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COEFFICIENT INEQUALITY FOR A SUB-CLASS OF GENERALIZED SAKAGUCHI TYPE FUNCTIONS

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Abstract. In this paper, we investigated the upper bounds of $|a_3 - \mu a_2^2|$, μ real, for a subclass of generalized Sakaguchi type functions of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, in the unit disc $E = \{z : |z| < 1\}$. Results due to various authors follows as special cases.
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§1. Introduction and preliminaries

Let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the unit disc $E = \{z : |z| < 1\}$. Let S be the class of functions of the form (1) which are analytic univalent in E.

Let U be the class of analytic bounded functions of the form

$$w(z) = \sum_{n=1}^{\infty} c_n z^n,\ z \in E, \tag{2}$$

and satisfying the conditions $w(0) = 0, |w(z)| < 1$. It is known (see [15]) that

$$|c_1| \leq 1,\ |c_2| \leq 1 - |c_1|^2. \tag{3}$$

We shall apply the subordination principle due to Rogosinski [17], which states that if $f(z) \prec F(z)$, then $f(z) = F(w(z))$, $w(z) \in U$ (where $\prec$ stands for subordination).

Recently Frasin [4] introduced and studied, the subclasses of generalized Sakaguchi type functions as follows:

$S(\alpha, s, t)$ is the class of functions $f(z) \in A$ which satisfies the following conditions

$$Re\left[\frac{(s-t)zf'(z)}{f(sz)-f(tz)}\right] > \alpha, 0 \leq \alpha < 1, s, t \in C, s \neq t, z \in E.$$

$T(\alpha, s, t)$ is the class of functions $f(z) \in A$ which satisfies the following conditions

$$Re\left[\frac{(s-t)(zf'(z))'}{(f(sz)-f(tz))'}\right] > \alpha, 0 \leq \alpha < 1, s, t \in C, s \neq t, z \in E.$$

Obviously $f(z) \in T(\alpha, s, t)$ if and only if $zf'(z) \in S(\alpha, s, t)$.

These classes generalise the classes introduced by Sakaguchi [18] and Das and Singh [3]. Various Sakaguchi type functions were investigated and studied by various authors including ([6],[8],[9],[13],[16],[20]). In this paper, we define the following new generalized class $M_s(\delta, \lambda, A, B, s, t)$, which is the generalization of the above defined classes and the classes studied by various authors.

A function $f(z) \in A$ is said to be in the class $M_s(\delta, \lambda, A, B, s, t)$ if it satisfies the following conditions

$$(1-\lambda)\left[\frac{(s-t)zf'(z)}{f(sz)-f(tz)}\right] + \lambda\left[\frac{(s-t)(zf'(z))'}{(f(sz)-f(tz))'}\right] \prec \left(\frac{1+Az}{1+Bz}\right)^{\delta}, \tag{4}$$

where $0 < \delta \leq 1$, $\lambda \geq 0$, $-1 \leq B < A \leq 1$, $s, t \in C, s \neq t$, $z \in E$.

The following observations are obvious:

(i) $M_s(1, \lambda, A, B, s, t) \equiv M_s(\lambda, A, B, s, t)$.

(ii) $M_s(1, \lambda, 1-2\alpha, -1, s, t) \equiv M_s(\lambda, \alpha, s, t)$.

(iii) $M_s(1, \lambda, 1, -1, s, t) \equiv M_s(\lambda, s, t)$.

(iv) $M_s(1, \lambda, A, B, 1, t) \equiv M_s(\lambda, A, B, t)$.

(v) $M_s(1, \lambda, 1-2\alpha, -1, s, t) \equiv M_s(\lambda, \alpha, t)$, the class introduced by Keerthi et al.[9].

(vi) $M_s(1, 0, A, B, 1, t) \equiv S^*(A, B, t)$, the class introduced by Goyal and Goswami [8].

(vii) $M_s(1, 1, A, B, 1, t) \equiv T(A, B, t)$, the class introduced by Goyal and Goswami [8].

(viii) $M_s(1, 0, 1-2\alpha, -1, 1, t) \equiv S^*(\alpha, t)$, the class introduced and studied by Owa et al.[16].

(ix) $M_s(1, 1, 1-2\alpha, -1, 1, t) \equiv T(\alpha, t)$, the class introduced and studied by Owa et al. [16].

(x) $M_s(1, 0, A, B, s, t) \equiv S^*(A, B, s, t)$, the class introduced by Shilpa and Latha [20].

