

$\theta p(\Lambda, p)$ -OPEN FUNCTIONS AND  $\theta p(\Lambda, p)$ -CLOSED FUNCTIONS

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ABSTRACT. This article deals with the concepts of  $\theta p(\Lambda, p)$ -open functions and  $\theta p(\Lambda, p)$ -closed functions. Moreover, several characterizations of  $\theta p(\Lambda, p)$ -open functions and  $\theta p(\Lambda, p)$ -closed functions are considered. 2020 Mathematics Subject Classification. 54A05, 54C10.

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## 1. INTRODUCTION

The concept of weakly open functions was first introduced by Rose [12]. Rose and Janković [11] investigated some of the fundamental properties of weakly closed functions. Caldas and Navalagi [5] introduced two new classes of functions called weakly preopen functions and weakly preclosed functions as generalization of weak openness and weak closedness due to [12] and [11], respectively. Moreover, Caldas and Navalagi [4] introduced and investigated the concepts of weakly semi-open functions and weakly semi-closed functions as a new generalization of weakly open functions and weakly closed functions, respectively. Noiri and Popa [8] studied a new class of functions called  $M$ -closed functions as functions defined between sets satisfying some conditions. Pal et al. [10] introduced and studied the notion of pre- $\theta$ -closed sets in topological spaces. Caldas et al. [3] introduced the notions of pre- $\theta$ -derided, pre- $\theta$ -border, pre- $\theta$ -frontier and pre- $\theta$ -exterior of a set. Noiri [9] introduced and investigated the notion of  $\theta$ -precontinuous functions. Furthermore, Caldas et al. [3] defined the concepts of  $\theta$ -preopenness and  $\theta$ -preclosedness as a natural dual to the  $\theta$ -precontinuity due to Noiri [9]. In [2], the present authors investigated some properties of  $(\Lambda, sp)$ -closed sets and  $(\Lambda, sp)$ -open sets. Boonpok and Viriyapong [1] introduced and studied the notions of  $(\Lambda, p)$ -closed sets and  $(\Lambda, p)$ -open sets. Srisarakham and Boonpok [13] investigated several properties of  $\delta p(\Lambda, s)$ -closed sets and the

$\delta p(\Lambda, s)$ -closure operator. In this article, we introduce the concepts of  $\theta(\Lambda, p)$ -open functions and  $\theta(\Lambda, p)$ -closed functions. Moreover, several characterizations of  $\theta(\Lambda, p)$ -open functions and  $\theta(\Lambda, p)$ -closed functions are investigated.

## 2. PRELIMINARIES

Throughout the present paper, spaces  $(X, \tau)$  and  $(Y, \sigma)$  (or simply  $X$  and  $Y$ ) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset  $A$  of a topological space  $(X, \tau)$ ,  $\text{Cl}(A)$  and  $\text{Int}(A)$ , represent the closure and the interior of  $A$ , respectively. A subset  $A$  of a topological space  $(X, \tau)$  is said to be *preopen* [7] if  $A \subseteq \text{Int}(\text{Cl}(A))$ . The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space  $(X, \tau)$  is denoted by  $PO(X, \tau)$ . A subset  $\Lambda_p(A)$  [6] is defined as follows:  $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$ . A subset  $A$  of a topological space  $(X, \tau)$  is called a  $\Lambda_p$ -set [1] (*pre- $\Lambda$ -set* [6]) if  $A = \Lambda_p(A)$ . A subset  $A$  of a topological space  $(X, \tau)$  is called  $(\Lambda, p)$ -closed [1] if  $A = T \cap C$ , where  $T$  is a  $\Lambda_p$ -set and  $C$  is a preclosed set. The complement of a  $(\Lambda, p)$ -closed set is called  $(\Lambda, p)$ -open. The family of all  $(\Lambda, p)$ -open (resp.  $(\Lambda, p)$ -closed) sets in a topological space  $(X, \tau)$  is denoted by  $\Lambda_p O(X, \tau)$  (resp.  $\Lambda_p C(X, \tau)$ ). Let  $A$  be a subset of a topological space  $(X, \tau)$ . A point  $x \in X$  is called a  $(\Lambda, p)$ -cluster point [1] of  $A$  if  $A \cap U \neq \emptyset$  for every  $(\Lambda, p)$ -open set  $U$  of  $X$  containing  $x$ . The set of all  $(\Lambda, p)$ -cluster points of  $A$  is called the  $(\Lambda, p)$ -closure [1] of  $A$  and is denoted by  $A^{(\Lambda, p)}$ . The union of all  $(\Lambda, p)$ -open sets of  $X$  contained in  $A$  is called the  $(\Lambda, p)$ -interior [1] of  $A$  and is denoted by  $A_{(\Lambda, p)}$ .

