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WEAKLY $\theta s(\Lambda, p)$-OPEN FUNCTIONS AND WEAKLY $\theta s(\Lambda, p)$-CLOSED FUNCTIONS

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Abstract. This paper is concerned with the concepts of weakly $\theta s(\Lambda, p)$-open functions and weakly $\theta s(\Lambda, p)$-closed functions. Moreover, some characterizations of weakly $\theta s(\Lambda, p)$-open functions and weakly $\theta s(\Lambda, p)$-closed functions are investigated.

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1. Introduction

In topology, there has been recently significant interest in characterizing and investigating the characterizations of some weak forms of open functions and closed functions. Semi-open sets, preopen sets, α-open sets, β-open sets, δ-open sets and θ-open sets play an important role in the researches of generalizations of open functions and closed functions. By using these sets, many authors introduced and studied various types of open functions and closed functions. In 1984, Rose [15] introduced and studied the notions of weakly open functions and almost open functions. In 1987, Rose and Janković [14] investigated some of the fundamental properties of weakly closed functions. Caldas and Navalagi [7] introduced and studied the notions of weakly semi-open functions and weakly semi-closed functions as a new generalization of weakly open functions and weakly closed functions. In 1987, Di Maio and Noiri [10] investigated the concepts of semi-θ-open sets and semi-θ-closed sets which provide a formulation of semi-θ-closure of a set in a topological space. Noiri [13] introduced and studied the concept of θ-semicontinuous functions by involving these sets. In 1991, Mukherjee and Basu [12] continued the work of Di Maio and Noiri and defined the notions of semi-θ-connectedness, semi-θ-components and semi-θ-quasi components. In 2006, Caldas et al. [8] introduced and studied two new classes of functions by utilizing the notions of semi-θ-open sets and the semi-θ-closure operator called

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weakly semi-θ-open functions and weakly semi-θ-closed functions. The class of weakly semi-θ-openness (resp. weakly semi-θ-closedness) as a new generalization of semi-θ-openness (semi-θ-closedness). The notions of (Λ, s)-open sets, $s(\Lambda, s)$-open sets, $p(\Lambda, s)$-open sets, $\alpha(\Lambda, s)$-open sets, $\beta(\Lambda, s)$-open sets and $b(\Lambda, s)$-open sets were studied in [4]. Furthermore, several properties of (Λ, sp)-open sets, $s(\Lambda, sp)$-open sets, $p(\Lambda, sp)$-open sets, $\alpha(\Lambda, sp)$-open sets, $\beta(\Lambda, sp)$-open sets and $b(\Lambda, sp)$-open sets were established in [5]. Srisarakham and Boonpok [16] studied some properties of $\delta p(\Lambda, s)$-closed sets and the $\delta p(\Lambda, s)$-closure operator. In [2], the authors investigated several characterizations of $\theta p(\Lambda, p)$-open functions and $\theta p(\Lambda, p)$-closed functions. Boonpok and Thongmoon [1] introduced and studied the notions of weakly $p(\Lambda, p)$-open functions and weakly $p(\Lambda, p)$-closed functions. In this paper, we introduce the notions of weakly $\theta s(\Lambda, p)$-open functions and $\theta s(\Lambda, p)$-closed functions. In particular, some characterizations of weakly $\theta s(\Lambda, p)$-open functions and $\theta s(\Lambda, p)$-closed functions are investigated.

2. Preliminaries

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ), Cl(A) and Int(A), represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [11] if $A \subseteq \text{Int}(\text{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [9] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a Λ_p*-set* [6] (*pre-*Λ*-set* [9]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called (Λ, p)*-closed* [6] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called (Λ, p)*-open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_pO(X, \tau)$ (resp. $\Lambda_pC(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a (Λ, p)*-cluster point* [6] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the (Λ, p)*-closure* [6] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the (Λ, p)*-interior* [6] of A and is denoted by $A_{(\Lambda,p)}$. The $\theta(\Lambda, p)$*-closure* [6] of A, $A^{\theta(\Lambda,p)}$, is defined as follows:

$$A^{\theta(\Lambda,p)} = \{x \in X \mid A \cap U^{(\Lambda,p)} \neq \emptyset \text{ for each } (\Lambda, p)\text{-open set } U \text{ containing } x\}.$$

