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ANTI-HOMOMORPHISMS IN BIPOLAR FUZZY IDEALS AND BI-IDEALS OF Γ-NEAR RINGS

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Abstract. This article explored anti-homomorphism's impact on the domains and codomains of bipolar fuzzy ideals and bi-ideals of Γ-near rings.

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1. Introduction

Pilz [7] pioneered near-ring theory. Nobusawa [6] proposed the concept of Γ-rings, which are generalisations of rings. Satyanarayana [9] defined Γ-near rings (GNRs). Satyanarayana [9] and Booth [1] investigated the ideal theory in GNRs. Several authors also investigated various algebraic structures on GNRs, such as ideals, weak ideals, bi-ideals, quasi-ideals, and normal ideals. Zhang [14] proposed the concept of bipolar-valued fuzzy sets (BFSs), which is an extension of Zadeh's [13] fuzzy set (FS) theory to BFSs. Taking this into account, numerous authors utilised fuzzifications on crisp sets, such as Satyanarayana explored and established the concept of fuzzy ideals and prime ideals of GNRs. Jun [3] discusses several results and properties on fuzzy ideals of GNRs. Jun and Lee [4] developed the application of BFSs, which is a generalisation of FSs, to analyse uncertainty. Several academicians, including Ragamayi [8,10–12], have researched the development of BFS theory on various algebraic structures such as semigroups, groups, semirings, rings, Γ-near rings, etc.

As a continuation of all these, we have introduced the concepts of bipolar fuzzy ideals and bi-ideals of GNRs in 2023. Now, we are studying the anti-homomorphisms of bipolar fuzzy ideals and bi-ideals.

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2. Preliminaries

Important definitions for the study in this paper are reviewed in this section.

Definition 2.1. [9] A Γ-*near ring* (GNR) is a triple $(M_R, +, \Gamma)$, where
(i) $(M_R, +)$ is a group,
(ii) Γ is a nonempty set of binary operators on M_R such that for each $\alpha \in \Gamma$, $(M_R, +, \alpha)$ is a near ring,
(iii) $\dot{\psi}\alpha(\dot{\omega}\beta\dot{\kappa}) = (\dot{\psi}\alpha\dot{\omega})\beta\dot{\kappa}, \forall \dot{\psi}, \dot{\omega}, \dot{\kappa} \in M_R, \alpha, \beta \in \Gamma$.

Definition 2.2. [10] A BFS $B_R = (\xi^+_{B_R}, \xi^-_{B_R})$ of a GNR M_R is called a *bipolar fuzzy ideal* (BFI) of M_R if
(i) $\xi^+_{B_R}(\dot{\psi} - \dot{\omega}) \geq \min\{\xi^+_{B_R}(\dot{\psi}), \xi^+_{B_R}(\dot{\omega})\}, \forall \dot{\psi}, \dot{\omega} \in M_R$,
(ii) $\xi^+_{B_R}(\dot{\omega} + \dot{\psi} - \dot{\omega}) \geq \xi^+_{B_R}(\dot{\psi}), \forall \dot{\psi}, \dot{\omega} \in M_R$,
(iii) $\xi^+_{B_R}(\dot{\omega}\alpha(\dot{\psi} + \dot{\kappa}) - \dot{\omega}\alpha\dot{\kappa}) \geq \xi^+_{B_R}(\dot{\psi}), \forall \dot{\psi}, \dot{\omega}, \dot{\kappa} \in M_R, \alpha \in \Gamma$,
(iv) $\xi^+_{B_R}(\dot{\psi}\alpha\dot{\omega}) \geq \xi^+_{B_R}(\dot{\psi}), \forall \dot{\psi}, \dot{\omega} \in M_R, \alpha \in \Gamma$,
(v) $\xi^-_{B_R}(\dot{\psi} - \dot{\omega}) \leq \max\{\xi^-_{B_R}(\dot{\psi}), \xi^-_{B_R}(\dot{\omega})\}, \forall \dot{\psi}, \dot{\omega} \in M_R$,
(vi) $\xi^-_{B_R}(\dot{\omega} + \dot{\psi} - \dot{\omega}) \leq \xi^-_{B_R}(\dot{\psi}), \forall \dot{\psi}, \dot{\omega} \in M_R$,
(vii) $\xi^-_{B_R}(\dot{\omega}\alpha(\dot{\psi} + \dot{\kappa}) - \dot{\omega}\alpha\dot{\kappa}) \leq \xi^-_{B_R}(\dot{\psi}), \forall \dot{\psi}, \dot{\omega}, \dot{\kappa} \in M_R, \alpha \in \Gamma$,
(viii) $\xi^-_{B_R}(\dot{\psi}\alpha\dot{\omega}) \leq \xi^-_{B_R}(\dot{\psi}), \forall \dot{\psi}, \dot{\omega} \in M_R, \alpha \in \Gamma$.

