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CONSISTENT IDEALS OF PSEUDO-COMPLEMENTED LATTICES

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Abstract. The notions of consistent ideals, fully consistent $\mathcal{F}^{\nabla}-$ideals, and closed ideals in a pseudo-complemented distributive lattice are introduced, and their characterization theorems are obtained. We also derived a set of equivalent conditions for every ideal of a pseudo-complemented distributive lattice to make it a consistent ideal. The concept of ornate prime $\mathcal{F}^{\nabla}-$ideal is introduced, and established equivalent conditions for every maximal $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented distributive lattice to make it an ornate prime $\mathcal{F}^{\nabla}-$ideal.

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1. Introduction

In 1937, Stone [16] established a theory of a topological duality for distributive lattice. Katriňák [10] investigated the characterization of stone lattices. Many researchers [2–4, 8, 9] studied the properties of stone lattices extensively. In 1963, Varlet characterized pseudo-complemented lattice in terms of principal ideals. Further he established a concept of quasicomplemented lattice in 1968. In 1969, T.P. Speed [13] provided several ways to characterize A^*, and established some properties of congruences. After that he [14, 15] developed important results on homeomorphism and isomorphism. In 1972, Cornish [6] explored the properties of prime ideals and annihilator ideals and later he studied the properties of minimal prime ideals [5]. In 1972, B.A. Davey [7] introduced the concept of $m-$stone lattices and investigated its properties. In 2018, Badawy [1] introduced and studied normal filters in the class of stone lattices. Phaneedra Kumar et al. [11] introduced the concept of $D-$filters in a distributive lattice and derived one-to-one correspondence between the class of all minimal prime $D-$filters of a distributive lattice and the class of all minimal prime $D-$filters of the corresponding

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quotient algebra with respect to this congruence. Recently, in 2022, M.S.Rao [12] defined and studied the concepts of coherent ideals and strongly coherent ideals in pseudo-complemented distributive lattices. This paper introduces and defines the concepts of consistent ideals, fully consistent $\mathcal{F}^{\nabla}-$ideals and ornate prime $\mathcal{F}^{\nabla}-$ideal with in pseudo-complemented distributive lattices. We establishes a set of equivalent conditions that determine an ideal in a pseudo-complemented distributive lattices to be consistent. It demonstrates that every fully consistent $\mathcal{F}^{\nabla}-$ideal and every closed ideal in a pseudo complemented distributive lattices is a consistent ideal. Later, we introduces the concept of quasi $\mathcal{F}-$stone pseudo complemented distributive lattices, which is a generalization of $\mathcal{F}-$stone lattices and it characterizes in terms of fully consistent ideals. We presents a notion of ornate prime $\mathcal{F}^{\nabla}-$ideal and study its characterizations. Our paper further establishes a set of equivalent conditions for a maximal $\mathcal{F}^{\nabla}-$ideal in a pseudo complemented distributive lattices to be fully consistent prime $\mathcal{F}^{\nabla}-$ideal. This characterization leads to better understanding of $\mathcal{F}^{\nabla}-$complemented lattices. Finally, the paper provides a set of equivalent conditions for a prime $\mathcal{F}^{\nabla}-$ideal in a pseudo complemented distributive lattice to be considered as ornate $\mathcal{F}^{\nabla}-$ideal.

2. Preliminaries

Definition 2.1. [3] An algebra $(\mathcal{L}, \wedge, \vee)$ of type $(2, 2)$ is called a distributive lattice if for all $\beta_1, \beta_2, \beta_3 \in \mathcal{L}$, it satisfies the following properties $(1), (2), (3)$ and (4) along with (5) or $(5')$

(1) $\beta_1 \wedge \beta_1 = \beta_1$, $\beta_1 \vee \beta_1 = \beta_1$,

(2) $\beta_1 \wedge \beta_2 = \beta_2 \wedge \beta_1$, $\beta_1 \vee \beta_2 = \beta_2 \vee \beta_1$,

(3) $(\beta_1 \wedge \beta_2) \wedge \beta_3 = \beta_1 \wedge (\beta_2 \wedge \beta_3)$, $(\beta_1 \vee \beta_2) \vee \beta_3 = \beta_1 \vee (\beta_2 \vee \beta_3)$,

(4) $(\beta_1 \wedge \beta_2) \vee \beta_1 = \beta_1$, $(\beta_1 \vee \beta_2) \wedge \beta_1 = \beta_1$,

(5) $\beta_1 \wedge (\beta_2 \vee \beta_3) = (\beta_1 \wedge \beta_2) \vee (\beta_1 \wedge \beta_3)$,

(5′) $\beta_1 \vee (\beta_2 \wedge \beta_3) = (\beta_1 \vee \beta_2) \wedge (\beta_1 \vee \beta_3)$.

A non-empty subset $\mathcal{A}$ of a lattice $\mathcal{L}$ is called an ideal(filter) of $\mathcal{L}$ if $\gamma_1 \vee \gamma_2 \in \mathcal{A}(\gamma_1 \wedge \gamma_2 \in \mathcal{A})$ and $\gamma_1 \wedge \beta_1 \in \mathcal{A}(\gamma_1 \vee \beta_1 \in \mathcal{A})$ whenever $\gamma_1, \gamma_2 \in \mathcal{A}$ and $\beta_1 \in \mathcal{L}$. The set $\mathcal{I}(\mathcal{L})$ of all ideals of $(\mathcal{L}, \vee, \wedge, 0)$ forms a complete distributive lattice as well as the set $\mathcal{F}(\mathcal{L})$ of all filters of $(\mathcal{L}, \vee, \wedge, 1)$ forms a complete distributive lattice. A proper ideal (filter) $\mathcal{M}$ of a lattice is called *maximal* if there exists no proper ideal(filter) $\mathcal{N}$ such that $\mathcal{M} \subset \mathcal{N}$.

The set $(\gamma_1] = \{\beta_1 \in \mathcal{L} \mid \beta_1 \leq \gamma_1\}$ is called a principal ideal generated by γ_1 and the set of all principal ideals is a sublattice of $\mathcal{I}(\mathcal{L})$. Dually the set $[\gamma_1) = \{\beta_1 \in \mathcal{L} \mid \gamma_1 \leq \beta_1\}$ is called a *principal filter* generated by γ_1 and the set of all principal filters is a sublattice of $\mathcal{F}(\mathcal{L})$. A proper ideal (proper filter) $\mathcal{P}$ of a lattice $\mathcal{L}$ is called *prime* if for all $\gamma_1, \gamma_2 \in \mathcal{L}$, $\gamma_1 \wedge \gamma_2 \in \mathcal{P}$ $(\gamma_1 \vee \gamma_2 \in \mathcal{P})$ then $\gamma_1 \in \mathcal{P}$ or $\gamma_2 \in \mathcal{P}$. Every maximal ideal (filter) is prime.

Theorem 2.2. [6] *A prime ideal $\mathcal{P}$ of a distributive lattice $\mathcal{L}$ is minimal if and only if to each $\beta_1 \in \mathcal{P}$, there exists $\beta_2 \notin \mathcal{P}$ such that $\beta_1 \wedge \beta_2 = 0$.*

The pseudo-complement γ_2^* of an element γ_2 is the greatest element disjoint from γ_2, if such an element exists. The defining property of γ_2^* is:

$$\gamma_1 \wedge \gamma_2 = 0 \Leftrightarrow \gamma_1 \wedge \gamma_2^* = \gamma_1 \Leftrightarrow \gamma_1 \leq \gamma_2^*$$

where $\leq$ is a partial ordering relation on the lattice $\mathcal{L}$.

A distributive lattice $\mathcal{L}$ in which every element has a pseudo-complement is called a pseudo-complemented distributive lattice. For any two elements γ_1, γ_2 of a pseudo-complemented lattice, we have the following:

(1) $\gamma_1 \leq \gamma_2 \Rightarrow (\gamma_2)^* \subseteq (\gamma_1)^*$,

(2) $\gamma_1 \leq \gamma_1^{**}$,

(3) $\gamma_1^{***} = \gamma_1^*$,

(4) $(\gamma_1 \vee \gamma_2)^* = \gamma_1^* \wedge \gamma_2^*$,

(5) $(\gamma_1 \wedge \gamma_2)^{**} = \gamma_1^{**} \wedge \gamma_2^{**}$.

An element γ_1 of a pseudo-complemented distributive lattice $\mathcal{L}$ is called a dense element if $\gamma_1^* = 0$ and the set $\mathcal{D}$ of all dense elements of $\mathcal{L}$ forms a filter in $\mathcal{L}$. An element $\gamma_1 \in \mathcal{L}$ is called a closed element if $\gamma_1^{**} = \gamma_1$.

Definition 2.3. [3] A pseudo-complemented distributive lattice $\mathcal{L}$ is called a stone lattice if $\beta_1^* \vee \beta_1^{**} = 1$ for all $\beta_1 \in \mathcal{L}$.

Theorem 2.4. [3] *The following conditions are equivalent in a pseudo-complemented distributive lattice $\mathcal{L}$:*

(1) *$\mathcal{L}$ is a stone lattice,*

(2) *for $\beta_1, \beta_2 \in \mathcal{L}$, $(\beta_1 \wedge \beta_2)^* = \beta_1^* \vee \beta_2^*$,*

(3) *for $\beta_1, \beta_2 \in \mathcal{L}$, $(\beta_1 \vee \beta_2)^{**} = \beta_1^{**} \vee \beta_2^{**}$.*

For any non-empty subset $\mathcal{A}$ of a distributive lattice $\mathcal{L}$, the annihilator [13] of $\mathcal{A}$ is defined as $\mathcal{A} = \{\beta_1 \in \mathcal{L} \mid \gamma_1 \wedge \beta_1 = 0, \text{ for all } \gamma_1 \in \mathcal{A}\}$. For any $\emptyset \neq \mathcal{A} \subseteq \mathcal{L}$, $\mathcal{A}^*$ is an ideal of $\mathcal{L}$ such that $\mathcal{A} \cap \mathcal{A}^* = \{0\}$. In case of $\mathcal{A} = \{\gamma_1\}$, we simply denote $\{\gamma_1\}^*$ by $(\gamma_1)^*$. Throughout the paper, $\mathcal{L}$ represents a pseudo-complemented distributive lattice and $\mathcal{F}$ represents a filter of $\mathcal{L}$.

3. Consistent ideals

This section presents the introduction of consistent ideals and fully consistent $\mathcal{F}^\nabla$-ideals within pseudo-complemented distributive lattices. We establishes a set of equivalent conditions that determine an ideal in a pseudo-complemented distributive lattice to be consistent. It demonstrates that every fully consistent $\mathcal{F}^{\nabla}-$ideal and every closed ideal in a pseudo complemented distributive lattice is a consistent ideal. We introduces the concept of $\mathcal{F}-$stone lattices and establishes its characterization results. We define the notion of quasi $\mathcal{F}-$stone lattices and it characterizes in terms of fully consistent $\mathcal{F}^{\nabla}-$ideals.

Definition 3.1. Let $\mathcal{I}$ be any ideal of distributive lattice $\mathcal{L}$. An ideal $\mathcal{G}$ of $\mathcal{L}$ is said to be an $\mathcal{I}-$ideal of $\mathcal{L}$ if $\mathcal{I} \subseteq \mathcal{G}$.

Definition 3.2. Let $\mathcal{S}$ be a non empty subset of a pseudo-complemented lattice $\mathcal{L}$. For any filter $\mathcal{F}$ of $\mathcal{L}$, define

$$\mathcal{S}^{\mathcal{F}} = \{\gamma_1 \in \mathcal{L} \mid \mu_1^* \vee \gamma_1^* \in \mathcal{F}, \text{ for all } \mu_1 \in \mathcal{S}\}$$
and
$$\mathcal{F}^{\nabla} = \{\beta_1 \mid \beta_1^* \in \mathcal{F}\}.$$

Clearly, we have that $\mathcal{F}^{\nabla}$ is an ideal of $\mathcal{L}$.

