Asia Pac. J. Math. 2024 **11**:44

ON PERFECT RINGS DOMINATIONS IN GRAPHS

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Received Feb. 20, 2024

Abstract. A dominating set $D_p \subset V(G)$ is said to be a perfect dominating set of G if for every $v \in V(G)\setminus D$, there exists only one $u \in D$ such that $uv \in E(G)$. A dominating set $D_{ri} \subset V(G)$ is said to be a rings dominating set if each vertex $v \in V(G)\setminus D_{ri}$ is adjacent to atleast two vertices $V(G)\setminus D_{ri}$. In this paper, we introduce the concept of perfect rings domination in graphs and graphs formed by binary operations and show the existence of such dominating set in graph. We also provide the cases for graphs when the perfect domination number and the rings domination number can be equal.

2020 Mathematics Subject Classification. 05C69; 05C38.

Key words and phrases. domination; perfect domination; rings domination; perfect rings domination.

1. Introduction

For many years, the concepts in graph theory and graph domination have always been one of the most celebrated studies in mathematics mainly because it has a lot of research ideas due to its extensive applications to different disciplines like economics and physics. It is evident because there are many papers who study more on these [7], [11].

The concept of domination was introduced in 1962 by Berge [3] and many mathematicians have introduced a lot of variants of dominations up until today. One of the variants of domination is the perfect domination in graphs introduced by Livingston and Stout in [13]. This concept arose from the idea of involving resource allocation and placement in parallel computers [14].

This research idea has caught the attention of many mathematicians and so Caay and Arugay in [5] introduced the concept of perfect equitable domination in graphs. This is an extended concept of the perfect domination by joining the concept of equitable domination introduced by Deepak, et.al. in [10].

DOI: 10.28924/APJM/11-44

Also, Caay and Palahang in [6] introduced the concept of independent perfect domination in graphs. There are also many variants of perfect dominations in graphs which are also found in [2,4,15].

One of the newest developed variants of domination is the concept of rings domination in graph in 2022 introduced by Saja Abed and M.N. Al-Harere in [1]. Since it is one of the newest concepts of dominations, it has no further extension of studies yet until the following year, Caay in [4] studied equitable rings domination in graphs.

In this study, we introduce the concept of perfect rings domination in graphs. This means that the dominating set is perfect dominating and rings dominating at the same time. The layout of this paper is as follows. Section 2 contains some preliminaries of graph theoretic notions that are used in the study of this paper. In Section 3 we compare the perfect dominations and the rings dominations and derive some important results of this comparison. In Section 4 we investigate the perfect rings dominations and provide some important results of this paper while in Section 5, we provide important results on the binary operations of graphs.

2. Preliminaries

Throughout this paper, the graph we consider here is a connected simple graph. That means, there are no loops and multiple edges. A pair $G = (V(G), E(G))$ is called a graph (on V). The elements of $V(G)$ are called the vertices of G and the elements of $E(G)$ are called the edges of G. If no confusion arises, we can use V and E to denote the set of vertices and set of edges of G, respectively. Suppose $v \in V$, the neighborhood of v is the set $N_G(v) = \{u \in V : uv \in E.\}$. Given $D \subseteq V$, the set $N_G(D) = N(D) = \bigcup_{v \in D} N_G(v)$ and the set $N_G[D] = N[D] = D \bigcup N(D)$ are the *open neighborhood* and the *closed neighborhood* of D respectively.

We denote $\Delta(G)$ and $\delta(G)$ to be the maximum and minimum degree of G, respectively. We denote P_n, C_n, K_n, T_n and W_n for the path graph, cycle graph, complete graph, trees and wheel graph of order n, respectively.

Theorem 2.1. [8] *A graph G is a cycle graph if and only if every vertex of G is adjacent to two other vertices.*

Definition 2.2. [12] A **spanning subgraph** of a graph G is a subgraph obtained by deleting some edges of G with the same vertex set.

Example 2.3. A cycle C_n is a spanning subgraph of a complete graph K_n.

The following are the definitions of the binary operations in graphs used in this study: join, corona and cartesian product.

Definition 2.4. [12] The **join** $G+H$ of the two graphs G and H is the graph with vertex set

$$V(G+H)=V(G)+V(H)$$

and the edge set

$$E(G+H)=E(G)\cup E(H)\cup\{uv\ :\ u\in V(G),\ v\in V(H)\}.$$

Definition 2.5. [9] The **corona** $G\circ H$ of two graphs G and H is the graph obtained by taking one copy of G of order n and n copies of H, and then joining the ith vertex of G to every vertex in the ith copy of H.