Definition 2.3. [12] A BFS $B_R = (\xi^+_{B_R}, \xi^-_{B_R})$ of a GNR M_R is called a *bipolar fuzzy bi-ideal* (BFBI) of M_R if
(i) $\xi^+_{B_R}(\dot{\psi} - \dot{\omega}) \geq \min\{\xi^+_{B_R}(\dot{\psi}), \xi^+_{B_R}(\dot{\omega})\}, \forall \dot{\psi}, \dot{\omega} \in M_R$,
(ii) $\xi^+_{B_R}(\dot{\omega} + \dot{\psi} - \dot{\omega}) \geq \xi^+_{B_R}(\dot{\psi}), \forall \dot{\psi}, \dot{\omega} \in M_R$,
(iii) $\xi^+_{B_R}((\dot{\psi}\alpha\dot{\omega}\beta\dot{\kappa}) \wedge (\dot{\psi}\alpha(\dot{\omega} + \dot{\kappa}) - \dot{\psi}\alpha\dot{\omega})) \geq \min\{\xi^+_{B_R}(\dot{\psi}), \xi^+_{B_R}(\dot{\kappa})\}, \forall \dot{\psi}, \dot{\omega}, \dot{\kappa} \in M_R, \alpha, \beta \in \Gamma$,
(iv) $\xi^-_{B_R}(\dot{\psi} - \dot{\omega}) \leq \max\{\xi^-_{B_R}(\dot{\psi}), \xi^-_{B_R}(\dot{\omega})\}, \forall \dot{\psi}, \dot{\omega} \in M_R$,
(v) $\xi^-_{B_R}(\dot{\omega} + \dot{\psi} - \dot{\omega}) \leq \xi^-_{B_R}(\dot{\psi}), \forall \dot{\psi}, \dot{\omega} \in M_R$,
(vi) $\xi^-_{B_R}((\dot{\psi}\alpha\dot{\omega}\beta\dot{\kappa}) \wedge (\dot{\psi}\alpha(\dot{\omega} + \dot{\kappa}) - \dot{\psi}\alpha\dot{\omega})) \leq \max\{\xi^-_{B_R}(\dot{\psi}), \xi^-_{B_R}(\dot{\kappa})\}, \forall \dot{\psi}, \dot{\omega}, \dot{\kappa} \in M_R, \alpha, \beta \in \Gamma$.

Definition 2.4. A function $\phi : M_{R1} \to M_{R2}$ of GNRs is called an *anti-homomorphism* if
(i) $\phi(\dot{\psi} + \dot{\omega}) = \phi(\dot{\omega}) + \phi(\dot{\psi}), \forall \dot{\psi}, \dot{\omega} \in M_{R1}$,
(ii) $\phi(\dot{\psi}\alpha\dot{\omega}) = \phi(\dot{\omega})\alpha\phi(\dot{\psi}), \forall \dot{\psi}, \dot{\omega} \in M_{R1}, \alpha \in \Gamma$.

3. Anti-Homomorphisms in BFIs and BFBIs of GNRs

In this session, we will explore the anti-homomorphic image and pre-image of BFIs and BFBIs of GNRs.

Theorem 3.1. A GNR anti-homomorphic image of a BFI possessing both supremum and infimum properties is a BFI.