Proposition 3.3. *Let $\mathcal{S}$ be a non empty subset of a pseudo-complemented lattice $\mathcal{L}$. Then $\mathcal{S}^{\mathcal{F}}$ is an $\mathcal{F}^{\nabla}-$ideal of $\mathcal{L}$.*

Proof. For any $\mu_1 \in \mathcal{S}$, we have that $\mu_1^* \vee 0^* = 0^* \in \mathcal{F}$ and hence $0 \in \mathcal{S}^{\mathcal{F}}$. Therefore $\mathcal{S}^{\mathcal{F}} \neq \emptyset$. Let $\gamma_1, \gamma_2 \in \mathcal{S}^{\mathcal{F}}$. Then $\mu_1^* \vee \gamma_1^* \in \mathcal{F}$ and $\mu_1^* \vee \gamma_2^* \in \mathcal{F}$, for all $\mu_1 \in \mathcal{S}$. That implies $(\mu_1^* \vee \gamma_1^*) \wedge (\mu_1^* \vee \gamma_2^*) \in \mathcal{F}$ and hence $\mu_1^* \vee (\gamma_1 \vee \gamma_2)^* \in \mathcal{F}$, for all $\mu_1 \in \mathcal{S}$. Therefore $\gamma_1 \vee \gamma_2 \in \mathcal{S}^{\mathcal{F}}$. Let $\gamma_1 \in \mathcal{S}^{\mathcal{F}}$. Then $\mu_1^* \vee \gamma_1^* \in \mathcal{F}$, for all $\mu_1 \in \mathcal{S}$. Let μ_2 be any element of $\mathcal{L}$. Since $\mu_2 \wedge \gamma_1 \leq \gamma_1$, we get $\gamma_1^* \leq (\mu_2 \wedge \gamma_1)^*$. Then $\mu_1^* \vee \gamma_1^* \leq \mu_1^* \vee (\gamma_1 \wedge \mu_2)^*$. Since $\mu_1^* \vee \gamma_1^* \in \mathcal{F}$, we get $\mu_1^* \vee (\gamma_1 \wedge \mu_2)^* \in \mathcal{F}$. Therefore $\gamma_1 \wedge \mu_2 \in \mathcal{S}^{\mathcal{F}}$. Thus $\mathcal{S}^{\mathcal{F}}$ is an ideal of $\mathcal{L}$. Let $\beta_1 \in \mathcal{F}^{\nabla}$. Then $\beta_1^* \in \mathcal{F}$ and hence $\mu_1^* \vee \beta_1^* \in \mathcal{F}$, for all $\mu_1 \in \mathcal{S}$. Therefore $\beta_1 \in \mathcal{S}^{\mathcal{F}}$. Thus $\mathcal{F}^{\nabla} \subseteq \mathcal{S}^{\mathcal{F}}$. □

The following lemma comes directly from the above definition.

Lemma 3.4. *Let $\mathcal{S}$ and $\mathcal{T}$ be two non-empty subsets of a pseudo-complemented lattice $\mathcal{L}$. Then*

(1) $\{0\}^{\mathcal{F}} = \mathcal{L}$ *and* $\mathcal{L}^{\mathcal{F}} = \mathcal{F}^{\nabla}$,

(2) $\mathcal{S} \subseteq \mathcal{T}$ *implies* $\mathcal{T}^{\mathcal{F}} \subseteq \mathcal{S}^{\mathcal{F}}$,

(3) $\mathcal{S} \subseteq \mathcal{S}^{\mathcal{F}\mathcal{F}}$,

(4) $\mathcal{S}^{\mathcal{F}\mathcal{F}\mathcal{F}} = \mathcal{S}^{\mathcal{F}}$,

(5) $\mathcal{S}^{\mathcal{F}} = \mathcal{L} \Leftrightarrow \mathcal{S} = \mathcal{F}^{\nabla}$.

Proposition 3.5. *Let $\mathcal{S}$ and $\mathcal{T}$ be any two ideals of a pseudo-complemented lattice $\mathcal{L}$. Then $(\mathcal{S} \vee \mathcal{T})^{\mathcal{F}} = \mathcal{S}^{\mathcal{F}} \cap \mathcal{T}^{\mathcal{F}}$.*

Proof. Let $\gamma_1 \in \mathcal{S}^{\mathcal{F}} \cap \mathcal{T}^{\mathcal{F}}$. Then $\mu_1^* \vee \gamma_1^* \in \mathcal{F}$, for all $\mu_1 \in \mathcal{S}$ and $\mu_3^* \vee \gamma_1^* \in \mathcal{F}$, for all $\mu_3 \in \mathcal{T}$. That implies $(\mu_1^* \vee \gamma_1^*) \wedge (\mu_3^* \vee \gamma_1^*) \in \mathcal{F}$, which gives $(\mu_1^* \wedge \mu_3^*) \vee \gamma_1^* \in \mathcal{F}$. Therefore $(\mu_1 \vee \mu_3)^* \vee \gamma_1^* \in \mathcal{F}$, for all $\mu_1 \vee \mu_3 \in \mathcal{S} \vee \mathcal{T}$. Hence $\gamma_1 \in (\mathcal{S} \vee \mathcal{T})^{\mathcal{F}}$. Thus $\mathcal{S}^{\mathcal{F}} \cap \mathcal{T}^{\mathcal{F}} \subseteq (\mathcal{S} \vee \mathcal{T})^{\mathcal{F}}$. Since $(\mathcal{S} \vee \mathcal{T})^{\mathcal{F}} \subseteq \mathcal{S}^{\mathcal{F}} \cap \mathcal{T}^{\mathcal{F}}$, we get $(\mathcal{S} \vee \mathcal{T})^{\mathcal{F}} = \mathcal{S}^{\mathcal{F}} \cap \mathcal{T}^{\mathcal{F}}$. □

For any $\mu_1 \in \mathcal{L}$, we denote $(\{\mu_1\})^{\mathcal{F}}$ by $(\mu_1)^{\mathcal{F}}$.

From the above results we get the following result easily.

Corollary 3.6. *For any elements β_1, β_2 in pseudo-complemented lattice $\mathcal{L}$, we have the following:*

(1) $\beta_1 \leq \beta_2 \Rightarrow (\beta_2)^{\mathcal{F}} \subseteq (\beta_1)^{\mathcal{F}}$,

(2) $(\beta_1 \vee \beta_2)^{\mathcal{F}} = (\beta_1)^{\mathcal{F}} \cap (\beta_2)^{\mathcal{F}}$,

(3) $\beta_1 \in (\beta_2)^{\mathcal{F}} \Rightarrow \beta_1 \wedge \beta_2 \in \mathcal{F}^{\nabla}$,

(4) $\beta_1^* = \beta_2^* \Rightarrow (\beta_1)^{\mathcal{F}} = (\beta_2)^{\mathcal{F}}$,

(5) $\beta_1 \in \mathcal{F} \Rightarrow (\beta_1)^{\mathcal{F}} = \mathcal{F}^{\nabla}$.

It can be verified easily that $\mathcal{S}^* \subseteq (\mathcal{S}, \mathcal{F}^{\nabla})$ and $\mathcal{S}^{\mathcal{F}} \subseteq (\mathcal{S}, \mathcal{F}^{\nabla})$, where $(\mathcal{S}, \mathcal{F}^{\nabla}) = \{\beta_1 \in \mathcal{L} \mid \beta_1 \wedge \mu_1 \in \mathcal{F}^{\nabla},$ for all $\mu_1 \in \mathcal{S}\}$. Converse need not to be true in the following example.

Example 3.7. Let $\mathcal{L} = \{0, 1, 2, 3, 4, 5\}$ be a set with binary operations $\vee, \wedge$ and unary operation $*$ given in the following tables

Table-1. Cayley table for the binary operation " $\wedge$ ".

$\wedge$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	2	0	2	2
3	0	3	0	3	3	3
4	0	4	2	3	4	4
5	0	5	2	3	4	5

Table-2. Cayley table for the binary operation " $\vee$ ".

$\vee$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	1	1	1	1	1
2	2	1	2	4	4	5
3	3	1	4	3	4	5
4	4	1	4	4	4	5
5	5	1	5	5	5	5

Table-3. Cayley table for the unary operation " $*$ ".

$*$	0	1	2	3	4	5
	1	0	3	2	0	0

Then $(\mathcal{L}, \vee, \wedge, ^*, 0, 1)$ is a pseudo-complemented distributive lattice. Consider a filter $\mathcal{F}_1 = \{1, 2, 4, 5\}$ and an ideal $\mathcal{F}_1^\nabla = \{0, 3\}$ of $\mathcal{L}$. Take $\mathcal{S} = \{2, 3\}$. Clearly, we have that $\mathcal{S}^* = \{0\}$, $\mathcal{S}^{\mathcal{F}_1} = \{0, 3\}$, and Hence $\mathcal{S}^* \subsetneqq (\mathcal{S}, \mathcal{F}_1^\nabla)$. Consider a filters $\mathcal{F}_2 = \{1\}$ and an ideal $\mathcal{F}_2^\nabla = \{0\}$ of $\mathcal{L}$. Take $\mathcal{S} = \{2\}$. Clearly, we have that $\mathcal{S}^{\mathcal{F}_2} = \{0\}$ and $(\mathcal{S}, \mathcal{F}_2^\nabla) = \{0, 3\}$. Hence $\mathcal{S}^{\mathcal{F}_2} \subsetneqq (\mathcal{S}, \mathcal{F}_2^\nabla)$.

Definition 3.8. A pseudo-complemented lattice $\mathcal{L}$ is said to be $\mathcal{F}-$stone if $\beta_1^* \vee \beta_1^{**} \in \mathcal{F}$, for all $\beta_1 \in \mathcal{L}$(i.e., $\beta_1^* \in (\beta_1)^{\mathcal{F}}$, for all $\beta_1 \in \mathcal{L}$).

Example 3.9. Let $\mathcal{L} = \{0, 1, 2, 3, 4\}$ be a set with binary operations $\vee, \wedge$ and unary operation $*$ given in the following tables

Table-1. Cayley table for the binary operation " $\wedge$ ".

$\wedge$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	2	0	2
3	0	3	0	3	3
4	0	4	2	3	4

Table-2. Cayley table for the binary operation " $\vee$ ".

$\vee$	0	1	2	3	4
0	0	1	2	3	4
1	1	1	1	1	1
2	2	1	2	4	4
3	3	1	4	3	4
4	4	1	4	4	4

Table-3. Cayley table for the unary operation " $*$ ".

$*$	0	1	2	3	4
	1	0	3	2	0

Then $(\mathcal{L}, \vee, \wedge, ^*, 0, 1)$ is a pseudo-complemented distributive lattice. If $\mathcal{F} = \{1, 4\}$ then $\mathcal{F}$ is a filter of $\mathcal{L}$. Also $\beta_1^* \vee \beta_1^{**} \in \mathcal{F}$ for all $\beta_1 \in \mathcal{L}$. Hence $\mathcal{L}$ is $\mathcal{F}-$stone. But $\mathcal{L}$ is not a stone lattice since $2^* \vee (2^*)^* = 3 \vee 3^* = 3 \vee 2 = 4 \neq 1$.

Example 3.10. Let $\mathcal{L} = \{0, 1, 2, 3, 4, 5, 6\}$ be a set with binary operations $\vee, \wedge$ and unary operation $*$ given in the following tables

Table-1. Cayley table for the binary operation " $\wedge$ ".