A subset A of a topological space (X, τ) is called $\theta(\Lambda, p)$*-closed* [6] if $A = A^{\theta(\Lambda,p)}$. The complement of a $\theta(\Lambda, p)$-closed set is said to be $\theta(\Lambda, p)$*-open*. A point $x \in X$ is called a $\theta(\Lambda, p)$*-interior point* [17] of A if $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$ for some $U \in \Lambda_pO(X, \tau)$. The set of all $\theta(\Lambda, p)$-interior points of A is called the $\theta(\Lambda, p)$*-interior* [17] of A and is denoted by $A_{\theta(\Lambda,p)}$. A subset A of a topological space (X, τ) is said to be $s(\Lambda, p)$*-open* [6] (resp. $p(\Lambda, p)$*-open* [6], $\alpha(\Lambda, p)$*-open* [18], $r(\Lambda, p)$*-open* [6]) if $A \subseteq [A_{(\Lambda,p)}]^{(\Lambda,p)}$ (resp.

$A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)}$, $A \subseteq [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$, $A = [A^{(\Lambda,p)}]_{(\Lambda,p)}$). The complement of a $s(\Lambda,p)$-open (resp. $p(\Lambda,p)$-open, $\alpha(\Lambda,p)$-open, $r(\Lambda,p)$-open) set is called $s(\Lambda,p)$-*closed* (resp. $p(\Lambda,p)$-*closed*, $\alpha(\Lambda,p)$-*closed*, $r(\Lambda,p)$-*closed*). The intersection of all $s(\Lambda,p)$-closed sets of X containing A is called the $s(\Lambda,p)$-*closure* of A and is denoted by $A^{s(\Lambda,p)}$. The union of all $s(\Lambda,p)$-open sets of X contained in A is called the $s(\Lambda,p)$-*interior* of A and is denoted by $A_{s(\Lambda,p)}$. Let A be a subset of a topological space (X,τ). A point $x \in X$ is called a $\theta s(\Lambda,p)$-*cluster point* [3] of A if $A \cap U^{s(\Lambda,p)} \neq \emptyset$ for every $s(\Lambda,p)$-open set U of X containing x. The set of all $\theta s(\Lambda,p)$-cluster points of A is called the $\theta s(\Lambda,p)$-*closure* [3] of A and is denoted by $A^{\theta s(\Lambda,p)}$. If $A = A^{\theta s(\Lambda,p)}$, then A is called $\theta s(\Lambda,p)$-*closed* [3]. The complement of a $\theta s(\Lambda,p)$-closed set is called $\theta s(\Lambda,p)$-*open*. The $\theta s(\Lambda,p)$-*interior* of A is defined by the union of all $\theta s(\Lambda,p)$-open sets of X contained in A and is denoted by $A_{\theta s(\Lambda,p)}$.

Lemma 1. [17] *For subsets A and B of a topological space (X,τ), the following properties hold:*

(1) $X - A^{\theta(\Lambda,p)} = [X - A]_{\theta(\Lambda,p)}$ *and* $X - A_{\theta(\Lambda,p)} = [X - A]^{\theta(\Lambda,p)}$.
(2) *A is $\theta(\Lambda,p)$-open if and only if $A = A_{\theta(\Lambda,p)}$.*
(3) $A \subseteq A^{(\Lambda,p)} \subseteq A^{\theta(\Lambda,p)}$ *and* $A_{\theta(\Lambda,p)} \subseteq A_{(\Lambda,p)} \subseteq A$.
(4) *If $A \subseteq B$, then $A^{\theta(\Lambda,p)} \subseteq B^{\theta(\Lambda,p)}$ and $A_{\theta(\Lambda,p)} \subseteq B_{\theta(\Lambda,p)}$.*
(5) *If A is (Λ,p)-open, then $A^{(\Lambda,p)} = A^{\theta(\Lambda,p)}$.*

Lemma 2. *For subsets A and B of a topological space (X,τ), the following properties hold:*

