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THE FUZZY LINEAR REGRESSION

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Abstract. This paper aimed to study fuzzy regression models including Tanaka model, Tanaka modified model and other models, and build a fuzzy regression model using fuzzy and non-fuzzy data. In this paper, a prediction model was constructed based on the application of the Sanli and Apaydin idea which adopted the Shapiro proposal based on the square distances provided by Diamond. It compared fuzzy regression model and regression model by using normal least square model. The research found that the fuzzy regression is clearer and easier to calculate, and does not differ much from the classical regression, which supports the idea of fuzzy regression prediction, especially with regard to fuzzy data.

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1. Introduction

The probabilistic concept of an event depends on an expectation on previous information, so uncertainty from a probabilistic perspective depends on the prediction, while the perception of uncertainty depends on the inaccuracy of meaning for many concepts or measurements. These include the results of medical tests for any disease or monitoring of any measurements by electronic devices, the accuracy of the results may vary from one device to another due to the efficiency and specifications of the devices. The study of this type of uncertain data and the construction of predictive models for fuzzy regression model is the efficient model and for any similar data.The regression is a common methodology for expressing the relationship

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between two or more independent variables and a dependent variable. Through the mathematical model, the value of the dependent variable expected using the values of independent variables. This makes the regression model the most appropriate method in many applications such as process research, and complex systems such as agricultural, biological, technical and engineering systems [8].

With such systems, it could be difficult to obtain accurate digital data because of the complexity of their systems, the ambiguity of people's thinking and judgment on the nature of phenomena, and the influence of uncertain factors present in a border environment around the systems involved.

The regression model using the traditional least-squares method may not be useful for dealing with fuzzy data types. This encouraged researchers to find highly efficient ways to deal with such data and to deal with their uncertain nature.

In 1982, Tanaka et al. Used linear programming to develop a fuzzy linear regression model, relying on the deviation between the observed value and the estimated value of the dependent variable as "ambiguity" (fuzzy) and dependent on the ambiguity (fuzzy) of the system architecture. This is the main idea of the linear programming method (LP) [16].

In 1987 and 1989 Tanaka modified his first model where the total ambiguity of the parameters in the first model was reduced $\min Z = c_1 + c_2 + ... + c_n$ where c_i is the opacity of the blurry parameters, and in the second model tried to minimize the overall mystery of the model. It is called the fuzzy linear regression $\min Z = \sum_{i=1}^{p}(c_\circ|X_\circ| + c_1|X_1 i| + c_2|X_2 i| + ... + c_n|X_n i|)$ (Possibilistic Fuzzy Linear Regression) (PFLR) [14] , [15] . To solution of the problem of the output of the second Tanaka model (PFLR), Peters modified the Tanaka method for the input data that is not fuzzy where he presents a new variable and constants of this variable ? represents the degree of membership of the solution in a set of good solutions so the Peters' model tries to maximize λ, [12]. Another problem in the Peters' Model (PFLR) was the conflicting trends in the interpretation of the overall trend of the model due to shrinking or expanding view trends. To avoid this problem, Lee and Chang proposed deleting the signal restriction in the (PFLR) model. The new form is called Unrestricted in Sign Fuzzy Linear Regression (UFLR) [9]. To develop the (UFLR) Chen proposed (Lee and Chang model), a three-step procedure:

(1) Is the detection of abnormal data.
(2) Determine the expected number of extreme value.

(3) The model is redrawn after modifying extreme data.