The  $\theta(\Lambda, p)$ -closure [1] of  $A$ ,  $A^{\theta(\Lambda, p)}$ , is defined as follows:  
 $A^{\theta(\Lambda, p)} = \{x \in X \mid A \cap U^{(\Lambda, p)} \neq \emptyset \text{ for each } (\Lambda, p)\text{-open set } U \text{ containing } x\}$ . A subset  $A$  of a topological space  $(X, \tau)$  is called  $\theta(\Lambda, p)$ -closed [1] if  $A = A^{\theta(\Lambda, p)}$ . The complement of a  $\theta(\Lambda, p)$ -closed set is said to be  $\theta(\Lambda, p)$ -open. A point  $x \in X$  is called a  $\theta(\Lambda, p)$ -interior point [14] of  $A$  if  $x \in U \subseteq U^{(\Lambda, p)} \subseteq A$  for some  $U \in \Lambda_p O(X, \tau)$ . The set of all  $\theta(\Lambda, p)$ -interior points of  $A$  is called the  $\theta(\Lambda, p)$ -interior [14] of  $A$  and is denoted by  $A_{\theta(\Lambda, p)}$ .

**Lemma 1.** [14] For subsets  $A$  and  $B$  of a topological space  $(X, \tau)$ , the following properties hold:

- (1)  $X - A^{\theta(\Lambda, p)} = [X - A]_{\theta(\Lambda, p)}$  and  $X - A_{\theta(\Lambda, p)} = [X - A]^{\theta(\Lambda, p)}$ .
- (2)  $A$  is  $\theta(\Lambda, p)$ -open if and only if  $A = A_{\theta(\Lambda, p)}$ .
- (3)  $A \subseteq A^{(\Lambda, p)} \subseteq A^{\theta(\Lambda, p)}$  and  $A_{\theta(\Lambda, p)} \subseteq A_{(\Lambda, p)} \subseteq A$ .
- (4) If  $A \subseteq B$ , then  $A^{\theta(\Lambda, p)} \subseteq B^{\theta(\Lambda, p)}$  and  $A_{\theta(\Lambda, p)} \subseteq B_{\theta(\Lambda, p)}$ .
- (5) If  $A$  is  $(\Lambda, p)$ -open, then  $A^{(\Lambda, p)} = A^{\theta(\Lambda, p)}$ .

A subset  $A$  of a topological space  $(X, \tau)$  is said to be  $p(\Lambda, p)$ -open [1] (resp.  $\alpha(\Lambda, p)$ -open [15],  $r(\Lambda, p)$ -open [1]) if  $A \subseteq [A^{(\Lambda, p)}]_{(\Lambda, p)}$  (resp.  $A \subseteq [[A_{(\Lambda, p)}]^{(\Lambda, p)}]_{(\Lambda, p)}$ ,  $A = [A^{(\Lambda, p)}]_{(\Lambda, p)}$ ). The complement of a

$p(\Lambda, p)$ -open (resp.  $\alpha(\Lambda, p)$ -open,  $r(\Lambda, p)$ -open) set is called  $p(\Lambda, p)$ -closed (resp.  $\alpha(\Lambda, p)$ -closed,  $r(\Lambda, p)$ -closed). The intersection of all  $p(\Lambda, p)$ -closed sets of  $X$  containing  $A$  is called the  $p(\Lambda, p)$ -closure of  $A$  and is denoted by  $A^{p(\Lambda, p)}$ . Let  $A$  be a subset of a topological space  $(X, \tau)$ . A point  $x \in X$  is called a  $\theta p(\Lambda, p)$ -cluster point of  $A$  if  $A \cap U^{p(\Lambda, p)} \neq \emptyset$  for every  $p(\Lambda, p)$ -open set  $U$  of  $X$  containing  $x$ . The set of all  $\theta p(\Lambda, p)$ -cluster points of  $A$  is called the  $\theta p(\Lambda, p)$ -closure of  $A$  and is denoted by  $A^{\theta p(\Lambda, p)}$ . If  $A = A^{\theta p(\Lambda, p)}$ , then  $A$  is called  $\theta p(\Lambda, p)$ -closed. The complement of a  $\theta p(\Lambda, p)$ -closed set is called  $\theta p(\Lambda, p)$ -open. The  $\theta p(\Lambda, p)$ -interior of  $A$  is defined by the union of all  $\theta p(\Lambda, p)$ -open sets of  $X$  contained in  $A$  and is denoted by  $A_{\theta p(\Lambda, p)}$ .

