A DERIVED NUMERICAL SCHEME FOR SOLVING DOMESTIC VIOLENCE MODEL

S.E. FADUGBA[1,2,3,*], M. ELKAF[4,5], M.C. KEKANA[6]

[1]Department of Mathematics, Ekiti State University, Ado Ekiti, Nigeria

[2]Department of Physical Sciences, Mathematics Programme, Landmark University, Omu-Aran, Nigeria

[3]SDG 4: Quality Education Research Group, Landmark University, Omu-Aran, Nigeria

[4]Laboratory of Mathematics, Computer science and Applications, Faculty of Sciences and Techniques Mohammedia, University Hassan-II Casablanca, Mohammedia, Morocco

[5]Royal Naval School, Casablanca, Morocco

[6]Department of Mathematics, Tshwane University of Technology, Pretoria, South Africa

*Corresponding author: fadugba.sunday@lmu.edu.ng, sunday.fadugba@eksu.edu.ng

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Abstract. The main aim of the article is to construct a new numerical scheme for the solution of domestic violence model. A comprehensive examination of the properties for the scheme shall be worked on. The proposed scheme was tested on domestic violence model that comprises of six state variables. The results generated by the scheme are analyzed and plotted.

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1. Introduction

In numerical analysis, solving differential equations have become a major problem. Most of the models which are formulated by means of differential equations are so complicated/ tedious to determine their exact solutions; therefore, there is a need for numerical methods. There had been a lot of numerical methods developed for solving IVPs of ODEs such as [1–18], just to mention a few. According to [19], accuracy property of any numerical method determines its order and convergence by means of the local truncation error and Taylor's series expansion. In this article, a numerical scheme constructed will be tested on the domestic violence model using fitted parameters. Domestic violence can take many forms, including physical abuse such as hitting, kicking, or choking, as well as emotional abuse, such as manipulation, verbal attacks, and isolation from friends and family. It can also involve sexual abuse, including forced sexual acts or unwanted sexual advances, as well as financial abuse,

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such as controlling the victim's finances or preventing them from working. This is a serious issue that can have long-lasting physical and psychological effects on victims, including physical injuries, depression, anxiety, and post-traumatic stress disorder (PTSD). It is important for victims to seek help and support from family, friends, and professional organizations such as domestic violence hotlines and shelters; [20]- [21].

This article will describe in subsequent sections, the construction of the scheme and the fundamental properties are presented in section 2. Application of the constructed scheme on domestic violence model is presented in section 3. Thereafter, the section 4 following will discuss the results obtained and then conclude the article.

2. Construction of the Scheme and its Properties

2.1. **Construction of the Scheme.** This subsection presents the construction of the scheme for the solution of physical model of the form:

$$\begin{cases} y' &= f(t,y), \quad t \in [t_0, b] \\ y(t_0) &= y_0 \end{cases} \tag{1}$$

The scheme is constructed via the interpolating function (combination of the polynomial and exponential types) given by:

$$F(t) = \sum_{j=0}^{5} \rho_j t^j + \rho_6 e^t \tag{2}$$

Where the constants $\rho_1, ..., \rho_6$ are to be determined.

To define these parameters, we will discretize the equation (2) as follow:

(i) Mesh size

$$Nh = b - t_0$$

(ii) Mesh point

$$t_{n+1} = t_0 + (n+1)h$$

Evaluating (2) at the points $t = t_n$, one obtains:

$$F(t_n) = \sum_{j=0}^{5} \rho_j {t_n}^j + \rho_6 e^{t_n} \tag{3}$$

and at the points $t = t_{n+1}$

$$F(t_{n+1}) = \sum_{j=0}^{5} \rho_j t_{n+1}^j + \rho_6 e^{t_{n+1}} \tag{4}$$

Differentiating (3) yields

$$\begin{cases} f_n = \rho_1 + 2\rho_2 t_n + 3\rho_3 t_n^2 + 4\rho_4 t_n^3 + 5\rho_5 t_n^4 + \rho_6 e^{t_n} \\ f_n^{(1)} = 2\rho_2 + 6\rho_3 t_n + 12\rho_4 t_n^2 + 20\rho_5 t_n^3 + \rho_6 e^{t_n} \\ f_n^{(2)} = 6\rho_3 + 24\rho_4 t_n + 60\rho_5 t_n^2 + \rho_6 e^{t_n} \\ f_n^{(3)} = 24\rho_4 + 120\rho_5 t_n + \rho_6 e^{t_n} \\ f_n^{(4)} = 120\rho_5 + \rho_6 e^{t_n} \\ f_n^{(5)} = \rho_6 e^{t_n} \end{cases} \tag{5}$$