Proof. Let $\phi : M_{R1} \to M_{R2}$ be a GNR anti-homomorphism and let $A_R = (M_{R1}, \xi^+_{A_R}, \xi^-_{A_R})$ be a BFI of M_{R1}. Let $\xi^+_{A_R}$ of M_{R1} possess supremum property. Let $B_R = (M_{R2}, \xi^+_{B_R}, \xi^-_{B_R})$ be the image of $A_R = (M_{R1}, \xi^+_{A_R}, \xi^-_{A_R})$ in M_{R2} with $\xi^+_{B_R}$ is the image of $\xi^+_{A_R}$, and $\xi^-_{B_R}$ is the image of $\xi^-_{A_R}$. Let $\phi(\dot{\psi}), \phi(\dot{\omega}), \phi(\dot{\kappa}) \in M_{R2}$ and $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned}
\xi^+_{B_R}(\phi(\dot{\psi}) - \phi(\dot{\omega})) &= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\psi}) - \phi(\dot{\omega}))} \xi^+_{A_R}(a_\gamma) \\
&= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\psi}) + \phi(-\dot{\omega}))} \xi^+_{A_R}(a_\gamma) \\
&= \sup_{a_\gamma \in \phi^{-1}(\phi(-\dot{\omega} + \dot{\psi}))} \xi^+_{A_R}(a_\gamma) \\
&= \sup_{a_\gamma \in \phi^{-1}(\phi(-\dot{\omega} - (-\dot{\psi})))} \xi^+_{A_R}(a_\gamma) \\
&= \xi^+_{A_R}(-\dot{\omega}_0 - (\dot{\psi}_0)) \\
&\geq \min\{ \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\psi}))} \xi^+_{A_R}(a_\gamma), \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\omega}))} \xi^+_{A_R}(a_\gamma) \} \\
&= \min\{ \xi^+_{B_R}(\phi(\dot{\psi})), \xi^+_{B_R}(\phi(\dot{\omega})) \},
\end{aligned}$$

$$\begin{aligned}
\xi^+_{B_R}(\phi(\dot{\psi}) + \phi(\dot{\omega}) - \phi(\dot{\psi})) &= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\psi}) + \phi(\dot{\omega}) - \phi(\dot{\psi}))} \xi^+_{A_R}(a_\gamma) \\
&= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\omega} + \dot{\psi}) + \phi(-\dot{\psi}))} \xi^+_{A_R}(a_\gamma) \\
&= \sup_{a_\gamma \in \phi^{-1}(\phi(-\dot{\psi} + \dot{\omega} + \dot{\psi}))} \xi^+_{A_R}(a_\gamma) \\
&= \xi^+_{A_R}(-\dot{\psi}_0 + \dot{\omega}_0 + \dot{\psi}_0) \\
&\geq \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\omega}) + \phi(\dot{\omega}) - \phi(\dot{\psi}))} \xi^+_{A_R}(a_\gamma) \\
&= \xi^+_{B_R}(\phi(\dot{\omega})),
\end{aligned}$$

$$\begin{aligned}
\xi^+_{B_R}(\phi(\dot{\psi})\alpha(\phi(\dot{\omega})\beta\phi(\dot{\kappa})) - \phi(\dot{\psi})\alpha\phi(\dot{\omega})) &= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\psi})\alpha(\phi(\dot{\omega})\beta\phi(\dot{\kappa})) - \phi(\dot{\psi})\alpha\phi(\dot{\omega}))} \xi^+_{A_R}(a_\gamma) \\
&= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\psi})\alpha(\phi(\dot{\kappa}\beta\dot{\omega}) - \phi(\dot{\omega}\alpha\dot{\psi}))} \xi^+_{A_R}(a_\gamma) \\
&= \sup_{a_\gamma \in \phi^{-1}(\phi((\dot{\kappa}\beta\dot{\omega})\alpha\dot{\psi}) - \phi(\dot{\omega}\alpha\dot{\psi}))} \xi^+_{A_R}(a_\gamma) \\
&\geq \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\psi}))} \xi^+_{A_R}(a_\gamma) \\
&= \xi^+_{B_R}(\phi(\dot{\psi})),
\end{aligned}$$

$$\begin{aligned}
\xi^+_{B_R}(\phi(\dot\psi)\alpha\phi(\dot\omega)) &= \sup_{a_\gamma\in\phi^{-1}(\phi(\dot\psi)\alpha\phi(\dot\omega))} \xi^+_{A_R}(a_\gamma) \\
&= \sup_{a_\gamma\in\phi^{-1}(\phi(\dot\omega\alpha\dot\psi))} \xi^+_{A_R}(a_\gamma) \\
&\geq \sup_{a_\gamma\in\phi^{-1}(\phi(\dot\omega))} \xi^+_{A_R}(a_\gamma) \\
&= \xi^+_{B_R}(\phi(\dot\omega)).
\end{aligned}$$

Similarly, we can prove it in the case of $\xi^-_{B_R}$. Hence, B_R is a BFI of M_{R2}. □

Theorem 3.2. A GNR anti-homomorphic pre-image of a BFI is a BFI.

