PROPERTIES OF NEW CLASSES OF ω-μ-SETS IN GTS

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Received Sep. 14, 2023

Abstract. The notions of ω-μ-closed sets and ω-μ-open sets in generalized topological space were introduced and studied by Al Ghour and Zareer. In this paper, we introduce new classes of ω-μ-open sets and continuous functions in topological spaces and study the characterizations of such ω-μ-open sets and continuous functions in detail. Finally we obtain some decomposition theorems.

2020 Mathematics Subject Classification. 54A05, 54C05, 54C08, 54C10.

Key words and phrases. ω-μ-open set; $\alpha(\omega,\mu)$-open set; $\pi(\omega,\mu)$ set; $\beta(\omega,\mu)$-open set; $t(\omega,\mu)$-set; $\delta(\omega,\mu)$-open set; semi$*$-ω-μ-closed set.

1. Introduction

In 2002, Császár [4] defined generalized topological spaces as follows: the pair (X,μ) is a generalized topological space if X is a nonempty set and μ is a collection of subsets of X such that $\emptyset \in \mu$ and μ is μ-closed under arbitrary unions. For a generalized topological space (X,μ), the elements of μ are called μ-open sets, the complements of μ-open sets are called μ-closed sets. The notions of ω-μ-closed sets and ω-μ-open sets in generalized topological space were introduced and studied by Al Ghour and Zareer [1]. Several characterizations and properties of ω-closed sets were provided in [2,3]. In this paper, we introduce new classes of ω-μ-open sets and continuous functions in topological spaces and study the characterizations of such sets and continuous functions in detail. Finally we obtain some decomposition theorems.

DOI: 10.28924/APJM/10-34

2. Preliminaries

Definition 2.1. [1] *Let H be a subset of a GT (X, μ), a point p in X is called a condensation point of H if for each μ-open set U containing p, $U \cap H$ is uncountable.*

Definition 2.2. [1] *A subset H of a GT (X, μ) is called ω-μ-closed if it contains all its condensation points. The complement of an ω-μ-closed set is called ω-μ-open. It is well known that a subset W of a GT (X, μ) is ω-μ-open if and only if for each $x \in W$, there exists $U \in \mu$ such that $x \in U$ and $U - W$ is countable. The family of all ω-μ-open sets, denoted by μ_ω, is a generalized topology on X, which is finer than μ. The interior and closure operator in (X, μ_ω) are denoted by i_ω and c_ω, respectively.*

Lemma 2.3. [1] *Let H be a subset of a GT (X, μ). Then*

(1) *H is ω-μ-closed in X if and only if $H = c_\omega(H)$.*
(2) *$c_\omega(X \setminus H) = X \setminus i_\omega(H)$.*
(3) *$c_\omega(H)$ is ω-μ-closed in X.*
(4) *$x \in c_\omega(H)$ if and only if $H \cap G \neq \emptyset$ for each ω-μ-open set G containing x.*
(5) *$c_\omega(H) \subset c(H)$.*
(6) *$i(H) \subset i_\omega(H)$.*

Remark 2.4. *For a subset of a GT (X, μ), the following property holds: Every μ-closed set is ω-μ-closed but not conversely.*

3. Some Subsets in GT

Using the notions of interior and closure operator in (X, μ) and the interior and closure operator in (X, μ_ω), we introduce the $\alpha(\omega, \mu)$-open set, $\pi(\omega, \mu)$-open set, $\beta(\omega, \mu)$-open set, $\sigma(\omega, \mu)$-open set, $\delta(\omega, \mu)$-open set, $\beta(\omega, \mu)$-open set, $\sigma(\omega^\star, \mu)$-open set, ω^*-t-set, semi-ω-regular set, $r(\omega, \mu)$-closed set, and find some of its properties and some relation between them.

Definition 3.1. *A subset H of a GT (X, μ) is said to be*

(1) *$\alpha(\omega, \mu)$-open if $H \subset i_\omega(c(i_\omega(H)))$;*
(2) *$\pi(\omega, \mu)$-open if $H \subset i_\omega(c(H))$;*
(3) *$\beta(\omega, \mu)$-open if $H \subset c(i_\omega(c(H)))$.*

Definition 3.2. *A subset H of a GT (X, μ) is called an $t(\omega, \mu)$-set if $i(H) = i_\omega(c(H))$.*

Definition 3.3. *A GT (X, μ) is called μ-submaximal if every μ-dense subset is μ-open.*

Definition 3.4. *A subset H of a GT (X, μ) is said to be*

(1) *$\sigma(\omega, \mu)$-open if $H \subset c(i_\omega(H))$.*

(2) *$\sigma(\omega,\mu)$-closed if $i(c_\omega(H)) \subset H$.*

The complement of a $\sigma(\omega,\mu)$-open set is called $\sigma(\omega,\mu)$-closed.

Example 3.5. *Let $X = \mathbb{R}$ and $\mu = \{\emptyset$, all possible unions of members of $\mathcal{B}\}$, where $\mathcal{B} = \{(a,b) : a, b \in \mathbb{R}\}$. Let $H = \mathbb{R} \setminus \mathbb{Q}$. Observe that $i_\omega(H) = H$, $c_\omega(H) = \mathbb{R}$, $i(H) = \emptyset$ and $c(H) = \mathbb{R}$. In consequence:*

(1) *H is $\alpha(\omega,\mu)$-open set, because $H \subset i_\omega(c(i_\omega(H))) = \mathbb{R}$.*

(2) *$W = \mathbb{Q}$ is $\pi(\omega,\mu)$-open set, because $W \subset i_\omega(c(W)) = i_\omega(\mathbb{R}) = \mathbb{R}$.*

(3) *H is $\beta(\omega,\mu)$-open set, because $H \subset c(i_\omega(\mathbb{R})) = c(\mathbb{R}) = \mathbb{R}$.*

(4) *H is $\sigma(\omega,\mu)$-open set, because $H \subset c(i_\omega(H)) = c(H) = c(i_(H) = \mathbb{R}$.*

(5) *$W = \mathbb{Q}$ is $\sigma(\omega,\mu)$-closed set, because $\emptyset = i(W) = i(c_\omega(W)) \subset W$.*

Theorem 3.6. *Let H be a subset of a GT (X,μ). Then H is $\alpha(\omega,\mu)$-open if and only if it is $\sigma(\omega,\mu)$-open and $\pi(\omega,\mu)$-open.*

Theorem 3.7. *For a subset H of a submaximal GT (X,μ), the following properties are equivalent.*

(1) *H is $\sigma(\omega,\mu)$-open,*

(2) *H is $\beta(\omega,\mu)$-open.*

Definition 3.8. *A subset H of a GT (X,μ) is said to be*

(1) *$\delta(\omega,\mu)$-open if $i_\omega(c(H)) \subset c(i_\omega(H))$.*

(2) *$\delta(\omega,\mu)$-closed if $i(c_\omega(H)) \subset c_\omega(i(H))$.*

The complement of a $\delta(\omega,\mu)$-open set is called $\delta(\omega,\mu)$-closed.