(1) $X - A^{\theta s(\Lambda,p)} = [X - A]_{\theta s(\Lambda,p)}$ *and* $X - A_{\theta s(\Lambda,p)} = [X - A]^{\theta s(\Lambda,p)}$.
(2) *A is $\theta s(\Lambda,p)$-open if and only if $A = A_{\theta s(\Lambda,p)}$.*
(3) $A \subseteq A^{s(\Lambda,p)} \subseteq A^{\theta s(\Lambda,p)}$ *and* $A_{\theta s(\Lambda,p)} \subseteq A_{s(\Lambda,p)} \subseteq A$.
(4) *If $A \subseteq B$, then $A^{\theta s(\Lambda,p)} \subseteq B^{\theta s(\Lambda,p)}$ and $A_{\theta s(\Lambda,p)} \subseteq B_{\theta s(\Lambda,p)}$.*
(5) *If A is $s(\Lambda,p)$-open, then $A^{s(\Lambda,p)} = A^{\theta s(\Lambda,p)}$.*

3. Weakly $\theta s(\Lambda,p)$-Open Functions

We begin this section by introducing the notion of weakly $\theta s(\Lambda,p)$-open functions.

Definition 1. *A functions $f : (X,\tau) \to (Y,\sigma)$ is said to be weakly $\theta s(\Lambda,p)$-open if $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$ for each (Λ,p)-open set U of X.*

Theorem 1. *For a function $f : (X,\tau) \to (Y,\sigma)$, the following properties are equivalent:*

(1) *f is weakly $\theta s(\Lambda,p)$-open;*
(2) $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{\theta s(\Lambda,p)}$ *for every subset A of X;*
(3) $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\theta s(\Lambda,p)})$ *for every subset B of Y;*
(4) $f^{-1}(B^{\theta s(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$ *for every subset B of Y;*
(5) *for each $x \in X$ and each (Λ,p)-open set U of X containing x, there exists a $\theta s(\Lambda,p)$-open set V of Y containing $f(x)$ such that $V \subseteq f(U^{(\Lambda,p)})$;*

(6) $f(K_{(\Lambda,p)}) \subseteq [f(K)]_{\theta s(\Lambda,p)}$ *for every* (Λ,p)*-closed set* K *of* X*;*

(7) $f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$ *for every* (Λ,p)*-open set* U *of* X*;*

(8) $f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$ *for every* $p(\Lambda,p)$*-open set* U *of* X*;*

(9) $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$ *for every* $\alpha(\Lambda,p)$*-open set* U *of* X.

Proof. (1) ⇒ (2): Let A be any subset of X and $x \in A_{\theta(\Lambda,p)}$. Then, there exists a (Λ,p)-open set U of X such that

$$x \in U \subseteq U^{(\Lambda,p)} \subseteq A.$$

Then, $f(x) \in f(U) \subseteq f(U^{(\Lambda,p)}) \subseteq f(A)$. Since f is weakly $\theta s(\Lambda,p)$-open, $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)} \subseteq [f(A)]_{\theta s(\Lambda,p)}$. It implies that

$$f(x) \in [f(A)]_{\theta s(\Lambda,p)}.$$

Thus, $x \in f^{-1}([f(A)]_{\theta s(\Lambda,p)})$ and hence $A_{\theta(\Lambda,p)} \subseteq f^{-1}([f(A)]_{\theta s(\Lambda,p)})$. This shows that $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{\theta s(\Lambda,p)}$.

(2) ⇒ (1): Let U be any (Λ,p)-open set of X. As $U \subseteq [U^{(\Lambda,p)}]_{\theta(\Lambda,p)}$ implies $f(U) \subseteq f([U^{(\Lambda,p)}]_{\theta(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$. Thus, f is weakly $\theta s(\Lambda,p)$-open.

(2) ⇒ (3): Let B be any subset of Y. Then by (2),

$$f([f^{-1}(B)]_{\theta(\Lambda,p)}) \subseteq B_{\theta s(\Lambda,p)}.$$

Thus, $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\theta s(\Lambda,p)})$.

(3) ⇒ (2): This is obvious.