Definition 2.6. [9,12] The **cartesian product** $G\times H$ of two graphs G and H is the graph with vertex set $V(G\times H)=V(G)\times V(H)$ and e is an edge of $G\times H$ if and only if $e=(u_i,v_j)\,(u_k,v_l)$ where either

i. $i=k$ and $v_jv_l\in E(H)$

ii. $j=l$ and $u_iu_k\in E(G)$.

We will now introduce the concept of domination.

Definition 2.7. [3] A subset X of V is a **dominating set** of G if for every $v\in V\backslash X$, there exists $x\in X$ such that $xv\in E$. That is, $N[X]=V$. The minimum cardinality of the dominating set X of G is called a **domination number** of G and is denoted by $\gamma(G)$.

Definition 2.8. [13] A dominating set S of G is said to be a **perfect dominating set** of G if when every element $v\in V$ is dominated by exactly one element in S. The cardinality of the smallest S of G is called **perfect domination number** of G and is denoted by $\gamma_p(G)$.

Definition 2.9. [1] A dominating set S of G is a **rings dominating set** if each vertex $v\in V\backslash S$ is adjacent to atleast two vertices $V\backslash S$. The cardinality of the minimal rings dominating set is called the **rings domination number** and is denoted by $\gamma_{ri}(G)$.

Remark 2.10. In this paper, if a graph G has a perfect dominating set or a rings dominating set, then G has a γ_p-set or G has a γ_{ri}-set, respectively.

Remark 2.11. For a rings dominating set S of any graph G of order n, we have

(1) the order of G is $n\geq 4$.

(2) for each $v\in V\backslash S$, $\deg(v)\geq 3$.

(3) $1\leq |S|\leq n-3$.

(4) $3\leq |V\backslash S|\leq n-1$.

(5) $1\leq \gamma_{ri}(G)\leq |S|\leq n-3$.

3. The Perfect Dominations and the Rings Dominations in Graphs

We present some important results of the perfect domination and the rings domination.

Proposition 3.1. *For any integer $n \geq 2$, $\gamma_p(P_n) = \left\lceil \frac{n}{2} \right\rceil$.*

Remark 3.2. In a path P_n, consecutive vertices of γ_p-set are either adjacent or at a distance 3 apart.

The proof of Theorem 3.1 and the observation of Remark 3.2 are similar from the paper of Caay and Arugay in [5]. In fact, the result of γ_p-set of P_n is equal to the perfect equitable dominating set in [5] but this concept is out of the study of this paper.

Lemma 3.3. *Let $m, n \geq 3$. Then $\gamma_p(K_n) = \gamma_p(W_m)$.*

Lemma 3.4. [1] *Let $m \geq 4$ and $n \geq 3$. Then $\gamma_{ri}(K_n) = \gamma_{ri}(W_m)$.*

Lemma 3.5. [1] *Trees have no rings dominating set.*

Dealing with the new concept domination which is the perfect rings domination in graphs, it is natural to ask how the perfect domination and rings domination relate to each other in terms of number. The result however is negative in general, but for some specific graphs, there is a specific relationship of this two concepts.

Theorem 3.6. *Let G be a graph of order $n \geq 4$ that is not formed by a binary operations of graphs. If γ_p-set and the γ_{ri}-set of G are equal, then for such dominating set S, every vertex of $V(G) \setminus S$ has degree atleast 3. Conversely, if every vertex of $V(G) \setminus S$ has degree atleast 3 and is adjacent to exactly one vertex in S, then S is the γ_p-set and the γ_{ri}-set of G.*

Proof. Let $\gamma_p(G) = \gamma_{ri}(G)$. If S be such dominating set, then by the definition, every vertex of $V(G) \setminus S$ is adjacent to atleast 2 vertices in $V(G) \setminus S$. Since S is also a γ_p-set, every vertex of $V(G) \setminus S$ is adjacent to exactly one vertex in S. Thus, every vertex in $V(G) \setminus S$ has degree atleast 3. Conversely, suppose every vertex of $V(G) \setminus S$ has degree atleast 3 and is adjacent to exactly one vertex in S. Then every vertex in $V(G) \setminus S$ is adjacent to atleast 2 vertices in $V(G) \setminus S$. By the definition, S is γ_{ri}-set and is also a γ_p-set. Hence, $\gamma_p(G) = \gamma_{ri}(G)$. □

The necessary condition of Theorem 3.6 now introduces the concept of perfect rings domination. We now introduce formally the concept.

