcal{B}_0(\mathfrak{H}))$ defined by

$$(3.5) \qquad \mathcal{F}^\sigma m(\xi \otimes \eta) = \int_G \langle \sigma(x^{-1})\xi, \eta \rangle dm(x), \; \xi \in H_\sigma, \eta \in \overline{H}_\sigma.$$

In the same way, the transformed of a function $f \in L^1(G, \mathcal{B}_0(\mathfrak{H}))$ is given by

$$(3.6) \qquad \mathcal{F}^\sigma f(\xi \otimes \eta) = \int_G \langle \sigma(x^{-1})\xi, \eta \rangle f(x) dx.$$

A little computation as in the proof of Proposition 3.1 shows that $\mathcal{F}^\sigma m$ and $\mathcal{F}^\sigma f$ are bounded operators from $H_\sigma \otimes \overline{H}_\sigma$ into $\mathcal{B}_0(\mathfrak{H})$ with

$$(3.7) \qquad \|\mathcal{F}^\sigma m\| \leq \|m\| \text{ and } \|\mathcal{F}^\sigma f\| \leq \|f\|_{L^1}.$$

By the density of $H_\sigma \otimes \overline{H}_\sigma$ in $H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma$ the operators $\mathcal{F}^\sigma m$ and $\mathcal{F}^\sigma f$ are uniquely extended to bounded operators from $H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma$ into $\mathcal{B}_0(\mathfrak{H})$.

For $i, j \in \Lambda$ and $x \in G$ set

$$(3.8) \qquad u_{ij}^\sigma(x) = \langle \sigma(x)\xi_i^\sigma, \xi_i^\sigma \rangle.$$

Proposition 3.2. *Let G be a locally compact unimodular group and let σ be a square integrable irreducible representation of G. Then*

$$(3.9) \qquad \mathcal{F}^\sigma u_{ij}^\sigma(\xi_k^\sigma \otimes \xi_l^\sigma) = \frac{\delta_{il}\delta_{jk}}{d_\sigma}.$$

Proof.

$$\mathcal{F}^\sigma u_{ij}^\sigma(\xi_k^\sigma \otimes \xi_l^\sigma) = \int_G \langle \sigma(x^{-1})\xi_k^\sigma, \xi_l^\sigma \rangle \langle \sigma(x)\xi_i^\sigma, \xi_j^\sigma \rangle dx$$

$$= \int_G \langle \sigma(x)\xi_i^\sigma, \xi_j^\sigma \rangle \overline{\langle \sigma(x)\xi_l^\sigma, \xi_k^\sigma \rangle} dx$$

$$= \frac{\langle \xi_i^\sigma, \xi_l^\sigma \rangle \overline{\langle \xi_j^\sigma, \xi_k^\sigma \rangle}}{d_\sigma}$$

$$= \frac{\delta_{il}\delta_{jk}}{d_\sigma}.$$

$\square$

Proposition 3.3. *Let G be a locally compact unimodular group and let σ be a square integrable irreducible representation of G. Consider the vectors $X = \sum_{k \in K} \alpha_k \xi_k^\sigma$ and $Y = \sum_{l \in L} \beta_l \xi_l^\sigma$ where K and L are finite subsets of Λ. Then*

(3.10)
$$\mathcal{F}^\sigma u_{ij}^\sigma(X \otimes Y) = \frac{\alpha_j \overline{\beta_i}}{d_\sigma}.$$

Proof.

$$\mathcal{F}^\sigma u_{ij}^\sigma(X \otimes Y) = \int_G \langle \sigma(x^{-1})X, Y \rangle \langle \sigma(x)\xi_i^\sigma, \xi_j^\sigma \rangle dx$$

$$= \int_G \langle \sigma(x^{-1}) \sum_{k \in K} \alpha_k \xi_k^\sigma, \sum_{l \in L} \beta_l \xi_l^\sigma \rangle \langle \sigma(x)\xi_i^\sigma, \xi_j^\sigma \rangle dx$$

$$= \int_G \sum_{k \in K} \sum_{l \in L} \alpha_k \overline{\beta_l} \langle \sigma(x^{-1})\xi_k^\sigma, \xi_l^\sigma \rangle \langle \sigma(x)\xi_i^\sigma, \xi_j^\sigma \rangle dx$$

$$= \int_G \sum_{k \in K} \sum_{l \in L} \alpha_k \overline{\beta_l} u_{kl}^\sigma(x^{-1}) u_{ij}^\sigma(x) dx$$

$$= \sum_{k \in K} \sum_{l \in L} \alpha_k \overline{\beta_l} \int_G u_{kl}^\sigma(x^{-1}) u_{ij}^\sigma(x) dx$$

$$= \sum_{k \in K} \sum_{l \in L} \alpha_k \overline{\beta_l} \int_G \langle \sigma(x)\xi_i^\sigma, \xi_j^\sigma \rangle \overline{\langle \sigma(x)\xi_l^\sigma, \xi_k^\sigma \rangle} dx$$