(xi) $M_s(1, 1, A, B, s, t) \equiv K(A, B, s, t)$, the class introduced by Shilpa and Latha [20].

(xii) $M_s(1, 0, 1-2\alpha, -1, s, t) \equiv S(\alpha, s, t)$, the class introduced and studied by Frasin [4].

(xiii) $M_s(1, 1, 1-2\alpha, -1, s, t) \equiv T(\alpha, s, t)$, the class introduced and studied by Frasin [4].

(xiv) $M_s(1,0,A,B,1,-1) \equiv S_s^*(A,B)$, the class introduced and studied by Goel and Mehrok [6].

(xv) $M_s(1,0,A,B,1,-1) \equiv K_s(A,B)$, the class studied by Mehrok and Singh [13].

(xvi) $M_s(1,0,1-2\alpha,-1,1,-1) \equiv S_s^*(\alpha)$, the class introduced by Cho et al. [2].

(xvii) $M_s(1,1,1-2\alpha,-1,1,-1) \equiv K_s(\alpha)$.

(xviii) $M_s(1,0,1,-1,1,-1) \equiv S_s^*$, the class introduced by Sakaguchi [18].

(xix) $M_s(1,1,1,-1,1,-1) \equiv K_s$, the class introduced by Das and Singh [3].

(xx) $M_s(1,\lambda,A,B,1,0) \equiv M(\lambda,A,B)$, the class studied by Kim and Jung [11].

(xxi) $M_s(1,\lambda,1,-1,1,0) \equiv M(\lambda)$, the class introduced by Mocanu [14].

(xxii) $M_s(1,0,1,-1,1,0) \equiv S^*$, the class of starlike functions.

(xxiii) $M_s(1,1,1,-1,1,0) \equiv K$, the class of convex functions.

(xxiv) $M_s(1,0,A,B,1,0) \equiv S^*(A,B)$, the class studied by Goel and Mehrok [5].

(xxv) $M_s(1,1,A,B,1,0) \equiv K(A,B)$, the class introduced by Goel and Mehrok [5].

§2. Main Result

Theorem 2.1. Let $f(z) \in M_s(\delta,\lambda,A,B,s,t)$. Then

(5)
$$|a_3 - \mu a_2^2| \leq \begin{cases} \dfrac{\delta^2(A-B)^2(\beta-\mu)}{(1+\lambda)^2(2-s-t)^2} & \text{if } \mu \leq \beta-\gamma, \\ \dfrac{\delta(A-B)}{(1+2\lambda)|3-s^2-st-t^2|} & \text{if } \beta-\gamma \leq \mu \leq \beta+\gamma, \\ \dfrac{\delta^2(A-B)^2(\mu-\beta)}{(1+\lambda)^2(2-s-t)^2} & \text{if } \mu \geq \beta+\gamma, \end{cases}$$

where

$$\beta = \frac{(1+\lambda)^2(2-s-t)^2[(\delta-1)(A-B)-2B] + 2\delta(A-B)(1+3\lambda)(s+t)(2-s-t)}{2\delta(A-B)(1+2\lambda)(3-s^2-st-t^2)}$$

and

$$\gamma = \frac{(1+\lambda)^2(2-s-t)^2}{\delta(A-B)(1+2\lambda)|3-s^2-st-t^2|}.$$

Proof. As $f(z) \in M_s(\delta,\lambda,A,B,s,t)$, so by principle of subordination, (4) yields

(6)
$$(1-\lambda)\left[\frac{(s-t)zf'(z)}{f(sz)-f(tz)}\right] + \lambda\left[\frac{(s-t)(zf'(z))'}{(f(sz)-f(tz))'}\right] = \left(\frac{1+Aw(z)}{1+Bw(z)}\right)^{\delta}, w(z) \in U.$$

On expanding and equating the coefficients of z and z^2 in (6), we get

(7)
$$a_2 = \frac{\delta(A-B)c_1}{(1+\lambda)(2-s-t)},$$

(8)
$$[(1+2\lambda)(3-s^2-st-t^2)]a_3 - (1+3\lambda)(s+t)(2-s-t)a_2^2 = \delta(A-B)\left[(c_2 - Bc_1^2) + \frac{(A-B)(\delta-1)c_1^2}{2}\right].$$

From (7) and (8), it is easily established that

$$(9) \qquad |a_3 - \mu a_2^2| \leq \frac{\delta(A-B)}{(1+2\lambda)|3-s^2-st-t^2|}|c_2| + \frac{\delta^2(A-B)^2}{(1+\lambda)^2(2-s-t)^2}|\beta-\mu||c_1|^2.$$