Definition 3.9. *A subset H of a GT (X,μ) is said to be $\beta(\omega,\mu)$-closed if $i(c_\omega(i(H))) \subset H$. The complement of a $\beta(\omega,\mu)$-closed set is called $\beta(\omega,\mu)$-open.*

Theorem 3.10. *For a subset H of a GT (X,μ), the following properties are equivalent:*

(1) *H is $\sigma(\omega,\mu)$-closed.*

(2) *H is $\beta(\omega,\mu)$-closed and $\delta(\omega,\mu)$-closed.*

Theorem 3.11. *A subset of X is $\alpha(\omega,\mu)$-open if and only if it is both $\delta(\omega,\mu)$-open and $\pi(\omega,\mu)$.*

Definition 3.12. *A subset H of a GT (X,μ) is said to be*

(1) *$\sigma(\omega^\star,\mu)$-open if $H \subset c_\omega(i(H))$.*

(2) *$\sigma(\omega^\star,\mu)$-closed if $i_\omega(c(H)) \subset H$.*

The complement of a $\sigma(\omega^\star,\mu)$-open set is called $\sigma(\omega^\star,\mu)$-closed.

Example 3.13. *According with Example 3.5,*

(1) *$W = \mathbb{Q}$ is $\beta(\omega,\mu)$-closed set, because $\emptyset = i((W) = i(c_\omega(W)) \subset W$.*

(2) *$H = \mathbb{R} \setminus \mathbb{Q}$ is not a $\sigma(\omega^\star, \mu)$-open set.*

Definition 3.14. *A subset H of a GT (X, μ) is said to be ω^*-t-set if $i_\omega(c(H)) = i_\omega(H)$.*

Theorem 3.15. *A subset H of a GT (X, μ) is $\sigma(\omega^\star, \mu)$-closed if and only if H is a ω^*-t-set.*

Definition 3.16. *A subset H of a GT (X, μ) is said to be semi-ω-regular if H is $\sigma(\omega, \mu)$-open and a ω^*-t-set.*

Theorem 3.17. *A subset H of a GT (X, μ) is semi-ω-regular if and only if H is both $\beta(\omega, \mu)$-open and $\sigma(\omega^\star, \mu)$-closed.*

Definition 3.18. *A subset H of a GT (X, μ) is called $r(\omega, \mu)$-closed if $H = c(i_\omega(H))$.*

Example 3.19. *Let $X = \mathbb{R}$ and $\mu = \{\emptyset$, all possible unions of members of $\mathcal{B}\}$, where $\mathcal{B} = \{(a, b) : a, b \in \mathbb{R}\}$. Let $H = (0, 1) \cap \mathbb{Q}$. Then H is not $\sigma(\omega, \mu)$-open, since $c(i_\omega(H)) = c(\emptyset) = \emptyset$.*

Theorem 3.20. *Let H be a subset of a GT (X, μ). Then the following properties are equivalent*

(1) *H is $r(\omega, \mu)$-closed.*
(2) *H is $\sigma(\omega, \mu)$-open and μ-closed.*
(3) *H is $\beta(\omega, \mu)$-open and μ-closed.*

Proof. (1)$\Rightarrow$ (2): If H is $r(\omega, \mu)$-closed, then $H = c(i_\omega(H))$ and $c(H) = c(i_\omega(H))$. Since $H \subset c(i_\omega(H))$, H is $\sigma(\omega, \mu)$-open. Also $H = c(H)$ and so H is μ-closed.
(2) $\Rightarrow$ (3): It follows from the fact that every $\sigma(\omega, \mu)$-open set is a $\beta(\omega, \mu)$-open set.
(3)$\Rightarrow$ (1): Suppose H is $\beta(\omega, \mu)$-open and μ-closed. Then $H \subset c(i_\omega(c(H)))$ and $H = c(H)$. Now $c(i_\omega(H)) \subset c(H) = H$. Also $H \subset c(i_\omega(H))$. Therefore $H = c(i_\omega(H))$ which implies that H is $r(\omega, \mu)$-closed. □

Example 3.21. *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $\{a\}$ is $\sigma(\omega, \mu)$-open.*

Example 3.22. *Let $X = \mathbb{R}$ and $\mu = \{\emptyset$, all possible unions of members of $\mathcal{B}\}$, where $\mathcal{B} = \{(a, b) : a, b \in \mathbb{R}\}$. Let $H = (0, 1) \cap \mathbb{Q}$. Then H is not $\sigma(\omega, \mu)$-open, since $c(i_\omega(H)) = c(\emptyset) = \emptyset$.*

Proposition 3.23. *In a GT (X, μ), every μ-semiopen subset is $\sigma(\omega, \mu)$-open.*

Proof. Let H be semi-open in (X, μ). Then $H \subset c(i(H)) \subset c(i_\omega(H))$. This proves that H is $\sigma(\omega, \mu)$-open. □

Example 3.24. *Consider (X, μ) as in Example 3.22. Let $H = \mathbb{R} \setminus \mathbb{Q}$ is $\sigma(\omega, \mu)$-open for $c(i_\omega(H)) = c(H) = \mathbb{R}$ and $H \subset c(i_\omega(H))$. But H is not μ-semiopen.*

Theorem 3.25. *In a μ-antilocally countable space, an ω-μ-closed and a $\sigma(\omega, \mu)$-open subset is μ-semiopen.*