$\wedge$	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	2	0	2	0	2
3	0	3	0	3	3	5	5
4	0	4	2	3	4	5	6
5	0	5	0	5	5	5	5
6	0	6	2	5	6	5	6

Table-2. Cayley table for the binary operation " $\vee$ ".

$\vee$	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	1	1	1	1	1	1
2	2	1	2	4	4	6	6
3	3	1	4	3	4	3	4
4	4	1	4	4	4	4	4
5	5	1	6	3	4	5	6
6	6	1	6	4	4	6	6

Table-3. Cayley table for the unary operation " $*$ ".

$*$	0	1	2	3	4	5	6
	1	0	3	2	0	2	0

Then $(\mathcal{L}, \vee, \wedge, ^*, 0, 1)$ is a pseudo-complemented distributive lattice. If $\mathcal{F} = \{1, 4\}$ then $\mathcal{F}$ is a filter of $\mathcal{L}$. Also $\beta_1^* \vee \beta_1^{**} \in \mathcal{F}$ for all $\beta_1 \in \mathcal{L}$. Hence $\mathcal{L}$ is $\mathcal{F}-$stone. But $\mathcal{L}$ is not a stone lattice since $2^* \vee (2^*)^* = 3 \vee 3^* = 3 \vee 2 = 4 \neq 1$.

Theorem 3.11. *In a pseudo-complemented lattice $\mathcal{L}$, The following are equivalent:*

(1) *$\mathcal{L}$ is an $\mathcal{F}-$stone lattice,*

(2) *for any ideal $\mathcal{S}$ of $\mathcal{L}$, $\mathcal{S}^{\mathcal{F}} = (\mathcal{S}, \mathcal{F}^{\nabla})$,*

(3) *for any $\beta_1 \in \mathcal{L}$, $(\beta_1)^{\mathcal{F}} = (\beta_1, \mathcal{F}^{\nabla})$,*

(4) *for any two ideals $\mathcal{S}, \mathcal{T}$ of $\mathcal{L}$, $\mathcal{S} \cap \mathcal{T} \subseteq \mathcal{F}^{\nabla} \Leftrightarrow \mathcal{S} \subseteq \mathcal{T}^{\mathcal{F}}$,*

(5) *for* $\beta_1, \beta_2 \in \mathcal{L}, \beta_1 \wedge \beta_2 \in \mathcal{F}^{\nabla} \Rightarrow \beta_1^* \vee \beta_2^* \in \mathcal{F}$.

Proof. (1) $\Rightarrow$ (2): Assume $\mathcal{L}$ is an $\mathcal{F}-$stone lattice. Then $\beta_1 \in (\beta_1^*)^{\mathcal{F}}$ for all $\beta_1 \in \mathcal{L}$. Let $\mathcal{S}$ be an ideal of $\mathcal{L}$ and $\gamma_1 \in (\mathcal{S}, \mathcal{F}^{\nabla})$. Then $\gamma_1 \wedge \gamma_2 \in \mathcal{F}^{\nabla}$, for all $\gamma_2 \in \mathcal{S}$. That implies $(\gamma_1 \wedge \gamma_2)^* \in \mathcal{F}$, which gives $(\gamma_1 \wedge \gamma_2)^{***} \in \mathcal{F}$. That implies $(\gamma_1^{**} \wedge \gamma_2^{**})^* \in \mathcal{F}$. Therefore $\gamma_1^{**} \wedge \gamma_2^{**} \in \mathcal{F}^{\nabla}$ and hence $(\gamma_1^* \vee \gamma_2^*)^* \in \mathcal{F}^{\nabla}$, for all $\gamma_2 \in \mathcal{S}$. That implies $\gamma_1^* \vee \gamma_2^* \in \mathcal{F}$, for all $\gamma_2 \in \mathcal{S}$. Therefore $\gamma_1 \in \mathcal{S}^{\mathcal{F}}$ and hence $\mathcal{S}^{\mathcal{F}} = (\mathcal{S}, \mathcal{F}^{\nabla})$.

(2) $\Rightarrow$ (3): It is obvious.

(3) $\Rightarrow$ (4): Assume (3). Let $\mathcal{S}, \mathcal{T}$ be two ideals of $\mathcal{L}$ with $\mathcal{S} \cap \mathcal{T} \subseteq \mathcal{F}^{\nabla}$. Let $\gamma_1 \in \mathcal{S}$. Clearly, we have that $\gamma_1 \wedge \gamma_2 \in \mathcal{S} \cap \mathcal{T}$, for all $\gamma_2 \in \mathcal{T}$. That implies $\gamma_1 \wedge \gamma_2 \in \mathcal{F}^{\nabla}$. Therefore $\gamma_1 \in (\gamma_2, \mathcal{F}^{\nabla}) = (\gamma_2)^{\mathcal{F}}$ and hence $\gamma_2^* \vee \gamma_1^* \in \mathcal{F}$. Thus $\gamma_1 \in \mathcal{T}^{\mathcal{F}}$. Conversely, assume that $\mathcal{S} \subseteq \mathcal{T}^{\mathcal{F}}$. Let $\gamma_1 \in \mathcal{S} \cap \mathcal{T}$. Then $\gamma_1 \in \mathcal{S} \subseteq \mathcal{T}^{\mathcal{F}}$ and $\gamma_1 \in \mathcal{T}$. That implies $\gamma_1 \in \mathcal{T} \cap \mathcal{T}^{\mathcal{F}} \subseteq \mathcal{F}^{\nabla}$. Hence $\mathcal{S} \cap \mathcal{T} \subseteq \mathcal{F}^{\nabla}$.

(4) $\Rightarrow$ (5): Assume condition (4). Let $\beta_1 \wedge \beta_2 \in \mathcal{F}^{\nabla}$. Then $(\beta_1] \cap (\beta_2] \subseteq \mathcal{F}^{\nabla}$. By our assumption, we have that $(\beta_1] \subseteq (\beta_2]^{\mathcal{F}}$ and hence $\beta_1 \in (\beta_2]^{\mathcal{F}}$. Therefore $\beta_2^* \vee \beta_1^* \in \mathcal{F}$.

(5) $\Rightarrow$ (1): Assume condition (5). We have that $\beta_1 \wedge \beta_1^* \in \mathcal{F}^{\nabla}$, for all $\beta_1 \in \mathcal{L}$. By our assumption we get that $\beta_1^* \vee \beta_1^{**} \in \mathcal{F}$.

□

Theorem 3.12. *A pseudo-complemented lattice is $\mathcal{F}$-stone if and only if $(\beta_1)^{\mathcal{F}\mathcal{F}} = (\beta_1^*)^{\mathcal{F}}$, for all $\beta_1 \in \mathcal{L}$.*

Proof. Assume that $\mathcal{L}$ is $\mathcal{F}-$stone. Let $\beta_1 \in \mathcal{L}$. Clearly, we have that $(\beta_1)^* \subseteq (\beta_1, \mathcal{F}^{\nabla}) = (\beta_1)^{\mathcal{F}}$ and hence $(\beta_1)^{\mathcal{F}\mathcal{F}} \subseteq (\beta_1^*)^{\mathcal{F}}$. Let $\gamma_1 \in (\beta_1)^{\mathcal{F}}$. Then $\gamma_1^* \vee \beta_1^* \in \mathcal{F}$ and hence $\gamma_1^{**} \wedge \beta_1^{**} \in \mathcal{F}^{\nabla}$. Let $\gamma_2 \in (\beta_1^*)^{\mathcal{F}}$. Then $\gamma_2^* \vee \beta_1^{**} \in \mathcal{F}$ and hence $\gamma_2^{**} \wedge \beta_1^* \in \mathcal{F}^{\nabla}$. Since $\gamma_1^{**} \wedge \beta_1^{**} \in \mathcal{F}^{\nabla}$, we get that $(\gamma_1^{**} \wedge \beta_1^{**}) \vee (\beta_1^* \wedge \gamma_2^{**}) \in \mathcal{F}^{\nabla}$. That implies $(\gamma_1^{**} \wedge \gamma_2^{**}) \wedge ((\gamma_1^{**} \wedge \beta_1^{**}) \vee (\beta_1^* \wedge \gamma_2^{**})) \in \mathcal{F}^{\nabla}$. That implies $({\gamma_1}^{**} \wedge \gamma_2^{**} \wedge \beta_1^{**}) \vee ({\gamma_1}^{**} \wedge \gamma_2^{**} \wedge \beta_1^*) \in \mathcal{F}^{\nabla}$. That implies $({\gamma_1}^{**} \wedge \gamma_2^{**}) \wedge (\beta_1^{**} \vee \beta_1^*) \in \mathcal{F}^{\nabla}$. By our assumption, we get that $({\gamma_1}^{**} \wedge \gamma_2^{**})^* \vee (\beta_1^{**} \vee \beta_1^*)^* \in \mathcal{F}$. That implies $({\gamma_1}^{**} \wedge \gamma_2^{**})^* \vee (\beta_1^{***} \wedge \beta_1^{**}) \in \mathcal{F}$. That implies $({\gamma_1}^{**} \wedge \gamma_2^{**})^* \in \mathcal{F}$. That implies $\gamma_1^{**} \wedge \gamma_2^{**} \in \mathcal{F}^{\nabla}$. By our assumption, we get that $\gamma_1^{***} \vee \gamma_2^{***} \in \mathcal{F}$, which gives $\gamma_1^* \vee \gamma_2^* \in \mathcal{F}$, for all $\gamma_1 \in (\beta_1)^{\mathcal{F}}$. Therefore $\gamma_2 \in (\beta_1)^{\mathcal{F}\mathcal{F}}$ and hence $(\beta_1^*)^{\mathcal{F}} \subseteq (\beta_1)^{\mathcal{F}\mathcal{F}}$. Thus $(\beta_1^*)^{\mathcal{F}} = (\beta_1)^{\mathcal{F}\mathcal{F}}$. Conversely assume that $(\beta_1)^{\mathcal{F}\mathcal{F}} = (\beta_1^*)^{\mathcal{F}}$, for all $\beta_1 \in \mathcal{L}$. Since $\beta_1 \in (\beta_1)^{\mathcal{F}\mathcal{F}}$, we get that $\beta_1 \in (\beta_1^*)^{\mathcal{F}}$ and hence $\beta_1^* \vee \beta_1^{**} \in \mathcal{F}$, for all $\beta_1 \in \mathcal{L}$. Thus $\mathcal{L}$ is $\mathcal{F}-$stone. □

The concept of consistent ideals is introduced in pseudo-complemented lattices.

Definition 3.13. An ideal $\mathcal{S}$ of a pseudo-complemented lattice $\mathcal{L}$ is called a *consistent ideal* if for all $\gamma_1, \gamma_2 \in \mathcal{L}, (\gamma_1)^{\mathcal{F}} = (\gamma_2)^{\mathcal{F}}$ and $\gamma_1 \in \mathcal{S}$ imply that $\gamma_2 \in \mathcal{S}$.

Example 3.14. From the Example-3.7, consider a filter $\mathcal{F} = \{1, 3, 4, 5\}$ and an ideal $\mathcal{I} = \{0, 2\}$. It is easy to verify that $\mathcal{I}$ is a consistent ideal.

Lemma 3.15. *In a pseudo-complemented lattice $\mathcal{L}$, we have the following:*

(1) *for any* $\gamma_1 \in \mathcal{L}$, $(\gamma_1)^{\mathcal{F}}$ *is a consistent ideal,*

(2) *for any element* γ_1 *of an ideal* $\mathcal{S}$, *with* $(\gamma_1)^{\mathcal{F}\mathcal{F}} \subseteq \mathcal{S}$, $\mathcal{S}$ *is a consistent ideal of* $\mathcal{L}$.