Definition 3.7. A dominating set $S \subset V$ is said to be **perfect rings dominating set** of G if for every $u \in V \backslash S$, there exists only one $v \in S$ such that $uv \in E$ and there exist at least two vertices u_s and u_t in $V \backslash S$ such that $uu_s, uu_t \in E$. The minimum cardinality of S is called **perfect rings domination number**

of G and is denoted by $\gamma_{pri}(G)$. Moreover, if $S \subseteq V(G)$ is a perfect rings dominating set of G, then S is a γ_{pri}-set of G.

Remark 3.8. In this study, if u belongs to a γ_{pri}-set S of G, then for any $v \in V(G) \setminus S$ such that $uv \in E(G)$, we say that u perfect rings dominates the vertex v, or v is perfect rings dominated by u.

Example 3.9. Consider the graph in Figure 1 with the vertex set

$$V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\},$$

Consider the set $S = \{u_1, u_8\}$. Note that u_2, u_3 and u_4 are perfect rings dominated by u_1, and u_5, u_6 and u_7 are perfect rings dominated by u_8. This means that S is a γ_{pri}-set of G. Hence, $\gamma_{pri}(G) = 2$.

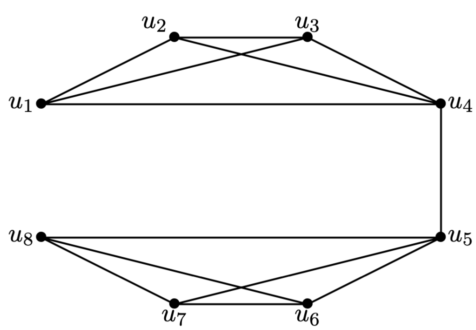


Figure 1. Example of γ_{pe0}-set in a graph G.

Remark 3.10. For any given graph G, the following is obvious observation from the definition.

i. $\gamma_{pri}(G) \leq \gamma_p(G)$; and
ii. $\gamma_{pri}(G) \leq \gamma_{ri}(G)$.

4. The Perfect Rings Domination in Graphs

In this section, we present the results of perfect rings domination of simple graphs or the graphs that are not formed by binary operation.

Theorem 4.1. *Let G be any graph of order $n \geq 4$. Then $\gamma_{pri}(G) = 1$ if and only if $\Delta(G) = n - 1$ and $\delta(G) \geq 3$.*

Proof. Suppose $\gamma_{pri}(G) = 1$. Then if S is γ_{pri}-set of G, $|S| = 1$. Let $v \in S$. Then $\deg(v) = n - 1$. Now for every $u \in V \backslash S$, u is adjacent to v and to at least 2 vertices not equal to v. This means that $\deg(u) \geq 3$. Conversely, suppose $v \in V$ with $\deg(v) = n - 1$. Let $u \in V$ with $\deg(u) \geq 3$. Then $uv \in E$. Thus, v dominates u and all other vertices of G. Also $\deg(u) \geq 3$ implies that u is adjacent to at least two vertices dominated by v. Thus, $v \in S$, the γ_{pri}-set of G. Now suppose there exists $w \in S$ with $v \neq w$. Then there exists $\overline{w} \in V$ such that $w\overline{w} \in E$. But $v\overline{w} \in E$ since $\deg(v) = n-1$. This contradicts the definition. Thus, $v = w$ and so $|S| = 1$ implying $\gamma_{pri}(G) = 1$. □

Theorem 4.2. *Let S be a subset of the vertex set V of a graph G. Then S is a perfect dominating set if and only if $V\backslash S$ which forms a cycle of order at least 3 and every element $x \in V\backslash S$ is adjacent to exactly one element of S*

Proof. Let $S \subset V$ be a perfect rings dominating set. By definition, every $x \in V\backslash S$ is adjacent to exactly one element in S and there exist at least two vertices in $V\backslash S$ that are adjacent to x. By Theorem 2.1, x and its adjacent vertices form a cycle. The converse directly follows from Theorem 2.1 and Definition 3.7. □

Theorem 4.3. *Let G be a graph. If G is either a complete graph or a wheel graph of order at least 4, then $\gamma_{pri}(G) = 1$.*

The proof of Theorem 4.3 is very obvious that it follows directly from the definition of a perfect domination and the fact that the number of vertices is at least 4 shows directly the message of rings domination.