Proof. Let (X, μ) be a μ-antilocally countable space and H an ω-μ-closed and a $\sigma(\omega, \mu)$-open subset of X. Since H is $\sigma(\omega, \mu)$-open, $H \subset c(i_\omega(H))$. Since (X, μ) is μ-antilocally countable and H is ω-μ-closed, $i_\omega(H) = i(H)$. Hence $H \subset c(i_\omega(H)) = c(i(H))$ and thus $H$is μ-semiopen. □

Definition 3.26. *A subset H of a GT (X, μ) is called $b(\omega, \mu)$-open if $H \subset i_\omega(c(H)) \cup c(i_\omega(H))$.*

Theorem 3.27. *For a GT (X, μ), the following properties hold:*

(1) *Every ω-μ-open set is $\sigma(\omega, \mu)$-open.*
(2) *Every $\alpha(\omega, \mu)$-open set is $\sigma(\omega, \mu)$-open.*
(3) *Every $\sigma(\omega, \mu)$-open set is $\beta(\omega, \mu)$-open.*
(4) *Every $\sigma(\omega, \mu)$-open set is $b(\omega, \mu)$-open.*

Proof. (1). If H is an ω-μ-open set, then $H = i_\omega(H) \subset c(i_\omega(H))$. Therefore H is $\sigma(\omega, \mu)$-open.
(2). If H is an $\alpha(\omega, \mu)$-open set, then $H \subset i_\omega(c(i_\omega(H))) \subset c(i_\omega(H))$. Therefore H is $\sigma(\omega, \mu)$-open.
(3). If H is an $\sigma(\omega, \mu)$-open set, then $H \subset c(i_\omega(H)) \subset c(i_\omega(c(H)))$. Therefore H is $\beta(\omega, \mu)$-open.
(4). If H is an $\sigma(\omega, \mu)$-open set, then $H \subset c(i_\omega(H)) \subset i_\omega(c(H)) \cup c(i_\omega(H))$. Therefore H is $b(\omega, \mu)$-open. □

The following Example support that the separate converses of the above Theorem are not true in general.

Example 3.28. *Consider (X, μ) as in Example 3.22.*

(1) *Let $H = (0, 1]$. Then H is $\sigma(\omega, \mu)$-open set but not ω-μ-open, since $H = (0, 1] \neq (0, 1) = i_\omega(H)$.*

(2) *Let $H = (0, 1]$. Then H is $\sigma(\omega, \mu)$-open set but not $\alpha(\omega, \mu)$-open, since $i_\omega(c(i_\omega(H))) = i_\omega(c(0, 1)) = i_\omega([0, 1]) = (0, 1)$.*

(3) *Let $H = [0, 1] \cap \mathbb{Q}$. Then H is $\beta(\omega, \mu)$-open set but not $\sigma(\omega, \mu)$-open, since $c(i_\omega(H)) = c(\emptyset) = \emptyset$.*

(4) *Let $H = \mathbb{Q}$. Then H is $b(\omega, \mu)$-open set but not $\sigma(\omega, \mu)$-open, since $c(i_\omega(H)) = c(\emptyset) = \emptyset$.*

Theorem 3.29. *A subset H of a GT (X, μ) is $\sigma(\alpha, \mu)$-open if and only if it is $\sigma(\omega, \mu)$-open and $\pi(\omega, \mu)$-open.*

Proof. Let H be an $\alpha(\omega, \mu)$-open. Then $H \subset i_\omega(c(i_\omega(H)))$. It implies that $H \subset i_\omega(c(i_\omega(H))) \subset c(i_\omega(H))$ and $H \subset i_\omega(c(i_\omega(H))) \subset i_\omega(c(H))$. Thus H is $\sigma(\omega, \mu)$-open and $\pi(\omega, \mu)$-open. Conversely, let H be $\sigma(\omega, \mu)$-open and $\pi(\omega, \mu)$-open. Then we have $H \subset c(i_\omega(H))$ and $H \subset i_\omega(c(H))$. Hence $H \subset i_\omega(c(H)) \subset i_\omega(c(i_\omega(H)))$ which implies that H is $\alpha(\omega, \mu)$-open. □

Remark 3.30. *The concepts of $\sigma(\omega,\mu)$-openness and $\pi(\omega,\mu)$-openness are independent. Consider (X,μ) as in Example 3.22. Then the set $H=(0,1]$ is $\sigma(\omega,\mu)$-open but not $\pi(\omega,\mu)$-open, since $i_\omega(c(H))=i_\omega([0,1])=(0,1)$. Let $H=\mathbb{Q}$. Then H is $\pi(\omega,\mu)$-open but not $\sigma(\omega,\mu)$-open, since $c(i_\omega(H))=c(\emptyset)=\emptyset$.*

Proposition 3.31. *The intersection of a $\sigma(\omega,\mu)$-open set and an open set is $\sigma(\omega,\mu)$-open.*

Proof. Let H be a $\sigma(\omega,\mu)$-open and U a μ-open set in X. Then $H\subset c(i_\omega(H))$ and $i(U)=U$. Then we have $U\cap H\subset U\cap c(i_\omega(H))\subset c(U\cap i_\omega(H))=c(i(U)\cap i_\omega(H))\subset c(i_\omega(U)\cap i_\omega(H))=c(i_\omega(U\cap H))$. Therefore $U\cap H$ is $\sigma(\omega,\mu)$-open. □

Remark 3.32. *The intersection of two $\sigma(\omega,\mu)$-open sets need not be $\sigma(\omega,\mu)$-open. Consider (X,μ) as in Example 3.22. Let $A=(0,1]$ and $B=[1,2)$, then A and B are $\sigma(\omega,\mu)$-open, but $A\cap B=\{1\}$ which is not $\sigma(\omega,\mu)$-open, since $c(i_\omega(A\cap B))=c(\emptyset)=\emptyset$.*

Theorem 3.33. *Let H be a subset of a GT (X,μ). If H is both μ-closed and $\beta(\omega,\mu)$-open, then H is $\sigma(\omega,\mu)$-open.*

Proof. Since H is a $\beta(\omega,\mu)$-open set, $H\subset c(i_\omega(c(H)))=c(i_\omega(H))$, H being μ-closed. Therefore H is $\sigma(\omega,\mu)$-open. □

