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A STUDY OF r-IDEALS, k-IDEALS AND $m-k$ IDEALS OF ORDERED SEMIRINGS

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Abstract. In this paper, we introduce the notion of k-ideal, r-ideal and m-k ideal in ordered semirings. We study the properties of ideals in ordered semirings, the relations between them and characterize m-k ideals using derivation of ordered semirings. We prove that if I is a maximal ideal of an ordered semiring M with unity satisfying $a+b \neq 1$, for all $a, b \in M$ then I is a m-k ideal of M.

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1. Introduction

The notion of semiring was introduced by an American mathematician H. S. Vandiver [33] in 1934. The non trivial example of semiring first appeared in the work of German mathematician Richard Dedikind in 1894 in connection with the study of algebra of ideals of a commutative ring. A semiring is an algebraic structure with two associative binary operations where one distributes over the other. In particular, if I is the unit interval on the real line then (I, max, min) is a semiring in which 0 is the additive identity and 1 is the multiplicative identity. Though semiring is a generalization of a ring, ideals of semiring do not coincide with ring ideals. Henriksen [3] defined $k-$ideals in semirings to obtain analogous of ring results for semiring. In structure, semirings lie between semigroups and rings. The results which hold in rings but not in semigroups hold in semirings, since semiring is a generalization of a ring. Semirings play an important role in studying matrices and determinants. Semirings are useful in the areas of theoretical computer science as well as in graph theory, optimization theory, in particular for studying automata,coding theory and formal languages. Semiring theory has many applications in other branches of mathematics. Ahn.et al. studied ideals, r-ideals in incline algebras. Murali Krishna Rao et al. [24,25] studied derivations of ordered semirings and incline. Ideals play an important role in advance studies and uses of algebraic structures. Generalization of ideals in algebraic structures is necessary for further study of algebraic structures. Many mathematicians proved important results and charecterization of algebraic structures by using the concept and the properties of generalization of ideals in algebraic structures.

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The notion of ideals was introduced by Dedekind for the theory of algebraic numbers and it was generalized by Noether for associative rings. The one and two sided ideals introduced by her, are still central concepts in ring theory and the notion of a one sided ideal of any algebraic structure is a generalization of notion of an ideal. In 1952, the concept of bi-ideals was introduced by Good and Hughes [2] for semigroups. The notion of bi-ideals in rings and semigroups were introduced by Lajos and Szasz [8,9]. Quasi ideals are generalization of right ideals and left ideals whereas bi-ideals are generalization of quasi ideals.

Steinfeld[32] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki [4,5,6] introduced the concept of quasi ideal for semirings. Murali Krishna Rao [13, 14] introduced the concept of bi-interior ideal for a semigroup and a $\Gamma-$semiring as a generalization of quasi ideal, bi-ideal and interior ideal Shabir et al. [31] studied ideals in semigroups.Murali Krishna Rao [10,11,12] studied ideals of $\Gamma-$semirings. Murali Krishna Rao [20-22] introduced the notion of left (right) bi-quasi ideal of semiring, $\Gamma-$semiring,$\Gamma-$semigroup and studied the properties of left bi-quasi ideals and characterized the left bi-quasi simple $\Gamma-$semigroup and regular $\Gamma-$semigroup using left bi-quasi ideals of $\Gamma-$semigroup Murali Krishna Rao[17,18] introduced the notion of bi- quasi-interior ideal as a generalization of quasi ideal, bi-ideal and interior ideal of $\Gamma-$semiring and studied their properties. In this paper, we introduce the notion of k-ideal, r-ideal and m-k ideal in ordered semirings, study the properties of these ideals in ordered semirings, the relations between them and characterize m-k ideals using derivation of ordered semirings.

2. Preliminaries

In this section, we will recall some of the fundamental concepts and definitions necessary for this paper.

Definition 2.1. A semiring $(M,+,\cdot)$ is an algebraiac structure with two binary operations $+$ and $\cdot$ such that $(M,+)$ is a commutative semigroup and $(M,\cdot)$ is a semigroup and the following distributive laws hold.
$x(y+z) = xy + xz$
$(x+y)z = xz + yz,$ for all $x, y, z \in M$.