Following the definition of the perfect rings domination, it is obvious that using the Theorem 3.5, the following proposition hold.

Proposition 4.4. *There does not exist a γ_{pri}-set in P_n. In general, there does not exist a γ_{pri}-set in T_n. Furthermore, There does not exist a γ_{pri}-set in C_n.*

5. The Perfect Rings Dominations in Some Binary Operations

In this section, we investigate the perfect rings domination in graphs formed by some binary operations. Let us briefly recall some binary operations.

Theorem 5.1. *Let G and H be graphs. Then $G + H$ has γ_{pri}-set and its γ_{pri}-set is the γ_{pri}-set of G if either of the following holds:*

(i) *G is a complete graph; or*
(ii) *G is a wheel graph.*

Moreover, $\gamma_{pri}(G + H) = 1$.

Proof. Suppose G is a complete graph. Then by Theorem 4.3, $\gamma_{pri}(G) = 1$. This means that if S is a $\gamma_{pri}(G)$, $|S| = 1$. Let $u \in S$. Let $\overline{u}$ be the vertex in $G + H$ corresponding to u in G. Then $\overline{u} \in \overline{S}$ where $\overline{S}$ is the set corresponding to the set S. By the definition of the join of graphs, $\overline{u}$ is adjacent to all vertices of H. Similar argument follows for the case when G is a wheel graph. □

The following theorem will show that the converse of Theorem 5.1 holds true but for a special case of complete graph which is a trivial graph.

Theorem 5.2. *Let G and H be graphs. If $G+H$ has γ_{pri}-set equal to the γ_{pri}-set of G, then $\gamma_{pri}(G+H)=1$ if and only if G is a trivial graph.*

Proof. Suppose $G+H$ has γ_{pri}-set equal to the γ_{pri}-set of G. If G is a trivial graph, then the result follows. Conversely, if $\gamma_{pri}(G+H)=1$, then the γ_{pri}-set of G has a cardinality of 1. Using Theorem 5.1 the result directly follows in particular. □

Remark 5.3. Theorem 5.1 and Theorem 5.2 hold if we swap the rule for G and H since the join of graph is "*commutative*" in nature.

Note that Proposition 4.4 tells that trees don't have γ_{pri}-set but the following corollary shows that when tree is added by either a complete graph or a wheel graph, the γ_{pri}-set exists.

Corollary 5.4. *Let G be any graph or order n. Then $G+T_n$ has γ_{pri}-set and its γ_{pri}-set is the γ_{pri}-set of G if and only if G is either a complete graph or a wheel graph. Moreover, $\gamma_{pri}(G+T_n)=1$.*

Proof. Let S be a γ_{pri}-set of $G+T_n$. By Proposition 4.4, $S \nsubseteq V(T_n)$. Thus, $S \subseteq V(G)$. Now by Theorem 5.1 the result follows. The converse follows by Theorem 5.1. □

Theorem 5.5. *Let G and H be nontrivial graphs. Then $\gamma_{pri}(G \circ H)=|V(G)|$.*

Proof. Let $v_i \in V(G)$ and $u_j \in V(H)$. Let $S \subseteq V(G \circ H)$ be the γ_{pri}-set of $G \circ H$.

If $S \subseteq V(G)\backslash V(H)$. Suppose $|S|<|V(G)|$. Then there exists $v_k \in V(G)\backslash S$. Let H^i be the ith copy of H joined in the ith vertex of G. Then H^k is the kth copy of H joined in $v_k \in V(G)$. Let $u_j^k \in V\left(H^k\right)$. Then $v_k u_j^k \in E(G \circ H)$ for all j. Thus $N[S] \neq V(G \circ H)$. This is a contradiction. If $S>|V(G)|$, then this is all but contradiction by the assumption of the case. Therefore, $|S|=|V(G)|$.