$$= \sum_{k \in K} \sum_{l \in L} \alpha_k \overline{\beta_l} \frac{\langle \xi_i^\sigma, \xi_l^\sigma \rangle \overline{\langle \xi_j^\sigma, \xi_k^\sigma \rangle}}{d_\sigma}$$

$$= \frac{\alpha_j \overline{\beta_i}}{d_\sigma}.$$

$\square$

Proposition 3.4. *Let m be a bounded $\mathcal{B}_0(\mathfrak{H})$-valued measure on a locally compact unimodular group G. Let σ be a square integrable irreducible representation of G. Then there exists a family $(T_{kl})_{k\in K, l\in L}$ of elements of $\mathcal{B}_0(\mathfrak{H})$ such that*

$$(3.11) \qquad \qquad \mathcal{F}^\sigma m = d_\sigma \sum_{k\in K} \sum_{l\in L} T_{kl} \mathcal{F}^\sigma u_{kl}^\sigma.$$

Proof. Let $X = \sum\limits_{k\in K} \alpha_k \xi_k^\sigma$ and $Y = \sum\limits_{l\in L} \beta_l \xi_l^\sigma$. Then we have

$$\mathcal{F}^\sigma m(X \otimes Y) = \int_G \langle \sigma(x^{-1}) \sum_{k\in K} \alpha_k \xi_k^\sigma, \sum_{l\in L} \beta_l \xi_l^\sigma \rangle dm(x)$$

$$= \sum_{k\in K} \sum_{l\in L} \alpha_k \overline{\beta_l} \mathcal{F}^\sigma m(\xi_k^\sigma \otimes \xi_l^\sigma)$$

$$= d_\sigma \sum_{k\in K} \sum_{l\in L} T_{kl} \mathcal{F}^\sigma u_{kl}^\sigma (X \otimes Y) \,\text{(use Proposition 3.3)}$$

where we have set $T_{kl} = \mathcal{F}^\sigma m(\xi_k^\sigma \otimes \xi_l^\sigma)$. Therefore $\mathcal{F}^\sigma m = d_\sigma \sum\limits_{k\in K} \sum\limits_{l\in L} T_{kl} \mathcal{F}^\sigma u_{kl}^\sigma.$ $\qquad \square$

We denote by $\mathcal{B}^\sigma(G, \mathcal{B}_0(\mathfrak{H}))$ the set of functions f defined on G by

$$(3.12) \qquad \qquad f(x) = \sum_{k\in\Lambda} \sum_{l\in\Lambda} T_{kl} u_{kl}^\sigma(x)$$

where the T_{ij} are compact operators on $\mathfrak{H}$.

Proposition 3.5. *Let G be a locally compact unimodular group. Let σ be a square integrable irreducible representation of G. If $T \in \mathcal{B}(H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma, \mathcal{B}_0(\mathfrak{H}))$ then there exists $f \in \mathcal{B}^\sigma(G, \mathcal{B}_0(\mathfrak{H}))$ such that $\mathcal{F}^\sigma f = T$.*

Proof. Let T be an element of $\mathcal{B}(H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma, \mathcal{B}_0(\mathfrak{H}))$. Set $T_{kl} = d_\sigma T(\xi_l^\sigma \otimes \xi_k^\sigma)$, $k, l \in \Lambda$. Then the function $f(\cdot) = \sum\limits_{k\in\Lambda} \sum\limits_{l\in\Lambda} T_{kl} u_{kl}^\sigma(\cdot)$ belongs to $\mathcal{B}^\sigma(G, \mathcal{B}_0(\mathfrak{H}))$. Moreover

$$\mathcal{F}^\sigma f(\xi_i^\sigma \otimes \xi_j^\sigma) = \sum_{k\in\Lambda} \sum_{l\in\Lambda} T_{kl} \mathcal{F}^\sigma u_{kl}^\sigma(\xi_i^\sigma \otimes \xi_j^\sigma)$$

$$= \sum_{k\in\Lambda} \sum_{l\in\Lambda} \frac{\delta_{kj}\delta_{li}}{d_\sigma} T_{kl} \quad \text{(use Proposition 3.2)}$$

$$= \frac{1}{d_\sigma} T_{ji}$$

$$= T(\xi_i^\sigma \otimes \xi_j^\sigma).$$

Hence $\mathcal{F}^\sigma f = T$. $\qquad \square$

We derive from the above proposition the following consequence.

Corollary 3.1. *Let G be a locally compact unimodular group. Let σ be a square integrable irreducible representation of G. If $T \in \mathcal{B}(H_\sigma \widehat{\otimes}_\pi \overline{H}_\sigma, \mathcal{B}_0(\mathfrak{H}))$ then the following integral representation of T holds:*

$$(3.13) \qquad T(\xi_i^\sigma \otimes \xi_j^\sigma) = \sum_{k \in \Lambda} \sum_{l \in \Lambda} T_{kl} \int_G \langle \sigma(x^{-1})\xi_i^\sigma, \xi_j^\sigma \rangle \langle \sigma(x)\xi_k^\sigma, \xi_l^\sigma \rangle dx$$

where the T_{kl} are compact operators on $\mathfrak{H}$.

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