(3) ⇒ (4): Let B be any subset of Y. Using (3), we have

$$\begin{aligned} X - [f^{-1}(B)]^{\theta(\Lambda,p)} &= [X - f^{-1}(B)]_{\theta(\Lambda,p)} \\ &= [f^{-1}(Y-B)]_{\theta(\Lambda,p)} \\ &\subseteq f^{-1}([Y-B]_{\theta s(\Lambda,p)}) \\ &= f^{-1}(Y - B^{\theta s(\Lambda,p)}) \\ &= X - f^{-1}(B^{\theta s(\Lambda,p)}) \end{aligned}$$

and hence $f^{-1}(B^{\theta s(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$.

(4) ⇒ (3): Let B be any subset of Y. By (4),

$$X - f^{-1}(B_{\theta s(\Lambda,p)}) \subseteq X - [f^{-1}(B)]_{\theta(\Lambda,p)}.$$

Thus, $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{\theta s(\Lambda,p)})$.

(1) $\Rightarrow$ (5): Let $x \in X$ and U be any (Λ,p)-open set of X containing x. Since f is weakly $\theta s(\Lambda,p)$-open, $f(x) \in f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$. Put $V = [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$. Then, V is a $\theta s(\Lambda,p)$-open set of Y containing $f(x)$ such that $V \subseteq f(U^{(\Lambda,p)})$.

(5) $\Rightarrow$ (1): Let U be any (Λ,p)-open set of X and $y \in f(U)$. It following from (5) $V \subseteq f(U^{(\Lambda,p)})$ for some $\theta s(\Lambda,p)$-open set V of Y containing y. Thus, $y \in V \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$ and hence

$$f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}.$$

This shows that f is weakly $\theta s(\Lambda,p)$-open.

(1) $\Rightarrow$ (6) $\Rightarrow$ (7) $\Rightarrow$ (8) $\Rightarrow$ (9) $\Rightarrow$ (1): This is obvious. □

Theorem 2. *Let $f : (X,\tau) \to (Y,\sigma)$ be a bijective function. Then, the following properties are equivalent:*

(1) *f is weakly $\theta s(\Lambda,p)$-open;*
(2) *$[f(U)]^{\theta s(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every (Λ,p)-open set U of X;*
(3) *$[f(K_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \subseteq f(K)$ for every (Λ,p)-closed set K of X.*

Proof. (1) $\Rightarrow$ (3): Let K be any (Λ,p)-closed set of X. Then, we have

$$\begin{aligned} f(X-K) &= Y - f(K) \\ &\subseteq [f([X-K]^{(\Lambda,p)})]_{\theta s(\Lambda,p)} \end{aligned}$$

and hence $Y - f(K) \subseteq Y - [f(K_{(\Lambda,p)})]^{\theta s(\Lambda,p)}$. Thus,

$$[f(K_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \subseteq f(K).$$

(3) $\Rightarrow$ (2): Let U be any (Λ,p)-open set of X. Since $U^{(\Lambda,p)}$ is (Λ,p)-closed and $U \subseteq [U^{(\Lambda,p)}]_{(\Lambda,p)}$, by (3) we have

$$\begin{aligned} [f(U)]^{\theta s(\Lambda,p)} &\subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \\ &\subseteq f(U^{(\Lambda,p)}). \end{aligned}$$

(2) $\Rightarrow$ (1): Let U be any (Λ,p)-open set of X. By (2), we have

$$[f(X - U^{(\Lambda,p)})]^{\theta s(\Lambda,p)} \subseteq f([X - U^{(\Lambda,p)}]^{(\Lambda,p)}).$$

Since f is bijective, $[f(X - U^{(\Lambda,p)})]^{\theta s(\Lambda,p)} = Y - [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$ and

$$\begin{aligned} f([X - U^{(\Lambda,p)}]^{(\Lambda,p)}) &= f(X - [U^{(\Lambda,p)}]_{(\Lambda,p)}) \\ &\subseteq f(X - U) \\ &= Y - f(U). \end{aligned}$$

Thus, $f(U) \subseteq [f(U^{(\Lambda,p)})]_{\theta s(\Lambda,p)}$ and hence f is weakly $\theta s(\Lambda,p)$-open. □

4. Weakly $\theta s(\Lambda, p)$-Closed Functions

In this section, we introduce the notion of weakly $\theta s(\Lambda, p)$-closed functions. Moreover, some characterizations of weakly $\theta s(\Lambda, p)$-closed functions are investigated.