**Lemma 2.** For subsets  $A$  and  $B$  of a topological space  $(X, \tau)$ , the following properties hold:

- (1)  $X - A^{\theta p(\Lambda, p)} = [X - A]_{\theta p(\Lambda, p)}$  and  $X - A_{\theta p(\Lambda, p)} = [X - A]^{\theta p(\Lambda, p)}$ .
- (2)  $A$  is  $\theta p(\Lambda, p)$ -open if and only if  $A = A_{\theta p(\Lambda, p)}$ .
- (3)  $A \subseteq A^{p(\Lambda, p)} \subseteq A^{\theta p(\Lambda, p)}$  and  $A_{\theta p(\Lambda, p)} \subseteq A_{p(\Lambda, p)} \subseteq A$ .
- (4) If  $A \subseteq B$ , then  $A^{\theta p(\Lambda, p)} \subseteq B^{\theta p(\Lambda, p)}$  and  $A_{\theta p(\Lambda, p)} \subseteq B_{\theta p(\Lambda, p)}$ .
- (5) If  $A$  is  $p(\Lambda, p)$ -open, then  $A^{p(\Lambda, p)} = A^{\theta p(\Lambda, p)}$ .

### 3. CHARACTERIZATIONS OF $\theta p(\Lambda, p)$ -OPEN FUNCTIONS

In this section, we introduce the concept of  $\theta p(\Lambda, p)$ -open functions. Moreover, some characterizations of  $\theta p(\Lambda, p)$ -open functions are discussed.

**Definition 1.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be  $\theta p(\Lambda, p)$ -open if  $f(U) \subseteq [f(U^{\Lambda, p})]_{\theta p(\Lambda, p)}$  for each  $(\Lambda, p)$ -open set  $U$  of  $X$ .

**Theorem 1.** For a function  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the following properties are equivalent:

- (1)  $f$  is  $\theta p(\Lambda, p)$ -open;
- (2)  $f(A_{\theta p(\Lambda, p)}) \subseteq [f(A)]_{\theta p(\Lambda, p)}$  for every subset  $A$  of  $X$ ;
- (3)  $[f^{-1}(B)]_{\theta p(\Lambda, p)} \subseteq f^{-1}(B_{\theta p(\Lambda, p)})$  for every subset  $B$  of  $Y$ ;
- (4)  $f^{-1}(B^{\theta p(\Lambda, p)}) \subseteq [f^{-1}(B)]^{\theta p(\Lambda, p)}$  for every subset  $B$  of  $Y$ ;
- (5)  $f(K_{(\Lambda, p)}) \subseteq [f(K)]_{\theta p(\Lambda, p)}$  for each  $(\Lambda, p)$ -closed set  $K$  of  $X$ ;
- (6)  $f([U^{\Lambda, p}]_{(\Lambda, p)}) \subseteq [f(U^{\Lambda, p})]_{\theta p(\Lambda, p)}$  for each  $(\Lambda, p)$ -open set  $U$  of  $X$ ;
- (7)  $f(U) \subseteq [f(U^{\Lambda, p})]_{\theta p(\Lambda, p)}$  for each  $r(\Lambda, p)$ -open set  $U$  of  $X$ ;
- (8)  $f(U) \subseteq [f(U^{\Lambda, p})]_{\theta p(\Lambda, p)}$  for each  $\alpha(\Lambda, p)$ -open set  $U$  of  $X$ .

*Proof.* The proofs of (5)  $\Rightarrow$  (6)  $\Rightarrow$  (7)  $\Rightarrow$  (8)  $\Rightarrow$  (1) are straightforward and are omitted.