Solving (5), one gets

$$\begin{cases} \rho_6 = \dfrac{f_n^{(5)}}{e^{t_n}} \\ \rho_5 = \dfrac{f_n^{(4)} - f_n^{(5)}}{120} \\ \rho_4 = \dfrac{1}{24}\left(f_n^{(3)} - t_n f_n^{(4)} + (t_n - 1) f_n^{(5)}\right) \\ \rho_3 = \dfrac{1}{12}\left(2f_n^{(2)} - 2t_n f_n^{(3)} + f_n^{(4)} - (t_n^2 - 2t_n + 2) f_n^{(5)}\right) \\ \rho_2 = \dfrac{1}{12}\left(6f_n^{(1)} - 6t_n f_n^{(2)} + 3t_n^2 f_n^{(3)} - t_n^3 f_n^{(4)} + (t_n^3 - 3t_n^2 + 6t_n - 6) f_n^{(5)}\right) \\ \rho_1 = \dfrac{1}{24}\left(f_n - t_n f_n^{(1)} + 12t_n^2 f_n^{(2)} - 4t_n^3 f_n^{(3)} + t_n^4 f_n^{(4)} + (-t_n^4 + 4t_n^3 - 12t_n^2 + t_n - 1) f_n^{(5)}\right) \end{cases} \tag{6}$$

Subtracting (3) from (4), one gets

$$F(t_{n+1}) - F(t_n) \equiv y_{n+1} - y_n = \sum_{j=0}^{5} \rho_j \left(t_{n+1}^j - {t_n}^j\right) + \rho_6 \left(e^{t_{n+1}} - e^{t_n}\right) \tag{7}$$

Therefore,

$$y_{n+1} = y_n + \sum_{j=0}^{5} \rho_j \left(t_{n+1}^j - {t_n}^j\right) + \rho_6 \left(e^{t_{n+1}} - e^{t_n}\right) \tag{8}$$

Where $t_0 = 0$ and $\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6$ are given by (6) with

$$\begin{cases} t_{n+1} - t_n = h, \\ t_{n+1}^2 - t_n^2 = (2n+1)h^2, \\ t_{n+1}^3 - t_n^3 = (3n^2 + 3n + 1)h^3, \\ t_{n+1}^4 - t_n^4 = (4n^3 + 6n^2 + 4n + 1)h^4, \\ t_{n+1}^5 - t_n^5 = (5n^4 + 10n^3 + 10n^2 + 5n + 1)h^5. \end{cases} \tag{9}$$

(8) is the newly constructed scheme.

Algorithm 1 Algorithmic scheme

Initiation of parameters and setting of initial conditions
for $i \leftarrow 0$ to $n-1$ **do**
  $T_1 \leftarrow h$
  $T_2 \leftarrow (2i+1)h^2$
  $T_3 \leftarrow (3i^2+3i+1)h^3$
  $T_4 \leftarrow (4i^3+6i^2+4i+1)h^4$
  $T_5 \leftarrow (5i^4+10i^3+10i^2+5i+1)h^5$
  $K_1 \leftarrow 0$
  for $j \leftarrow 1$ to 5 **do**
    $K_1 \leftarrow K_1 + \rho_j T_j$
  end for
  $K_2 \leftarrow \rho_6(e^{t_{i+1}} - e^{t_i})$
  $y_{i+1} \leftarrow y_i + K_1 + K_2$
end for

2.2. **Properties of the Scheme.** The properties of the scheme are summarized in the following remarks
Remark 1: The scheme has sixth order of accuracy.
Remark 2: The consistency property of the scheme is confirmed as follows. The increment function of the proposed scheme $\Phi(t_n, y_n; h)= f(t_n, y_n)$ as $h = 0$.
Remark 3: The scheme is zero stable since the first characteristic polynomial of the scheme satisfies the Dahlquist root condition.
Remark 4: The stability and convergence analyses of the derived scheme is given by the following result.
Theorem 1: [5] The necessary and sufficient condition that (1) is stable and convergent is

$$|y_{n+1} - m_{n+1}| < L|\rho - \rho^*| \tag{10}$$

where L is a constant,

$$\begin{cases} y_{n+1} &= y_n + h\, f(t_n, y_n; h), \\ y_n &= y(t_n), \\ y(t_0) &= \rho. \end{cases} \tag{11}$$

$$\begin{cases} m_{n+1} &= m_n + h\, g(t_n, l_n; h), \\ m_n &= m(t_n), \\ m(t_0) &= \rho^*. \end{cases} \tag{12}$$

and

$$|\rho - \rho^*| < \epsilon,\ \epsilon > 0 \tag{13}$$

3. Application of the Proposed Scheme to Domestic Violence Model

3.1. **Domestic Violence Model.** Consider the domestic violence model in which two interacting populations namely violent individuals and victims are involved in a host community of six classes [20]; $S(t), V(t), R(t), S_v(t), V_v(t)$ and $R_v(t)$.