Definition 2.2. A semiring $(M,+,\cdot)$ is said to be division semiring if $(M \setminus 0, \cdot)$ is a group.

Example 2.3. Let M be the set of all natural numbers. Then $(M,\ max,\ min)$ is a semiring.

Definition 2.4. A semiring M is called an ordered semiring if it admits a compatible relation $\leq$. i.e. $\leq$ is a partial ordering on M satisfies the following conditions. If $a \leq b$ and $c \leq d$ then

(i) $a + c \leq b + d$
(ii) $ac \leq bd$
(iii) $ca \leq db$, for all $a, b, c, d \in M$

Example 2.5. Let $M = [0, 1]$. A binary operation $+$ is defined as $a + b = \max\{a, b\}$, for all $a, b \in M$ and $x \cdot y = \min\{x, y\}$, for all $x, y \in M$. Then M is an ordered semiring M with usual ordering . All ideals of M are closed intervals, $[0, a]$ for some $a \in M$.

Definition 2.6. An ordered semiring M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0x = x0 = 0,$ for all $x \in M$.

Definition 2.7. An ordered semiring M is said to be commutative semiring if $xy = yx$, for all $x, y \in M$

Definition 2.8. A non-zero element a in an ordered semiring M is said to be a zero divisor if there exists non zero element $b \in M$, such that $ab = ba = 0$.

Definition 2.9. An ordered semiring M with unity 1 and zero element 0 is called an integral ordered semiring if it has no zero divisors.

Definition 2.10. A non-empty subset A of an ordered semiring M is called a subsemiring M if $(A, +)$ is a subsemigroup of $(M, +)$ and $ab \in A$ for all $a, b \in A$.

Definition 2.11. Let M and N be ordered semirings. A mapping $f : M \to N$ is called a homomorphism if

(i) $f(a + b) = f(a) + f(b)$

(ii) $f(ab) = f(a)f(b)$, for all $a, b \in M$.

Definition 2.12. Let M be an ordered semiring. A mapping $d : M \to M$ is called a derivation if it satisfies
(i) $d(x + y) = d(x) + d(y)$
(ii) $d(xy) = d(x)y + xd(y)$ for all $x, y \in M$.

3. Ideals in ordered semirings

In this section, we introduce the notion of ideal, k-ideal, r-ideal and m-k ideal in ordered semirings.Throughout this paper,if $a \leq b$ then $a + b = b$. for all $a, b \in M$.

Definition 3.1. A subsemiring I of an ordered semiring M is called an ideal (filter) if it is a lower set. i.e., for any $x \in I, y \in M$ and $y \leq x \Rightarrow y \in I$.

Definition 3.2. A proper ideal P of an ordered semiring M is said to be prime ideal if for all $x, y \in M, xy \in P \Rightarrow x \in P$ or $y \in P$.

Definition 3.3. A subsemiring I of an ordered semiring M is said to be a left (right) $r-$ideal of M if $MI \subseteq I$ $(IM \subseteq I)$.

Definition 3.4. If I is both a left $r-$ideal and a right $r-$ideal then I is called a $r-$ideal of an ordered semiring M.

Definition 3.5. A subsemiring I of an ordered semiring M is said to be $k-$ideal if $x + y \in I, x \in M$ and $y \in I$ then $x \in I$.

Definition 3.6. An ideal I of an ordered Γ-semiring M is said to be $m - k$ ideal if $xy \in I, x \in I, 1 \neq y \in M$ then $y \in I$.

Definition 3.7. An ideal K of an ordered semiring M is said to be maximal ideal if $K \neq M$ and for every ideal I of M with $K \subseteq I \subseteq M$ then either $I = K$ or $I = M$.

Definition 3.8. An ordered semiring M is said to be $r-$ simple if it has no proper $r-$ideals of M.