Theorem 3.34. *Let H be a subset of a space (X,μ). If H is both $\beta(\omega,\mu)$-open and $t(\omega,\mu)$-set, then H is $\sigma(\omega,\mu)$-open.*

Proof. Since H is a $t(\omega,\mu)$-set, $i(H)=i_\omega(c(H))$. Since H is $\beta(\omega,\mu)$-open also, $H\subset c(i_\omega(c(H)))\subset c(i(H))\subset c(i_\omega(H))$. Therefore H is $\sigma(\omega,\mu)$-open. □

Theorem 3.35. *Let H be a subset of a space (X,μ). If H is both $b(\omega,\mu)$-open and $t(\omega,\mu)$-set, then H is $\sigma(\omega,\mu)$-open.*

Proof. Since H is $t(\omega,\mu)$-set, $i_\omega(c(H))=i(H)\subset i_\omega(H)$. Since H is $b(\omega,\mu)$-open, $H\subset i_\omega(c(H))\cup c(i_\omega(H))\subset i_\omega(H)\cup c(i_\omega(H))=c(i_\omega(H))$. Therefore H is $\sigma(\omega,\mu)$-open. □

Proposition 3.36. *A subset H of a GT (X,μ) is $\sigma(\omega,\mu)$-open if, and only if $c(H)=c(i_\omega(H))$.*

Proof. Let H be $\sigma(\omega,\mu)$-open. Then $H\subset c(i_\omega(H))$ and $c(H)\subset c(i_\omega(H))$. But always $c(i_\omega(H))\subset c(H)$. Thus $c(H)=c(i_\omega(H))$. Conversely, let the condition hold. We have $H\subset c(H)=c(i_\omega(H))$, by the given condition. Thus $H\subset c(i_\omega(H))$ and hence H is $\sigma(\omega,\mu)$-open. □

Theorem 3.37. *For a subset H of a μ-submaximal space (X,μ), the following properties are equivalent.*

(1) *H is $\sigma(\omega,\mu)$-open,*
(2) *H is $\beta(\omega,\mu)$-open.*

Proof. (1) $\Rightarrow$ (2): It follows from the fact that every $\sigma(\omega,\mu)$-open set is $\beta(\omega,\mu)$-open.

(2) $\Rightarrow$ (1): Let H be a $\beta(\omega,\mu)$-open set in X. Then $H \subset c(i_\omega(c(H)))$ and $c(H) \subset c(i_\omega(c(H)))$. Thus, $c(H)$ is $\sigma(\omega,\mu)$-open. Put $A = c(H)$ and $K = H \cup (X \backslash c(H))$. We have $H = c(H) \cap K$ and $c(K) = X$. This implies that $H = A \cap K$, where A is $\sigma(\omega,\mu)$-open and K is μ-dense. Since X is μ-submaximal, then K is μ-open. Then $H = A \cap K$ is $\sigma(\omega,\mu)$-open. □

Theorem 3.38. *A subset H of a GT (X,μ) is $\sigma(\omega,\mu)$-open if and only if there exists an ω-μ-open set U such that $U \subset H \subset c(U)$.*

Proof. Let H be $\sigma(\omega,\mu)$-open. Then $H \subset c(i_\omega(H))$. Take $i_\omega(H) = U$. Then we have $U \subset H \subset c(U)$. Conversely, let $U \subset H \subset c(U)$ for some $U \in \mu_\omega$. Since $U \subset H$, $U \subset i_\omega(H)$ and hence $c(U) \subset c(i_\omega(H))$. Thus $H \subset c(i_\omega(H))$ and H is $\sigma(\omega,\mu)$-open. □

Corollary 3.39. *If A is a $\sigma(\omega,\mu)$-open set in a GT (X,μ) and $A \subset B \subset c(A)$, then B is $\sigma(\omega,\mu)$-open in X.*

Proof. Since A is $\sigma(\omega,\mu)$-open, $A \subset c(i_\omega(A)) \subset c(i_\omega(B))$ for $A \subset B$. So $c(A) \subset c(i_\omega(B))$. Since $B \subset c(A)$, $B \subset c(i_\omega(B))$. Thus B is $\sigma(\omega,\mu)$-open. □

Example 3.40.
(1) *Consider (X,μ) as in Example 3.22. Let $H = \mathbb{Q}$. Then H is not $\delta(\omega,\mu)$-open, since $i_\omega(c(\mathbb{Q})) = i_\omega(\mathbb{R}) = \mathbb{R}$ and $c(i_\omega(\mathbb{Q})) = c(\emptyset) = \emptyset$.*
(2) *Let $X = \mathbb{R}$ and $\mu = \{\emptyset$, all possible unions of members of $\mathcal{B}\}$, where $\mathcal{B} = \{(a,b) : a,b \in \mathbb{R}\}$. Let $H = (0,1]$. Then H is $\delta(\omega,\mu)$-open, since $i_\omega(c((0,1])) = i_\omega([0,1]) = (0,1)$ and $c(i_\omega(H)) = c(0,1) = [0,1]$.*

Proposition 3.41. *For a subset of a GT (X,μ), the following properties hold:*

(1) *Every $\alpha(\omega,\mu)$-open set is $\delta(\omega,\mu)$-open.*
(2) *Every $t(\omega,\mu)$-set is $\delta(\omega,\mu)$-open.*

Proof. (1). Since H is $\alpha(\omega,\mu)$-open, $H \subset i_\omega(c(i_\omega(H))) \subset c(i_\omega(H))$. Then $c(H) \subset c(i_\omega(H))$ and $i_\omega(c(H)) \subset c(H) \subset c(i_\omega(H))$. Hence H is $\delta(\omega,\mu)$-open.