$$\begin{cases} \dfrac{dS}{dt}(t) &= a - cS(t)V(t) - (h+u)S(t) \\ \dfrac{dS_v}{dt}(t) &= b - rmS_v(t)V(t) - (k+u)S_v(t) \\ \dfrac{dV}{dt}(t) &= cS(t)V(t) - (e+u)V(t) \\ \dfrac{dV_v}{dt}(t) &= rS_v(t) - (g+n+u)V_v(t) \\ \dfrac{dR}{dt}(t) &= eV(t) + hS(t) - uR(t) \\ \dfrac{dR_v}{dt}(t) &= gV_v(t) + kS_v(t) - uR_v(t) \end{cases} \tag{14}$$

With initial conditions $S(0) = 800, V(0) = 350, R(0) = 80$, $S_v(0)$ = 1000, $V_v(0)$ = 520 and $R_v\,(0) = 200$. Where $S(t)$ is the strongly or potentially violent individual of the population, $V(t)$ is the violent individual, $R(t)$ is the recovered violent individual, $S_v(t)$ is the susceptible victims, $V_v(t)$ is the victims and $R_v(t)$ is the recovered victims, t is the time, a is the recruitment rate into violent population, b is the recruitment rate into victim population, c is the rate at which potentially violent becomes violent, u is the natural mortality rate, n is the violence induced death rate, m is the probability that a contact between violent individual and a victim leads to domestic violence, k is the recovered rate for potential victim individual, h is the recovered rate for violent individual, g is the recovered rate for victims and e is the recovered rate for violent individual by reformation and r is the rate at which susceptible individual becomes victimised.

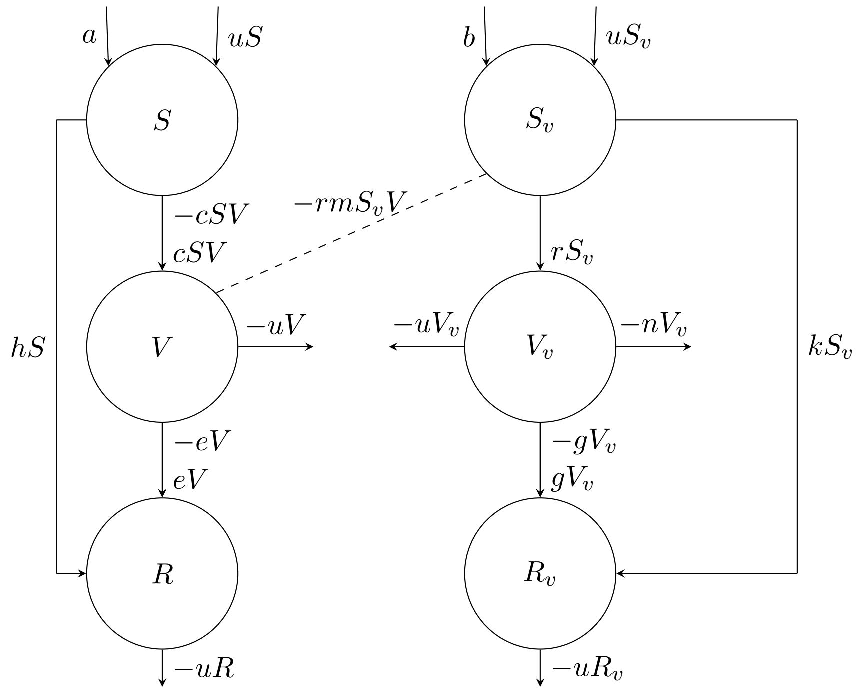

FIGURE 1. Graphical diagram of the model (14)

The accompanying schematic diagram at Figure 1 demonstrates this interplay between potentially violent, violent individual and recovered one, to help better understand (14). The assumed parameters are given as [20]; $a = 0.290$, $b = 0.600$, $c = 0.040$, r = 0.0032, $e = 0.0166$, $g = 0.0430$, $h = 0.00066$, $k = 0.0014$, $m = 0.6$, $n = 0.003$, $u = 0.0124$.

The plots of the six state variables for different values of time t are displayed in Figures 2-8. The behaviours of the six state variables for different values of natural mortality rate u are captured in Figures 9-14.