Using (3), (9) yields

$$(10) \qquad |a_3 - \mu a_2^2| \leq \frac{\delta(A-B)}{(1+2\lambda)|3-s^2-st-t^2|} + \frac{\delta^2(A-B)^2}{(1+\lambda)^2(2-s-t)^2}[|\beta-\mu|-\gamma]|c_1|^2.$$

Case I. For $\mu \leq \beta$, from (10), we have

$$(11) \qquad |a_3 - \mu a_2^2| \leq \frac{\delta(A-B)}{(1+2\lambda)|3-s^2-st-t^2|} + \frac{\delta^2(A-B)^2}{(1+\lambda)^2(2-s-t)^2}[(\beta-\gamma)-\mu]|c_1|^2.$$

If $\mu \leq (\beta-\gamma)$, using (3), (11) gives

$$|a_3 - \mu a_2^2| \leq \frac{\delta(A-B)}{(1+2\lambda)|3-s^2-st-t^2|} + \frac{\delta^2(A-B)^2}{(1+\lambda)^2(2-s-t)^2}[\beta-\mu-\gamma].$$

$|a_3 - \mu a_2^2| \leq \frac{\delta^2(A-B)^2}{(1+\lambda)^2(2-s-t)^2}[\beta-\mu]$.

If $\beta-\gamma \leq \mu \leq \beta$, (11) yields

$$|a_3 - \mu a_2^2| \leq \frac{\delta(A-B)}{(1+2\lambda)|3-s^2-st-t^2|}.$$

Case II. For $\mu \geq \beta$, from (10), we have

$$(12) \qquad |a_3 - \mu a_2^2| \leq \frac{\delta(A-B)}{(1+2\lambda)|3-s^2-st-t^2|} + \frac{\delta^2(A-B)^2}{(1+\lambda)^2(2-s-t)^2}[\mu-\beta-\gamma]|c_1|^2.$$

If $\mu \leq (\beta+\gamma)$, using (3), (12) gives second inequality of (5).
If $\mu \geq (\beta+\gamma)$, (12) gives third inequality of (5).
This completes the proof of the theorem.

For $\delta = 1, \lambda = 0$, Theorem 2.1 gives the following result due to Mathur and Mathur [12].

Corollary 2.1. Let $f(z) \in S(A,B,s,t)$. Then

$$|a_3 - \mu a_2^2| \leq \beta(A-B) \begin{cases} C^* & \text{if } \mu \leq \sigma_1^*, \\ 1 & \text{if } \sigma_1^* \leq \mu \leq \sigma_2^*, \\ -C^* & \text{if } \mu \geq \sigma_2^*, \end{cases}$$

where $\beta = \frac{1}{|3-s^2-st-t^2|}$, $C^* = -B + (A-B)\left[\left(\frac{s+t}{2-s-t}\right) - \mu\frac{(3-s^2-st-t^2)}{(2-s-t)^2}\right]$,

$\sigma_1^* = \frac{(2-s-t)^2}{(3-s^2-st-t^2)} \left[\left(\frac{s+t}{2-s-t}\right) - \left(\frac{B+1}{A-B}\right)\right]$ and $\sigma_2^* = \frac{(2-s-t)^2}{(3-s^2-st-t^2)} \left[\left(\frac{s+t}{2-s-t}\right) - \left(\frac{B-1}{A-B}\right)\right]$.

For $\delta = 1, \lambda = 1$, Theorem 2.1 gives the following result due to Mathur and Mathur [12].

Corollary 2.2. Let $f(z) \in T(A, B, s, t)$. Then

$$|a_3 - \mu a_2^2| \leq \frac{\beta}{3}(A-B) \begin{cases} C^{**} & \text{if } \mu \leq \sigma_1^{**}, \\ 1 & \text{if } \sigma_1^{**} \leq \mu \leq \sigma_2^{**}, \\ -C^{**} & \text{if } \mu \geq \sigma_2^{**}, \end{cases}$$

where $\beta = \frac{1}{|3-s^2-st-t^2|}$, $C^{**} = -B + (A-B)\left[\left(\frac{s+t}{2-s-t}\right) - \frac{3\mu}{4}\frac{(3-s^2-st-t^2)}{(2-s-t)^2}\right]$,

$\sigma_1^{**} = \frac{4(2-s-t)^2}{3(3-s^2-st-t^2)} \left[\left(\frac{s+t}{2-s-t}\right) - \left(\frac{B+1}{A-B}\right)\right]$ and $\sigma_2^{**} = \frac{4(2-s-t)^2}{3(3-s^2-st-t^2)} \left[\left(\frac{s+t}{2-s-t}\right) - \left(\frac{B-1}{A-B}\right)\right]$.