Theorem 3.9. *If I is a $r-$ideal of an idempotent ordered semiring M with identity $x+yx=x$,for all $x,y\in M$, then I is a $k-$ideal of M.*

Proof. Suppose $x+y\in I, y\in I, x\in M$.We have $x+xy=x$, $xx=x$. Then

$$x+y\in I,\ x\in M$$

$$(x+y)x\in I$$

$$\Rightarrow xx+yx\in I$$

$$\Rightarrow x+yx\in I$$

$$\Rightarrow x\in I.$$

Hence I is a $k-$ideal of the ordered semiring M. □

Theorem 3.10. *Let M be an ordered semiring in which semigroup $(M,\cdot)$ is negatively ordered. If I is an ideal of an ordered semiring M then I is a $r-$ideal of M.*

Proof. Suppose I is an ideal of the ordered semiring M, $x\in I, y\in M$.
Then $xy\leq x$ and $yx\leq x$.
Since I is an ideal, $xy,\ yx\in I$.
Hence I is a $r-$ideal of the ordered semiring M □

The following example shows that converse of theorem 3.10 need not be true.

Example 3.11. Let $I=[0,1]$ be a set of reals between 0 and 1 with $x+y=\max\{x,y\}$ and $x\cdot y=xy$, where $\cdot$ is a usual multiplication , for all $x,y\in I$. Then I is an ordered semiring. Let M be the set of all 2×2 matrices whose elements be in I. be additive abeliean semigroup with respect to usual addition of matrices. A multiplication operation is defined as usual matrix multiplication.

Let $A=(a_{ij})$ and $B=(b_{ij})\in M$. We define $A\leq B$ if and only if $a_{ij}\leq b_{ij}$, for all i,j. Then M is an ordered semiring.

Let $B=\left\{\begin{pmatrix} a & b\\ 0 & 0\end{pmatrix} \mid a,b\in I\right\}$. Suppose $\begin{pmatrix} a & b\\ 0 & 0\end{pmatrix}\in B$, $\begin{pmatrix} x & y\\ z & w\end{pmatrix}\in M$. Then $\begin{pmatrix} a & b\\ 0 & 0\end{pmatrix}\begin{pmatrix} p & q\\ r & s\end{pmatrix}=\begin{pmatrix} ax & by\\ 0 & 0\end{pmatrix}\in B$.

Suppose $A=(a_{ij})$ and $B=(b_{ij})\in M$.
We define $A\leq B$ if and only if $a_{ij}\leq b_{ij}$, for all i,j.
We have $\begin{pmatrix} 0.5 & 0.5\\ 0 & 0\end{pmatrix}\leq\begin{pmatrix} 0.5 & 0.6\\ 0 & 0\end{pmatrix}\in B$ but $\begin{pmatrix} 0.5 & 0.5\\ 0 & 0\end{pmatrix}\notin B$.
Hence B is a $r-$ ideal but not an ideal of the ordered semiring M

Theorem 3.12. *Every m-k ideal of an ordered semiring M in which semigroup $(M,\cdot)$ is negatively ordered. Then m-k ideal is a k-ideal of M.*

Proof. Let I be a m-k ideal of an ordered semiring M. Suppose $x+y\in I$, $x\in I, y\in M$ then by Theorem 3.10, $(x+y)y\in I$.
Therefore $y\in I$. Since I is a m-k ideal.
Hence I is a k- ideal of M.

□

Converse of the theorem need not be true.

Example 3.13. Let M be a the set of all non-negative integers. Then$M$ be additive abelian semigroup. A multiplication operation is defined as usual multiplication of integers. Then M is an
ordered semiring. A subset $I = 3M\setminus\{3\}$ of M is an ideal of M but not a $k-$ideal of M.

Example 3.14. Let M be the set of all natural numbers. Then $(M,\ max,\ min)$ with usual ordering is an ordered semiring.Then M is an ordered $\Gamma-$semiring. If $I_n = \{1,2,\cdot,\cdot,\cdot,n\}$ then I_n forms a $k-$ideal but not $m-k-$ ideal of ordered $\Gamma-$semiring.

Example 3.15. Let $\mathcal{N}$ set of all non-negative integers and $M = \{\begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \mid a,b,c\in\mathcal{N}\}$ be additive abeliean semigroup. A multiplication operation is defined as usual matrix multiplication for all $x,y\in M$. Let $A=(a_{ij})$ and $B=(b_{ij})\in M$. We define $A\leq B$ if and only if $a_{ij}\leq b_{ij}$, for all i,j. Then M is an ordered semiring. Define a derivation $d: M\longrightarrow M$ by $d\begin{pmatrix} a & 0 \\ b & c \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix}$, for all $\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\in M$. And define $Ker\ d = \{A\mid A\in M$ and $d(A) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\}$. Then $Ker\ d$ is a $m-k$ ideal of the ordered semiring M.