Proof. Let $\phi : M_{R1} \to M_{R2}$ be a GNR anti-homomorphism. Let $B_R = (M_{R2}, \xi^+_{B_R}, \xi^-_{B_R})$ be a BFI of M_{R2}. Let $A_R = (M_{R1}, \xi^+_{A_R}, \xi^-_{A_R})$ be the pre-image of $B_R = (M_{R2}, \xi^+_{B_R}, \xi^-_{B_R})$ in M_{R1}. Let $\dot\psi, \dot\omega, \dot\kappa \in M_{R1}$ and $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned}
\xi^+_{A_R}(\dot\psi - \dot\omega) &= \xi^+_{B_R}(\phi(\dot\psi - \dot\omega)) \\
&= \xi^+_{B_R}(\phi(\dot\psi + (-\dot\omega)) \\
&= \xi^+_{B_R}(\phi(-\dot\omega) + \phi(\dot\psi)) \\
&= \xi^+_{B_R}(-\phi(\dot\omega) + \phi(\dot\psi)) \\
&\geq \min\{\xi^+_{B_R}(\phi(\dot\omega), \xi^+_{B_R}(\phi(\dot\psi))\} \\
&= \min\{\xi^+_{A_R}(\dot\omega), \xi^+_{A_R}(\dot\psi)\},
\end{aligned}$$

$$\begin{aligned}
\xi^+_{A_R}(\dot\psi + \dot\omega - \dot\psi) &= \xi^+_{B_R}(\phi(\dot\psi + \dot\omega - \dot\psi)) \\
&= \xi^+_{B_R}(\phi((\dot\psi + \dot\omega) + (-\dot\psi)) \\
&= \xi^+_{B_R}(\phi(-\dot\psi) + \phi(\dot\psi + \dot\omega)) \\
&= \xi^+_{B_R}(\phi(-\dot\psi) + \phi(\dot\omega) + \phi(\dot\psi)) \\
&\geq \xi^+_{A_R}(\phi(\dot\omega)),
\end{aligned}$$

$$\begin{aligned}
\xi^+_{A_R}(\dot\psi\alpha(\dot\omega\beta\dot\kappa) - \dot\psi\alpha\dot\omega) &= \xi^+_{B_R}(\phi(\dot\psi\alpha(\dot\omega\beta\dot\kappa) - \dot\psi\alpha\dot\omega)) \\
&= \xi^+_{B_R}(\phi(\dot\omega\beta\dot\kappa)\alpha\phi(\dot\psi) - \phi(\dot\psi\alpha\dot\omega)) \\
&= \xi^+_{B_R}(\phi(\dot\kappa)\beta\phi(\dot\omega))\alpha\phi(\dot\psi) - \phi(\dot\omega)\alpha\phi(\dot\psi)) \\
&\geq \xi^+_{B_R}(\phi(\dot\psi)) \\
&= \xi^+_{A_R}(\phi(\dot\psi)),
\end{aligned}$$

$$\begin{aligned}\xi^+_{A_R}(\dot\psi\alpha\dot\omega) &= \xi^+_{B_R}(\phi(\dot\psi\alpha\dot\omega))\\ &= \xi^+_{B_R}(\phi(\dot\omega)\alpha\phi(\dot\psi))\\ &\geq \xi^+_{B_R}(\phi(\dot\omega))\\ &= \xi^+_{A_R}(\phi(\dot\omega)).\end{aligned}$$

Similarly, we can prove it in the case of $\xi^-_{A_R}$. Hence, A_R is a BFI of M_{R1}. □

Theorem 3.3. A GNR anti-homomorphic image of a BFBI possessing both supremum and infimum properties is BFBI.

