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OPTIMAL CONTROL AND COST-EFFECTIVENESS ANALYSIS FOR CASHEW FUSARIUM WILT DISEASE TRANSMISSION IN TANZANIA

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Abstract. This paper assesses the effectiveness of optimal control theory in determining control strategies for fusarium wilt disease in cashew plants. Control strategies such as thermos-sanitation, advertisement, and organic soil amendments were incorporated into the model to minimise disease transmission. The goal is achieved using Pontryagin's principle. The optimal control is characterised by an optimality system and solved numerically. The data collected from Lindi and Mtwara regions of Tanzania was used to validate and estimate model parameters using maximum likelihood. The cost-effective analysis focuses on the strategies mentioned to obtain effective control with minimum cost. The findings show that continuous implementation of thermos-sanitation and organic soil amendments are the best strategies to minimize disease transmission.

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Key words and phrases. fusarium wilt; organic soil amendments; thermos-sanitation; chlamydospores; macroconidia; cashew plants; optimal control; cost-effective.

1. Introduction

Cashew plants are seriously threatened by fusarium wilt disease, especially in the southern-eastern region of Tanzania, where the disease was initially discovered in 2012 [1]. Fusarium wilt disease is caused by the fungus called Fusarium oxysporum that grows in soil [2]. This pathogen can seriously harm cashew plants. The capacity of fusarium wilt disease to harm cashew plants has a cascading effect on individual families and the country's total cashew crop production [3]. Decreases in production impacts the earnings of individual farming households and potentially lower the country's total cashew exports and foreign exchange profits. In mitigating the disease, an efficient control strategies are needed.

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The intervention measures have been investigated by variety of researchers, for example Organic soil amendments that aims to reduce the impact of disease by introducing beneficial organisms and plant extracts to suppress the growth and activity of pathogenic fusarium fungi [4]. This approach increases the number of healthy plants compared to untreated areas [5–9]. Most organic soil amendment control strategies, including plant extracts, are easy to apply and affordable [8]. Basing on field experience, it is important for the farmers to be aware on using organic soil amendments even though they are not examined in the studies.

Awareness is a critical component in epidemiology, that every farmer must have accurate knowledge and information about the disease, which can aid in developing a suitable control plan during an outbreak. Furthermore, advertisements enhance farmers' awareness of managing fusarium wilt disease [10]. Increasing awareness campaigns raise crop production by decreasing disease prevalence [11]. Note that the recovery of plant disease resulting from early detection and proper choice of control method tends to increase production [12]. Advertisements help to minimise the spread of the disease due to adequate information about the transmission, impact, and control of fusarium wilt [13,16]. Effective control techniques will be adopted among those who are aware of the disease and control method [11].

On the other hand, the thermos-sanitation method was found to play a significant role in controlling fusarium wilt disease [17–19]. Basing on the debris from deceased infected cashew plants contributes to disease infection transmission [20,21]. Further, it was investigated that decomposed diseased plants produce chlamydospores and increase fungus saturation rate in the soil, leading to outbreaks [22–25]. Thermos-sanitation help to reduce the production and transmission of chlamydospores in the field by burning debris and sanitising farming tools [21].

The investigated control strategies can be utilized through mathematical tools to evaluate modelling has emerged as a powerful tool for understanding disease dynamics and devising effective control measures for various infectious diseases. It helps to simulate the interactions between the disease pathogen and cashew plants to assess the effectiveness of different control interventions. Furthermore, a cost-effectiveness analysis examines the cost of implementing each control strategy and the potential reduction in disease spread.

This paper intends to investigate the effect of farming awareness and the contribution of debris from dead plants on the spread of disease. The particular study aims to minimize the cost of interventions and the spread of the fusarium wilt disease by implementing optimal control theory, with control strategies such as thermos-sanitation, advertising, and organic soil additives.

2. Model Formulation and Description

The fusarium wilt disease model was created following the disease transmission dynamics described by Rana and Sahgal [26]. The modelled populations involve plants and the fungus population. Cashew plants change classes due to the dynamics of the disease. The cashew plant, which is disease-free yet vulnerable to infection when the fungus comes in touch with it, is the susceptible plant (B). Plants become exposed (L) when their roots or wounds come into contact with infectious plants or contaminated soil [27,28]. The infected plants from infectious class (X) may recover after being treated and join a treated class (Y) or die due to disease and join the dead class (G) [12]. Also, the fungal classes are represented by (A) for macroconidia spores and (F) for chlamydospores.

The model assumes that the cashew plant population's growth rate follows a logistic function by considering the carrying capacity s and intrinsic growth rate a . It is assumed that when plants and fungus come into contact, a force of infection is represented by

$$\delta = \left(\frac{\varepsilon_1 F}{b + F} + \varepsilon_2 X \right). \tag{1}$$

The ε_1 indicates the frequency of contact between chlamydospores and susceptible while ε_2 is the frequency of contact rate of susceptible plants with diseased plants [25].