(2). Since H is an $t(\omega,\mu)$-set, $i_\omega(c(H)) = i(H) \subset H$. Then $i_\omega(c(H)) \subset i_\omega(H) \subset c(i_\omega(H))$. Therefore H is $\delta(\omega,\mu)$-open. □

Example 3.42. *Consider (X,μ) as in Example 3.22.*

(1) *Let $H = (0,1]$. Then H is $\delta(\omega,\mu)$-open but not $\alpha(\omega,\mu)$-open, since $i_\omega(c(H)) = (0,1)$ and $c(i_\omega(H)) = [0,1]$.*
(2) *Let $H = \mathbb{Q}$. Then H is $\delta(\omega,\mu)$-open but not $t(\omega,\mu)$-set, since $i(\mathbb{Q}) = \emptyset$, $i_\omega(c(\mathbb{Q})) = \mathbb{R}$ and $c(i_\omega(\mathbb{Q})) = c(\mathbb{Q}) = \mathbb{R}$.*

Proposition 3.43. *Let H be a subset of a GT (X,μ). Then H is $\beta(\omega,\mu)$-closed if and only if $i(c_\omega(i(H))) = i(H)$.*

Proof. Since H is $\beta(\omega,\mu)$-closed set, $i(c_\omega(i(H))) \subset H$ and then we obtain $i(c_\omega(i(H))) \subset i(H)$. But $i(H) \subset i(c_\omega(i(H)))$. Thus we have $i(H) = i(c_\omega(i(H)))$. Conversely, let the condition hold. We have $i(c_\omega(i(H))) = i(H) \subset H$. Therefore H is $\beta(\omega,\mu)$-closed. □

Theorem 3.44. *For a subset H of a GT (X,μ), the following properties are equivalent:*

(1) *H is $\sigma(\omega,\mu)$-closed.*
(2) *H is $\beta(\omega,\mu)$-closed and $\delta(\omega,\mu)$-closed.*

Proof. (1) $\Rightarrow$ (2): Let H be $\sigma(\omega,\mu)$-closed. Then H is $\beta(\omega,\mu)$-closed. Since H is $\sigma(\omega,\mu)$-closed, $i(c_\omega(H)) \subset H$ and $i(c_\omega(H)) \subset i(H)$. Then $c_\omega(i(c_\omega(H))) \subset c_\omega(i(H))$. Thus $i(c_\omega(H)) \subset c_\omega(i(c_\omega(H))) \subset c_\omega(i(H))$ and so H is $\delta(\omega,\mu)$-closed.
(2) $\Rightarrow$ (1): Since H is $\delta(\omega,\mu)$-closed, $i(c_\omega(H)) \subset c_\omega(i(H))$ and $i(c_\omega(H)) \subset i(c_\omega(i(H)))$. Since H is $\beta(\omega,\mu)$-closed, $i(c_\omega(i(H))) \subset H$. Then $i(c_\omega(H)) \subset H$ and so H is $\sigma(\omega,\mu)$-closed. □

Remark 3.45. *The concepts of $\beta(\omega,\mu)$-closedness and $\delta(\omega,\mu)$-closedness are independent.*

Example 3.46. *Let $X = \mathbb{R}$ and $\mu = \{\emptyset, X, \mathbb{Q}\}$.*

(1) *Let $H = \mathbb{Q}$. Then H is $\delta(\omega,\mu)$-closed but not $\beta(\omega,\mu)$-closed, since $\mathbb{Q}$ is not $\beta(\omega,\mu)$-open.*

(2) *Let $X = \mathbb{R}$ and $\mu = \{\emptyset$, all possible unions of members of $\mathcal{B}\}$, where $\mathcal{B} = \{(a,b) : a,b \in \mathbb{R}\}$. Let $H = \mathbb{Q}$. Then H is $\beta(\omega,\mu)$-closed but not $\delta(\omega,\mu)$-closed, since $\mathbb{Q}$ is not $\delta(\omega,\mu)$-open.*

Theorem 3.47. *A subset H of X is $\alpha(\omega,\mu)$-open if and only if it is both $\delta(\omega,\mu)$-open and $\pi(\omega,\mu)$-open.*

Proof. Let H be an $\alpha(\omega,\mu)$-open set. Then $H \subset i_\omega(c(i_\omega(H)))$. It implies that $c(H) \subset c(i_\omega(H))$ and $i_\omega(c(H)) \subset i_\omega(c(i_\omega(H))) \subset c(i_\omega(H))$. Hence H is a $\delta(\omega,\mu)$-open set. On the other hand, since H is an $\alpha(\omega,\mu)$-open set, H is a $\pi(\omega,\mu)$-open set. Conversely, let H be both $\delta(\omega,\mu)$-open and $\pi(\omega,\mu)$-open. Since H is $\delta(\omega,\mu)$-open, $i_\omega(c(H)) \subset c(i_\omega(H))$ and hence $i_\omega(c(H)) \subset i_\omega(c(i_\omega(H)))$. Since H is $\pi(\omega,\mu)$-open, $H \subset i_\omega(c(H))$. Hence $H \subset i_\omega(c(i_\omega(H)))$ which proves that H is an $\alpha(\omega,\mu)$-open set. □

Remark 3.48. *The concepts of $\delta(\omega,\mu)$-openness and $\pi(\omega,\mu)$-openness are independent. Consider (X,μ) as in Example 3.22. Clearly, $H = (0,1]$ is $\delta(\omega,\mu)$-open but not $\pi(\omega,\mu)$-open and $H = \mathbb{Q}$ is $\pi(\omega,\mu)$-open but not $\delta(\omega,\mu)$-open.*

Proposition 3.49. *Let A and B be subsets of a GT (X,μ). If $A \subset B \subset c(A)$ and A is $\delta(\omega,\mu)$-open in X, then B is $\delta(\omega,\mu)$-open in X.*

Proof. Suppose that $A \subset B \subset c(A)$ and A is $\delta(\omega,\mu)$-open in X. Then $i_\omega(c(A)) \subset c(i_\omega(A))$. Since $A \subset B$, $c(i_\omega(A)) \subset c(i_\omega(B))$ and $i_\omega(c(A)) \subset c(i_\omega(B))$. Since $B \subset c(A)$, we have $c(B) \subset c(c(A)) = c(A)$ and $i_\omega(c(B)) \subset i_\omega(c(A))$. Hence $i_\omega(c(B)) \subset c(i_\omega(B))$. This shows that B is a $\delta(\omega,\mu)$-open set. □

Corollary 3.50. *Let (X,μ) be a GT. If $A \subset X$ is $\delta(\omega,\mu)$-open and dense in (X,μ), then every subset of X containing A is $\delta(\omega,\mu)$-open.*