Proof. Let $\phi : M_{R1} \to M_{R2}$ be a GNR anti-homomorphism and let $A_R = (M_{R1}, \xi^+_{A_R}, \xi^-_{A_R})$ be a BFBI of M_{R1}. Let $\xi^+_{A_R}$ of M_{R1} possess supremum property. Let $B_R = (M_{R2}, \xi^+_{B_R}, \xi^-_{B_R})$ be the image of $A_R = (M_{R1}, \xi^+_{A_R}, \xi^-_{A_R})$ in M_{R2} with $\xi^+_{B_R}$ is the image of $\xi^+_{A_R}$, and $\xi^-_{B_R}$ is the image of $\xi^-_{A_R}$. Let $\phi(\dot\psi), \phi(\dot\omega), \phi(\dot\kappa) \in M_{R2}$ and $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned}\xi^+_{B_R}(\phi(\dot\psi) - \phi(\dot\omega)) &= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot\psi)-\phi(\dot\omega))} \xi^+_{A_R}(a_\gamma)\\ &= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot\psi)+\phi(-\dot\omega))} \xi^+_{A_R}(a_\gamma)\\ &= \sup_{a_\gamma \in \phi^{-1}(\phi(-\dot\omega+\dot\psi))} \xi^+_{A_R}(a_\gamma)\\ &= \sup_{a_\gamma \in \phi^{-1}(\phi(-\dot\omega-(-\dot\psi)))} \xi^+_{A_R}(a_\gamma)\\ &= \xi^+_{A_R}(-\dot\omega_0 - (\dot\psi_0))\\ &\geq \min\{\sup_{a_\gamma \in \phi^{-1}(\phi(\dot\psi))} \xi^+_{A_R}(a_\gamma), \sup_{a_\gamma \in \phi^{-1}(\phi(\dot\omega))} \xi^+_{A_R}(a_\gamma)\}\\ &= \min\{\xi^+_{B_R}(\phi(\dot\psi)), \xi^+_{B_R}(\phi(\dot\omega))\},\end{aligned}$$

$$\begin{aligned}\xi^+_{B_R}(\phi(\dot\psi) + \phi(\dot\omega) - \phi(\dot\psi)) &= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot\psi)+\phi(\dot\omega)-\phi(\dot\psi))} \xi^+_{A_R}(a_\gamma)\\ &= \sup_{a_\gamma \in \phi^{-1}(\phi(\dot\omega+\dot\psi)+\phi(-\dot\psi))} \xi^+_{A_R}(a_\gamma)\\ &= \sup_{a_\gamma \in \phi^{-1}(\phi(-\dot\psi+\dot\omega+\dot\psi))} \xi^+_{A_R}(a_\gamma)\\ &= \xi^+_{A_R}(-\dot\psi_0 + \dot\omega_0 + \dot\psi_0)\\ &\geq \sup_{a_\gamma \in \phi^{-1}(\phi(\dot\omega)+\phi(\dot\omega)-\phi(\dot\psi))} \xi^+_{A_R}(a_\gamma)\\ &= \xi^+_{B_R}(\phi(\dot\omega)),\end{aligned}$$

$$\begin{aligned}
&\xi^{+}_{B_R}((\phi(\dot{\psi})\alpha\phi(\dot{\omega})\beta\phi(\dot{\kappa})) \wedge (\phi(\dot{\psi})\alpha(\phi(\dot{\omega}) + \phi(\dot{\kappa})) - \phi(\dot{\psi})\alpha\phi(\dot{\omega}))) \\
&= \sup_{a_\gamma \in \phi^{-1}((\phi(\dot{\psi})\alpha\phi(\dot{\omega})\beta\phi(\dot{\kappa}))\wedge(\phi(\dot{\psi})\alpha(\phi(\dot{\omega})+\phi(\dot{\kappa}))-\phi(\dot{\psi})\alpha\phi(\dot{\omega})))} \xi^{+}_{A_R}(a_\gamma) \\
&= \sup_{a_\gamma \in \phi^{-1}((\phi(\dot{\kappa}\beta\dot{\omega}\alpha\dot{\psi})\wedge(\phi(\dot{\kappa}+\dot{\omega})\alpha\dot{\psi})-\phi(\dot{\omega}\alpha\dot{\psi})))} \xi^{+}_{A_R}(a_\gamma) \\
&\geq \min\{\sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\kappa}))} \xi^{+}_{A_R}(a_\gamma), \sup_{a_\gamma \in \phi^{-1}(\phi(\dot{\psi}))} \xi^{+}_{A_R}(a_\gamma)\} \\
&= \min\{\xi^{+}_{B_R}(\phi(\dot{\kappa})), \xi^{+}_{B_R}(\phi(\dot{\psi}))\}.
\end{aligned}$$

Similarly, we can prove it in the case of $\xi^{-}_{B_R}$. Hence, $B_R = (M_{R2}, \xi^{+}_{B_R}, \xi^{-}_{B_R})$ is a BFBI of M_{R2}. □

Theorem 3.4. A GNR anti-homomorphic pre-image of a BFBI is a BFBI.