If $S \subseteq V(H)\backslash V(G)$. Consider $V(H)^k$, the kth copy of the $V(H)$. Let $S^k \subseteq V(H)^k$ be the kth disjoint subset of $V(H)$ of the γ_{pri}-set S. Suppose $\left|S^k\right|>1$. Then there exist $u_r^k, u_s^k \in V(H)$ with $r \neq s$ that dominates the other vertex u_t^k. Since every u_i^k is adjacent to $v_k \in V(G)$, u_r^k and u_s^k both dominate v_k. This is a contradiction to the definition. Thus $\left|S^k\right|=1$. Since there are $|V(G)|$ copies of S^i and every S^i are disjoint, it follows that $|S|=\left|S^k\right|\ |V(G)|=|V(G)|$.

Therefore either of the case, $\gamma_{pri}(G \circ H)=|V(G)|$. This proves the claim. □

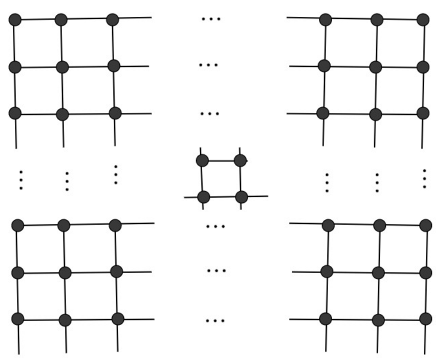
Corollary 5.6. *Let G be any graph of order $n \geq 3$. If H is a trivial graph, K_n or W_n, $n \geq 4$, then $\gamma_{pri}(G \circ H)=|V(G)|$.*

Remark 5.7. Note that Proposition 4.4 tells us that γ_{pri}-set of P_n does not exist. However, when a cartesian product is done, there is an exception. Consider the following propositions below.

Proposition 5.8. *Let n and m be positive integers such that $m \leq n$. Then*

$$\gamma_{pri}\left(P_n \times P_m\right) \leq m\left\lceil\frac{n}{3}\right\rceil.$$

Proof. We assume that the vertices of $P_n \times P_m$ are arranged in a manner as rows and columns positions as shown below.

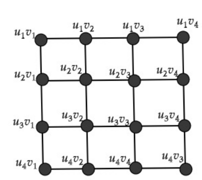


For instance consider the vertex u_iv_j where $u_i \in V(P_n)$ and $v_j \in V(P_m)$, the vertex that is placed in the ith row and jth column position in the figure. The given vertex is adjacent to a vertex in the $(i-1)$th row that is not dominated by any vertex in the ith row. In particular, u_iv_j is adjacent to $u_{i-1}v_j$, but u_iv_{j-1} or u_iv_{j+1} is not adjacent to $u_{i-1}v_j$. Also, such $u_{i-1}v_j$ is adjacent to at least two vertices not in the ith row. Thus, this arrangement qualifies for γ_{ri}-set and so our focus is to form the γ_p-set. Since $m \leq n$, we consider the arrangement of n for choosing the possible elements of γ_p-set to be multiplied m times since our goal is to choose the smallest possible value. Without loss of generality, we may assume that n represents the number of rows. Then we have m number of columns. Consider the first column. This is P_n. By Theorem 3.1, we have $\left\lceil\frac{n}{3}\right\rceil$. Using the fact that there are m columns, the result automatically follows. □

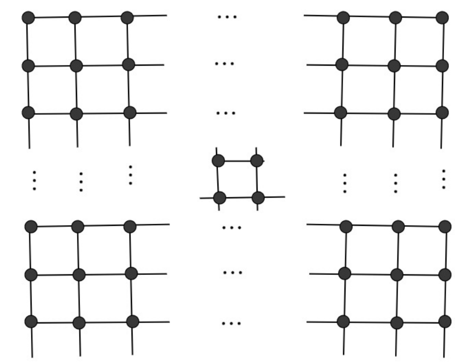
Proposition 5.9. *Let n be even positive integer. Then*

$$\gamma_{pri}\left(P_{4n} \times P_{4n}\right) \leq \left(4\sum_{k=0}^{n/2} 4(k+1)+1\right)\left\lceil\frac{2n}{3}\right\rceil.$$

Proof. Consider $P_4 \times P_4$ and label the vertices as shown below.



Note that the graph formed a central cycle of order 4 with vertices u_2v_2, u_2v_3, u_3v_3 and u_3v_2. The same things goes if we consider $P_8 \times P_8$. We inductively extend the graph so that we have $P_{4n} \times P_{4n}$ for a very large n. Assume that the vertices are arranged in the same manner as rows and column positions as shown below. For instance, the vertex u_iv_j is located in the ith row and jth column position.



