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A BIVARIATE REPLACEMENT POLICY (U, N) UNDER PARTIAL PRODUCT PROCESS

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Abstract. Considering an extreme shock maintenance model for a degenerative simple repairable system, explicit expression for the long run average cost under the bivariate replacement policy (U, N) has been obtained. Comparison of this bivariate optimal replacement policy $(U, N)^*$ with the univariate optimal replacement policies U^* and N^* is also carried out.

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1. Introduction

The study of replacement model for a simple repairable system is a fundamental and important problem in classical reliability theory. R.E. Barlow and Hunter (1960) proposed and studied the basic replacement policies. R.E. Barlow and F. Proschan (1965) have introduced age replacement model [3]. P. Govindaraju, U. Rizwan and V. Thangaraj (2011) have studied an extreme shock maintenance model under a bivariate replacement policy [6]. D. Babu, P. Govindaraju and U. Rizwan (2018) introduced and studied replacement models where the consecutive repair time follow an increasing partial product process [1]. A common assumption in replacement problems is that the repair of a failed system may yield a functions system, which may be either as good as new or as old as just prior to failure.

In this paper, we study an extreme shock maintenance model, we present the maintenance problem using the bivariate replacement policy (U, N). We also show that the bivariate optimal replacement policy $(U, N)^*$ is better than the univariate optimal replacement U^* and N^* policies.

2. Model Description

Definition 1. Partial product process (Babu, Govindaraju and Rizwan (2019))
Let $\{X_n, n = 1, 2, 3, \ldots\}$ be a sequence of independent and non-negative random variables and let

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$F(x)$ be the distribution function of X_1. Then $\{X_n, n = 1, 2, 3 \ldots\}$ is called partial product process, if the distribution function of X_{k+1} is $F(\alpha_k x)\,(k = 1, 2, 3 \cdots)$, where $\alpha_k > 0$ are real constants and $\alpha_k = \alpha_0 \alpha_1 \alpha_2 \ldots \alpha_{k-1}$. In what follows, $F(x)$ denotes the distribution function of non-negative random variable X_1.

Definition 2. A partial product process is called a decreasing partial product process, if $\alpha_0 > 1$ and is called an increasing partial product process, if $0 < \alpha_0 < 1$. It is clear that if $\alpha_0 = 1$, then the partial product process is a renewal process.

Lemma 1. Let $E(Y_1) = \mu, \operatorname{var}(Y_1) = \sigma^2$. Then for $k = 1, 2, 3, \ldots$. $E(Y_{k+1}) = \frac{\mu}{\beta_0^{2^{k-1}}}$ and $\operatorname{var}(Y_{k+1}) = \frac{\sigma^2}{{\beta_0}^{2^k}}$, where $\beta_0 > 0$.

Definition 3. A bivariate replacement policy (U, N) is a replacement model under which the system is replaced at the time of N-th failure or the total repair time exceeds U, whichever occurs earlier.

We make the following assumptions about the model for a simple degenerative reparable system subject to shocks.

Assumption 2.1: At time $t = 0$, a new system is installed . Whenever the system fails it will be repaired. The System will be replaced by an identical new one, sometimes later.

Assumption 2.2: Once the system is operating , the shocks from the environment arrive according to a renewal process. Let $X_{ni},\ i = 1, 2, \ldots$ be the intervals between the $(i-1)$-st and i-th shock, after the $(n-1)$-st repair. Let $E(X_{11}) = \lambda$. Assume that $X_{ni},\ i$=1,2,.... are independent and identically distributed random variables, for all $n \in \mathbb{N}$.

Assumption 2.3: Let $Y_{ni}, i = 1, 2, \ldots$ be the sequence of the random amount of damage produced by the i-th shock, after the $(n-1)$-st repair. Let $E(Y_{11}) = \mu$. Then $\{Y_{ni}, i = 1, 2, \ldots\}$ are iid sequences, for all $n \in \mathbb{N}$. If the system fails, it is closed, so that the random shocks have no effect on the system during the repair time. In the n-th operating stage, that is, after the $(n-1)$-st repair, the system will fail, if the amount of the shock damage first exceed $\alpha_0^{2^{n-1}} M$, where $0 < \alpha_0 \leq 1$ and M is a positive constant.