Theorem 3.16. *In a pseudo-complemented lattice* $\mathcal{L}$, *the following are equivalent:*

(1) *for any* $\gamma_1 \in \mathcal{L}$, $(\gamma_1]$ *is a consistent ideal,*

(2) *every ideal is a consistent ideal,*

(3) *every prime ideal is a consistent ideal,*

(4) *for* $\beta_1, \beta_2 \in \mathcal{L}$, $(\beta_1)^{\mathcal{F}} = (\beta_2)^{\mathcal{F}}$ *implies* $\beta_1 = \beta_2$.

Proof. (1) $\Rightarrow$ (2): Assume condition (1). Let $\mathcal{S}$ be an ideal of $\mathcal{L}$. Choose $\beta_1, \beta_2 \in \mathcal{L}$. Suppose $(\beta_1)^{\mathcal{F}} = (\beta_2)^{\mathcal{F}}$ and $\beta_1 \in \mathcal{S}$. Then clearly $(\beta_1] \subseteq \mathcal{S}$. Since $\beta_1 \in (\beta_1]$ and $(\beta_1]$ is a consistent ideal, we have $\beta_2 \in (\beta_1] \subseteq \mathcal{S}$. Therefore $\mathcal{S}$ is a consistent ideal.

(2) $\Rightarrow$ (3): It's obvious.

(3) $\Rightarrow$ (4): Assume (4). Let $\beta_1, \beta_2 \in \mathcal{L}$ such that $(\beta_1)^{\mathcal{F}} = (\beta_2)^{\mathcal{F}}$. Suppose $\beta_1 \neq \beta_2$. Then there exists a prime ideal $\mathcal{N}$ of $\mathcal{L}$ such that $\beta_1 \subseteq \mathcal{N}$ and $\beta_2 \notin \mathcal{N}$. By our assumption, $\mathcal{N}$ is a consistent ideal of $\mathcal{L}$. Since $(\beta_1)^{\mathcal{F}} = (\beta_2)^{\mathcal{F}}$ and $\beta_1 \in \mathcal{N}$, we get $\beta_2 \in \mathcal{N}$, which is a contradiction. Hence $\beta_1 = \beta_2$.

(4) $\Rightarrow$ (1): Assume (4). Let $(\beta_1]$ be a principal ideal ideal of $\mathcal{L}$. Let $\gamma_1, \gamma_2 \in \mathcal{L}$ such that $(\gamma_1)^{\mathcal{F}} = (\gamma_2)^{\mathcal{F}}$ and $\gamma_1 \in (\beta_1]$. By our assumption, we get that $\gamma_1 = \gamma_2$ and hence $\gamma_2 \in (\beta_1]$. Thus $(\beta_1]$ is a consistent ideal of $\mathcal{L}$. □

Definition 3.17. Let $\mathcal{S}$ be an $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice $\mathcal{L}$. Define $\chi(\mathcal{S})$ as follows:

$$\chi(\mathcal{S}) = \{\gamma_1 \in \mathcal{L} \mid (\gamma_1)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}\}$$

Lemma 3.18. *Let* $\mathcal{S}, \mathcal{T}$ *be two* $\mathcal{F}^{\nabla}-$*ideals of a pseudo-complemented lattice* $\mathcal{L}$. *We have the following:*

(1) $\mathcal{S} \subseteq \mathcal{T} \Rightarrow \chi(\mathcal{S}) \subseteq \chi(\mathcal{T})$,

(2) $\chi(\mathcal{S} \cap \mathcal{T}) = \chi(\mathcal{S}) \cap \chi(\mathcal{T})$,

(3) $\chi(\mathcal{S}) \subseteq \mathcal{S}$.

Proof. (1) and (2) are verified easily.

(3) Let $\gamma_1 \in \chi(\mathcal{S})$. Then $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}$. Since $\gamma_1 \in \mathcal{L}$, there exist elements $\beta_2 \in (\gamma_1)^{\mathcal{F}}$ and $\mu_1 \in \mathcal{S}$ such that $\gamma_1 = \beta_2 \vee \mu_1$. Since $\beta_2 \in (\gamma_1)^{\mathcal{F}}$, we have that $\beta_2^* \vee \gamma_1^* \in \mathcal{F}$ and hence $\gamma_1 \wedge \beta_2 \in \mathcal{F}^{\nabla} \subseteq \mathcal{S}$. That implies $\gamma_1 \wedge \mu_1 \in \mathcal{S}$. Therefore $\gamma_1 = \gamma_1 \wedge \gamma_1 = \gamma_1 \wedge (\beta_2 \vee \mu_1) = (\gamma_1 \wedge \beta_2) \vee (\gamma_1 \wedge \mu_1) = \gamma_1 \wedge \mu_1 \in \mathcal{S}$. Thus $\chi(\mathcal{S}) \subseteq \mathcal{S}$. □

Proposition 3.19. *Let* $\mathcal{S}$ *be an* $\mathcal{F}^{\nabla}-$*ideal of a pseudo-complemented lattice* $\mathcal{L}$. *Then* $\chi(\mathcal{S})$ *is an ideal* $\mathcal{L}$.

Proof. Since $(0)^{\mathcal{F}} = \mathcal{L}$, we have that $(0)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}$. Then $0 \in \chi(\mathcal{S})$ and hence $\chi(\mathcal{S}) \neq \emptyset$. Let $\gamma_1, \gamma_2 \in \chi(\mathcal{S})$. Then $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} = (\gamma_2)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}$. Now, $(\gamma_1 \vee \gamma_2)^{\mathcal{F}} \vee \mathcal{S} = \{(\gamma_1)^{\mathcal{F}} \cap (\gamma_2)^{\mathcal{F}}\} \vee \mathcal{S} = \{(\gamma_1)^{\mathcal{F}} \vee \mathcal{S}\} \cap \{(\gamma_2)^{\mathcal{F}} \vee \mathcal{S}\} = \mathcal{L}$.

Therefore $\gamma_1 \vee \gamma_2 \in \chi(\mathcal{S})$. Let $\gamma_1 \in \chi(\mathcal{S})$. Then $\mathcal{L} = (\gamma_1)^{\mathcal{F}} \vee \mathcal{S}$. Let μ_2 be any element of $\mathcal{L}$. Since $(\gamma_1)^{\mathcal{F}} \subseteq (\gamma_1 \wedge \mu_2)^{\mathcal{F}}$, we get that $\mathcal{L} = (\gamma_1)^{\mathcal{F}} \vee \mathcal{S} \subseteq (\gamma_1 \wedge \mu_2)^{\mathcal{F}} \vee \mathcal{S}$ and hence $(\gamma_1 \wedge \mu_2)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}$. Therefore $\gamma_1 \wedge \mu_2 \in \chi(\mathcal{S})$. Hence $\chi(\mathcal{S})$ is an ideal of $\mathcal{L}$. □

Definition 3.20. An $\mathcal{F}^{\nabla}-$ideal $\mathcal{S}$ of a pseudo-complemented lattice $\mathcal{L}$ is said to be *fully consistent* if $\mathcal{S} = \chi(\mathcal{S})$.

Proposition 3.21. *Every fully consistent $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice is a consistent ideal.*

Proof. Let $\mathcal{S}$ be a fully consistent ideal of a pseudo-complemented lattice $\mathcal{L}$. Then $\mathcal{S} = \chi(\mathcal{S})$. Let $\gamma_1, \gamma_2 \in \mathcal{L}$ with $(\gamma_1)^{\mathcal{F}} = (\gamma_2)^{\mathcal{F}}$ and $\gamma_1 \in \mathcal{S} = \chi(\mathcal{S})$. That implies $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}$. Therefore $(\gamma_2)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}$ and hence $\gamma_2 \in \chi(\mathcal{S}) = \mathcal{S}$. Thus $\mathcal{S}$ is a consistent ideal of $\mathcal{L}$. □

Definition 3.22. An ideal $\mathcal{S}$ of a pseudo-complemented lattice $\mathcal{L}$ is said to be closed if $\mathcal{S} = \mathcal{S}^{\mathcal{F}\mathcal{F}}$.

Example 3.23. From the Example-3.7, consider a filter $\mathcal{F} = \{1, 4, 5\}$ and an ideal $\mathcal{S} = \{0, 3\}$ of $\mathcal{L}$. Clearly, we have that $\mathcal{S}^{\mathcal{F}\mathcal{F}} = \mathcal{S}$. Therefore $\mathcal{S}$ is closed.

Clearly $\mathcal{F}^{\nabla}$ is the smallest closed ideal and $\mathcal{L}$ is the largest closed ideal.

Proposition 3.24. *Every closed ideal of a pseudo-complemented lattice $\mathcal{L}$ is a consistent ideal.*

Proof. Let $\mathcal{S}$ be a closed ideal of a pseudo-complemented lattice $\mathcal{L}$. Then $\mathcal{S} = \mathcal{S}^{\mathcal{F}\mathcal{F}}$. Let $\gamma_1, \gamma_2 \in \mathcal{L}$ with $(\gamma_1)^{\mathcal{F}} = (\gamma_2)^{\mathcal{F}}$ and $\gamma_1 \in \mathcal{S}$. Then $\gamma_2 \in (\gamma_2)^{\mathcal{F}\mathcal{F}} = (\gamma_1)^{\mathcal{F}\mathcal{F}} \subseteq \mathcal{S}^{\mathcal{F}\mathcal{F}} = \mathcal{S}$. Hence $\mathcal{S}$ is a consistent ideal of $\mathcal{L}$. □

Definition 3.25. A pseudo-complemented lattice $\mathcal{L}$ is said to be quasi $\mathcal{F}$-stone if $(\gamma_1)^{\mathcal{F}} \vee (\gamma_1)^{\mathcal{F}\mathcal{F}} = \mathcal{L}$, for all $\gamma_1 \in \mathcal{L}$.

Theorem 3.26. *Every $\mathcal{F}-$stone lattice is a quasi $\mathcal{F}-$stone lattice.*

Proof. Assume that $\mathcal{L}$ is a $\mathcal{F}-$stone lattice. Then $\gamma_1^* \vee \gamma_1^{**} \in \mathcal{F}$, for all $\gamma_1 \in \mathcal{L}$. Therefore $(\gamma_1^* \vee \gamma_1{}^{**}] \subseteq (\mathcal{F}]$ and hence $(\gamma_1^*] \vee (\gamma_1^{**}] = \mathcal{L}$. By our assumption, we have that $(\gamma_1)^* = (\gamma_1)^{\mathcal{F}}$. Therefore $(\gamma_1)^{\mathcal{F}} \vee (\gamma_1)^{\mathcal{F}\mathcal{F}} = \mathcal{L}$. Thus $\mathcal{L}$ is quasi $\mathcal{F}-$stone. □

Generally, the converse of the above result doesn't hold. We can illustrate this with the subsequent example.

Example 3.27. Let $\mathcal{L} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be a set with binary operations $\vee, \wedge$ and unary operation $*$ given in the following tables

Table-1. Cayley table for the binary operation " $\wedge$ ".

$\wedge$	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	2	0	2	2	0	2	0	2
3	0	3	0	3	3	8	6	6	8	8
4	0	4	2	3	4	9	6	7	8	9
5	0	5	2	8	9	5	0	2	8	9
6	0	6	0	6	6	0	6	6	0	0
7	0	7	2	6	7	2	6	7	0	2
8	0	8	0	8	8	8	0	0	8	8
9	0	9	2	8	9	9	0	2	8	9

Table-2. Cayley table for the binary operation " $\vee$ ".