Proof. Clear. □

Definition 3.51. *A subset H of a GT (X,μ) is said to be*

(1) *$\sigma(\omega^\star,\mu)$-open if $H \subset c_\omega(i(H))$.*

(2) *$\sigma(\omega^\star,\mu)$-closed if $i_\omega(c(H)) \subset H$.*

Example 3.52. *Let $X = \{a,b,c\}$ and $\mu = \{\emptyset, X, \{a\}, \{a,b\}\}$. (1) Let $H = \{a\}$. Then H is $\sigma(\omega^\star,\mu)$-open, since $i(H) = \{a\}$ and $c_\omega(i(H)) = \{a\}$. (2) Let $H = \{c\}$. Then H is not $\sigma(\omega^\star,\mu)$-open, since $i(H) = \emptyset$ and $c_\omega(i(H)) = \emptyset$.*

Proposition 3.53. *For a subset of a GT (X,μ), every $\sigma(\omega^\star,\mu)$-open set is $\sigma(\omega,\mu)$-open.*

Proof. If H is $\sigma(\omega^\star,\mu)$-open set, then $H \subset c_\omega(i(H)) \subset c(i_\omega(H))$. Therefore H is $\sigma(\omega,\mu)$-open. □

Example 3.54. *Consider (X,μ) as in Example 3.22. Let $H = \mathbb{Q}$. Then H is $\sigma(\omega,\mu)$-open but not $\sigma(\omega,\mu)$-open, since $c(i_\omega(H)) = c(\mathbb{Q}) = \mathbb{R}$ and $c_\omega(i(H)) = c_\omega(\emptyset) = \emptyset$.*

Proposition 3.55. *A subset H of a GT $(X,\mu)$is $\sigma(\omega^\star,\mu)$-open if and only if $c_\omega(H) = c_\omega(i(H))$.*

Proof. If H is $\sigma(\omega^\star,\mu)$-open set, then $H \subset c_\omega(i(H))$ and $c_\omega(H) \subset c_\omega(i(H))$. But $c_\omega(i(H)) \subset c_\omega(H)$. Hence $c_\omega(H) = c_\omega(i(H))$. Conversely, let the condition hold. We have $H \subset c_\omega(H)$ and $c_\omega(H) = c_\omega(i(H))$. Therefore H is $\sigma(\omega^\star,\mu)$-open. □

Definition 3.56. *A subset H of a GT (X,μ) is said to be $t(\omega^\star,\mu)$-set if $i_\omega(c(H)) = i_\omega(H)$.*

Example 3.57. *Consider (X,μ) as in Example 3.22. Then $(0,1]$ is a $t(\omega^\star,\mu)$-set and $\mathbb{Q}$ is not a $t(\omega^\star,\mu)$-set.*

Proposition 3.58. *In a GT (X,μ), every μ-closed set is a $t(\omega^\star,\mu)$-set.*

Proof. Let H be a μ-closed set. Then $H = c(H)$ and we have $i_\omega(c(H)) = i_\omega(H)$ which proves that H is a $t(\omega^\star,\mu)$-set. □

Example 3.59. *Consider (X,μ) as in Example 3.22. Then $(0,1]$ is $t(\omega^\star,\mu)$-set but not μ-closed.*

Proposition 3.60. *In a GT (X,μ), every $t(\omega,\mu)$-set is a $t(\omega^\star,\mu)$-set.*

Proof. If H is a $t(\omega, \mu)$-set, then $i_\omega(c(H)) = i(H) \subset i_\omega(H) \subset i_\omega(c(H))$. Thus we have $i_\omega(c(H)) = i_\omega(H)$ and hence H is a $t(\omega^\star, \mu)$-set. $\square$

Example 3.61. *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then $H = \{c\}$ is a $t(\omega^\star, \mu)$-set but not a $t(\omega, \mu)$-set. Since $i_\omega(H) = H$, $i(H) = \emptyset$ and $i_\omega(c(H)) = i_\omega(H) = H$, we have $i_\omega(c(H)) = i_\omega(H)$ and $i_\omega(c(H)) \neq i(H)$. This proves that H is a $t(\omega^\star, \mu)$-set but not a $t(\omega, \mu)$-set.*

Theorem 3.62. *A subset H of a GT (X, μ) is $\sigma(\omega^\star, \mu)$-closed if and only if H is a $t(\omega^\star, \mu)$-set.*

Proof. Let H be a $\sigma(\omega^\star, \mu)$-closed set in X. Then $X \backslash H$ is $\sigma(\omega^\star, \mu)$-open. Then we have $c_\omega(X \backslash H) = c_\omega(i(X \backslash H))$. It follows that $X \backslash i_\omega(H) = c_\omega(X \backslash c(H)) = X \backslash i_\omega(c(H))$. Thus, $i_\omega(c(H)) = i_\omega(H)$ and hence H is a $t(\omega^\star, \mu)$-set in X. Conversely, let H be a $t(\omega^\star, \mu)$-set. Then $i_\omega(c(H)) = i_\omega(H) \subset H$. Therefore H is $\sigma(\omega^\star, \mu)$-closed. $\square$

Proposition 3.63. *If A and B are $t(\omega^\star, \mu)$-sets of a GT (X, μ), then $A \cap B$ is a $t(\omega^\star, \mu)$-set.*

Proof. Let A and B be $t(\omega^\star, \mu)$-sets. Then $i_\omega(A \cap B) \subset i_\omega(c(A \cap B)) \subset i_\omega(c(A) \cap c(B)) = i_\omega(c(A)) \cap i_\omega(c(B)) = i_\omega(A) \cap i_\omega(B) = i_\omega(A \cap B)$. Then $i_\omega(A \cap B) = i_\omega(c(A \cap B))$ and hence $A \cap B$ is an $t(\omega^\star, \mu)$-set. $\square$

Definition 3.64. *A subset H of a GT (X, μ) is said to be $\sigma(\omega, \mu)$-regular if H is $\sigma(\omega, \mu)$-open and a $t(\omega^\star, \mu)$-set.*

Example 3.65. *Consider (X, μ) as in Example 3.22. Then $(0, 1]$ is $\sigma(\omega, \mu)$-regular and $\mathbb{R} \backslash \mathbb{Q}$ is not $\sigma(\omega, \mu)$-regular, since $\mathbb{R} \backslash \mathbb{Q}$ is not $t(\omega^\star, \mu)$-set.*