Assumption 2.4: Let $Z_n,\ n = 1, 2, \ldots$ be the repair time after the n-th repair and $\{Z_n, n = 1, 2, \ldots\}$ constitute a non decreasing Partial product process with $E(Z_1) = \delta$ and ratio β_0, such that $0 < \beta_0 < 1$. $N_n(t)$ is the counting process denoting the number of shocks after the $(n-1)$-st repair. It is clear that $E(Z_n) = \frac{\mu}{\beta_0^{2^{n-1}}}$.

Assumption 2.5: Let r be the reward rate per unit time of the system, when it is operating and c be the repair cost rate per unit time of the system and the replacement cost is R. The replacement time is a random variable Z with $E(Z) = \tau$.

Assumption 2.6: The sequences $\{X_{ni}, i = 1, 2, \ldots\}$, $\{Y_{ni}, i = 1, 2, \ldots\}$, $\{Z_n, n = 1, 2, \ldots\}$ and Z are independent.

Assumption 2.7: The replacement policy (U, N) is adapted.

3. The Bivariate Replacement Policy (U, N)

In this section, we study an extreme shock model for the maintenance problem of a simple repairable system under (U, N) policy. Let

$$L_n = \min\{l : Y_{nl} > \alpha_0^{2^{n-1}} M\}$$

and

$$W_n = \sum_{i=1}^{L_n} X_{ni}.$$

Thus L_n is the number of shocks until the first deadly shock occurred following $(n-1)$-st failure and L_n has a geometric distribution with $P[L_n = k] = p_n q_n^{k-1},\ k = 1, 2, \ldots$, where $p_n = P[Y_{nl} > \alpha_0^{2^{n-1}} M]$ and $q_n = 1 - p_n$. We have $E(L_n) = \dfrac{1}{p_n}$. Since $\{X_{ni},\ i = 1, 2, \ldots\}$ and $\{Y_{ni},\ i = 1, 2, \ldots\}$ are independent, it is clear that L_n and $\{X_{ni}\}$are independent. By Wald's equation

$$\begin{aligned} E(W_n) &= E\left(\sum_{i=1}^{L_n} X_{ni}\right) \\ &= E(L_n)\ E(X_{n1}) \\ &= \frac{\lambda}{p_n}. \end{aligned}$$

The distribution function of W_n is $F_n(\,\cdot\,)$.

The working age T of the system at time t is the cumulative life time given by

$$T(t) = \begin{cases} t - V_n, & U_n + V_n \le t < U_{n+1} + V_n \\ U_{n+1}, & U_{n+1} + V_n \le t < U_{n+1} + V_{n+1}, \end{cases}$$

where $U_n = \sum_{i=1}^{n} W_i$ and $V_n = \sum_{i=1}^{n} Z_i$ and $U_0 = V_0 = 0$.

Let U_1 be the first replacement time; in general for $n = 2, 3, \ldots$, let U_n be the time between the $(n-1)$-st replacement and the n-th replacement. Thus the sequence $\{U_n,\ n = 1, 2, \ldots\}$ forms a renewal process. A cycle is completed, if a replacement is done. A cycle is actually the time interval between the installation of the system and the first replacement or the time interval between two consecutive replacements. Finally, the successive cycles together with the cost incurred in each cycle will constitute a renewal reward process.

The length of the cycle under the replacement policy (U, N) is

$$W = \left[U + \sum_{n=1}^{\eta} Y_n\right] \chi_{(M_n > U)} + \left[\sum_{n=1}^{N} W_n + \sum_{n=1}^{N-1} Y_n\right] \chi_{(M_n \le U)} + Z,$$

where $\eta = 0, 1, 2, \dots N-1$ is the number of failures before the total repair time of the system exceeds U and $\chi_{(A)}$ denotes the indicator function. The expected length of a cycle is