$\vee$	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1	1
2	2	1	2	4	4	5	7	7	9	9
3	3	1	4	3	4	1	3	4	3	4
4	4	1	4	4	4	1	4	4	4	4
5	5	1	5	1	1	5	1	1	5	5
6	6	1	7	3	4	1	6	7	3	4
7	7	1	7	4	4	1	7	7	4	4
8	8	1	9	3	4	5	3	4	8	9
9	9	1	9	4	4	5	4	4	9	9

Table-3. Cayley table for the unary operation " $*$ ".

$*$	0	1	2	3	4	5	6	7	8	9
	1	0	3	2	0	6	5	8	7	6

Then $(\mathcal{L}, \vee, \wedge, ^*, 0, 1)$ is a pseudo-complemented distributive lattice. If $\mathcal{F} = \{1, 5\}$ then $\mathcal{F}$ is a filter of $\mathcal{L}$. Clearly, we have that $(\gamma_1)^{\mathcal{F}} \vee (\gamma_1)^{\mathcal{F}\mathcal{F}} = \mathcal{L}$, for all $\gamma_1 \in \mathcal{L}$. Hence $\mathcal{L}$ is a quasi $\mathcal{F}-$stone. But $\mathcal{L}$ is not $\mathcal{F}-$stone, since $2^* \vee (2^*)^* = 3 \vee 3^* = 3 \vee 2 = 4 \notin \mathcal{F}$.

Theorem 3.28. *In a pseudo-complemented lattice $\mathcal{L}$, the following are equivalent:*

(1) *$\mathcal{L}$ is a quasi $\mathcal{F}-$stone lattice,*

(2) *every closed* $\mathcal{F}^{\nabla}-$*ideal is fully consistent,*

(3) *for each* $\gamma_1 \in \mathcal{L}$, $(\gamma_1)^{\mathcal{F}\mathcal{F}}$ *is fully consistent.*

Proof. (1) $\Rightarrow$ (2): Assume (1). Let $\mathcal{S}$ be a closed $\mathcal{F}^{\nabla}-$ideal of $\mathcal{L}$. Then $\mathcal{S}^{\mathcal{F}\mathcal{F}} = \mathcal{S}$. Let $\gamma_1 \in \mathcal{S}$. Then $(\gamma_1)^{\mathcal{F}\mathcal{F}} \subseteq \mathcal{S}^{\mathcal{F}\mathcal{F}}$. That implies $\mathcal{L} = (\gamma_1)^{\mathcal{F}} \vee (\gamma_1)^{\mathcal{F}\mathcal{F}} \subseteq (\gamma_1)^{\mathcal{F}} \vee \mathcal{S}^{\mathcal{F}\mathcal{F}} = (\gamma_1)^{\mathcal{F}} \vee \mathcal{S}$ and hence $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}$. Therefore $\gamma_1 \in \chi(\mathcal{S})$. Thus $\mathcal{S} \subseteq \chi(\mathcal{S})$. Since $\chi(\mathcal{S}) \subseteq \mathcal{S}$, we have that $\chi(\mathcal{S}) = \mathcal{S}$. Hence $\mathcal{S}$ is fully consistent.

(2) $\Rightarrow$ (3): Assume (2). Clearly, we have that for any $\gamma_1 \in \mathcal{L}$, $(\gamma_1)^{\mathcal{F}\mathcal{F}}$ is closed. By our assumption, we get that, $(\gamma_1)^{\mathcal{F}\mathcal{F}}$ is fully consistent.

(3) $\Rightarrow$ (1): Assume (3). Let $\gamma_1 \in \mathcal{L}$. Then $\chi((\gamma_1)^{\mathcal{F}\mathcal{F}}) = (\gamma_1)^{\mathcal{F}\mathcal{F}}$. Since $\gamma_1 \in (\gamma_1)^{\mathcal{F}\mathcal{F}}$, we have that $(\gamma_1)^{\mathcal{F}} \vee (\gamma_1)^{\mathcal{F}\mathcal{F}} = \mathcal{L}$. Hence $\mathcal{L}$ is a quasi $\mathcal{F}-$stone lattice. □

Example 3.29. In Example-3.27, if $\mathcal{F} = \{1, 5\}$ then $\mathcal{F}$ is a filter of $\mathcal{L}$ and $\mathcal{F}^{\nabla} = \{0, 6\}$ is an ideal of $\mathcal{L}$. Also, $(0)^{\mathcal{F}} = (6)^{\mathcal{F}} = \mathcal{L}$, $(0)^{\mathcal{F}\mathcal{F}} = (6)^{\mathcal{F}\mathcal{F}} = \mathcal{L}^{\mathcal{F}} = \{0, 6\} = \mathcal{F}^{\nabla}$ and $(1)^{\mathcal{F}} = (2)^{\mathcal{F}} = (3)^{\mathcal{F}} = (4)^{\mathcal{F}} = (5)^{\mathcal{F}} = (7)^{\mathcal{F}} = (8)^{\mathcal{F}} = (9)^{\mathcal{F}} = \{0, 6\}$. Hence $(1)^{\mathcal{F}\mathcal{F}} = (2)^{\mathcal{F}\mathcal{F}} = (3)^{\mathcal{F}\mathcal{F}} = (4)^{\mathcal{F}\mathcal{F}} = (5)^{\mathcal{F}\mathcal{F}} = (7)^{\mathcal{F}\mathcal{F}} = (8)^{\mathcal{F}\mathcal{F}} = (9)^{\mathcal{F}\mathcal{F}} = \{0, 6\}^{\mathcal{F}} = \mathcal{L}$.

Now, $\chi((0)^{\mathcal{F}\mathcal{F}}) = \{0, 6\} = (0)^{\mathcal{F}\mathcal{F}}$,
$\chi((1)^{\mathcal{F}\mathcal{F}}) = \mathcal{L} = (1)^{\mathcal{F}\mathcal{F}}$,
$\chi((2)^{\mathcal{F}\mathcal{F}}) = \mathcal{L} = (2)^{\mathcal{F}\mathcal{F}}$,
$\chi((3)^{\mathcal{F}\mathcal{F}}) = \mathcal{L} = (3)^{\mathcal{F}\mathcal{F}}$,
$\chi((4)^{\mathcal{F}\mathcal{F}}) = \mathcal{L} = (4)^{\mathcal{F}\mathcal{F}}$,
$\chi((5)^{\mathcal{F}\mathcal{F}}) = \mathcal{L} = (5)^{\mathcal{F}\mathcal{F}}$,
$\chi((6)^{\mathcal{F}\mathcal{F}}) = \{0, 6\} = (6)^{\mathcal{F}\mathcal{F}}$,
$\chi((7)^{\mathcal{F}\mathcal{F}}) = \mathcal{L} = (7)^{\mathcal{F}\mathcal{F}}$,
$\chi((8)^{\mathcal{F}\mathcal{F}}) = \mathcal{L} = (8)^{\mathcal{F}\mathcal{F}}$,
$\chi((9)^{\mathcal{F}\mathcal{F}}) = \mathcal{L} = (9)^{\mathcal{F}\mathcal{F}}$.
Therefore, $\chi((\beta_1)^{\mathcal{F}\mathcal{F}}) = (\beta_1)^{\mathcal{F}\mathcal{F}}$, for all $\beta_1 \in \mathcal{L}$ and $(\gamma_1)^{\mathcal{F}} \vee (\gamma_1)^{\mathcal{F}\mathcal{F}} = \mathcal{L}$ for all $\gamma_1 \in \mathcal{L}$. Hence, for any $x \in \mathcal{L}$, $(\beta_1)^{\mathcal{F}\mathcal{F}}$ is a fully consistent ideal of $\mathcal{L}$ and every closed $\mathcal{F}^{\nabla}-$ideal is fully consistent ideal of $\mathcal{L}$.

4. Ornate prime $\mathcal{F}^{\nabla}-$ideals

In this section, the notion of ornate prime $\mathcal{F}^{\nabla}-$ideals is introduced in pseudo-complemented distributive lattices. Characterization theorems of ornate prime $\mathcal{F}^{\nabla}-$ideals are derived for every prime $\mathcal{F}^{\nabla}-$ideal turn into an ornate prime and every maximal $\mathcal{F}^{\nabla}-$ideal come to be an ornate prime $\mathcal{F}^{\nabla}-$ideal. The concept of $\mathcal{I}-$complemented lattice is introduced and $\mathcal{F}^{\nabla}-$complemented lattices are characterized. Finally, a set of equivalent conditions is derived for every maximal $\mathcal{F}^{\nabla}-$ideal to qualify as ornate.

Lemma 4.1. *For any prime $\mathcal{F}^{\nabla}-$ideal $\mathcal{N}$ of a pseudo-complemented lattice $\mathcal{L}$ and $\gamma_1 \notin \mathcal{N}$, we have $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$.*

Proof. Let $\mathcal{N}$ be a prime ideal of $\mathcal{L}$ with $\gamma_1 \notin \mathcal{N}$. Also, let $\gamma_2 \in (\gamma_1)^{\mathcal{F}}$. Then $\gamma_2^* \vee \gamma_1^* \in \mathcal{F}$. That implies $\gamma_2^{**} \wedge \gamma_1^{**} \in \mathcal{F}^{\nabla}$. Therefore $\gamma_2 \wedge \gamma_1 \in \mathcal{F}^{\nabla} \subseteq \mathcal{N}$. Since $\gamma_1 \notin \mathcal{N}$, we get that $\gamma_2 \in \mathcal{N}$. Hence $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$. □

Definition 4.2. A prime $\mathcal{F}^{\nabla}-$ideal $\mathcal{N}$ of a pseudo-complemented lattice $\mathcal{L}$ is said to be *ornate* if to each $\gamma_1 \in \mathcal{N}$, there exists $\gamma_2 \notin \mathcal{N}$ such that $\gamma_1^* \vee \gamma_2^* \in \mathcal{F}$.

Example 4.3. Let $\mathcal{L} = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ be a set with binary operations $\vee, \wedge$ and unary operation $*$ given in the following tables

Table-1. Cayley table for the binary operation " $\wedge$ ".

$\wedge$	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	9
2	0	2	2	0	2	2	2	2	2
3	0	3	0	3	3	3	3	3	3
4	0	4	2	3	4	4	4	4	4
5	0	5	2	3	4	5	6	7	8
6	0	6	2	3	4	6	6	6	6
7	0	7	2	3	4	7	6	7	7
8	0	8	2	3	4	8	6	7	8

Table-2. Cayley table for the binary operation " $\vee$ ".

$\vee$	0	1	2	3	4	5	6	7	8
0	0	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1	1
2	2	1	2	4	4	5	6	7	8
3	3	1	4	3	4	5	6	7	8
4	4	1	4	4	4	5	6	7	8
5	5	1	5	5	5	5	5	5	5
6	6	1	6	6	6	5	6	7	8
7	7	1	7	7	7	5	7	7	8
8	8	1	8	8	8	5	8	8	8

Table-3. Cayley table for the unary operation " $*$ ".

$*$	0	1	2	3	4	5	6	7	8
	1	0	3	2	0	0	0	0	0

Then $(\mathcal{L}, \vee, \wedge, ^*, 0, 1)$ is a pseudo-complemented distributive lattice. Consider a filter $\mathcal{F} = \{1, 2, 4, 5, 6, 7, 8\}$ and an ideal $\mathcal{F}^{\nabla} = \{0, 3\}$ of $\mathcal{L}$. Clearly $\mathcal{P} = \{0, 2, 3, 4, 6\}$ is a prime $\mathcal{F}^{\nabla}-$ideal, but not ornate, because for the element $2 \in \mathcal{P}$ there is no element $\beta_1 \notin \mathcal{P}$ such that $2^* \vee \beta_1^* \in \mathcal{F}$.