(1)  $\Rightarrow$  (2): Let  $A$  be any subset of  $X$  and  $x \in A_{\theta p(\Lambda, p)}$ . Then, there exists a  $(\Lambda, p)$ -open set  $U$  of  $X$  such that

$$x \in U \subseteq U^{\Lambda, p} \subseteq U.$$

Then,  $f(x) \in f(U) \subseteq f(U^{(\Lambda,p)}) \subseteq f(A)$ . Since  $f$  is  $\theta_p(\Lambda,p)$ -open,  $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta_p(\Lambda,p)} \subseteq [f(A)]_{\theta_p(\Lambda,p)}$ . It implies that

$$f(x) \in [f(A)]_{\theta_p(\Lambda,p)}.$$

Therefore,  $x \in f^{-1}([f(A)]_{\theta_p(\Lambda,p)})$ . Thus,  $A_{\theta(\Lambda,p)} \subseteq f^{-1}([f(A)]_{\theta_p(\Lambda,p)})$  and hence  $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{\theta_p(\Lambda,p)}$ .

(2)  $\Rightarrow$  (3): Let  $B$  be any subset of  $Y$ . Then by (2),

$$f([f^{-1}(B)]_{\theta(\Lambda,p)}) \subseteq B_{\theta_p(\Lambda,p)}.$$

Thus,  $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\theta_p(\Lambda,p)})$ .

(3)  $\Rightarrow$  (4): Let  $B$  be any subset of  $Y$ . Using (3), we have

$$\begin{aligned} X - [f^{-1}(B)]^{\theta(\Lambda,p)} &= [X - f^{-1}(B)]_{\theta(\Lambda,p)} \\ &= [f^{-1}(Y - B)]_{\theta(\Lambda,p)} \\ &\subseteq f^{-1}([Y - B]_{\theta_p(\Lambda,p)}) \\ &= f^{-1}(Y - B^{\theta_p(\Lambda,p)}) \\ &= X - f^{-1}(B^{\theta_p(\Lambda,p)}) \end{aligned}$$

and hence  $f^{-1}(B^{\theta_p(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$ .

(4)  $\Rightarrow$  (5): Let  $K$  be any  $(\Lambda,p)$ -closed set of  $X$ . Thus, by (4),

$$f^{-1}([Y - f(K)]^{\theta_p(\Lambda,p)}) \subseteq [f^{-1}(Y - f(K))]^{\theta(\Lambda,p)}.$$

We have

$$\begin{aligned} f^{-1}([Y - f(K)]^{\theta_p(\Lambda,p)}) &= f^{-1}(Y - [f(K)]_{\theta_p(\Lambda,p)}) \\ &= X - f^{-1}([f(K)]_{\theta_p(\Lambda,p)}). \end{aligned}$$

On the other hand,

$$\begin{aligned} [f^{-1}(Y - f(K))]^{\theta(\Lambda,p)} &= [X - f^{-1}(f(K))]^{\theta(\Lambda,p)} \\ &\subseteq [X - K]^{\theta(\Lambda,p)} \\ &= X - K_{\theta(\Lambda,p)} \\ &= X - K_{(\Lambda,p)}, \end{aligned}$$

since  $K$  is  $(\Lambda,p)$ -closed. Thus,  $K_{(\Lambda,p)} \subseteq f^{-1}([f(K)]_{\theta_p(\Lambda,p)})$  and hence  $f(K_{(\Lambda,p)}) \subseteq [f(K)]_{\theta_p(\Lambda,p)}$ .  $\square$

**Theorem 2.** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a bijective function. Then, the following properties are equivalent:

- (1)  $f$  is  $\theta p(\Lambda, p)$ -open;
- (2)  $[f(U)]^{\theta p(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$  for every  $(\Lambda, p)$ -open set  $U$  of  $X$ ;
- (3)  $[f(K_{(\Lambda, p)})]^{\theta p(\Lambda, p)} \subseteq f(K)$  for every  $(\Lambda, p)$ -closed set  $K$  of  $X$ .