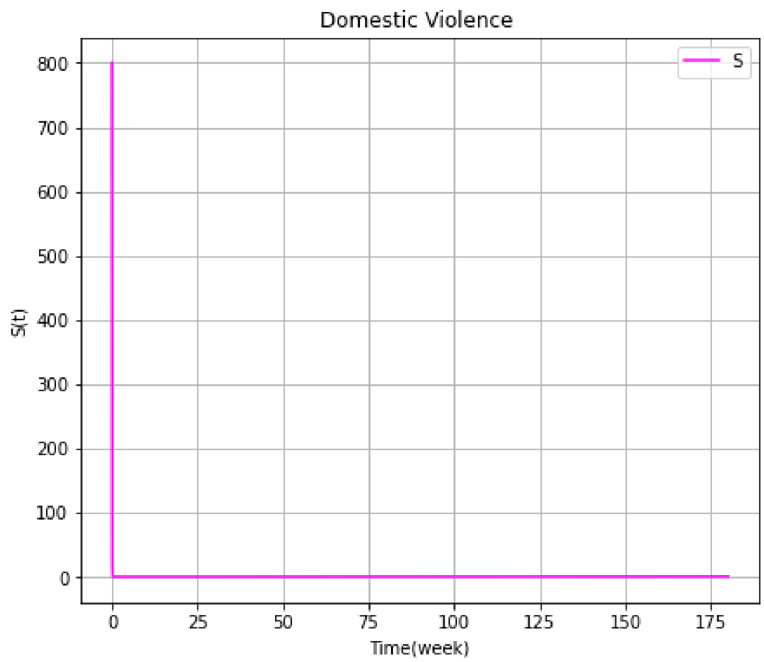

Figure 2. The plot potentially violent individual, $S(t)$.

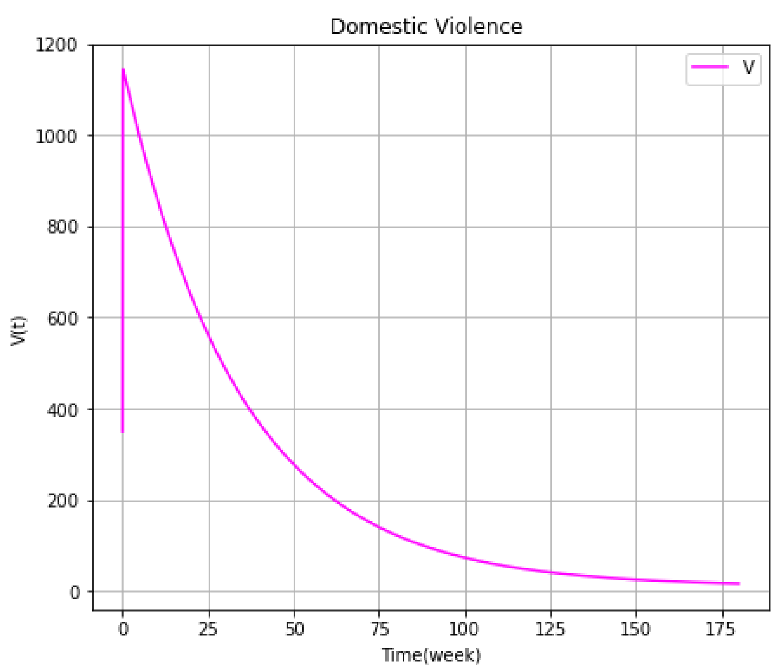

Figure 3. The plot violent individual, $V(t)$

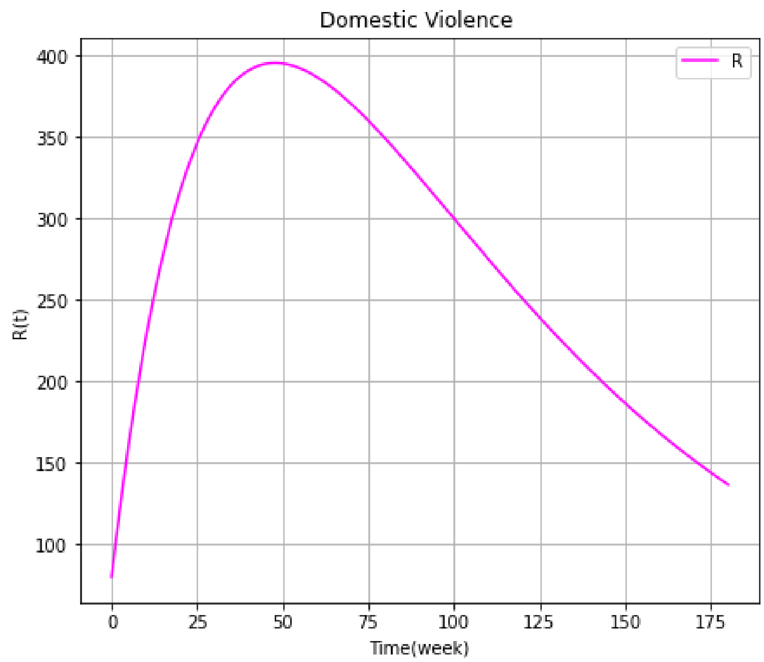

FIGURE 4. The plot of recovered violent individual, $R(t)$.