Lemma 4.4. *For any ornate prime $\mathcal{F}^{\nabla}-$ideal $\mathcal{N}$ of a pseudo-complemented lattice $\mathcal{L}$ and $\gamma_1 \in \mathcal{L}$, we have $\gamma_1 \in \mathcal{N}$ if and only if $(\gamma_1)^{\mathcal{F}\mathcal{F}} \subseteq \mathcal{N}$.*

Proof. Assume that $\gamma_1 \in \mathcal{N}$. Let $\mu_1 \in (\gamma_1)^{\mathcal{F}\mathcal{F}}$. Then $(\gamma_1)^{\mathcal{F}} \subseteq (\mu_1)^{\mathcal{F}}$. Since $\gamma_1 \in \mathcal{N}$ and $\mathcal{N}$ is ornate, $\gamma_1^* \vee \gamma_2^* \in \mathcal{F}$, for some $\gamma_2 \notin \mathcal{N}$. Then $\gamma_2 \in (\gamma_1)^{\mathcal{F}} \subseteq (\mu_1)^{\mathcal{F}}$. Since $\gamma_2 \notin \mathcal{N}$, we have that $(\gamma_2)^{\mathcal{F}} \subseteq \mathcal{N}$. Therefore $\mu_1 \in (\mu_1)^{\mathcal{F}\mathcal{F}} \subseteq (\gamma_2)^{\mathcal{F}} \subseteq \mathcal{N}$. Hence $(\gamma_1)^{\mathcal{F}\mathcal{F}} \subseteq \mathcal{N}$. □

Theorem 4.5. *A prime $\mathcal{F}^{\nabla}-$ideal $\mathcal{N}$ of a pseudo-complemented lattice $\mathcal{L}$ is ornate if and only if it satisfies*

$$\gamma_1 \notin \mathcal{N} \;\Leftrightarrow\; (\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$$

Proof. Let $\mathcal{N}$ be a prime $\mathcal{F}^{\nabla}-$ideal of $\mathcal{L}$. Assume that $\mathcal{N}$ is ornate and $\gamma_1 \in \mathcal{L}$. Suppose $\gamma_1 \notin \mathcal{N}$. By Lemma 4.1, we get $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$. Conversely, assume that $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$. Suppose $\gamma_1 \in \mathcal{N}$. Since $\mathcal{N}$ is ornate, there exists $\gamma_2 \notin \mathcal{N}$ such that $\gamma_1^* \vee \gamma_2^* \in \mathcal{F}$. Hence $\gamma_2 \in (\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$, which is a contradiction. Therefore $\gamma_1 \notin \mathcal{N}$.

Conversely, assume that $\gamma_1 \notin \mathcal{N} \;\Leftrightarrow\; (\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$. Suppose $\gamma_1 \in \mathcal{N}$. By our assumption, we have $(\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}$. Then $\gamma_2 \notin \mathcal{N}$, for some $\gamma_2 \in (\gamma_1)^{\mathcal{F}}$. Therefore $\gamma_1^* \vee \gamma_2^* \in \mathcal{F}$ and hence $\mathcal{N}$ is ornate. □

Theorem 4.6. *Every ornate prime $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice is a consistent ideal.*

Proof. Let $\mathcal{N}$ be an ornate prime $\mathcal{F}^{\nabla}-$ideal of $\mathcal{L}$. Suppose $\gamma_1, \gamma_2 \in \mathcal{L}$ with $(\gamma_1)^{\mathcal{F}} = (\gamma_2)^{\mathcal{F}}$ and $\gamma_1 \in \mathcal{N}$. Then $\gamma_1^* \vee \mu_1^* \in \mathcal{F}$, for some $\mu_1 \notin \mathcal{N}$. That implies $\mu_1 \in (\gamma_1)^{\mathcal{F}} = (\gamma_2)^{\mathcal{F}}$. Since $\mu_1 \in (\gamma_2)^{\mathcal{F}}$, we have that $\gamma_2 \wedge \mu_1 \in \mathcal{F}^{\nabla} \subseteq \mathcal{N}$. Since $\mathcal{N}$ is prime and $\mu_1 \notin \mathcal{N}$, it gives that $\gamma_2 \in \mathcal{N}$. Hence $\mathcal{N}$ is a consistent ideal. □

Definition 4.7. Let $\mathcal{N}$ be a prime $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice $\mathcal{L}$. Define $\lambda(\mathcal{N}) = \{\gamma_1 \in \mathcal{L} \mid (\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}\}$.

Lemma 4.8. *Let $\mathcal{N}$ be a prime $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice $\mathcal{L}$. Then $\lambda(\mathcal{N})$ is an ideal contained in $\mathcal{N}$.*

Proof. Let $\mathcal{N}$ be a prime ideal of a pseudo-complemented lattice $\mathcal{L}$. Then $(0)^{\mathcal{F}} = \mathcal{L} \nsubseteq \mathcal{N}$ and hence $0 \in \lambda(\mathcal{N})$. Therefore $\lambda(\mathcal{N}) \neq \emptyset$. Let $\gamma_1, \gamma_2 \in \lambda(\mathcal{N})$. Then $(\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}$ and $(\gamma_2)^{\mathcal{F}} \nsubseteq \mathcal{N}$. Since $\mathcal{N}$ is prime, we get $(\gamma_1 \vee \gamma_2)^{\mathcal{F}} = (\gamma_1)^{\mathcal{F}} \cap (\gamma_2)^{\mathcal{F}} \nsubseteq \mathcal{N}$. Therefore $\gamma_1 \vee \gamma_2 \in \lambda(\mathcal{N})$. Let $\gamma_1 \in \lambda(\mathcal{N})$ and $\gamma_2 \leq \gamma_1$. Then $(\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}$ and $(\gamma_1)^{\mathcal{F}} \subseteq (\gamma_2)^{\mathcal{F}}$. Since $(\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}$, we get $(\gamma_2)^{\mathcal{F}} \nsubseteq \mathcal{N}$. Therefore $\gamma_2 \in \lambda(\mathcal{N})$ and hence $\lambda(\mathcal{N})$ is an ideal of $\mathcal{L}$. Now, let $\gamma_1 \in \lambda(\mathcal{N})$. Then, we get $(\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}$. Hence there exists $\mu_1 \in (\gamma_1)^{\mathcal{F}}$ such that $\mu_1 \notin \mathcal{N}$. Since $\mu_1 \in (\gamma_1)^{\mathcal{F}}$, we get $\mu_1 \wedge \gamma_1 \in \mathcal{F}^{\nabla} \subseteq \mathcal{N}$. Since $\mu_1 \notin \mathcal{N}$, we must have $\gamma_1 \in \mathcal{N}$. Therefore $\lambda(\mathcal{N}) \subseteq \mathcal{N}$. □

The set of all prime $\mathcal{F}^{\nabla}-$ideals of a pseudo-complemented lattice $\mathcal{L}$ is denoted by $Spec^{\mathcal{F}^{\nabla}}\mathcal{L}$. For any $\beta_1 \in \mathcal{L}$, we define $\Re(\beta_1) = \{\mathcal{N} \in Spec^{\mathcal{F}^{\nabla}}\mathcal{L} \mid \beta_1 \notin \mathcal{N}\}$.

Theorem 4.9. *For any $\beta_1 \in \mathcal{L}$, we have $(\beta_1)^{\mathcal{F}} \subseteq \bigcap_{\mathcal{N}\in\Re(\beta_1)} \lambda(\mathcal{N})$.*

Proof. Let $\beta_2 \in (\beta_1)^{\mathcal{F}}$ and $\mathcal{N} \in \Re(\beta_1)$. Then $\beta_1^* \vee \beta_2^* \in \mathcal{F}$ and $\beta_1 \notin \mathcal{N}$. That implies $\beta_1 \in (\beta_2)^{\mathcal{F}}$ and $\beta_1 \notin \mathcal{N}$. It gives that $(\beta_2)^{\mathcal{F}} \nsubseteq \mathcal{N}$. Therefore $\beta_2 \in \lambda(\mathcal{N})$ and hence $(\beta_1)^{\mathcal{F}} \subseteq \lambda(\mathcal{N})$, for all $\mathcal{N} \in \Re(\beta_1)$. Thus $(\beta_1)^{\mathcal{F}} \subseteq \bigcap_{\mathcal{N}\in\Re(\beta_1)} \lambda(\mathcal{N})$. □

Corollary 4.10. *For any $\beta_1 \in \mathcal{L}$, we have $\beta_1 \notin \mathcal{P}$ implies $(\beta_1)^{\mathcal{F}} \subseteq \lambda(\mathcal{N})$.*

Proposition 4.11. *Every ornate prime $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice $\mathcal{L}$ is minimal.*

Proof. Let $\mathcal{N}$ be an ornate prime $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice $\mathcal{L}$ and $\gamma_1 \in \mathcal{L}$. Suppose $\gamma_1 \in \mathcal{N}$. Since $\mathcal{N}$ is ornate, $\gamma_1^* \vee \gamma_2^* \in \mathcal{F}$, for some $\gamma_2 \notin \mathcal{N}$. Then $\gamma_1 \wedge \gamma_2 \in \mathcal{F}^{\nabla}$. That is for any $\gamma_1 \in \mathcal{N}$, there is $\gamma_2 \notin \mathcal{N}$ such that $\gamma_1 \wedge \gamma_2 \in \mathcal{F}^{\nabla}$. Hence $\mathcal{N}$ is minimal. □

Example 4.12. From Example-4.3, Consider a filter $\mathcal{F} = \{1, 2, 4, 5, 6, 7, 8, \}$ and an ideal $\mathcal{F}^{\nabla} = \{0, 3\}$. Clearly, $\mathcal{P} = \{0, 2, 3, 4\}$ is a minimal prime $\mathcal{F}^{\nabla}-$ideal Clearly, $\mathcal{P}$ is not ornate, because for the element $2 \in \mathcal{P}$, there is no element $\beta_1 \notin \mathcal{P}$ such that $2^* \vee \beta_1^* \in \mathcal{F}$.

Theorem 4.13. *For any prime $\mathcal{F}^{\nabla}-$ideal $\mathcal{N}$ of an $\mathcal{F}-$stone lattice $\mathcal{L}$, we have $\mathcal{N}$ contains a unique ornate prime $\mathcal{F}^{\nabla}-$ideal $\lambda(\mathcal{N})$.*

Proof. Clearly, $\mathcal{N}$ contains a minimal prime $\mathcal{F}^{\nabla}-$ideal, say $\mathcal{M}$. Let $\gamma_1 \in \mathcal{M}$. Since $\mathcal{M}$ is minimal, $\gamma_1 \wedge \gamma_2 \in \mathcal{F}^{\nabla}$, for some $\gamma_2 \notin \mathcal{M}$. Since $\mathcal{L}$ is $\mathcal{F}-$stone, $\gamma_1^* \vee \gamma_2^* \in \mathcal{F}$. Therefore $\mathcal{M}$ is ornate and hence $\mathcal{N}$ contains an ornate prime $\mathcal{F}^{\nabla}-$ideal $\mathcal{M}$. Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be two ornate prime $\mathcal{F}^{\nabla}-$ideals with $\mathcal{M}_1 \subseteq \mathcal{N}$ and $\mathcal{M}_2 \subseteq \mathcal{N}$. Suppose $\mathcal{M}_1 \neq \mathcal{M}_2$. Choose $\mu_1 \in \mathcal{M}_1 \setminus \mathcal{M}_2$. Since $\mu_1 \wedge \mu_1^* = 0 \in \mathcal{M}_2$ and $\mu_1 \notin \mathcal{M}_2$, we must have $\mu_1^* \in \mathcal{M}_2$. Since $\mathcal{M}_1$ is minimal, we get that $\mathcal{L} \setminus \mathcal{M}_1$ is a maximal filter such that $\mu_1 \notin \mathcal{L} \setminus \mathcal{M}_1$. Since $\mathcal{L} \setminus \mathcal{M}_1$ is maximal, we get $(\mathcal{L} \setminus \mathcal{M}_1) \vee [\mu_1) = \mathcal{L}$. Then $0 = \mu_3 \wedge \mu_1$, for some $\mu_3 \notin \mathcal{M}_1$. That implies $\mu_1^* \wedge \mu_3 = \mu_3$. Since $\mu_3 \in \mathcal{L} \setminus \mathcal{M}_1$, we get that $\mu_1^* \in \mathcal{L} \setminus \mathcal{M}_1$. Since $\mu_1^* \wedge \mu_1^{**} = 0$, we get that $\mu_1^{**} \in \mathcal{M}_1$. That implies $0^* = \mu_1^* \vee \mu_1^{**} \in \mathcal{M}_2 \vee \mathcal{M}_1 \subseteq \mathcal{N}$, which is a contradiction. Hence $\mathcal{N}$ contains a unique ornate prime $\mathcal{F}^{\nabla}-$ideal. Since $\mathcal{L}$ is $\mathcal{F}-$stone and $\lambda(\mathcal{N}) \subseteq \mathcal{N}$, we get that $\mathcal{N}$ contains the unique ornate prime $\mathcal{F}^{\nabla}-$ideal, precisely $\lambda(\mathcal{N})$. □