$$\begin{aligned} E(W) &= E\left[U + \left(\sum_{n=1}^{\eta} Y_n\right)\chi_{(M_n>U)}\right] \\ &+E\left[\left(\sum_{n=1}^{N} W_n + \sum_{n=1}^{N-1} Y_n\right)\chi_{(M_n\leq U)}\right] + E(Z) \\ &= E\left[\left[U\chi_{(M_n>U)}\right] + E\left(\sum_{n=1}^{\eta} Y_n\right)\chi_{(M_n>U)}\right] \\ &+E\left[E\left(\sum_{n=1}^{N} W_n + \sum_{n=1}^{N-1} Y_n\right)\chi_{(M_n\leq U)}\middle| M_n = u\right] + E(Z) \end{aligned}$$

$$\begin{aligned} E(W) &= U\overline{G}_N(U) + \mu\left[G_2(U) + \sum_{n=2}^{\infty}\frac{G_{n+1}(U)}{\beta_0^{2^{n-1}}}\right] E[\chi_{(M_N\leq U<M_n)} \\ &+\int_0^U E\left[\sum_{n=1}^{N} W_n\right] udG_N(u) \\ &+\int_0^U \left[\sum_{n=1}^{N-1} E(Y_n)\right] dG_N(u) + \tau \\ &= U\overline{G}_N(U) + \mu\left[G_2(U) + \sum_{n=2}^{\infty}\frac{G_{n+1}(U)}{\beta_0^{2^{n-1}}}\right] P(M_N \leq U < M_n) \\ &+\sum_{n=1}^{N}\frac{\lambda}{p_n}\int_0^U udG_{N(u)} + \mu\left[1 + \sum_{n=2}^{N-1}\frac{1}{\beta_0^{2^{n-2}}}\right] G_N(U) + \tau \\ &= U\overline{G}_N(U) + \mu\left[G_2(U) + \sum_{n=2}^{\infty}\frac{G_{n+1}(U)}{\beta_0^{2^{n-1}}}\right]\left[G_n(U) - G_N(U)\right] \\ &+\sum_{n=1}^{N}\frac{\lambda}{p_n}\int_o^U udG_N(u) + \mu\left[1 + \sum_{n=2}^{N-1}\frac{1}{\beta_0^{2^{n-2}}}\right] G_N(U) + \tau \\ &= U\overline{G}_N(U) + \mu\left[G_2(U) + \sum_{n=2}^{\infty}\frac{G_{n+1}(U)}{\beta_0^{2^{n-1}}}\right] G_N(U) \\ &+\sum_{n=1}^{N}\frac{\lambda}{p_n}\int_o^U udG_N(u) + \mu\left[1 + \sum_{n=2}^{N-1}\frac{1}{\beta_0^{2^{n-2}}}\right] + \tau. \end{aligned} \tag{1}$$

Let $C(U, N)$ be the long run average cost per unit unit per time under the bivariate replacement policy (U, N).

By the elementary renewal theorem , the long run average cost per unit time under the replacement policy (U, N) is given by

$$C(U,N) = \frac{\text{expected cost incurred in a cycle}}{\text{expected length of a cycle}}$$
$$= \frac{\left[\begin{array}{c} E\left[\left[c\sum_{n=1}^{\eta} Y_n - rU\right]\chi_{(M_n>U)}\right] \\ +E\left[\left[c\sum_{n=1}^{N-1} Y_n - r\sum_{n=1}^{N} W_n\right]\chi_{(M_n\leq U)}\right] + R + c_p E(Z)\end{array}\right]}{E(W)}. \quad (2)$$

Consider

$$\begin{aligned} E\left[c\left(\sum_{n=1}^{\eta} Y_n\right)\chi_{(M_n>U)}\right] &= E\left[c\left(\sum_{n=1}^{\eta} Y_n\right)\chi_{(M_N\leq U<M_n)}\right] \\ &= c\sum_{n=1}^{\eta} E(Y_n)E[\chi_{(M_N\leq U<M_n)}] \\ &= c\sum_{n=1}^{\eta} E(Y_n)P[(M_N\leq U<M_n)] \\ &= c\mu\left(G_2(U) + \sum_{n=2}^{\infty}\frac{G_{n+1}(U)}{\beta_0^{2^{n-1}}}\right)\left[G_N(U)\right] \quad (3) \end{aligned}$$