Proof. Let $\phi : M_{R1} \to M_{R2}$ be a GNR anti-homomorphism. Let $B_R = (M_{R2}, \xi^{+}_{B_R}, \xi^{-}_{B_R})$ be a BFBI of M_{R2}. Let $A_R = (M_{R1}, \xi^{+}_{A_R}, \xi^{-}_{A_R})$ be the pre-image of $B_R = (M_{R2}, \xi^{+}_{B_R}, \xi^{-}_{B_R})$ in M_{R1}. Let $\dot{\psi}, \dot{\omega}, \dot{\kappa} \in M_{R1}$ and $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned}
\xi^{+}_{A_R}(\dot{\psi} - \dot{\omega}) &= \xi^{+}_{B_R}(\phi(\dot{\psi} - \dot{\omega})) \\
&= \xi^{+}_{B_R}(\phi(\dot{\psi} + (-\dot{\omega})) \\
&= \xi^{+}_{B_R}(\phi(-\dot{\omega}) + \phi(\dot{\psi})) \\
&= \xi^{+}_{B_R}(-\phi(\dot{\omega}) + \phi(\dot{\psi})) \\
&\geq \min\{\xi^{+}_{B_R}(\phi(\dot{\omega}), \xi^{+}_{B_R}(\phi(\dot{\psi}))\} \\
&= \min\{\xi^{+}_{A_R}(\dot{\omega}), \xi^{+}_{A_R}(\dot{\psi})\},
\end{aligned}$$

$$\begin{aligned}
\xi^{+}_{A_R}(\dot{\psi} + \dot{\omega} - \dot{\psi}) &= \xi^{+}_{B_R}(\phi(\dot{\psi} + \dot{\omega} - \dot{\psi})) \\
&= \xi^{+}_{B_R}(\phi((\dot{\psi} + \dot{\omega}) + (-\dot{\psi})) \\
&= \xi^{+}_{B_R}(\phi(-\dot{\psi}) + \phi(\dot{\psi} + \dot{\omega})) \\
&= \xi^{+}_{B_R}(\phi(-\dot{\psi}) + \phi(\dot{\omega}) + \phi(\dot{\psi})) \\
&\geq \xi^{+}_{A_R}(\phi(\dot{\omega})),
\end{aligned}$$

$$\begin{aligned}
&\xi^{+}_{A_R}((\dot{\psi}\alpha\dot{\omega}\beta\dot{\kappa}) \wedge (\dot{\psi}\alpha(\dot{\omega} + \dot{\kappa}) - \dot{\psi}\alpha\dot{\omega})) \\
&= \xi^{+}_{B_R}(\phi((\dot{\psi}\alpha\dot{\omega}\beta\dot{\kappa}) \wedge (\dot{\psi}\alpha(\dot{\omega} + \dot{\kappa}) - \dot{\psi}\alpha\dot{\omega})) \\
&= \xi^{+}_{B_R}(\phi(\dot{\kappa})\beta\phi(\dot{\omega})\alpha\phi(\dot{\psi})) \wedge ((\phi(\dot{\kappa}) + \phi(\dot{\omega}))\alpha\phi(\dot{\psi}) - \dot{\psi}(\dot{\omega})\alpha\phi(\dot{\psi}))) \\
&\geq \min\{\xi^{+}_{B_R}(\phi(\dot{\kappa}), \xi^{+}_{B_R}(\phi(\dot{\psi}))\} \\
&= \min\{\xi^{+}_{A_R}(\dot{\kappa}), \xi^{+}_{A_R}(\dot{\psi})\}.
\end{aligned}$$

Similarly, we can prove it in the case of $\xi^{-}_{A_R}$. Hence, A_R is a BFBI of M_{R1}. □

4. Conclusion

This article explored the anti-homomorphic image and pre-image of BFIs and BFBIs of GNRs. Soon we will extend these properties to weak BFBIs, prime BFIs, and bipolar fuzzy filters of GNRs.

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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