Theorem 4.14. *In a pseudo-complemented lattice $\mathcal{L}$, the following are equivalent:*

(1) *$\mathcal{L}$ is an $\mathcal{F}-$stone lattice,*

(2) *for any $\mathcal{N} \in Spec^{\mathcal{F}^{\nabla}}\mathcal{L}$, $\lambda(\mathcal{N})$ is prime,*

(3) *for any $\beta_1, \beta_2 \in \mathcal{L}$, $\beta_1 \wedge \beta_2 \in \mathcal{F}^{\nabla} \Rightarrow (\beta_1)^{\mathcal{F}} \vee (\beta_2)^{\mathcal{F}} = \mathcal{L}$.*

Proof. (1) $\Rightarrow$ (2): Assume (1). Let $\mathcal{N}$ be a prime $\mathcal{F}^{\nabla}-$ideal of $\mathcal{L}$. By Theorem 4.13, we have that $\lambda(\mathcal{N}) = \mathcal{N}$ is prime.

(2) $\Rightarrow$ (3): Assume (2). Let $\beta_1, \beta_2 \in \mathcal{L}$ with $\beta_1 \wedge \beta_2 \in \mathcal{F}^{\nabla}$. Suppose $(\beta_1)^{\mathcal{F}} \vee (\beta_2)^{\mathcal{F}} \neq \mathcal{L}$. Then $(\beta_1)^{\mathcal{F}} \vee (\beta_2)^{\mathcal{F}} \subseteq \mathcal{N}$, for some $\mathcal{N} \in Spec^{\mathcal{F}^{\nabla}}\mathcal{L}$. That implies $(\beta_1)^{\mathcal{F}} \subseteq \mathcal{N}$ and $(\beta_2)^{\mathcal{F}} \subseteq \mathcal{N}$. Therefore $\beta_1 \notin \lambda(\mathcal{N})$ and $\beta_2 \notin \lambda(\mathcal{N})$. Since $\lambda(\mathcal{N})$ is prime, we get that $0 = \beta_1 \wedge \beta_2 \notin \lambda(\mathcal{N})$, which is a contradiction. Therefore $(\beta_1)^{\mathcal{F}} \vee (\beta_2)^{\mathcal{F}} = \mathcal{L}$.

(3) $\Rightarrow$ (1): Assume for any $\beta_1, \beta_2 \in \mathcal{L}$, $\beta_1 \wedge \beta_2 \in \mathcal{F}^{\nabla} \Rightarrow (\beta_1)^{\mathcal{F}} \vee (\beta_2)^{\mathcal{F}} = \mathcal{L}$. Then $1 \in (\beta_1)^{\mathcal{F}} \vee (\beta_2)^{\mathcal{F}}$. That implies there exist $\gamma_1 \in (\beta_1)^{\mathcal{F}}$ and $\gamma_2 \in (\beta_2)^{\mathcal{F}}$ such that $\gamma_1 \vee \gamma_2 = 1$. That implies $\gamma_1^* \wedge \gamma_2^* = 0$. Since $\gamma_1 \in (\beta_1)^{\mathcal{F}}$ and $\gamma_2 \in (\beta_2)^{\mathcal{F}}$, we get $\gamma_1^* \vee \beta_1^* \in \mathcal{F}$ and $\gamma_2^* \vee \beta_2^* \in \mathcal{F}$. Now, $\beta_1^* \vee \beta_2^* = (\beta_1^* \vee \beta_2^*) \vee (\gamma_1^* \wedge \gamma_2^*) = (\beta_1^* \vee \beta_2^* \vee \gamma_1^*) \wedge (\beta_1^* \vee \beta_2^* \vee \gamma_2^*) \in \mathcal{F}$. Therefore $\beta_1^* \vee \beta_2^* \in \mathcal{F}$. By Theorem-3.11, we get that $\mathcal{L}$ is $\mathcal{F}-$stone. ☐

Theorem 4.15. *Let $\mathcal{F}$ be a filter of $\mathcal{L}$. Then $\mathcal{L}$ is $\mathcal{F}-$stone if and only if $(\beta_1)^{\mathcal{F}} \vee (\beta_1^*)^{\mathcal{F}} = \mathcal{L}$, for all $\beta_1 \in \mathcal{L}$.*

Proof. Assume that $(\beta_1)^{\mathcal{F}} \vee (\beta_1^*)^{\mathcal{F}} = \mathcal{L}$, for all $\beta_1 \in \mathcal{L}$. Since $(\beta_1^*)^{\mathcal{F}} \subseteq (\beta_1^*, \mathcal{F}^{\nabla})$, we get that $(\beta_1, \mathcal{F}^{\nabla}) \vee ((\beta_1^*], \mathcal{F}^{\nabla}) = \mathcal{L}$. Since $(\beta_1)^* \subseteq (\beta_1, \mathcal{F}^{\nabla})$, we get that $(\beta_1^*] \vee (\beta_1^{**}] = \mathcal{L}$. Then $1 \in (\beta_1^*] \vee (\beta_1^{**}]$. That implies there exist $\gamma_1 \in (\beta_1^*]$ and $\gamma_2 \in (\beta_1^{**}]$ such that $\gamma_1 \vee \gamma_2 = 1$. Since $\gamma_1 \in (\beta_1^*]$ and $\gamma_2 \in (\beta_1^{**}]$, we get that $\gamma_1 \leq \beta_1^*$ and $\gamma_2 \leq \beta_1^{**}$. That implies $1 = \gamma_1 \vee \gamma_2 \leq \beta_1^* \vee \beta_1^{**}$, which gives $\beta_1^* \vee \beta_1^{**} \in \mathcal{F}$. Hence $\mathcal{L}$ is $\mathcal{F}-$stone. Converse part is clear. ☐

The set of all maximal $\mathcal{F}^{\nabla}-$ideals of a pseudo-complemented lattice $\mathcal{L}$ by $Max^{\mathcal{F}^{\nabla}}\mathcal{L}$. For any $\mathcal{F}^{\nabla}-$ideal $\mathcal{S}$ of a pseudo-complemented lattice $\mathcal{L}$, define $\mathcal{M}_{\mathcal{S}} = \{\mathcal{N} \in Max^{\mathcal{F}^{\nabla}}\mathcal{L} \mid \mathcal{S} \subseteq \mathcal{N}\}$.

Theorem 4.16. *Let $\mathcal{S}$ be an $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice $\mathcal{L}$. Then $\chi(\mathcal{S}) = \bigcap_{\mathcal{N} \in \mathcal{M}_{\mathcal{S}}} \lambda(\mathcal{N})$.*

Proof. Let $\gamma_1 \in \chi(\mathcal{S})$ and $\mathcal{S} \subseteq \mathcal{N}$ where $\mathcal{N} \in Max^{\mathcal{F}^{\nabla}}\mathcal{L}$. Then $\mathcal{L} = (\gamma_1)^{\mathcal{F}} \vee \mathcal{S} \subseteq (\gamma_1)^{\mathcal{F}} \vee \mathcal{N}$. Suppose $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$, then $\mathcal{N} = \mathcal{L}$, which is a contradiction. That implies $(\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}$. Therefore $\gamma_1 \in \lambda(\mathcal{N})$, for all $\mathcal{N} \in \mathcal{M}_{\mathcal{S}}$. Hence $\chi(\mathcal{S}) \subseteq \bigcap_{\mathcal{N} \in \mathcal{M}_{\mathcal{S}}} \lambda(\mathcal{N})$.

Conversely, let $\gamma_1 \in \bigcap_{\mathcal{N} \in \mathcal{M}_{\mathcal{S}}} \lambda(\mathcal{N})$. Then, we get $\gamma_1 \in \lambda(\mathcal{N})$ for all $\mathcal{N} \in \mathcal{M}_{\mathcal{S}}$. Suppose $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} \neq \mathcal{L}$. Then there exists $\mathcal{N}_0 \in Max^{\mathcal{F}^{\nabla}}\mathcal{L}$ such that $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} \subseteq \mathcal{N}_0$. That implies $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}_0$ and $\mathcal{S} \subseteq \mathcal{N}_0$. Since $\mathcal{S} \subseteq \mathcal{N}_0$, by hypothesis, we get $\gamma_1 \in \lambda(\mathcal{N}_0)$. That implies $(\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}_0$, which is a contradiction. Therefore $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}$ and hence $\gamma_1 \in \chi(\mathcal{S})$. Thus $\bigcap_{\mathcal{N} \in \mathcal{M}_{\mathcal{S}}} \lambda(\mathcal{N}) \subseteq \chi(\mathcal{S})$. ☐

Definition 4.17. Let $\mathcal{I}$ be an ideal of $\mathcal{L}$. A distributive lattice $\mathcal{L}$ is said to be $\mathcal{I}-$complemented if for any $\gamma_1 \in \mathcal{L}$ there exists an element $\gamma_2 \in \mathcal{L}$ such that $\gamma_1 \wedge \gamma_2 \in \mathcal{I}$ and $\gamma_1 \vee \gamma_2 = 1$.

Clearly, we have that every Boolean algebra is an $\mathcal{I}-$complemented, but converse need not to be true.

Example 4.18. Let $\mathcal{L} = \{0, 1, 2, 3, 4\}$ be a set with binary operations $\vee, \wedge$ given in the following tables

Table-1. Cayley table for the binary operation " $\wedge$ ".

$\wedge$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	2	2	2
3	0	3	2	3	2
4	0	4	2	2	4

Table-2. Cayley table for the binary operation " $\vee$ ".

$\vee$	0	1	2	3	4
0	0	1	2	3	4
1	1	1	1	1	1
2	2	1	2	3	4
3	3	1	3	3	1
4	4	1	4	1	4

Then $(\mathcal{L}, \vee, \wedge, 0, 1)$ is a distributive lattice. If $\mathcal{I} = \{0, 2\}$ then $\mathcal{I}$ is an ideal of $\mathcal{L}$. Clearly, $\mathcal{L}$ is an $\mathcal{I}-$complemented lattice.

As there are no hidden difficulties to prove the following theorem we omit its proof.