Putting $\delta = 1, \lambda = 0, s = 1$ in Theorem 2.1, we obtain the following result due to Goyal and Goswami [8].

Corollary 2.3. Let $f(z) \in S^*(A, B, t)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{B-A}{(2+t)(1-t)}\left[B - (A-B)\left(\frac{1+t}{1-t}\right) + \mu(A-B)\left(\frac{2+t}{1-t}\right)\right] & \text{if } \mu \leq \sigma_1^*, \\ \frac{A-B}{(2+t)(1-t)} & \text{if } \sigma_1^* \leq \mu \leq \sigma_2^*, \\ \frac{A-B}{(2+t)(1-t)}\left[B - (A-B)\left(\frac{1+t}{1-t}\right) + \mu(A-B)\left(\frac{2+t}{1-t}\right)\right] & \text{if } \mu \geq \sigma_2^*, \end{cases}$$

where $\sigma_1^* = \frac{(1-t)}{(2+t)}\left[-\frac{1+B}{(A-B)} - \left(\frac{1+t}{1-t}\right)\right]$ and $\sigma_2^* = \frac{(1-t)}{(2+t)}\left[\frac{1-B}{(A-B)} - \left(\frac{1+t}{1-t}\right)\right]$.

Putting $\delta = 1, A = 1-2\alpha, B = -1, \lambda = 0, s = 1$ in Theorem 2.1, we obtain the following result due to Goyal and Goswami [8].

Corollary 2.4. Let $f(z) \in S^*(\alpha, t)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{2(1-\alpha)}{(2+t)(1-t)}\left[1 + 2(1-\alpha)\left(\frac{1+t}{1-t}\right) - 2\mu(1-\alpha)\left(\frac{2+t}{1-t}\right)\right] & \text{if } \mu \leq \sigma_1^{**}, \\ \frac{2(1-\alpha)}{(2+t)(1-t)} & \text{if } \sigma_1^{**} \leq \mu \leq \sigma_2^{**}, \\ \frac{2(1-\alpha)}{(2+t)(1-t)}\left[(1-2\alpha) - \alpha\left(\frac{1+t}{1-t}\right) - \mu\alpha\left(\frac{2+t}{1-t}\right)\right] & \text{if } \mu \geq \sigma_2^{**}, \end{cases}$$

where $\sigma_1^{**} = -\left(\frac{1+t}{2+t}\right)$ and $\sigma_2^{**} = \frac{(1-t)}{(2+t)}\left[\frac{1}{(1-\alpha)} - \left(\frac{1+t}{1-t}\right)\right]$.

For $\delta = 1, \lambda = 0, s = 1, t = -1$, Theorem 2.1 gives the following result due to Shanmugam et al. [19].

Corollary 2.5. Let $f(z) \in S_s^*(A, B)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{B-A}{2}\left[B + \frac{\mu}{2}(A-B)\right] & \text{if } \mu \leq -2\left[\frac{1+B}{A-B}\right], \\ \frac{A-B}{2} & \text{if } -2\left[\frac{1+B}{A-B}\right] \leq \mu \leq 2\left[\frac{1-B}{A-B}\right], \\ \frac{A-B}{2}\left[B + \frac{\mu}{2}(A-B)\right] & \text{if } \mu \geq 2\left[\frac{1-B}{A-B}\right]. \end{cases}$$

For $\delta = 1, \lambda = 0, A = 1 - 2\alpha, B = -1, s = 1, t = -1$, Theorem 2.1 gives the following result due to Shanmugam et al. [19].

Corollary 2.6. Let $f(z) \in S_s^*(\alpha)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} (1-\alpha)(1-\mu(1-\alpha)) & \text{if } \mu \leq 0, \\ (1-\alpha) & \text{if } 0 \leq \mu \leq \frac{2}{1-\alpha}, \\ -(1-\alpha)(1-\mu(1-\alpha)) & \text{if } \mu \geq \frac{2}{1-\alpha}. \end{cases}$$

For $\delta = 1, \lambda = 1, s = 1, t = -1$, Theorem 2.1 gives the following result due to Shanmugam et al. [19].