Theorem 3.16. *Let M be an ordered semiring with unity 1 and zero element 0. If I is a $r-$ideal containing an unit element then $I = M$.*

Proof. Let I be a $r-$ideal of the ordered semiring M containing a unit element u and $x\in M$. Then $x1 = x$. Since I is a $r-$ideal, $x,u\in I$. Since u is a unit, there exists $t\in M$ such that $ut = 1$.

$$\Rightarrow xut = x1 = x$$

$$\Rightarrow x\in I.$$

Hence $I = M$. □

Theorem 3.17. *An ordered division semiring M is a $r-$simple.*

Proof. Let I be a proper $r-$ideal of the ordered division semiring M. Every non-zero element of I is a unit. By Theorem 3.16, we have $I = M$. Hence ordered division semiring M is a simple. □

Theorem 3.18. *Let I be an ideal of an ordered semiring M. Then $I^* = \{x\in M\mid x+a\in I,$ for some $a\in I,\}$ is an ideal of M.*

Proof. Let $x, y \in I^*$. Then there exist $a, b \in I$ such that $x + a, y + b \in I$ and

$$(x + a) + (y + b) = x + y + a + b \in I$$
$$\Rightarrow x + y \in I^*.$$
$$(x + a)(y + b) = xy + xb + ay + ab$$
$$\Rightarrow xy \in I^*.$$

Therefore I^* is the ordered subsemiring of M.

Let $x, y \in M, x \leq y$ and $y \in I^*$. Then there exists $a \in I$ such that $y + a \in I$. We have

$$x + y = y$$
$$\Rightarrow x + y + a = y + a$$
$$\Rightarrow x + y + a \in I$$
$$\Rightarrow x \in I^*.$$

Hence I^* is an ideal of M. □

Theorem 3.19. *Let I be a subsemiring of an ordered semiring M in which semigroup (M,+) is band. Then I is an ideal of M if and only if I is a $k-$ideal of M.*

Proof. Let I be an ideal of the ordered semiring M and $x + y \in I, y \in I$.

$$x + y = (x + x) + y$$
$$= x + (x + y)$$
$$\Rightarrow x \leq x + y.$$

Therefore, by definition of ideal, $x \in I$. Hence I is a $k-$ideal.

Conversely suppose that I is a $k-$ideal of the ordered semiring M. Let $y \in M, x \in I$ and $y \leq x$.

$$\Rightarrow y + x = x$$
$$\Rightarrow y + x \in I$$
$$\Rightarrow y \in I, \text{ since } I \text{ is a } k-\text{ideal of the ordered semiring } M\ .$$

Hence I is an ideal of the ordered semiring M. □

Theorem 3.20. *Let $f : K \to L$ be a homomorphism of ordered semirings. If J is an ideal of L then $f^{-1}(J)$ is an ideal of an ordered semiring K.*

Proof. Suppose J is an ideal of L, $f : K \to L$ be a homomorphism of ordered semirings and $x, y \in f^{-1}(J)$.

$$\begin{aligned} &\Rightarrow f(x), f(y) \in J \\ &\Rightarrow f(x) + f(y) = f(x+y) \in J \\ &\Rightarrow x + y \in f^{-1}(J) \\ x, y \in f^{-1}(J) &\Rightarrow f(x), f(y) \in J \\ &\Rightarrow f(x)f(y) \in J \\ &\Rightarrow f(xy) \in J \\ &\Rightarrow xy \in f^{-1}(J). \end{aligned}$$

Hence $f^{-1}(J)$ is the ordered subsemiring of K. Let $x \in K, y \in f^{-1}(J)$ such that $x \leq y$.

$$\begin{aligned} &\Rightarrow x + y = y \\ &\Rightarrow f(x+y) = f(y) \\ &\Rightarrow f(x) + f(y) = f(y) \in J \\ &\Rightarrow f(x) \leq f(y) \\ &\Rightarrow f(x) \in J \\ &\Rightarrow x \in f^{-1}(J). \end{aligned}$$