Theorem 4.19. *In a pseudo-complemented lattice $\mathcal{L}$, the following are equivalent:*

(1) *$\mathcal{L}$ is a $\mathcal{F}^{\nabla}-$complemented lattice,*

(2) *every maximal $\mathcal{F}^{\nabla}-$ideal is ornate,*

(3) *for any $\mathcal{N} \in Max^{\mathcal{F}^{\nabla}}\mathcal{L}$, $\lambda(\mathcal{N})$ is maximal,*

(4) *for any $\mathcal{S}, \mathcal{T} \in \mathcal{I}(\mathcal{L})$, $\mathcal{S} \vee \mathcal{T} = \mathcal{L} \Rightarrow \chi(\mathcal{S}) \vee \chi(\mathcal{T}) = \mathcal{L}$,*

(5) *for any $\mathcal{S}, \mathcal{T} \in \mathcal{I}(\mathcal{L})$, $\chi(\mathcal{S}) \vee \chi(\mathcal{T}) = \chi(\mathcal{S} \vee \mathcal{T})$,*

(6) *for any $\mathcal{M}, \mathcal{N} \in Max^{\mathcal{F}^{\nabla}}\mathcal{L}$ with $\mathcal{M} \neq \mathcal{N}$, $\lambda(\mathcal{M}) \vee \lambda(\mathcal{N}) = \mathcal{L}$,*

(7) *for any $\mathcal{N} \in Max^{\mathcal{F}^{\nabla}}\mathcal{L}$, $\mathcal{N}$ is the unique member of $Max^{\mathcal{F}^{\nabla}}\mathcal{L}$ such that $\lambda(\mathcal{N}) \subseteq \mathcal{N}$.*

Proposition 4.20. *Every prime fully consistent $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice $\mathcal{L}$ is minimal.*

Proof. Let $\mathcal{N}$ be a prime fully consistent $\mathcal{F}^{\nabla}-$ideal of a pseudo-complemented lattice $\mathcal{L}$ and $\gamma_1 \in \mathcal{N}$. Then $\gamma_1 \in \chi(\mathcal{N})$. That implies $(\gamma_1)^{\mathcal{F}} \vee \mathcal{N} = \mathcal{L}$. There exist $\mu_1 \in (\gamma_1)^{\mathcal{F}}$ and $\mu_3 \in \mathcal{N}$ such that $\mu_1 \vee \mu_3 = 1$. Since $\mu_1 \in (\gamma_1)^{\mathcal{F}}$, we get $\mu_1 \wedge \gamma_1 \in \mathcal{F}^{\nabla}$. Suppose $\mu_1 \in \mathcal{N}$. Since $\mu_3 \in \mathcal{N}$, we get $1 = \mu_1 \vee \mu_3 \in \mathcal{N}$ which is a contradiction. Therefore $\mu_1 \notin \mathcal{N}$. That is, for any $\gamma_1 \in \mathcal{N}$, there exists $\mu_1 \notin \mathcal{N}$ such that $\mu_1 \wedge \gamma_1 \in \mathcal{F}^{\nabla}$. Hence $\mathcal{N}$ is minimal. $\square$

Example 4.21. From Example-4.3, consider a filter $\mathcal{F} = \{1, 2, 4, 5, 6, 7, 8\}$ and an ideal $\mathcal{F}^{\nabla} = \{0, 3\}$. Clearly, $\mathcal{P} = \{0, 2, 3, 4, 5, 6, 7, 8\}$ is a maximal $\mathcal{F}^{\nabla}-$ideal. Clearly, $\mathcal{P}$ is not a fully consistent ideal, because $(2)^{\mathcal{F}} \vee \mathcal{P} = \mathcal{F}^{\nabla} \vee \mathcal{P} \neq \mathcal{L}$.

For every maximal $\mathcal{F}^{\nabla}-$ideal in a pseudo-complemented lattice, a collection of equivalent conditions is established to transform it into a fully consistent $\mathcal{F}^{\nabla}-$ideal.

Theorem 4.22. *In a pseudo-complemented lattice $\mathcal{L}$, the following are equivalent:*

(1) *$\mathcal{L}$ is a $\mathcal{F}^{\nabla}-$complemented lattice,*

(2) *every maximal $\mathcal{F}^{\nabla}-$ideal is a fully consistent,*

(3) *every maximal $\mathcal{F}^{\nabla}-$ideal is minimal.*

Proof. (1) $\Rightarrow$ (2): Assume (1). Let $\mathcal{N}$ be a maximal $\mathcal{F}^{\nabla}-$ideal of $\mathcal{L}$. By the above Theorem-4.19, we have that $\lambda(\mathcal{N}) = \mathcal{N}$. Let $\gamma_1 \in \chi(\mathcal{N})$. Then $(\gamma_1)^{\mathcal{F}} \vee \mathcal{N} = \mathcal{L}$. If $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$, we get $\mathcal{N} = \mathcal{L}$, which is a contradiction. Therefore $(\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}$ and hence $\gamma_1 \in \lambda(\mathcal{N})$. Thus $\chi(\mathcal{N}) \subseteq \lambda(\mathcal{N})$. Let $\gamma_1 \in \lambda(\mathcal{N})$. Then $(\gamma_1)^{\mathcal{F}} \nsubseteq \mathcal{N}$. Since $\mathcal{N}$ is maximal, we have that $(\gamma_1)^{\mathcal{F}} \vee \mathcal{N} = \mathcal{L}$. Therefore $\gamma_1 \in \chi(\mathcal{N})$ and hence $\lambda(\mathcal{N}) = \chi(\mathcal{N}) = \mathcal{N}$. Thus $\mathcal{N}$ is fully consistent.
(2) $\Rightarrow$ (3): Assume (2). Then every maximal $\mathcal{F}^{\nabla}-$ideal of $\mathcal{L}$ is a prime fully consistent ideal. By Proposition 4.20, every maximal $\mathcal{F}^{\nabla}-$ideal is a minimal prime $\mathcal{F}^{\nabla}-$ideal.
(3) $\Rightarrow$ (1): Assume (3). Let $\gamma_1 \in \mathcal{L}$. Suppose $0^* \notin (\gamma_1] \vee (\gamma_1)^{\mathcal{F}}$. Then there exists $\mathcal{N} \in Max^{\mathcal{F}^{\nabla}}\mathcal{L}$ such that $(\gamma_1] \vee (\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$. That implies $\gamma_1 \in \mathcal{N}$ and $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$. By our assumption, $\mathcal{N}$ is a minimal prime $\mathcal{F}^{\nabla}-$ideal. Since $\mathcal{N}$ is minimal, and $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$, we get that $\gamma_1 \notin \mathcal{N}$ which is a contradiction. That implies $0^* \in (\gamma_1] \vee (\gamma_1)^{\mathcal{F}}$. Then there exist $\mu_1 \in (\gamma_1)^{\mathcal{F}}$ such that $0^* = \gamma_1 \vee \mu_1$. Hence $\gamma_1 \wedge \mu_1 \in \mathcal{F}^{\nabla}$ and $\gamma_1 \vee \mu_1 = 0^*$. Thus μ_1 is the complement of γ_1 in $\mathcal{L}$. Therefore $\mathcal{L}$ is an $\mathcal{F}^{\nabla}-$complemented lattice. □

Example 4.23. From Example-4.3, Consider a filter $\mathcal{F} = \{1, 2, 4, 5, 6, 7, 8\}$ and an ideal $\mathcal{F}^{\nabla} = \{0, 3\}$. Clearly, $\mathcal{P} = \{0, 2, 3, 4, 5, 6, 7, 8\}$ is a maximal $\mathcal{F}^{\nabla}-$ideal. Clearly, $\mathcal{P}$ is not ornate, because for $2 \in \mathcal{P}$ there is no $\beta_1 \notin \mathcal{P}$ such that $2^* \vee \beta_1^* \in \mathcal{F}$.

From the Example-3.27, consider a filter $\mathcal{F} = \{1, 3, 4, 6, 7\}$, an ideal $\mathcal{F}^{\nabla} = \{0, 2\}$ and a maximal $\mathcal{F}^{\nabla}-$ideal $\mathcal{M} = \{0, 2, 3, 4, 6, 7, 8, 9\}$. Clearly $\mathcal{M}$ is an ornate ideal of $\mathcal{L}$.

Theorem 4.24. *In a pseudo-complemented lattice $\mathcal{L}$, the following are equivalent :*

(1) *every maximal $\mathcal{F}^{\nabla}-$ideal is ornate,*

(2) *every $\mathcal{F}^{\nabla}-$ideal is fully consistent,*

(3) *every prime $\mathcal{F}^{\nabla}-$ideal is fully consistent,*

(4) *every prime $\mathcal{F}^{\nabla}-$ideal is ornate.*

Proof. (1) ⇒ (2): Assume that every maximal $\mathcal{F}^{\nabla}-$ideal is ornate. Let $\mathcal{S}$ be an ideal of $\mathcal{L}$. Clearly $\chi(\mathcal{S}) \subseteq \mathcal{S}$. Let $\gamma_1 \in \mathcal{S}$. If $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} \neq \mathcal{L}$, then there exists $\mathcal{N} \in Max^{\mathcal{F}^{\nabla}}\mathcal{L}$ such that $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} \subseteq \mathcal{N}$. That implies $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$ and $\gamma_1 \in \mathcal{S} \subseteq \mathcal{N}$. By our assumption, $\mathcal{N}$ is ornate. Since $(\gamma_1)^{\mathcal{F}} \subseteq \mathcal{N}$, we get $\gamma_1 \notin \mathcal{N}$, which is a contradiction. Hence $(\gamma_1)^{\mathcal{F}} \vee \mathcal{S} = \mathcal{L}$. Therefore $\mathcal{S}$ is a fully consistent $\mathcal{F}^{\nabla}-$ideal.

(2) ⇒ (3): It is obvious.

(3) ⇒ (4): Assume (3). Let $\mathcal{N}$ be a prime $\mathcal{F}^{\nabla}-$ideal of $\mathcal{L}$. By our assumption, we have that $\chi(\mathcal{N}) = \mathcal{N}$. Let $\gamma_1 \in \mathcal{N} = \chi(\mathcal{N})$. Then $(\gamma_1)^{\mathcal{F}} \vee \mathcal{N} = \mathcal{L}$. That implies $0^* \in (\gamma_1)^{\mathcal{F}} \vee \mathcal{N}$. There exist $\mu_1 \in (\gamma_1)^{\mathcal{F}}$ and $\mu_3 \in \mathcal{N}$ such that $0^* = \mu_1 \vee \mu_3$. Clearly, we have $\mu_1 \notin \mathcal{N}$, otherwise $0^* = \mu_1 \vee \mu_3 \in \mathcal{N}$. Therefore $\gamma_1^* \vee \mu_1^* \in \mathcal{F}$ for $\mu_1 \notin \mathcal{N}$. Hence $\mathcal{N}$ is ornate.

(4) ⇒ (1): It is obvious. □

5. Conclusion and Future Work

In this paper, we have introduced and defined the concepts of consistent ideals, fully consistent $\mathcal{F}^{\nabla}-$ideals, and closed ideals within pseudo-complemented distributive lattices. We have provided characterization theorems for these ideals. Specifically, we have established a set of equivalent conditions that must be satisfied for an ideal in a pseudo-complemented distributive lattice to be considered as consistent. We have presented the notion of quasi $\mathcal{F}-$stone pseudo complemented distributive lattices and it characterizes in terms of fully consistent ideals. Moreover, we have introduced the concept of ornate prime $\mathcal{F}^{\nabla}-$ideals and defined them within this lattice framework. We have also derived a set of equivalent conditions for a maximal $\mathcal{F}^{\nabla}-$ideal in a pseudo-complemented distributive lattice to qualify as an ornate prime $F^{\nabla}-$ideal. In future we may also introduce many concepts like ideal, filters, congruences etc. in an $\mathcal{F}-$stone and a quasi $\mathcal{F}-$stone lattice.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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