Observe that we have a central cycle of order 4 with vertices

$$u_{4n/2}v_{4n/2}, u_{4n/2}v_{4n/2+1}, u_{4n/2+1}v_{4n/2+1} \text{ and } u_{4n/2+1}v_{4n/2}$$

For instance, observe that $u_{4n/2}v_{4n/2+1}$ is adjacent to vertices $u_{4n/2}v_{4n/2+2}$ and $u_{4n/2-1}v_{4n/2+1}$ but not to $u_{4n/2-1}v_{4n/2+2}$, $u_{4n/2-2}v_{4n/2+3}$ and so on. We make an imaginary path P' from $u_{4n/2}v_{4n/2+1}$ to $u_{4n/2-1}v_{4n/2+2}$ to $u_{4n/2-2}v_{4n/2+3}$ and so on until u_1v_{4n}. The resulting P' is of length $2n$. Note that each vertices of P' have at least 4 adjacent vertices and so their adjacent vertices not belong to P' have also at least 4 degrees. This means we can only focus on the existence of γ_p-set. By Theorem 3.1, we have $\left\lceil \frac{2n}{3} \right\rceil$ to choose from P' for the possible members of γ_{pri}-set of $P_{4n} \times P_{4n}$. Since every element of P' is part of a cycle of different order and the entire cycle can dominate the neighboring cycle such that every vertices of the neighboring cycle is dominated once by the selected cycle, the selection of elements for γ_{pri}-set is uniquely determined so that the total number is as smallest as possible. In this case we select the central cycle since it is the smallest cycle in the pattern. This means, from the vertices of P', we choose $u_{4n/2}v_{4n/2+1}$. By Remark 3.2, the next vertex is $u_{4n/2}v_{4n/2+1}$. Continuing the process, we get the last vertex of P' as u_1v_{4n}. Also by Remark 3.2, it is possible that in γ_p-set for P', the vertices could be two consecutive vertices. If this case happens, then we may choose the next cycle after the central since it is the smallest cycle among the others. By this, we may have two cases.

- **Case 1**. Suppose the selection of vertices P' are all 3 distance apart. Then from $u_{4n/2}v_{4n/2+1}$ from the cycle of order 4, the next vertex is $u_{4n/2-3}v_{4n/2+4}$ and is part of the cycle of order $4(4)$. Thus, by induction, the chosen vertex from P' to dominate the rest of the vertices are

$u_{4n/2-(1+2k}v_{4n/2+2k}$ and is part of the cycle of order $4k$. Thus the sum of the sizes of the cycles containing those chosen vertices is less than or equal to $\sum_{k=0}^{n/2} 4(4k)$.

- **Case 2**. Suppose there exists two consecutive vertices by Remark 3.2. Then, choosing the cycle next to the central and such cycle is of order 8. In this case, the chosen vertex is move one unit away from the central. In this case, each cycle of the chosen vertex is of size $4k+4$. Thus the sum of the sizes of the cycles containing those chosen vertices is less than or equal to $\sum_{k=0}^{n/2} 4(4k+4)+4$.

Simplifying each of the case, we obtain the desired result.

□

Theorem 4.3 tells us that there exists a γ_{pri}-set of a complete graph and a wheel graph. However, performing the cartesian product of with K_n will result differently.

Proposition 5.10. *Let G be any graph of order n with $\delta(G)=1$ and $\Delta(G)<n-1$, then $K_n \times G$ does not have γ_{pri}-set.*

The following results are obvious and easy to show.

Theorem 5.11. *For any integer $n \geq 3$, $\gamma_{pri}(K_n \times K_n)=1$.*

Corollary 5.12. $\gamma_{pri}(P_2 \times P_2)=1$.

6. Conclusion and Recommendations

From the study of perfect domination introduced by Livingston and Stout in [**?**] as a response of the problem in resource allocation and placement in parallel computers, and the rings domination introduced by Saaja and Al-Harere in [1], the concept of perfect rings domination is introduced in this study. We provide the concept with existence of such domination and bounds on the number for simple graphs and graphs produced by binary operations like join, corona and cartesian product. This concept offers a wide range of applications especially in analyzing how the placement of computer networks is done properly to maximize the connection and yet to minimize the cost of the use of routers for internet source without compromising the speed of connectivity. To this concept we recommend that one can investigate this variant of domination to other binary binary operations including the strong product.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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