Hence $f^{-1}(J)$ is an ideal of the ordered semiring K. □

Theorem 3.21. *Let M be an ordered semiring with unity 1 in which semigroup $(M, \cdot)$ is negatively ordered. If I is an ideal containing a unit element then $I = M$.*

Proof. Let I be an ideal of the ordered semiring M containing a unit element u and $x \in M$. Then $x1 = x$. Since I is an ideal and by Theorem3.10 $xu \in I$. Since u is a unit, there exists $t \in M$ such that $ut = 1$.

$$\begin{aligned} &\Rightarrow xut = x1 = x \\ &\Rightarrow x \in I. \end{aligned}$$

Hence $I = M$. □

Theorem 3.22. *An ordered division semiring M is simple.*

Proof. Let I be a proper ideal of the ordered division semiring M. Every non-zero element of I is a unit. By Theorem 3.21, we have $I = M$. Hence ordered division semiring M is simple. □

Theorem 3.23. *Let M be an ordered $\Gamma-$semiring.If I is a m-k ideal of M then I is a maximal ideal of M.*

Proof. Let I be a m-k ideal of an ordered $\Gamma-$semiring M. Suppose J is an ideal of M such that $I \subseteq J,\ x \in J,\ y \in I$,.Therefore $xy \in I. Then \Rightarrow x \in I$, since I is a m-k ideal of M.

Therefore $I = J$. Hence m-k ideal I of M is a maximal ideal.

□

Theorem 3.24. *If I is a maximal ideal of an ordered semiring M with unity satisfying $a + b \neq 1$, for all $a, b \in M$ then I is a m-k ideal of M.*

Proof. Suppose I is a maximal ideal of the ordered semiring M, $xy \in I, x \in I$ and $y \notin I$. Then $I \subseteq I + (y)$, where (y) is a principal ideal generated by y.

I is a proper subset of $I + (y)$, since $y \in I + (y)$ and $I + (y) \neq M$, since $1 \notin I + (y)$. Which is a contradiction to maximality of I.

Hence I is a m-k ideal of the ordered semiring M. □

The following theorems are characterizations of m-k ideal of an ordered semiring M Let M be an ordered semiring. $E[+]$ denotes the set $\{x \in M \mid x + x = x\}$.

Theorem 3.25. *Let M be an ordered semiring in which $(M, \cdot)$ is cancellative semigroup. If $E[+] \neq \emptyset$ then $E[+]$ is a m- k ideal of an ordered semiring M.*

Proof. Let $x \in E[+], y \in M$. Then

$$x = x + x$$
$$c \Rightarrow xy = (x + x)y$$
$$= xy + xy$$

Therefore $xy \in E[+]$. Similarly $yx \in E[+]$. Suppose $x, y \in E[+]$. Then

$$x + x = x, y + y = y$$
$$\Rightarrow (x + y) + (x + y) = (x + x) + (y + y) = x + y$$
$$\Rightarrow x + y \in E[+].$$

Suppose $x \leq y, y \in E[+]$. Then x +y =y.

$$\Rightarrow x + x + y = x + y.$$
$$\Rightarrow x + x = x.$$

Therefore $x \in E[+]$ Hence $E[+]$ is an ideal of the ordered semiring. Suppose $x, x+y \in E[+]$. Then

$$\begin{aligned} & x+x=x, x+y+x+y=x+y \\ \Rightarrow & (x+y)+(x+y)=x+y \\ \Rightarrow & (x+x)+(y+y)=x+y \\ \Rightarrow & x+(y+y)=x+y \\ \Rightarrow & y+y=y \\ \Rightarrow & y \in E[+]. \end{aligned}$$

Hence $E[+]$ is a k- ideal of the ordered semiring M.

Suppose $xy \in E[+]$ and $x \in E[+]$. Then

$$\begin{aligned} & xy+xy=xy \\ \Rightarrow & x(y+y)=xy \\ \Rightarrow & y+y=y \\ \Rightarrow & y \in E[+]. \end{aligned}$$

Hence $E[+]$ is a m- k ideal of the ordered semiring M. □

Theorem 3.26. *Let d be a derivation of the ordered semiring M, where semigroup $(M, \cdot)$ is left cancellative semigroup, $(M,+)$ is positively ordered,right cancellative and band.*
Define a set $Fix_d(M) = \{x \in M / d(x) = x\}$.Then $Fix_d(M)$ is a k-ideal and a m-k ideal of an ordered semiring M.