Definition 2. *A functions $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly $\theta s(\Lambda, p)$-closed if $[f(K_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \subseteq f(K)$ for each (Λ, p)-closed set K of X.*

Theorem 3. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

1. *f is weakly $\theta s(\Lambda, p)$-closed;*
2. *$[f(U)]^{\theta s(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every $r(\Lambda, p)$-open set U of X;*
3. *for each subset B of Y and each (Λ, p)-open set U of X with $f^{-1}(B) \subseteq U$, there exists a $\theta s(\Lambda, p)$-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U^{(\Lambda,p)}$;*
4. *for each point $y \in Y$ and each (Λ, p)-open set U of X with $f^{-1}(y) \subseteq U$, there exists a $\theta s(\Lambda, p)$-open set V of Y containing y such that $f^{-1}(V) \subseteq U^{(\Lambda,p)}$;*
5. *$[f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for each (Λ, p)-open set U of X;*
6. *$[f([U^{\theta(\Lambda,p)}]_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \subseteq f(U^{\theta(\Lambda,p)})$ for each (Λ, p)-open set U of X;*
7. *$[f(U)]^{\theta s(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for each $p(\Lambda, p)$-open set U of X.*

Proof. It is clear that $(1) \Rightarrow (5) \Rightarrow (7) \Rightarrow (2) \Rightarrow (1)$, $(1) \Rightarrow (6)$ and $(3) \Rightarrow (4)$. To show that $(2) \Rightarrow (3)$: Let B be any subset of Y and U be any (Λ, p)-open set of X with $f^{-1}(B) \subseteq U$. Then,

$$f^{-1}(B) \cap [X - U^{(\Lambda,p)}]^{(\Lambda,p)} = \emptyset$$

and $B \cap f([X - U^{(\Lambda,p)}]^{(\Lambda,p)}) = \emptyset$. Since $X - U^{(\Lambda,p)}$ is $r(\Lambda, p)$-open, $B \cap [f(X - U^{(\Lambda,p)})]^{\theta s(\Lambda,p)} = \emptyset$ by (2). Put $V = Y - [f(X - U^{(\Lambda,p)})]^{\theta s(\Lambda,p)}$. Then, V is a $\theta s(\Lambda, p)$-open set of Y such that $B \subseteq V$ and

$$\begin{aligned} f^{-1}(V) &\subseteq X - f^{-1}([f(X - U^{(\Lambda,p)})]^{\theta s(\Lambda,p)}) \\ &\subseteq X - f^{-1}(f(X - U^{(\Lambda,p)})) \\ &\subseteq U^{(\Lambda,p)}. \end{aligned}$$

$(6) \Rightarrow (1)$: It is suffices see that $U^{\theta(\Lambda,p)} = U^{(\Lambda,p)}$ for every (Λ, p)-open set U of X.

$(4) \Rightarrow (1)$: Let K be any (Λ, p)-closed set of X and $y \in Y - f(K)$. Since $f^{-1}(y) \subseteq X - K$, there exists a $\theta s(\Lambda, p)$-open set V of Y such that $y \in V$ and $f^{-1}(V) \subseteq [X - K]^{(\Lambda,p)} = X - K_{(\Lambda,p)}$ by (4). Thus, $V \cap f(K_{(\Lambda,p)}) = \emptyset$ and hence $y \in Y - [f(K_{(\Lambda,p)})]^{\theta s(\Lambda,p)}$. Therefore, $[f(K_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \subseteq f(K)$. This shows that f is weakly $\theta s(\Lambda, p)$-closed.

$(6) \Rightarrow (7)$: This is obvious since $U^{\theta(\Lambda,p)} = U^{(\Lambda,p)}$ for every $p(\Lambda, p)$-open set U of X. □

The proof of the following result is mostly straightforward and is therefore omitted.