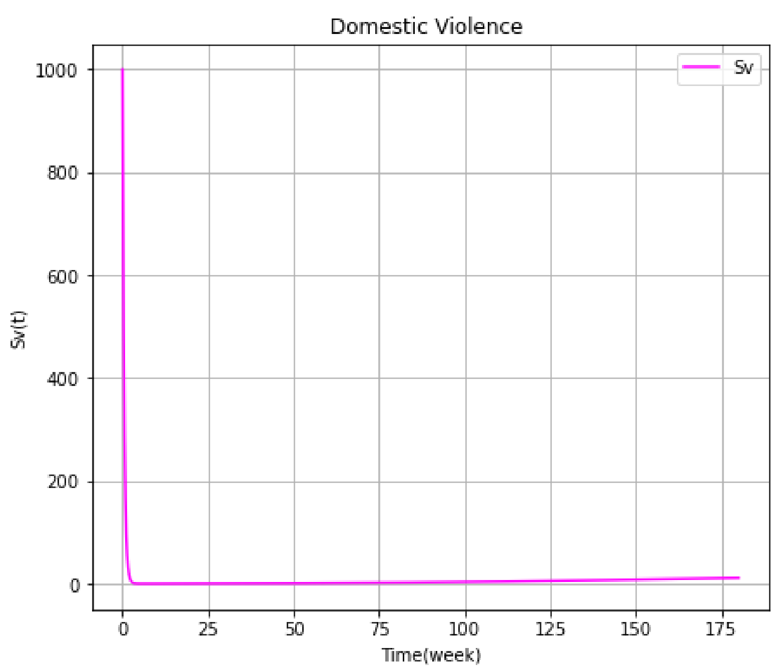

FIGURE 5. The plot of susceptible victim individual, $S_v(t)$.

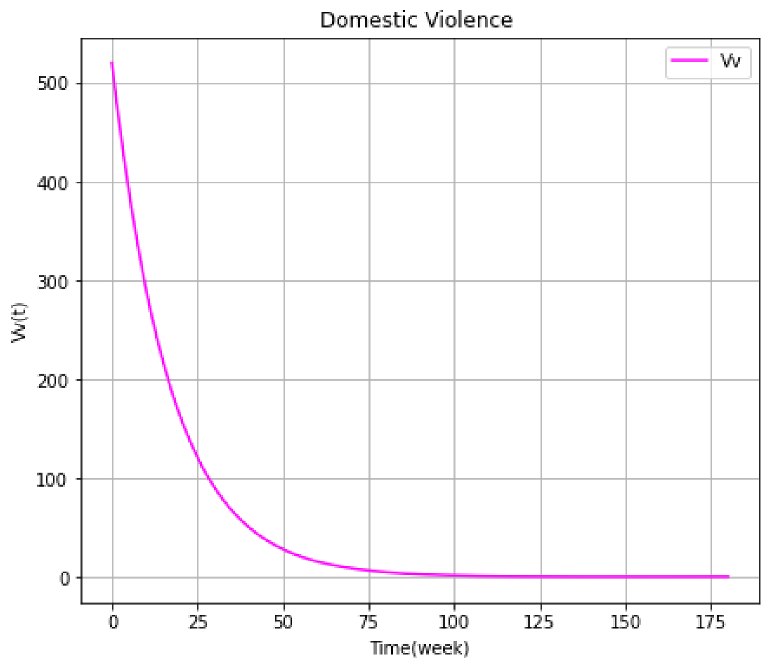

FIGURE 6. The plot of victim individual, $V_v(t)$.

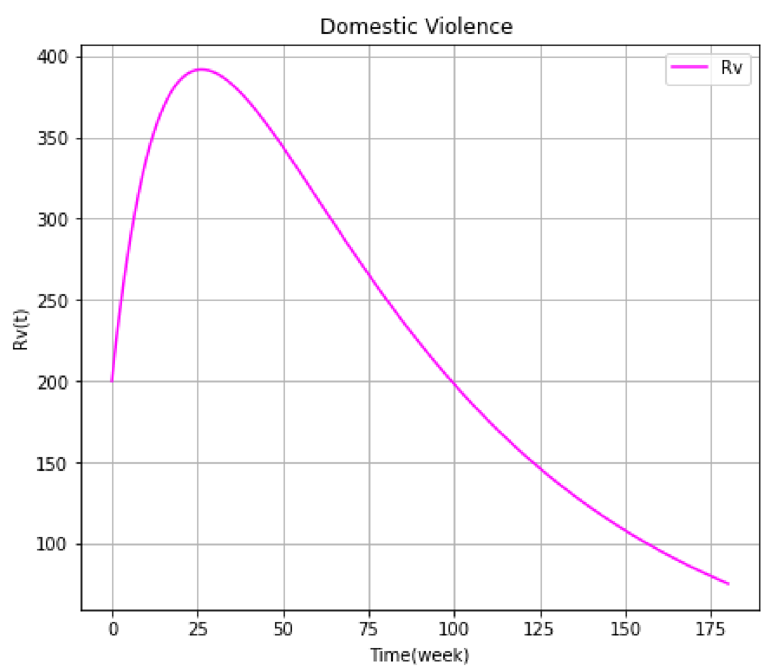

FIGURE 7. The plot of recovered victim individual, $R_v(t)$.