Corollary 2.7. Let $f(z) \in K_s(A, B)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{B-A}{6}\left[B + \frac{3\mu}{8}(A-B)\right] & \text{if } \mu \leq -\frac{8}{3}\left[\frac{1+B}{A-B}\right], \\ \frac{A-B}{6} & \text{if } -\frac{8}{3}\left[\frac{1+B}{A-B}\right] \leq \mu \leq \frac{8}{3}\left[\frac{1-B}{A-B}\right], \\ \frac{A-B}{6}\left[B + \frac{3\mu}{8}(A-B)\right] & \text{if } \mu \geq \frac{8}{3}\left[\frac{1-B}{A-B}\right]. \end{cases}$$

For $\delta = 1, \lambda = 1, A = 1 - 2\alpha, B = -1, s = 1, t = -1$, Theorem 2.1 gives the following result due to Shanmugam et al. [19].

Corollary 2.8. Let $f(z) \in K_s(\alpha)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{(1-\alpha)}{3}(1 - \frac{3\mu}{4}(1-\alpha)) & \text{if } \mu \leq 0, \\ \frac{(1-\alpha)}{3} & \text{if } 0 \leq \mu \leq \frac{8}{3(1-\alpha)}, \\ -\frac{(1-\alpha)}{3}(1 - \frac{3\mu}{4}(1-\alpha)) & \text{if } \mu \geq \frac{8}{3(1-\alpha)}. \end{cases}$$

For $\delta = 1, s = 1, t = 0$, Theorem 2.1 gives the following result due to Goel and Mehrok [7].

Corollary 2.9. Let $f(z) \in M(\lambda, A, B)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \dfrac{(A-B)^2(\beta-\mu)}{(1+\lambda)^2} & \text{if } \mu \leq \beta - \gamma, \\ \dfrac{(A-B)}{2(1+2\lambda)} & \text{if } \beta - \gamma \leq \mu \leq \beta + \gamma, \\ \dfrac{(A-B)^2(\mu-\beta)}{(1+\lambda)^2(2-s-t)^2} & \text{if } \mu \geq \beta + \gamma, \end{cases}$$

where

$$\beta = \frac{(A-B)(1+3\lambda) - B(1+\lambda)^2}{2(1+2\lambda)(A-B)}$$

and

$$\gamma = \frac{(1+\lambda)^2}{2(1+2\lambda)(A-B)}.$$

For $\delta = 1, A = 1 - 2\beta, B = -1, s = 1, t = 0$, Theorem 2.1 gives the following result due to Cho and Owa [1].

Corollary 2.10. Let $f(z) \in M_\lambda(\beta)$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{(\lambda^2+8\lambda+3-4(1+2\lambda)\mu)\beta^2}{(1+2\lambda)(1+\lambda)^2} & \text{if } \mu \leq \frac{(\lambda^2+8\lambda+3)\beta-(1+\lambda)^2}{4(1+2\lambda)\beta}, \\ \dfrac{\beta}{1+2\lambda} & \text{if } \frac{(\lambda^2+8\lambda+3)\beta-(1+\lambda)^2}{4(1+2\lambda)\beta} \leq \mu \leq \frac{(\lambda^2+8\lambda+3)\beta+(1+\lambda)^2}{4(1+2\lambda)\beta}, \\ \frac{(4(1+2\lambda)\mu-(\lambda^2+8\lambda+3))\beta^2}{(1+2\lambda)(1+\lambda)^2} & \text{if } \mu \geq \frac{(\lambda^2+8\lambda+3)\beta+(1+\lambda)^2}{4(1+2\lambda)\beta}. \end{cases}$$

Putting $\delta = 1, \lambda = 0, s = 1, t = 0, A = 1, B = -1$, Theorem 2.1 gives the following result due to Keogh and Merkes [10].

Corollary 2.11. Let $f(z) \in S^*$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu & \text{if } \mu \leq \frac{1}{2}, \\ 1 & \text{if } \frac{1}{2} \leq \mu \leq 1, \\ 4\mu - 3 & \text{if } \mu \geq 1. \end{cases}$$

Putting $\delta = 1, \lambda = 1, s = 1, t = 0, A = 1, B = -1$, Theorem 2.1 gives the following result due to Keogh and Merkes [10].

Corollary 2.12. Let $f(z) \in K$. Then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 1 - \mu & \text{if } \mu \leq \frac{2}{3}, \\ \frac{1}{3} & \text{if } \frac{2}{3} \leq \mu \leq \frac{4}{3}, \\ \mu - 1 & \text{if } \mu \geq \frac{4}{3}. \end{cases}$$

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