There are many other studies whose methods can be roughly categorized into two methods: linear programming based methods (Probabilistic Approach) and Least Squares Approach [3]. In both approaches, the concept of better estimation involves improving the function associated with the problem. When the observed values are identified by probable distributions or a mysterious organic function rather than classical probability distributions, the regression models associated with the probability or fuzzy regression model are called, [8], particularly in the Probabilistic Approach. For estimated outputs, either as a weighted linear sum comprising the coefficients estimated in the linear regression, or as a quadratic form in the case of exponential probability regression [5]. Thus, the Tanaka method of probabilistic regression analysis uses linear programming to incorporate relationships rather than measuring the least squares of errors $\sum e_i^2 = \sum (y_i - \hat{y}_i)^2$ yet the least-squares method is not desirable for probability regression analysis [17]. Many studies have shown that fuzzy regression may be better than probability regression in the following cases:

(1) When the data set is insufficient to support the probability regression analysis.
(2) When we cannot assume and justify statistical distribution.
(3) If the regression model is poorly represented.
(4) When human judgments are involved (e.g., input / output ambiguous values.
(5) If errors are associated with defining the model structure and dispelling the human perception of the model (unlike the statistical situation where the errors are associated with observations) [11].

There are many situations in which observations cannot be accurately described, for example, when they depend on environmental conditions or individual responses. In such cases, we can only provide a rough description of them, and feel uncertainty. This differs from randomness and is sometimes referred to as ambiguity. For example, when "we measure a current with a digital meter, we receive static data in theory but are actually discrete because the measurement procedures are not accurate [6]. Probability regression has been studied by many researchers as a integrated method to traditional regression, and the main objective of traditional regression is to capture the average behavior of the system. In the system under consideration, this view is useful for example in civil engineering - when building bridges, we usually prefer not to fall even in the worst case, rather than calculating the average life span [1].

2. Materials and working methods

2.1. Fuzzy Linear Regression. Regression analysis is a commonly used method for analyzing the relationship and correlations between two variables of the response variable, called the dependent variable, and one or more explanatory variables, called independent variables, called classical linear regression and takes the following form:

$$Y_i = \beta_\circ + \beta_1 X_{i1} + \beta_1 X_{i1} + \beta_2 X_{i2} + ... + \beta_i X_{ij} + \varepsilon_i \tag{2.1}$$

where: $i = 1, 2, 3, ..., n$ (represents the number of views)
and $j = 1, 2, 3, ..., p$ (represents the number of independent variables) and the model can be expressed by the system of matrices in the image. $\underline{Y} = X\underline{\beta} + \underline{U}$.

In classical linear regression (CLR) we express the deviations between the observed value and the estimated value of the dependent variable, where these errors are usually distributed with an average of zero $E(\varepsilon_i) = 0$ and variance $Var(\varepsilon_i) = \sigma_i^2$. This error is a kind of uncertainty and may be due to inaccuracies in measurement, or any other effects that are not accounted for during the collection of the phenomenon data. The fuzzy regression works in a fuzzy environment and is therefore called fuzzy regression, and can be divided into the following cases [3] as shown in Table (1).

Table 1. Fuzzy Regression Cases

Type	Sys- structure	Ou- Variable	In- Variable
1	Fuzzy	Non fuzzy	Non fuzzy
2	Fuzzy	fuzzy	Non fuzzy
3	Fuzzy	fuzzy	fuzzy

In 1982 Tanaka et al. used linear programming to develop a fuzzy linear regression model, relying on the deviation between the observed value and the estimated value of the dependent variable as "fuzzy" and dependent on the fuzzy of the system architecture. It has been studied as symmetric trigonometric functions where the formula for the linear fog model (FLR) is:

$$\tilde{Y}_i = \tilde{A}_\circ + \tilde{A}_1 x_{i1} + \tilde{A}_2 x_{i2} + \tilde{A}_3 x_{i3} + ... + \tilde{A}_p x_{ip} \quad , \quad \tilde{Y} = \tilde{A}' X \tag{2.2}$$

Where: $\tilde{A} = [\tilde{A}_\circ, \tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_p]'$ It is a fuzzy parameter vector. And that $X = [1, x_1, x_2, ..., x_p]$ is the vector of independent variables, and the function of membership of fuzzy parameter at form:

$$\mu_{\tilde{A}_i(a_i)} = \left\{ \begin{array}{ll} 1 - \frac{|\alpha_i - a_i|}{c_i}; & \alpha_i - c_i \leq a_i \leq \alpha_i + c_i \\ 0 & otherwise \end{array} \right\}; i = 1, 2, 3, ..., n, a_i > 0 \tag{2.3}$$

where: α_i, c_i are the center value and spread value for fuzzy parameter respectively. Using