Theorem 4. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is weakly $\theta s(\Lambda, p)$-closed;*

(2) $[f(U)]^{\theta s(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ *every (Λ, p)-open set U of X;*

(3) $[f(K_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \subseteq f(K)$ *every $p(\Lambda, p)$-closed set K of X;*

(4) $[f(K_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \subseteq f(K)$ *every $\alpha(\Lambda, p)$-closed set K of X.*

Theorem 5. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is weakly $\theta s(\Lambda, p)$-closed;*

(2) $[f(U)]^{\theta s(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ *for every (Λ, p)-open set U of X.*

Proof. $(1) \Rightarrow (2)$: Let U be any (Λ, p)-open set of X. By (1), we have

$$\begin{aligned}[f(U)]^{\theta s(\Lambda,p)} &= [f(U_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \\ &\subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{\theta s(\Lambda,p)} \\ &\subseteq f(U^{(\Lambda,p)}).\end{aligned}$$

$(2) \Rightarrow (1)$: Let K be any (Λ, p)-closed set of X. Using (2), we have

$$\begin{aligned}[f(K_{(\Lambda,p)})]^{\theta s(\Lambda,p)} &\subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)}) \\ &\subseteq f(K^{(\Lambda,p)}) \\ &= f(K).\end{aligned}$$

This shows that f is weakly $\theta s(\Lambda, p)$-closed. □

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] C. Boonpok, M. Thongmoon, Weakly $p(\Lambda, p)$-open functions and weakly $p(\Lambda, p)$-closed functions, Int. J. Anal. Appl. 22 (2024), 10.

[2] C. Boonpok, N. Srisarakham, $\theta p(\Lambda, p)$-open functions and $\theta p(\Lambda, p)$-closed functions, Asia Pac. J. Math. 10 (2023), 48.

[3] C. Boonpok, M. Thongmoon, Characterizations of some topological spaces, Eur. J. Pure Appl. Math. 16 (2023), 1434–1447.

[4] C. Boonpok, C. Viriyapong, On some forms of closed sets and related topics, Eur. J. Pure Appl. Math. 16 (2023), 336–362.

[5] C. Boonpok and J. Khampakdee, (Λ, sp)-open sets in topological spaces, Eur. J. Pure Appl. Math. 15 (2022), 572–588.

[6] C. Boonpok and C. Viriyapong, On (Λ, p)-closed sets and the related notions in topological spaces, Eur. J. Pure Appl. Math. 15 (2022), 415–436.

[7] M. Caldas, G. Navalagi, On weak forms of semi-open and semi-closed functions, Missouri J. Math. Sci. 18 (2006), 165–178. https://doi.org/10.35834/2006/1803165.

[8] M. Caldas, S. Jafari, G. Navalagi, Weak forms of open and closed functions via semi-θ-open sets, Carpathian J. Math. 22 (2006), 21–31. https://www.jstor.org/stable/43998903.

[9] M. Ganster, S. Jafari, T. Noiri, On pre-Λ-sets and pre-V-sets, Acta Math. Hungar. 95 (2002), 337–343.

[10] G. Di Maio and T. Noiri, On s-closed spaces, Indian J. Pure Appl. Math. 18 (1987), 226–233.

[11] A.S. Mashhour, M.E. Abd El-Monsef, S.N. El-Deeb, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, 53 (1982), 47–53.

[12] M.N. Mukherjee, C.K. Basu, On semi-θ-closed sets, semi-θ-connectedness and some associated mappings, Bull. Cal. Math. Soc. 83 (1991), 227–238.

[13] T. Noiri, On θ-semicontinuous functions, Indian J. Pure Appl. Math. 21 (1990), 410–415.

[14] D.A. Rose, D.S. Janković, Weakly closed functions and Hausdorff spaces, Math. Nachr. 130 (1987), 105–110. https://doi.org/10.1002/mana.19871300109.

[15] D.A. Rose, Weak openness and almost openness, Int. J. Math. Math. Sci. 7 (1984), 35–40. https://doi.org/10.1155/s0161171284000041.

[16] N. Srisarakham, C. Boonpok, On characterizations of $\delta p(\Lambda, s)$-$\mathscr{D}_1$ spaces, Int. J. Math. Comp. Sci. 18 (2023), 743–747.

[17] M. Thongmoon, C. Boonpok, Strongly $\theta(\Lambda, p)$-continuous functions, Int. J. Math. Comp. Sci. 19 (2024), 475–479.

[18] N. Viriyapong, C. Boonpok, On (Λ, p)-extremally disconnected spaces, Int. J. Math. Comp. Sci. 18 (2023), 289–293.