*Proof.* (1)  $\Rightarrow$  (3): Let  $K$  be any  $(\Lambda, p)$ -closed set of  $X$ . Then, we have

$$\begin{aligned} Y - f(K) &= f(X - K) \\ &\subseteq [f([X - K]^{(\Lambda, p)})]_{\theta p(\Lambda, p)} \end{aligned}$$

and hence  $Y - f(K) \subseteq Y - [f(K_{(\Lambda, p)})]^{\theta p(\Lambda, p)}$ . Thus,

$$[f(K_{(\Lambda, p)})]^{\theta p(\Lambda, p)} \subseteq f(K).$$

(3)  $\Rightarrow$  (2): Let  $U$  be any  $(\Lambda, p)$ -open set of  $X$ . Since  $U^{(\Lambda, p)}$  is  $(\Lambda, p)$ -closed and  $U \subseteq [U^{(\Lambda, p)}]_{(\Lambda, p)}$ , by (3) we have

$$\begin{aligned} [f(U)]^{\theta p(\Lambda, p)} &\subseteq [f([U^{(\Lambda, p)}]_{(\Lambda, p)})]^{\theta p(\Lambda, p)} \\ &\subseteq f(U^{(\Lambda, p)}). \end{aligned}$$

(2)  $\Rightarrow$  (1): Let  $U$  be any  $(\Lambda, p)$ -open set of  $X$ . By (2), we have

$$[f(X - U^{(\Lambda, p)})]^{\theta p(\Lambda, p)} \subseteq f([X - U^{(\Lambda, p)}]^{(\Lambda, p)}).$$

Since  $f$  is bijective,  $[f(X - U^{(\Lambda, p)})]^{\theta p(\Lambda, p)} = Y - [f(U^{(\Lambda, p)})]_{\theta p(\Lambda, p)}$  and

$$\begin{aligned} f([X - U^{(\Lambda, p)}]^{(\Lambda, p)}) &= f(X - [U^{(\Lambda, p)}]_{(\Lambda, p)}) \\ &\subseteq f(X - U) \\ &= Y - f(U). \end{aligned}$$

Thus,  $f(U) \subseteq [f(U^{(\Lambda, p)})]_{\theta p(\Lambda, p)}$  and hence  $f$  is  $\theta p(\Lambda, p)$ -open.  $\square$

#### 4. CHARACTERIZATIONS OF $\theta p(\Lambda, p)$ -CLOSED FUNCTIONS

In this section, we introduce the notion of  $\theta p(\Lambda, p)$ -closed functions. Furthermore, several characterizations of  $\theta p(\Lambda, p)$ -closed functions are considered.

**Definition 2.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be  $\theta p(\Lambda, p)$ -closed if  $[f(K_{(\Lambda, p)})]^{\theta p(\Lambda, p)} \subseteq f(K)$  for each  $(\Lambda, p)$ -closed set  $K$  of  $X$ .

**Theorem 3.** For a function  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the following properties are equivalent:

- (1)  $f$  is  $\theta p(\Lambda, p)$ -closed;
- (2)  $[f(U)]^{\theta p(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$  for every  $(\Lambda, p)$ -open set  $U$  of  $X$ ;
- (3)  $[f(U)]^{\theta p(\Lambda, p)} \subseteq f(U^{(\Lambda, p)})$  for every  $p(\Lambda, p)$ -open set  $U$  of  $X$ ;

- (4)  $[f(K_{(\Lambda,p)})]^{\theta p(\Lambda,p)} \subseteq f(K)$  for every  $p(\Lambda, p)$ -closed set  $K$  of  $X$ ;  
 (5)  $[f(K_{(\Lambda,p)})]^{\theta p(\Lambda,p)} \subseteq f(K)$  for every  $\alpha(\Lambda, p)$ -closed set  $K$  of  $X$ ;  
 (6)  $[f([A^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta p(\Lambda,p)} \subseteq f(A^{(\Lambda,p)})$  for every subset  $A$  of  $X$ .

*Proof.* (1)  $\Rightarrow$  (2): Let  $U$  be any  $(\Lambda, p)$ -open set of  $X$ . Then by (1),

$$[f(U)]^{\theta p(\Lambda,p)} = [f(U_{(\Lambda,p)})]^{\theta p(\Lambda,p)} \subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)}).$$

(2)  $\Rightarrow$  (3): Let  $U$  be any  $p(\Lambda, p)$ -open set of  $X$ . Using (2), we have

$$[f(U)]^{\theta p(\Lambda,p)} \subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta p(\Lambda,p)} \subseteq f([U^{(\Lambda,p)}]_{(\Lambda,p)})^{(\Lambda,p)} \subseteq f(U^{(\Lambda,p)}).$$

(3)  $\Rightarrow$  (4): Let  $K$  be any  $p(\Lambda, p)$ -closed set of  $X$ . Then, we have

$$[f(K_{(\Lambda,p)})]^{\theta p(\Lambda,p)} \subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)}) \subseteq f(K).$$

It is clear that (4)  $\Rightarrow$  (5)  $\Rightarrow$  (6)  $\Rightarrow$  (1). □

**Definition 3.** [1] A topological space  $(X, \tau)$  is said to be  $\Lambda_p$ -regular if for each  $(\Lambda, p)$ -closed set  $F$  and each  $x \notin F$ , there exist disjoint  $(\Lambda, p)$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $F \subseteq V$ .