Theorem 3.66. *A subset H is $\sigma(\omega, \mu)$-regular if and only if H is both $\beta(\omega, \mu)$-open and $\sigma(\omega^\star, \mu)$-closed.*

Proof. If H is $\sigma(\omega, \mu)$-regular, then H is both $\sigma(\omega, \mu)$-open and a $t(\omega^\star, \mu)$-set. Since every $\sigma(\omega, \mu)$-open set is $\beta(\omega, \mu)$-open, H is both $\beta(\omega, \mu)$-open and a $t(\omega^\star, \mu)$-set. Conversely, let H be $\sigma(\omega^\star, \mu)$-closed and $\beta(\omega, \mu)$-open. Since H is a $\sigma(\omega^\star, \mu)$-closed, by Theorem 11, H is a $t(\omega^\star, \mu)$-set. Since H is $\beta(\omega, \mu)$-open, $H \subset c(i_\omega(c(H))) = c(i_\omega(H))$. Therefore H is $\sigma(\omega, \mu)$-open. Since H is both $\sigma(\omega, \mu)$-open and a $t(\omega^\star, \mu)$-set, H is $\sigma(\omega, \mu)$-regular. $\square$

Remark 3.67. *The concepts of $\beta(\omega, \mu)$-openness and $\sigma(\omega^\star, \mu)$-closedness are independent.*

Example 3.68. *Let $X = \mathbb{R}$ and $\mu = \{\emptyset, X, \mathbb{R} \setminus \mathbb{Q}\}$.*

(1) *Then $H = \mathbb{Q}$ is $\sigma(\omega^\star, \mu)$-closed but not $\beta(\omega, \mu)$-open. Since $i_\omega(c(H)) = i_\omega(H) = \emptyset \subset H$, H is $\sigma(\omega^\star, \mu)$-closed. Also H is not $\beta(\omega, \mu)$-open.*

(2) *Let $X = \mathbb{R}$ and $\mu = \{\emptyset$, all possible unions of members of $\mathcal{B}\}$, where $\mathcal{B} = \{(a, b) : a, b \in \mathbb{R}\}$. Let $H = \mathbb{Q}$. Then H is $\beta(\omega, \mu)$-open but not $\sigma(\omega^\star, \mu)$-closed, since $i_\omega(c(H)) = i_\omega(\mathbb{R}) = \mathbb{R}$.*

Theorem 3.69. *Let (X, μ) be a GT and H a subset of X. Then the following properties are equivalent.*

(1) *$H \neq \emptyset$ is $r(\omega, \mu)$-closed.*

(2) *There exists a non-empty ω-μ-open set G such that $G \subset H = c(G)$.*

(3) *There exists a non-empty ω-μ-open set G such that $H = G \cup (c(G) \backslash G)$.*

Proof. (1) $\Rightarrow$ (2): Suppose $H \neq \emptyset$ is an $r(\omega, \mu)$-closed set. Then $H = c(i_\omega(H))$. Let $G = i_\omega(H)$. Then G is the required ω-μ-open set such that $G \subset H = c(G)$.
(2) $\Rightarrow$ (3): Since $H = c(G) = G \cup (c(G) \backslash G)$ where G is a nonempty ω-μ-open set, (3) follows.
(3) $\Rightarrow$ (1): $H = G \cup c(G) \backslash G)$ implies that $H = c(G) = c(i_\omega(G)) \subset c(i_\omega(H))$, since G is ω-μ-open and $G \subset H$. Again $i_\omega(H) \subset H$ implies that $c(i_\omega(H)) \subset c(H) = c(G) = H$. Therefore $H = c(i_\omega(H))$ which implies that H is $r(\omega, \mu)$-closed. □

Theorem 3.70. *Let H be a subset of a GT (X, μ). If H is $\beta(\omega, \mu)$-open, then $c(H)$ is $r(\omega, \mu)$-closed.*

Proof. Suppose H is $\beta(\omega, \mu)$-open. Then $H \subset c(i_\omega(c(H)))$ and so $c(H) \subset c(i_\omega(c(H))) \subset c(H)$ which implies that $c(H) = c(i_\omega(c(H)))$. Therefore $c(H)$ is $r(\omega, \mu)$-closed. □

Remark 3.71. (i) *The concepts of $\sigma(\omega, \mu)$-openness and μ-closedness are independent.* (ii) *The concepts of $\beta(\omega, \mu)$-openness and μ-closedness are independent.*

Example 3.72. *Let $X = \mathbb{R}$ and $\mu = \{\emptyset$, all possible unions of members of $\mathcal{B}\}$, where $\mathcal{B} = \{(a, b) : a, b \in \mathbb{R}\}$.*

(1) *Let $H = (0, 1]$. Then H is $\sigma(\omega, \mu)$-open but not μ-closed.*

(2) *Let $X = \mathbb{R}$ and $\mu = \{\emptyset, R, \mathbb{Q}\}$. Let $H = \mathbb{Q}$. Then H is μ-closed but not $\sigma(\omega, \mu)$-open.*

Example 3.73. *Let $X = \mathbb{R}$ and $\mu = \{\emptyset$, all possible unions of members of $\mathcal{B}\}$, where $\mathcal{B} = \{(a, b) : a, b \in \mathbb{R}\}$.*

(1) *Let $H = (0, 1]$. Then H is $\beta(\omega, \mu)$-open but not μ-closed.*

(2) *Let $X = \mathbb{R}$ and $\mu = \{\emptyset, R, \mathbb{Q}\}$. Let $H = \mathbb{Q}$. Then H is μ-closed but not $\beta(\omega, \mu)$-open.*

Definition 3.74. *A GT (X, μ) is called ω-μ-submaximal if every ω-μ-dense subset of X is ω-μ-open.*

Proposition 3.75. *Every μ-submaximal space is ω-μ-submaximal.*

Proof. Let $H \subset X$ be ω-μ-dense. Then $X = c_\omega(H) \subset c(H)$ and $X = c(H)$. Thus H is μ-dense in X. Since X is μ-submaximal, H is μ-open and hence ω-μ-open in X. Therefore, X is ω-μ-submaximal. □

Example 3.76. *Let $X = \{a, b, c\}$ and $\mu = \{\emptyset, X, \{c\}, \{b, c\}\}$. Set $H = \{a, c\}$. Then $c(H) = X$ and $H \notin \mu$. Hence X is not μ-submaximal but it is ω-μ-submaximal, since the only ω-μ-dense set is X.*