Now,

$$\begin{aligned} E\left[\left(c\sum_{n=1}^{N-1} Y_n\right)\chi_{(M_n\leq U)}\right] &= E\left[\left(c\sum_{n=1}^{N-1} Y_n \middle| M_N = U\chi_{(M_n\leq U)}\right)\right] \\ &= \int_o^U cE\left(\sum_{n=1}^{N-1} Y_n \middle| M_N = U\right)dG_n(U) \\ &= \int_o^U c\left(\sum_{n=1}^{N-1} E(Y_n)\right)dG_n(U) \\ &= \sum_{n=1}^{N-1} E(Y_n)\int_o^U cdG_nU \\ &= \sum_{n=1}^{N-1} E(Y_n)cG_n(U) \\ &= c\mu\left[1 + \sum_{n=2}^{N-1}\frac{1}{\beta_0^{2^{n-2}}}\right]G_n(U) \quad (4) \end{aligned}$$

$$\begin{aligned} E\left[\left(r\sum_{n=1}^{N} W_n\right)\chi_{(M_n\leq U)}\right] &= E\left[rE\left(\sum_{n=1}^{N} W_n\right)\middle| M_n = U\chi_{(M_N\leq U)}\right] \\ &= \int_o^U rE\left(\sum_{n=1}^{N} W_n \middle| M_N = U\right)dG_n(U) \end{aligned}$$

$$= \int_o^U r\left(\sum_{n=1}^{N} E(W_n)\right) dG_n(U)$$

$$\begin{aligned} E\left[\left(r\sum_{n=1}^{N} W_n\right)\chi_{(M_n\le U)}\right] &= r\sum_{n=1}^{N} E(W_n)\int_o^U dG_n(U) \\ &= r\sum_{n=1}^{N} E(W_n)G_n(U) \\ &= \sum_{n=1}^{N}\frac{r\lambda}{p_n}G_n(U) \end{aligned} \tag{5}$$

and

$$\begin{aligned} E\left[rU\chi_{(M_n>U)}\right] &= rE\left[U\chi_{(M_n>U)}\right] \\ &= rUE\left[\chi_{(M_n>U)}\right] \\ &= rU\overline{G}_N(U) \end{aligned} \tag{6}$$

On substituting (1), (3) ,(4), (5) and (6) in equation (2), we obtain the following.

Theorem 3.1 For the model described in section 2, the long run average cost per unit time under the bivariate replacement policy (U, N) for a simple degenerative repairable is given by

$$C(U,N) = \frac{\left[\begin{array}{c} \left[c\mu\left[G_2(U)+\sum_{n=2}^{\infty}\frac{G_{n+1}(U)}{\beta_0^{2^{n-1}}}\right][G_N(U)] - rU\overline{G}_N(U)\right] + \\ c\mu\left[1+\sum_{n=2}^{N-1}\frac{1}{\beta_0^{2^{n-2}}}\right]G_N(U) - \sum_{n=1}^{N}\frac{r\lambda}{p_n}rG_n(U) + R + c_p\tau \\ + \end{array}\right]}{\left[\begin{array}{c} U\overline{G}_N(U) + \mu\left[G_2(U)+\sum_{n=2}^{\infty}\frac{G_{n+1}(U)}{\beta_0^{2^{n-1}}}\right][G_N(U)] + \\ \sum_{n=1}^{N}\frac{r\lambda}{p_n}\int_o^U u\,dG_N(u) + \mu\left[1+\sum_{n=2}^{N-1}\frac{1}{\beta_0^{2^{n-2}}}\right] + \tau \end{array}\right]}. \tag{7}$$

Deductions. The long run average cost $C(U, N)$ is a bivariate function in U and N. Obviously, when N is fixed, $C(U, N)$ is a function of T.