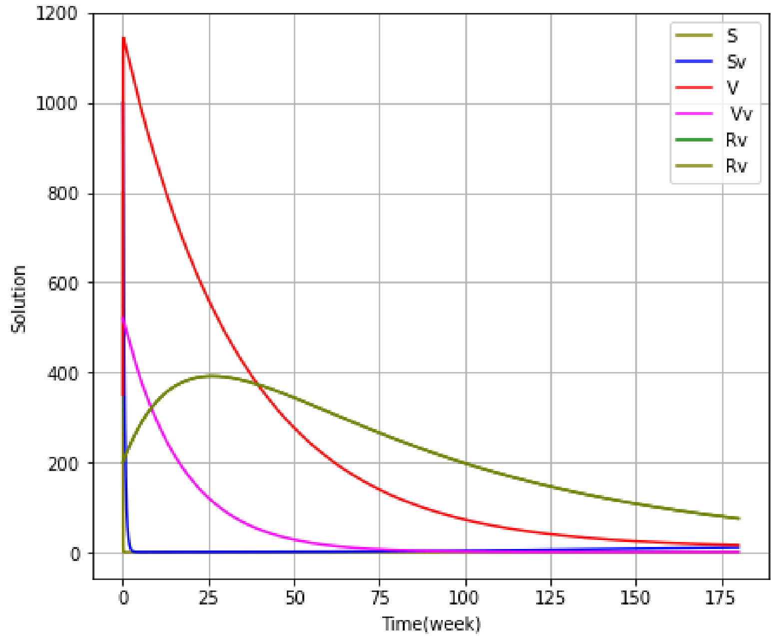

FIGURE 8. The plots of $S(t), V(t), R(t), S_v(t), V_v(t), R_v(t)$.

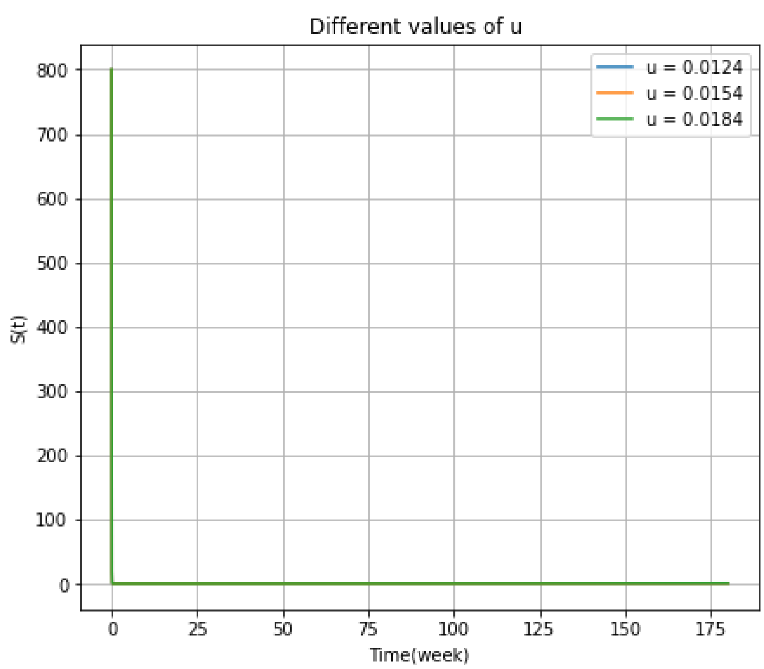

FIGURE 9. The behaviour of potentially violent individual, $S(t)$ for different values of u.

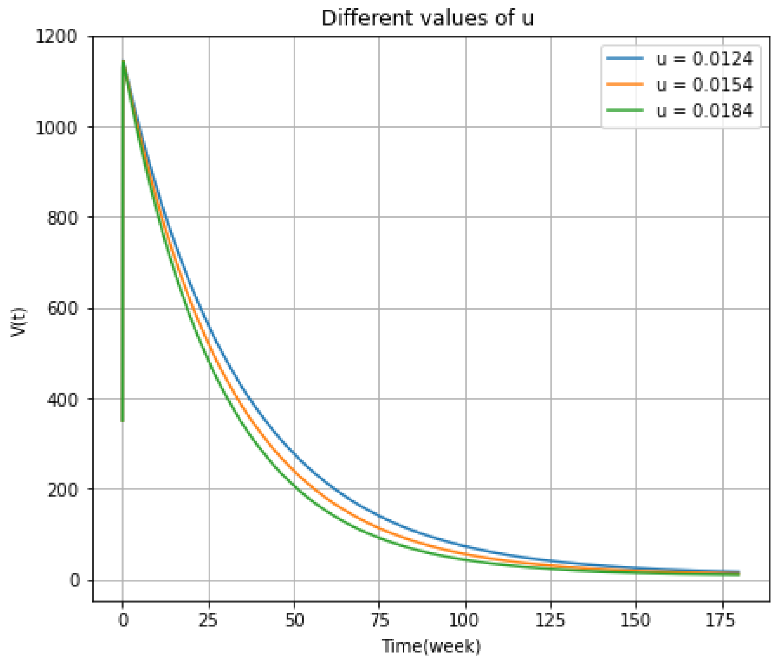

FIGURE 10. The behaviour of violent individual, $V(t)$ for different values of u.