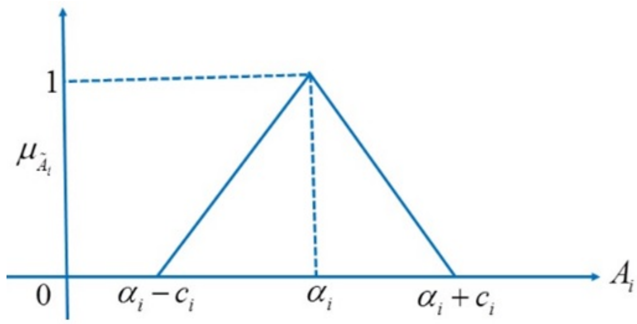

FIGURE 1. illustrates the parameter of the trigonometric regression model

the fuzzy parameters A_i as shown in equation (2.3) as fuzzy numbers, and equation (2.2) to obtain the membership function of Y in the image:

$$\mu_{Y_i(y)} = \left\{ \begin{array}{ll} 1 - \frac{|y - x'\alpha|}{c|x|}; & x \neq 0 \\ 1 & ; x, y = 0 \\ 0 & otherwise \end{array} \right\}; Where |x| = (|x_1|, |x_2|, ..., |x_p|)' \tag{2.4}$$

in this model, (Tanaka) and his colleagues hypothesized the second case shown in Table (1). Thus, the function of membership to the response variable (dependent variable) is:

$$\mu_{Y_i(y_i)} = \left\{ \begin{array}{ll} 1 - \frac{|y_i - y|}{e_i}; & y_i - e_i \leq y \leq y_i + e_i \\ 0 & otherwise \end{array} \right\} \tag{2.5}$$

Where y_i, e_i are the center vlue and *spread value* for fuzzy parameter respectively.

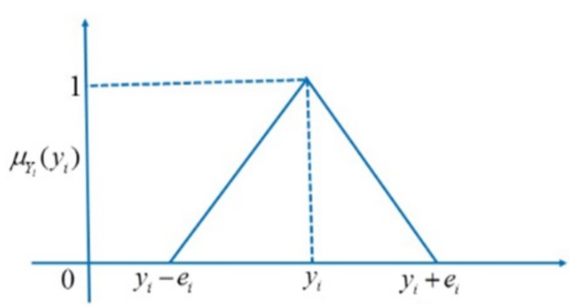

FIGURE 2. illustrates the function of membership to the dependent variable.

Tanaka through this model was seeking to reduce the total spread value through the model:

$$\text{Min}J = c_1 + c_2 + ... + c_p$$

subject to:

(2.6)
$$\alpha^t x_i + (1-h)\sum_j c_j|x_{ij}| \geq y_i + (1-h)e_i$$
$$-\alpha^t x_i + (1-h)\sum_j c_j|x_{ij}| \geq -y_i + (1-h)e_i$$
$$c_i \geq 0 \qquad ; i = 1, 2, 3, ..., n$$

h is an appropriate degree of fuzzy linear model [10], proposed by the decision maker, which is the same level of confidence, $(1-\alpha)$ in the hypothesis tests, and may take the same concept, $\alpha - level$, see Figure 3.