For fixed $N = m$, it can be written as

$$C(U,N) = C_m(U), \qquad m = 1, 2, \ldots.$$

Thus, for a fixed m, we can find U_m^* by analytical or numerical methods such that $C_m(U_m^*)$ is minimized. That is, when $N = 1, 2, \ldots, m, \ldots,$ we can find $U_1^*, U_2^*, U_3^*, \ldots, U_m^*, \ldots$ respectively, such that $C_1(U_1^*)$, $C_2(U_2^*), \ldots, C_m(U_m^*), \ldots$ are minimized. Because the total life -time of a multistate degenerative system is limited, the minimum of the long-run average cost per unit time exists. So we can determine the minimum of the long-run average cost per unit time based on $C_1(U_1^*), C_2(U_2^*), \ldots, C_m(U_m^*), \ldots$. Then, if the minimum is denoted by $C_n(U_n^*)$, we obtain the bivariate optimal replacement policy $(U,N)^*$ such that

$$\begin{aligned} C((U,N)^*) &= \min_N C_n(U_n^*) \\ &= \min_N[\min_U C(U,N)] \\ &\leq C(\infty, N) \\ &\equiv C(N^*) \end{aligned}$$

the optimal policy $(U,N)^*$ is better than the optimal policy N^*. Moreover, under some mild conditions, an optimal replacement policy N^* is better than the optimal policy U^*. so under the same conditions, an optimal policy $(U,N)^*$ is better than the optimal replacement policies N^* and U^*.

4. Numerical Example

In this section, we give an example to illustrate theoretical results. Assume that $\{X_i, i = 1, 2, 3, \ldots\}$ is a sequence of independent random variables and each X_i has an exponential distribution $\exp(\lambda_i)$ with $\lambda_i \neq \lambda_j$ for $i \neq j$. Then the probability density function of $\sum_{i=1}^n X_i$ is given by

$$f_n(t) = \begin{cases} (-1)^{n-1}\lambda_1\lambda_2\cdots\lambda_n \displaystyle\sum_{i=1}^n \frac{\exp(-\lambda_i t)}{\displaystyle\prod_{\substack{j=1 \\ j\neq i}}^n \lambda_{i-1} - \lambda_{j-1}}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Let $\lambda_i = \dfrac{\lambda}{\alpha_0 2^{i-1}}$ for $i = 1, 2, 3, \ldots$. Then the distribution function of $\sum_{i=1}^n X_i$ is

$$F_n(T) = (-1)^{n-1}\left(\frac{\lambda}{\alpha_0}\right)^n \left(\frac{1}{2}\right)^{\frac{n(n-1)}{2}} \sum_{i=1}^n \frac{1 - \exp\left(-\frac{\lambda}{\alpha_0 2^{i-1}}T\right)}{\displaystyle\prod_{\substack{j=1 \\ j\neq i}}^n \left(\frac{\lambda}{\alpha_0 2^{i-1}} - \frac{\lambda}{\alpha_0 2^{j-1}}\right)}.$$

The distribution function of $\sum_{i=1}^n Y_i$ is

$$G_n(T) = (-1)^{n-1}\left(\frac{\mu}{\beta_0}\right)^n \left(\frac{1}{2}\right)^{\frac{n(n-1)}{2}} \sum_{i=1}^n \frac{1 - \exp\left(-\frac{\mu}{\beta_0 2^{i-1}}T\right)}{\displaystyle\prod_{\substack{j=1 \\ j\neq i}}^n \left(\frac{\mu}{\beta_0 2^{i-1}} - \frac{\mu}{\beta_0 2^{j-1}}\right)}.$$

Let the parameter values be

$$\lambda = 500 \qquad c = 10 \qquad R = 50000$$

$$\mu = 35 \qquad r = 450 \qquad \tau = 15$$

$$\alpha_0 = 1.05 \qquad \beta_0 = 0.95 \qquad c_p = 3$$

In this case, assuming equation (7) and over passing numerical calculations, we arrive at $(U, N)^* = (210, 19)$, such that $C(U, N)$ is minimum at $(U, N)^*$ and the long run average cost per unit per unit time is $C(U, N) = C(210, 19) = -23.6798$ monetary units. The value of $C(U, N)$ for U ranging from 110 to 250 time units in steps of 10 and N ranging from 16 to 20 are evaluated and given in table 1. Further these values are plotted in the figure 1.