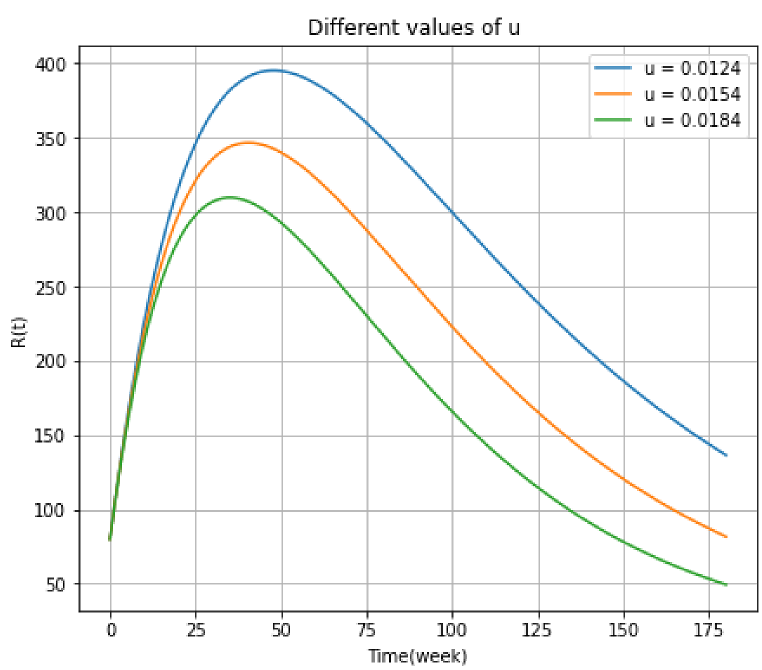

FIGURE 11. The behaviour of recovered violent individual, $R(t)$ for different values of u.

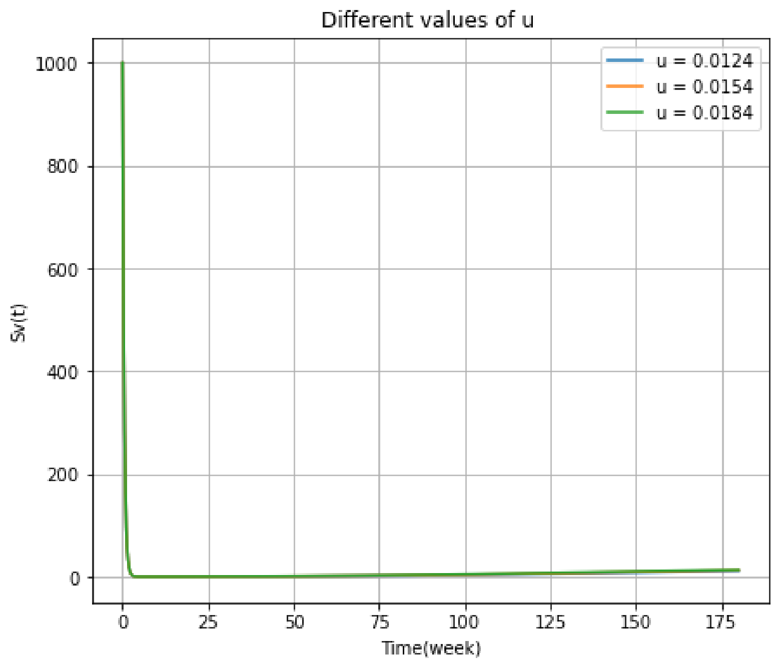

FIGURE 12. The behaviour of susceptible victim individual, $S_v(t)$ for different values of u.