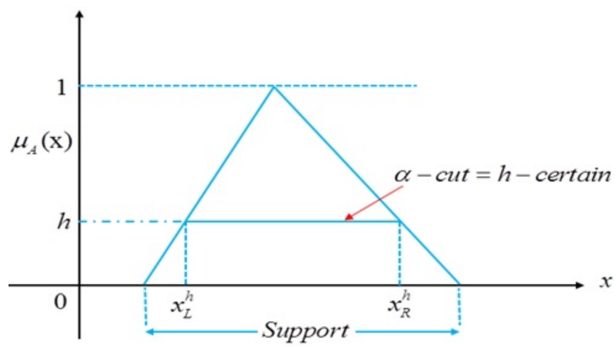

FIGURE 3. illustrates $\alpha - level = h - certain$.

In 1987 and 1989, Tanaka modified his first model and attempted in the second model to reduce the overall ambiguity of the model as:

$$\text{Minimize} \qquad J(c) = \sum_i c^t|x_i|$$

subject to

$$y_i + |L^{-1}(h)|e_i \geq \alpha^t x - |L^{-1}(h)|c^t|x_i|$$
$$y_i - |L^{-1}(h)|e_i \leq \alpha^t x + |L^{-1}(h)|c^t|x_i|$$
$$c^t \geq 0 \qquad ; i = 1, 2, 3, ..., n$$

It is called the foggy linear regression model and is given in three cases: micro-optimal solutions, super-optimal, simultaneous solutions, symbolized by:

$\hat{A}_j = (\hat{\alpha}_j, \hat{c}_j)_L$, $\underline{A}_j = (\underline{\alpha}_j, \underline{c}_j)_L$, $\overline{A}_j = (\overline{\alpha}_j, \overline{c}_j)_L$ respectively , [4].

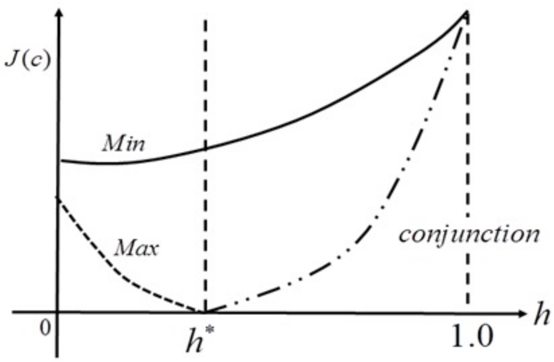

FIGURE 4. The relationship between the degree of suitability of the model and the propagation function $J(c)$.

2.2. **Fuzzy Linear Regression Model:** Diamond proposed the method of fuzzy least squares using the $(L^2 - metric)$ standard [4], which exceeded many of the problems of the Tanaka model such as the problems of linear multiplicity, where he proposed to estimate the parameters of the model: $\tilde{Y}_i = \tilde{A}_\circ + \tilde{A}_1 X_{i1} + \tilde{A}_2 X_{i2} + ... + \tilde{A}_n X_{ip}$. Using the fuzzy least squares (FLR) method based on squared distances, It was assumed that $\tilde{X}$ independent variables are fuzzy numbers within a given spread with a left L_x , right R_x , and central value $X = (x, L_x, R_x)$, Similarly to a $\tilde{Y}$ dependent variable, fuzzy numbers are within a given spread with a left L_y , right R_y , and central value $\tilde{Y} = (y, L_y, R_y)$, and a function of belonging to these variables. Based on the Diamond Quadrature Proposal, Shapiro suggested that if estimating the spread of upper points of data in a straight line Y_U and another line to estimate the spread of the lower points in a straight line Y_L and a third line midway between the upper and lower lines: $Y_h = \frac{1}{2}(Y_U + Y_L)$,Where h the coefficient of confidence (level of confidence), which explains the level of data concentration, if we have the two variables X, Y and a function of triangular belonging, the square distance is given by:

$$d(X,Y)^2 = [x - y - (L_x - L_y)]^2 + [x - y + (R_x - R_y)]^2 + [x - y]^2 \tag{2.7}$$

To obtain the best matching model, we reduce the squared distances between the spread of the variables. Example: Assuming we have two variables:

$X_i, Y_i \quad , \quad i = 1, 2, 3, ..., n$, let us have the following model: $Y = a + bX \quad ; a, b \in R$ It is using relationship (2.7): $\quad MinJ(a,b) = \sum d(a + bX_i - Y_i)^2$

Here are two cases to find the sum of squared distances:

when $b \geq 0, b < 0$ it:

$d(a + bX_i, Y_i)^2 = [a + bx_i - y_i - (L_x - L_y)]^2 + [a + bx_i - y_i + (R_x - R_y)]^2 + (a + bx_i - y_i)^2; b \geq 0$

And also:

$d(a+bX_i,Y_i)^2=[a+bx_i-y_i+(L_x-L_y)]^2+[a+bx_i-y_i-(R_x-R_y)]^2+(a+bx_i-y_i)^2; b<0$

Applying the idea of Sanli , Apaydin , [7] to reduce the squared distances using the normal squares method of the fuzzy model described in equation (2.2):

$\tilde{A}_i=\tilde{A}_\circ+\tilde{A}_1x_{i1}+\tilde{A}_2x_{i2}+\tilde{A}_3x_{i3}+...+\tilde{A}_px_{ip}$

Then:

$$MinJ(\tilde{A}_\circ+\tilde{A}_1+\tilde{A}_2+...+\tilde{A}_p)=\sum d(\tilde{A}_\circ+\tilde{A}_1x_{i1}+\tilde{A}_2x_{i2}+...+\tilde{A}_px_{ip}+\tilde{Y}_i)^2 \tag{2.8}$$

$$\begin{aligned}&MinJ(A^t)=\\&\sum[[\tilde{A}_\circ+\tilde{A}_1x_{i1}+\tilde{A}_2x_{i2}+...+\tilde{A}_px_{ip}-\tilde{Y}_i-(\tilde{A}_1L_{x_{i1}}+\tilde{A}_2L_{x_{i2}}+...+\tilde{A}_pL_{x_{ip}})]^2\\&\quad+[\tilde{A}_\circ+\tilde{A}_1x_{i1}+\tilde{A}_2x_{i2}+...+\tilde{A}_px_{ip}-\tilde{Y}_i+(\tilde{A}_1R_{x_{i1}}+\tilde{A}_2R_{x_{i2}}+...+\tilde{A}_pR_{x_{ip}})]^2\\&\quad+[\tilde{A}_\circ+\tilde{A}_1x_{i1}+\tilde{A}_2x_{i2}+...+\widetilde{A_p}x_{ip}-\tilde{Y}_i]^2]\end{aligned}$$

The partial differential of equation (2.8) for the parameters $\tilde{A}_j$ is obtained $p+1$ from the differential equations: By setting each of the resulting equations to zero, we obtain a set of equations that can be solved simultaneously using matrices to arrive at estimates of model parameters as follows , [15]:

$$\hat{\hat{A}}=({X_L}^tX_L+XX+{X_R}^tX_R)^{-1}({X_L}^tY_L+\acute{X}Y+{X_R}^tY_R) \tag{2.9}$$

Where: $X=\begin{pmatrix}1&x_{11}&x_{12}&\cdots&x_{1p}\\1&x_{21}&x_{22}&\cdots&x_{2p}\\\vdots&\vdots&\vdots&\vdots\vdots&\vdots\\1&x_{n1}&x_{n2}&\cdots&x_{np}\end{pmatrix}$, $Y=\begin{pmatrix}y_1\\y_2\\\vdots\\y_n\end{pmatrix}$

$$X_L=\begin{pmatrix}1&x_{11}-Lx_{11}&x_{12}-Lx_{12}&\cdots&x_{1p}-Lx_{1p}\\1&x_{21}-Lx_{21}&x_{22}-Lx_{22}&\cdots&x_{2p}-Lx_{2p}\\\vdots&\vdots&\vdots&\vdots\vdots&\vdots\\1&x_{n1}-Lx_{n1}&x_{n2}-Lx_{n2}&\cdots&x_{np}-Lx_{np}\end{pmatrix},Y_L=\begin{pmatrix}y_1-L_y\\y_2-L_y\\\vdots\\y_n-L_y\end{pmatrix}$$