Table 1. Values of $C(U, N)$ against (U, N)

(U,N)	$C(U,N)$	(U,N)	$C(U,N)$	(U,N)	$C(U,N)$
(110, 16)	42.2416	(210, 17)	31.8621	(160, 19)	12.6432
(120, 16)	38.1359	(220, 17)	33.7624	(170, 19)	5.7829
(130, 16)	36.9762	(230, 17)	34.5624	(180, 19)	−2.1468
(140, 16)	35.7201	(240, 17)	35.7891	(190, 19)	−11.5436
(150, 16)	35.0065	(250, 17)	36.1243	(200, 19)	−18.7651
(160, 16)	34.8756	(110, 18)	46.9832	(**210**, **19**)	(−**23.6798**)
(170, 16)	34.7543	(120, 18)	35.7413	(220, 19)	−15.3214
(180, 16)	34.6254	(130, 18)	34.6549	(230, 19)	−7.4926
(190, 16)	34.5544	(140, 18)	32.8461	(240, 19)	6.1374
(200, 16)	32.1368	(150, 18)	32.7528	(250, 19)	6.9387
(210, 16)	31.8465	(160, 18)	32.7109	(110, 20)	29.4235
(220, 16)	32.7892	(170, 18)	32.6655	(120, 20)	28.7952
(230, 16)	35.9184	(180, 18)	32.5041	(130, 20)	28.2134
(240, 16)	36.1325	(190, 18)	32.4438	(140, 20)	27.5436
(250, 16)	37.3587	(200, 18)	32.0987	(150, 20)	26.1243
(110, 17)	45.1789	(210,18)	−31.9345	(160, 20)	25.8759
(120, 17)	44.2982	(220, 18)	34.5297	(170, 20)	24.3357
(130, 17)	43.8972	(230, 18)	35.8765	(180, 20)	22.9768
(140, 17)	42.7965	(240, 18)	36.1243	(190, 20)	22.8641
(150, 17)	41.5438	(250, 18)	38.8456	(200, 20)	22.5708
(160, 17)	40.3187	(110, 19)	32.9807	(210, 20)	21.9543
(170, 17)	38.1374	(120, 19)	30.7643	(220, 20)	24.8769
(180, 17)	35.9013	(130, 19)	25.8463	(230, 20)	29.1496
(190, 17)	32.8877	(140, 19)	21.9325	(240, 20)	32.5675
(200, 17)	32.7894	(150, 19)	18.0651	(250, 20)	34.8977

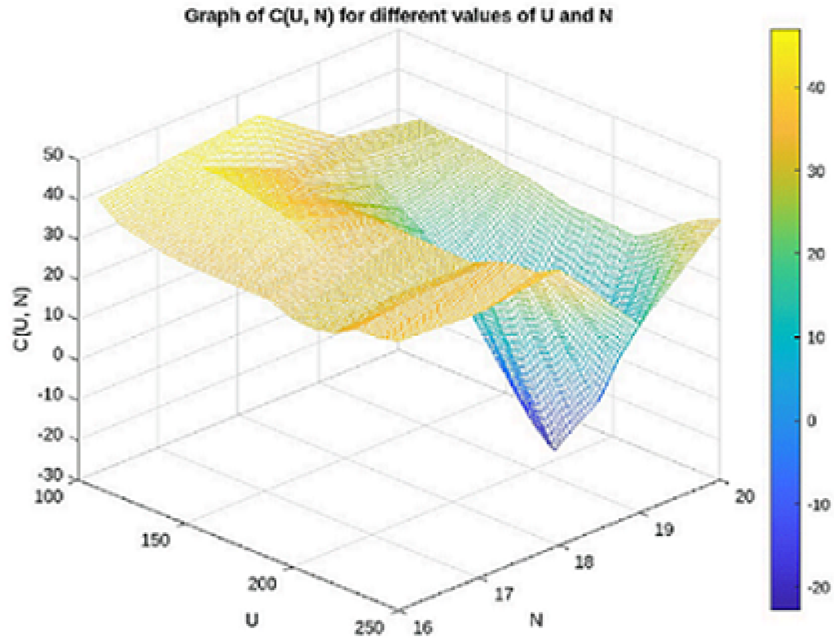

FIGURE 1. Plot of $C(U, N)$ against (U, N)

5. Conclusion

In this paper, we have considered an extreme shock maintenance model for a degenerative simple repairable system. Explicit expression for the long run average cost under the bivariate replacement policy (U, N) is derived. Comparison of this bivariate optimal replacement policy $(U, N)^*$ with the univariate optimal replacement policies U^* and N^* is also carried out.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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