Proof. Let d be a derivation of M. Suppose $x, y \in Fix_d(M)$. Then

$$d(x) = x, d(y) = y$$
$$d(x+y) = d(x) + d(y) = x + y.$$

Therefore $x + y \in Fix_d(M)$

$$\begin{aligned} d(xy) &= d(x)y + xd(y) \\ &= xy + xy \\ &= xy. \end{aligned}$$

Therefore $Fix_d(M)$ is a subsemiring of M.

Suppose $x \leq y$ and $y \in Fix_d(M)$.

$$x \leq y$$

$$\text{Then } x + y = y$$

$$\Rightarrow d(x + y) = x + y$$

$$\Rightarrow d(x) + d(y) = x + y$$

$$\Rightarrow d(x) + y = x + y.$$

$$\text{Therefore } d(x) = x.$$

Suppose $x + y, y \in Fix_d(M)$.

$$x + y \in Fix_d(M$$

$$\text{Then } d(x + y) = x + y$$

$$\Rightarrow d(x) + d(y) = x + y$$

$$\Rightarrow d(x) + y = x + y.$$

$$\text{Therefore } d(x) = x.$$

Therefore $d(x) = x$. Hence $Fix_d(M)$ is a $k-$ideal of M. Suppose $xy \in Fix_d(M), x \in Fix_d(M)$. Then $d(xy) = xy$

$$\Rightarrow d(x)y + xd(y) = xy$$

$$\Rightarrow xy + xd(y) = xy$$

$$\Rightarrow x[y + d(y)] = xy$$

$$\Rightarrow y + d(y) = y$$

$$\Rightarrow d(y) \leq y + d(y) = y$$

we have $y \leq d(y)$.
Hence $d(y) = y, y \in Fix_d(M)$.
Hence $Fix_d(M)$ is a m-k ideal of M. □

Theorem 3.27. *Let d be a derivatipn of an ordered semiring M.Define $ker\ d = \{x \in M/d(x) = 0\}$. Then $ker\ d$ is a k-ideal of an ordered semiring M.*

Proof. Let $x, y \in ker\ d$. Then

$$d(x) = 0, d(y) = 0$$

$$d(x + y) = d(x) + d(y) = 0.$$

Therefore $x + y \in ker\ d$.

$$d(xy) = d(x)y + xd(y)$$

$$= 0y + x0 = 0.$$

Therefore $xy \in ker\ d$.

Suppose $x \leq y$ and $y \in ker\ d$.

$$x \leq y$$

$$\text{Then } x + y = y$$

$$\Rightarrow d(x + y) = d(y)$$

$$\Rightarrow d(x) + d(y) = d(y)$$

$$\Rightarrow d(x) + 0 = 0.$$

$$\text{Therefore } d(x) = 0.$$

$$\Rightarrow x \in ker\ d.$$

Suppose $x + y \in ker\ d$ and $y \in ker\ d$.Then $d(x + y) = 0 \Rightarrow d(x) + d(y) = 0$, $\Rightarrow d(x) = 0$, $\Rightarrow x \in ker\ d$. Hence $ker\ d$ is a k- ideal of the ordered -semiring M.

□

Theorem 3.28. *Let d be a derivation of an integral ordered semiring M.*
Define $ker\ d = \{x \in M/d(x) = 0\}$. Then $ker\ d$ is a m- k ideal of M.

Proof. By Theorem 3.27, $ker\ d$ is an ideal. Let $0 \neq y \in ker\ d$, $x \in M \in$ and $xy \in ker\ d$ then $d(xy) = 0$

$$\Rightarrow d(x)y + xd(y) = 0$$

$$\Rightarrow d(x)y = 0$$

$$\Rightarrow d(x) = 0$$

since M is an integral ordered semiring. Therefore $ker\ d$ is a $m - k$ ideal of M. □

4. Conclusion

In this paper, we introduced the notion of k-ideal, r-ideal,m-k ideal, prime idea and maximal ideal in ordered semirings and studied the properties and the relations between them.We characterized m-k ideals using derivation of ordered semiring.

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