$$X_R = \begin{pmatrix} 1 & x_{11}+Rx_{11} & x_{12}+Rx_{12} & \cdots & x_{1p}+Rx_{1p} \\ 1 & x_{21}+Rx_{12} & x_{22}+Rx_{22} & \cdots & x_{2p}+Rx_{2p} \\ \vdots & \vdots & \vdots & \vdots\vdots & \vdots \\ 1 & x_{n1}+Rx_{1n} & x_{n2}+Rx_{n2} & \cdots & x_{np}+Rx_{np} \end{pmatrix}, Y_R = \begin{pmatrix} y_1+R_y \\ y_2+R_y \\ \vdots \\ y_n+R_y \end{pmatrix}$$

The estimated fuzzy model is: $\hat{\hat{\hat{Y}}} = \hat{\hat{\hat{A}}}X$

3. Results and discussion

The practical part presents an analysis and construction of the fuzzy model. The study relied on the data published in the M. Sc. thesis of the linear regression of the researcher Bekir [2], as shown in Table (2) which are meteorological data (humidity - wind speed - rain amount - temperature). And there are study variables Y, X_1, X_2, X_3 respectively, were simulated by the following relationships: $Y = 2X_1 + 3X_2 + 2X_3$ where:$X_1, X_2 \sim N(2,1)$ and: $X_3 \sim N(4,1)$ used the analytical method of the data based on the proposed misty ratio of 0.1 and 0.05 for the variables, and using MATLAB program was applied to the proposed model according to equation (2.9):

$\hat{\hat{\hat{A}}} = ({X_L}^t X_L + XX + {X_R}^t X_R)^{-1}({X_L}^t Y_L + \acute{X}Y + {X_R}^t Y_R)$

The data was fuzzed in two values:

$R_y = L_y = Rx_{ip} = Lx_{ip} = 0.1$, $R_y = L_y = Rx_{ip} = Lx_{ip} = 0.05$ These values are suggested by the researcher because the values of the variables cannot be adopted as actual accurate values because of the efficiencies and competencies of the instruments used in the measurements. Therefore, the fumigation has been proposed in the data values as a maximum or a minimum of the two proposed values.

Table 2. Study Variables

N	Y	X1	X2	X3	N	Y	X1	X2	X3
1	19.3	1.39	3.12	3.17	11	18.5	0.8	2.28	4.16
2	20.2	3	1.73	3.91	12	22.6	3.53	2.48	3.64
3	19.2	2.37	1.14	4.93	13	24.1	1.83	3.77	4.62
4	15.7	0.43	2.44	3.62	14	20.4	1.17	2.72	5.05
5	20.8	2.32	2.61	4.06	15	23.9	2.46	2.71	4.48
6	17.7	0.68	2.13	4.35	16	20.3	2.99	2.4	3.36
7	24.9	2.28	3.26	4.46	17	18.1	1.59	1.72	4.53
8	20.9	5.05	0.51	4.91	18	30.1	3.81	3.37	5.61
9	32.2	2.31	5.12	5.42	19	22.1	1.91	2.92	4.23
10	19.9	3.08	1.93	4.07	20	18.5	1.51	1.72	4.68