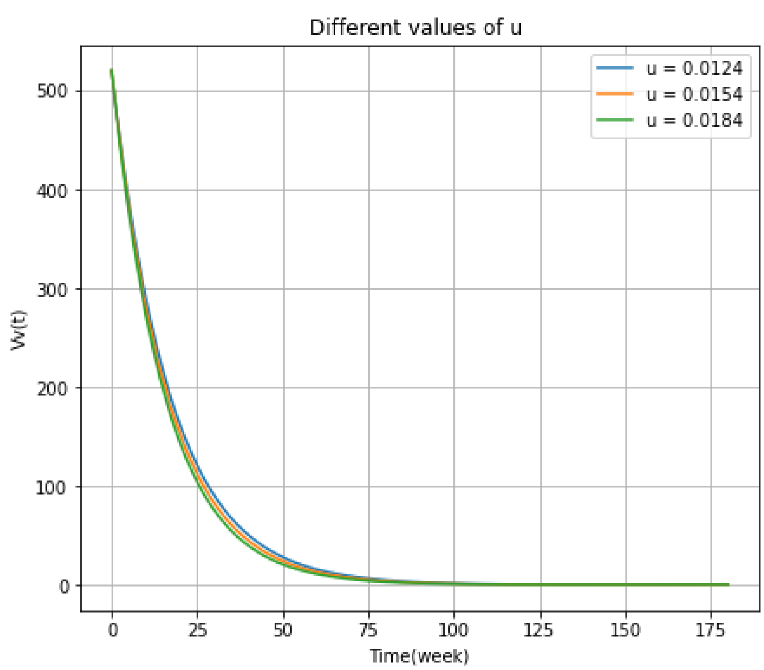

FIGURE 13. The behaviour of victim individual, $V_v(t)$ for different values of u.

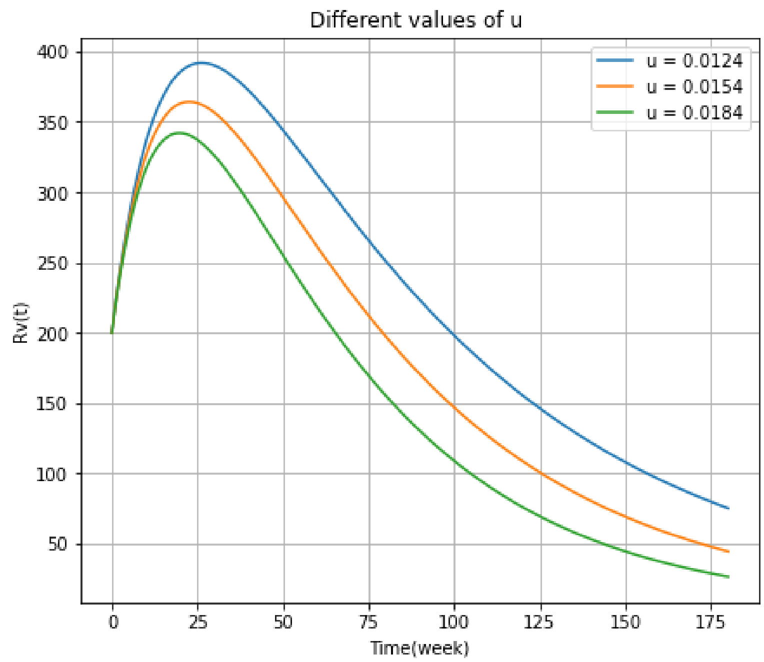

FIGURE 14. The behaviour of recovered victim individual, $R_v(t)$ for different values of u.

4. CONCLUDING REMARKS

In this article, a novel numerical method of order six has been proposed for the solution of domestic violence model. The method was devised via interpolating function comprises of both polynomial and exponential types. The derived method was tested on domestic violence model with six state variables using some fitted parameters. The plots of the state variables with different values of time t are displayed in Figures 2-7. It is observed from Figure 2, that the population of potentially violent individual decay for different values of time t. It is observed from Figures 3 and 4, there is a sharp increase in the violent and recovered violent individuals before it starts dropping down slowly as t increases. It is observed from Figure 5 that the susceptible victim individual decays sharply and later stabilizes below 10% of its initial value as time t increases. From Figure 6, it is observed that the victim individual dies out as time t increases. It is observed from Figure 7 that there is a sharp increase in recovered victim individual at $t = 25$ and sharp decrease at $t > 25$. Figure 8 captures the interaction between the state variables $S(t), V(t), R(t), S_v(t), V_v(t), R_v(t)$ for different values of t. The effects of natural mortality rate u with the fixed values of $a, b, c, r, e, g, h, k, m, n$ on the six state variables $S(t), V(t), R(t), S_v(t), V_v(t), R_v(t)$ are shown in Figures 9-14, respectively. It is observed from Figures 9, 12 and 13 that there is no significant effect of u on $S(t), S_v(t), V_v(t)$. It is observed from Figures 10, 11 and 14 that the values of $V(t), R(t), R_v(t)$ decreases as u increases. Moreover, it is noteworthy to say that the results generated via the derived scheme agreed with the results of [20]. Hence, The results generated by the scheme indicates that, when proper measures of changing attitudes of violent individuals were taken it can significantly eradicate domestic violence from the society. For future work, the proposed scheme shall be implemented on some special IVPs of ODEs and the results shall be compared with that of the existing developed methods in the literature.

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