Definition 3.77. *A subset H of a GT (X, μ) is called ω-μ-codense if $X \backslash H$ is ω-μ-dense.*

Theorem 3.78. *For a GT (X, μ), the following are equivalent.*

(1) *X is ω-μ-submaximal,*

(2) *Every ω-μ-codense subset H of X is ω-μ-closed.*

Proof. (1) $\Rightarrow$ (2): Let H be a ω-μ-codense subset of X. Then $X\backslash H$ is ω-μ-dense and therefore $X\backslash H$ is ω-μ-open, X being ω-μ-submaximal by assumption. Thus H is ω-μ-closed.
(2)$\Rightarrow$ (1): Let H be a ω-μ-dense subset of X. Then $X\backslash H$ is ω-μ-codense in X and by assumption $X\backslash H$ is ω-μ-closed. Hence H is ω-μ-open and thus X is ω-μ-submaximal. □

4. Some Decomposition Theorems

Using the sets defined in the above section, we define some new notions of continuous functions and find some characterizations of them.

Definition 4.1. *A function $f : (X, \mu) \to (Y, \lambda)$ is said to be*

(1) *pre-(ω, λ)-continuous if $f^{-1}(V)$ is $\pi(\omega, \mu)$-open in X for each $V \in \lambda$,*
(2) *α-(ω, λ)-continuous if $f^{-1}(V)$ is $\alpha(\omega, \mu)$-open in X for each $V \in \lambda$,*
(3) *semi-(ω, λ)-continuous if $f^{-1}(V)$ is $\sigma(\omega, \mu)$-open in X for each $V \in \lambda$,*
(4) *β-(ω, λ)-continuous if $f^{-1}(V)$ is $\beta(\omega, \mu)$-open in X for each $V \in \lambda$,*
(5) *δ-(ω, λ)-continuous if $f^{-1}(V)$ is $\delta(\omega, \mu)$-open in X for each $V \in \lambda$,*
(6) *contra-pre-(ω, λ)-continuous if $f^{-1}(V)$ is $\pi(\omega, \mu)$-closed in X for each $V \in \lambda$,*
(7) *contra-α-(ω, λ)-continuous if $f^{-1}(V)$ is $\alpha(\omega, \mu)$-closed in X for each $V \in \lambda$,*
(8) *contra-semi-(ω, λ)-continuous if $f^{-1}(V)$ is $\sigma(\omega, \mu)$-closed in X for each $V \in \lambda$,*
(9) *contra-β-(ω, λ)-continuous if $f^{-1}(V)$ is $\beta(\omega, \mu)$-closed in X for each $V \in \lambda$,*
(10) *contra-δ-(ω, λ)-continuous if $f^{-1}(V)$ is $\delta(\omega, \mu)$-closed in X for each $V \in \lambda$,*
(11) *$semi^{*}$-(ω, λ)-continuous $f^{-1}(V)$ is $\sigma(\omega^{\star}, \mu)$-closed in X for each μ-closed set V in Y,*
(12) *ω^{*}-t-continuous $f^{-1}(V)$ is ω^{*}-t-set in X for each μ-closed set V in Y,*
(13) *ω-R-continuous $f^{-1}(V)$ is ω-R-closed in X for each μ-closed set V in Y.*

Theorem 4.2. *A function $f : (X, \mu) \to (Y, \lambda)$ is α-(ω, λ)-continuous if and only if it is semi-(ω, λ)-continuous and pre-(ω, λ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.6. □

Theorem 4.3. *A function $f : (X, \mu) \to (Y, \lambda)$ is is contra-α-(ω, λ)-continuous if and only if it is contra-semi-(ω, λ)-continuous and contra-pre-(ω, λ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.6. □

Theorem 4.4. *Let X be a μ-submaximal space. A function $f : (X, \mu) \to (Y, \lambda)$ is semi-(ω, λ)-continuous if and only if it is β-(ω, λ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.7. □

Theorem 4.5. *Let X be a μ-submaximal space. A function $f : (X, \mu) \to (Y, \lambda)$ is contra-semi-(ω, λ)-continuous if and only if it is contra-β-(ω, λ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.7. □

Theorem 4.6. *A function $f : (X, \mu) \to (Y, \lambda)$ is semi-(ω, λ)-continuous if and only if it is β-(ω, λ)-continuous and δ-(ω, λ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.10. □

Theorem 4.7. *A function $f : (X, \mu) \to (Y, \lambda)$ is contra-semi-(ω, λ)-continuous if and only if it is contra-β-(ω, λ)-continuous and contra-δ-(ω, λ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.10. □

Theorem 4.8. *A function $f : (X, \mu) \to (Y, \lambda)$ is α-(ω, λ)-continuous if and only if it is δ-(ω, λ)-continuous and pre-(ω, λ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.11. □

Theorem 4.9. *A function $f : (X, \mu) \to (Y, \lambda)$ is contra-α-(ω, λ)-continuous if and only if it is contra-δ-(ω, λ)-continuous and contra-pre-(ω, λ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.11. □

Definition 4.10. *A function $f : (X, \mu) \to (Y, \lambda)$ is said to be semi-ω-regular continuous if $f^{-1}(V)$ is semi-ω-regular in X for each closed set V in Y.*

Theorem 4.11. *A function $f : (X, \mu) \to (Y, \lambda)$ is $semi^*$-(ω, λ)-continuous if and only if it is ω^*-t-continuous.*

Proof. This is an immediate consequence of Theorem 3.15. □

Theorem 4.12. *A function $f : (X, \mu) \to (Y, \lambda)$ is semi-ω-regular continuous if and only if it is contra-β-(ω, λ)-continuous and $semi^*$-(ω, λ)-continuous.*

Proof. This is an immediate consequence of Theorem 3.17. □

Theorem 4.13. *For a function $f : (X, \mu) \to (Y, \lambda)$, the following are equivalent:*

(1) *f is ω-R-continuous.*

(2) *f is contra-semi-(ω, λ)-continuous and continuous.*

(3) *f is contra-β-(ω, λ)-continuous and continuous.*

Proof. This is an immediate consequence of Theorem 3.20. □

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