$$X = \begin{pmatrix} 1 & 1.39 & 3.12 & 3.17 \\ 1 & 3.0 & 1.73 & 3.91 \\ 1 & 2.37 & 1.14 & 4.93 \\ 1 & 0.43 & 2.44 & 3.62 \\ 1 & 2.32 & 2.61 & 4.06 \\ 1 & 0.68 & 2.13 & 4.35 \\ 1 & 2.28 & 3.26 & 4.46 \\ 1 & 5.05 & 0.51 & 4.91 \\ 1 & 2.31 & 5.12 & 5.42 \\ 1 & 3.08 & 1.93 & 4.07 \\ 1 & 0.8 & 2.28 & 4.16 \\ 1 & 3.53 & 2.48 & 3.64 \\ 1 & 1.83 & 3.77 & 4.62 \\ 1 & 1.17 & 2.72 & 5.05 \\ 1 & 2.46 & 2.71 & 4.48 \\ 1 & 2.99 & 2.4 & 3.36 \\ 1 & 1.59 & 1.72 & 4.53 \\ 1 & 3.81 & 3.37 & 5.61 \\ 1 & 1.91 & 2.92 & 4.23 \\ 1 & 1.51 & 1.72 & 4.68 \end{pmatrix}, Y = \begin{pmatrix} 19.3 \\ 20.2 \\ 19.2 \\ 15.7 \\ 20.8 \\ 17.7 \\ 24.9 \\ 20.9 \\ 32.2 \\ 19.9 \\ 18.5 \\ 22.6 \\ 24.1 \\ 20.4 \\ 23.9 \\ 20.3 \\ 18.1 \\ 30.1 \\ 22.1 \\ 18.5 \end{pmatrix}, Y_L = \begin{pmatrix} 19.2 \\ 20.1 \\ 19.1 \\ 15.6 \\ 20.7 \\ 17.6 \\ 24.8 \\ 20.8 \\ 32.1 \\ 19.8 \\ 18.4 \\ 22.5 \\ 24.0 \\ 20.3 \\ 23.8 \\ 20.2 \\ 18.0 \\ 30.0 \\ 22.0 \\ 18.4 \end{pmatrix}, Y_R = \begin{pmatrix} 19.4 \\ 20.3 \\ 19.3 \\ 15.8 \\ 20.9 \\ 17.8 \\ 25.0 \\ 21.0 \\ 32.3 \\ 20.0 \\ 18.6 \\ 22.7 \\ 24.2 \\ 20.5 \\ 24.0 \\ 20.4 \\ 18.2 \\ 30.2 \\ 22.2 \\ 18.6 \end{pmatrix}$$

$$X_L = \begin{pmatrix} 1 & 1.29 & 3.02 & 3.07 \\ 1 & 2.9 & 1.63 & 3.81 \\ 1 & 2.27 & 1.04 & 4.83 \\ 1 & 0.33 & 2.34 & 3.52 \\ 1 & 2.22 & 2.51 & 3.96 \\ 1 & 0.58 & 2.03 & 4.25 \\ 1 & 2.17 & 3.16 & 4.36 \\ 1 & 4.95 & 0.41 & 4.81 \\ 1 & 2.21 & 5.02 & 5.32 \\ 1 & 2.98 & 1.83 & 3.97 \\ 1 & 0.7 & 2.18 & 4.06 \\ 1 & 3.43 & 2.38 & 3.54 \\ 1 & 1.73 & 3.67 & 4.52 \\ 1 & 1.07 & 2.62 & 4.95 \\ 1 & 2.36 & 2.61 & 4.38 \\ 1 & 2.89 & 2.3 & 3.26 \\ 1 & 1.49 & 1.62 & 4.43 \\ 1 & 3.71 & 3.27 & 5.51 \\ 1 & 1.81 & 2.82 & 4.13 \\ 1 & 1.41 & 1.62 & 4.58 \end{pmatrix}, X_R = \begin{pmatrix} 1 & 1.39 & 3.12 & 3.17 \\ 1 & 3.0 & 1.73 & 3.91 \\ 1 & 2.37 & 1.14 & 4.93 \\ 1 & 0.43 & 2.44 & 3.62 \\ 1 & 2.32 & 2.61 & 4.06 \\ 1 & 0.68 & 2.13 & 4.35 \\ 1 & 2.28 & 3.26 & 4.46 \\ 1 & 5.05 & 0.51 & 4.91 \\ 1 & 2.31 & 5.12 & 5.42 \\ 1 & 3.08 & 1.93 & 4.07 \\ 1 & 0.8 & 2.28 & 4.16 \\ 1 & 3.53 & 2.48 & 3.64 \\ 1 & 1.83 & 3.77 & 4.62 \\ 1 & 1.17 & 2.72 & 5.05 \\ 1 & 2.46 & 2.71 & 4.48 \\ 1 & 2.99 & 2.4 & 3.36 \\ 1 & 1.59 & 1.72 & 4.53 \\ 1 & 3.81 & 3.37 & 5.61 \\ 1 & 1.91 & 2.92 & 4.23 \\ 1 & 1.51 & 1.72 & 4.68 \end{pmatrix},$$