**Lemma 3.** [1] A topological space  $(X, \tau)$  is  $\Lambda_p$ -regular if and only if for each  $x \in X$  and each  $(\Lambda, p)$ -open set  $U$  containing  $x$ , there exists a  $(\Lambda, p)$ -open set  $V$  such that  $x \in V \subseteq V^{(\Lambda,p)} \subseteq U$ .

**Theorem 4.** Let  $(Y, \sigma)$  be a  $\Lambda_p$ -regular space. Then, for a function  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the following properties are equivalent:

- (1)  $f$  is  $\theta p(\Lambda, p)$ -closed;  
 (2)  $[f(U)]^{\theta p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$  for each  $r(\Lambda, p)$ -open set  $U$  of  $X$ ;  
 (3) for each subset  $B$  of  $Y$  and each  $(\Lambda, p)$ -open set  $U$  of  $X$  with  $f^{-1}(B) \subseteq U$ , there exists a  $\theta p(\Lambda, p)$ -open set  $V$  of  $Y$  such that  $B \subseteq V$  and  $f^{-1}(V) \subseteq U^{(\Lambda,p)}$ ;  
 (4) for each point  $y \in Y$  and each  $(\Lambda, p)$ -open set  $U$  of  $X$  with  $f^{-1}(y) \subseteq U$ , there exists a  $\theta p(\Lambda, p)$ -open set  $V$  of  $Y$  containing  $y$  and  $f^{-1}(V) \subseteq U^{(\Lambda,p)}$ .

*Proof.* (1)  $\Rightarrow$  (2) and (3)  $\Rightarrow$  (4): The proofs are obvious.

(2)  $\Rightarrow$  (3): Let  $B$  be any subset of  $Y$  and  $U$  be any  $(\Lambda, p)$ -open set of  $X$  with  $f^{-1}(B) \subseteq U$ . Then,  $f^{-1}(B) \cap [X - U^{(\Lambda,p)}]^{(\Lambda,p)} = \emptyset$  and hence  $B \cap f([X - U^{(\Lambda,p)}]^{(\Lambda,p)}) = \emptyset$ . Since  $X - U^{(\Lambda,p)}$  is  $r(\Lambda, p)$ -open,  $B \cap [f(X - U^{(\Lambda,p)})]^{\theta p(\Lambda,p)} = \emptyset$  by (2). Put  $V = Y - [f(X - U^{(\Lambda,p)})]^{\theta p(\Lambda,p)}$ . Then,  $V$  is a  $\theta p(\Lambda, p)$ -open set of  $Y$  such that  $B \subseteq V$  and

$$\begin{aligned} f^{-1}(V) &\subseteq X - f^{-1}([f(X - U^{(\Lambda,p)})]^{\theta p(\Lambda,p)}) \\ &\subseteq X - f^{-1}(f(X - U^{(\Lambda,p)})) \\ &\subseteq U^{(\Lambda,p)}. \end{aligned}$$

(4)  $\Rightarrow$  (1): Let  $K$  be any  $(\Lambda, p)$ -closed set of  $Y$  and  $y \in Y - f(K)$ . Since  $f^{-1}(y) \subseteq X - K$ , there exists a  $\theta p(\Lambda, p)$ -open set  $V$  of  $Y$  such that  $y \in V$  and  $f^{-1}(V) \subseteq [X - K]^{(\Lambda, p)} = X - K_{(\Lambda, p)}$  by (4). Thus,  $V \cap f(K_{(\Lambda, p)}) = \emptyset$  and hence  $y \in Y - [f(K_{(\Lambda, p)})]^{\theta p(\Lambda, p)}$ . It implies that  $[f(K_{(\Lambda, p)})]^{\theta p(\Lambda, p)} \subseteq f(K)$ . This shows that  $f$  is  $\theta p(\Lambda, p)$ -closed.  $\square$

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