The model parameters are estimated using equation (2.9):

$\hat{\hat{\acute{A}}} = \begin{pmatrix} 0.8164 & 1.8417 & 3.1020 & 2.0141 \end{pmatrix}$

Similarly when: $R_y = L_y = Rx_{pi} = Lx_{pi} = 0.05$

The model parameters are estimated according to the same model:

$\hat{\hat{\acute{A}}} = \begin{pmatrix} 0.4344 & 1.8645 & 3.1330 & 2.0722 \end{pmatrix}$

The parameters of the model were estimated in the traditional way (least squares) according to the model: $\underline{\hat{A}}_{OLS} = (\acute{X}X)^{-1}.\acute{X}\underline{\hat{Y}}$

His results were as follows:$\hat{\acute{A}}_{OLS} = \begin{pmatrix} 0.3450 & 1.8720 & 3.1470 & 2.0817 \end{pmatrix}$

The sum of the error boxes was calculated (see Table 3).

	predicted values by Ordinary squares OLS		Fuzzy model predicted values at: $R_y = L_y = Rx_{ip} = Lx_{ip} = .1$		Fuzzy model predicted values at: $R_y = L_y = Rx_{ip} = Lx_{ip} = .05$	
Y	$\hat{Y}_OLS$	e^2_{iOLS}	$\hat{Y}_{FL.1}$	$e^2_{iFL.1}$	$\hat{Y}_{FL.05}$	$e^2_{iFL.05}$
19.3	19.36471	0.004187255	0.01938	19.4392	0.0048845	19.369889
20.2	19.54476	0.429343389	0.3807	19.583	0.3805767	19.583091
19.2	18.632	0.322622864	0.3059	18.6469	0.3057847	18.647022
15.7	16.36439	0.441419387	0.59006	16.4682	0.5902127	16.468253
20.8	21.35341	0.306264842	0.31642	21.3625	0.31653	21.36261
17.7	17.37647	0.104674896	0.06904	17.4373	0.0689845	17.437351
24.9	24.15676	0.552402725	0.62287	24.1108	0.6227072	24.110882
20.9	21.62472	0.52521473	0.47353	21.5881	0.4736688	21.588236
32.2	32.06477	0.018286071	0.10937	31.8693	0.1093036	31.869389
19.9	20.65699	0.573032346	0.5975	20.673	0.5976573	20.673083
18.5	17.67763	0.676289127	0.57627	17.7409	0.5761174	17.740976
22.6	22.33511	0.070167772	0.06667	22.3418	0.0666234	22.341885
24.1	25.2524	1.328034979	1.18003	25.1863	1.1802498	25.186393
20.4	21.60767	1.458454752	1.39177	21.5797	1.3920083	21.579834
23.9	22.80451	1.200107104	1.26232	22.7765	1.262095	22.77657
20.3	20.48959	0.035945126	0.0553	20.5352	0.0553468	20.535259
18.1	18.16442	0.004150065	0.0108	18.2039	0.0108193	18.204016
30.1	29.76105	0.114889136	0.26418	29.586	0.2640747	29.586118
22.1	21.91535	0.034095253	0.03556	21.9114	0.0355209	21.91153
18.5	18.32692	0.029958071	0.01997	18.3587	0.0199389	18.358795

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