Chlamydospore saturation at a constant rate in the soil is presented by b. The exposed plants take two to six months to become infected, as represented by the parameter σ [22]. The rate at which the infected cashew plant naturally perishes is μ. The plants that are not highly diseased might recover at the rate of ρ and become vulnerable at the rate of τ [12]. The severely infected plants die from the disease at a rate of φ. In most cases, farmers get rid of dead plants for various uses while some dead plants are left to decay on the farm at a rate of η. The increased mortality rate of diseased plants as a result of farming awareness is presented by $\frac{eWX}{1+W}$.

Microconidia spores attack the tissue of drying plants to produce macroconidia spores from the sporodochia at a rate of π [29–33]. The parameter π is represented by the following equation

$$\pi = c_1 L + c_2 W + c_3 D, \tag{2}$$

where c_1, c_2 and c_3 are the production rate of macroconidia spores from latently, diseased and deceased plants, respectively. Chlamydospores are produced from the macroconidia spores after the breakdown of deceased cashew plants at a rate of x while leaving the survival compartment by decay at a rate of α [34]. The increased mortality rate of chlamydospores as a result of awareness is represented by $\frac{rWF}{1+W}$.

Awareness campaingns through mobile phones, radio, television, magazines, and social media increasing farming information on the disease transmission among farmers at a rate of n [35]. The amount of diseased plants visible in their environment will raise the farmers' local awareness at a rate

h [11]. The level of farming awareness may decrease due to its decreasing significance at a rate of y [36].

The time-dependent control was devised and implemented to achieve complete cashew plant protection. The preventative techniques considered in this study are organic soil amendments g_1, advertisement g_2, and thermos-sanitation g_3. Some dead plants may remain in the field after sanitation and contribute to the production of macroconidia spores at a rate of $1 - g_3$. The integrated control strategies of fusarium wilt disease are designed to reduce fungi by decreasing the concentration of pathogens in the field from the exposed, infected, and disease-induced dead plants by the use of advertisements for improving farming awareness, application of organic soil amendments using aqueous extracts to inhibit the growth and activity of fusarium oxysporum and thermos-sanitation by burning dead plants contaminated with fungus. Table 1, presents variables and their definitions, respectively.

Table 1. State variables and definitions for fusarium wilt disease model.

Variable	**Definition**
B	Susceptible cashew plants at a time t.
L	Exposed cashew plants at a time t.
X	Infected cashew plants at a time t.
Y	Recovered cashew plants at a time t.
G	Diseased induced dead plants at a time t.
A	Macroconidia spores at a time t.
F	Chlamydospores at a time t.
W	Farming awareness at a time t.

2.1. **Mathematical equations.** Considering the given assumptions, the following are equations generated.

$$\begin{cases} \frac{dB}{dt} &= aB\left(1 - \frac{B}{s}\right) + \tau Y - \left(\frac{\varepsilon_1 F}{b+F} + \varepsilon_2 X\right) g_1 B \\ \frac{dL}{dt} &= \left(\frac{\varepsilon_1 F}{b+F} + \varepsilon_2 X\right) g_1 B - (\sigma + \mu(1-g_3))L, \\ \frac{dX}{dt} &= \sigma L - \left(\rho g_1 + \varphi + \mu(1-g_3) + \frac{g_2 e W}{1+W}\right) X, \\ \frac{dY}{dt} &= \rho g_1 Y - (\mu + \tau) Y, \\ \frac{dG}{dt} &= \varphi X - \eta(1-g_3) G, \\ \frac{dA}{dt} &= c_1 L + c_2 X + c_3 G - x g_1 A, \\ \frac{dF}{dt} &= x g_1 A + \frac{r g_2 W F}{1+W} - \alpha F, \\ \frac{dW}{dt} &= n g_2 + hX - yW. \end{cases} \tag{3}$$

where,

g_1 are the organic soil amendments using aqueous extracts.

g_2 are the advertisements based on farming awareness.

g_3 are thermos-sanitation methods based on tools sanitation and burning of dead exposed, infected, and diseased-induced plants.

3. OPTIMAL CONTROL ANALYSIS

Implementing effective control strategies tends to minimize fusarium wilt disease transmission. As Pei et al. [37] emphasised, on the important of minimizing plant disease through control strategies. Introducing appropriate control measures can reduce the spread of diseases and minimize their impact on plant health and productivity. Therefore, it is essential to prioritize developing and implementing effective control strategies to safeguard plant health and promote agricultural sustainability. The inclusion of controls in the model aims to determine the optimal level of intervention that minimizes both the spread of disease and implementation costs. The control variables g_1, g_2 and g_3 are minimized subject to the model equations (3), and the minimization objective functional is given as

$$J = \min_{g_1,g_2,g_3} \int_0^f \left(Q_1 X + Q_2 F + \frac{C_1 g_1^2}{2} + \frac{C_2 g_2^2}{2} + \frac{C_3 g_3^2}{2} \right) dt. \tag{4}$$

where f is the final time, Q_1 and Q_2 is the weight related to the amount of chlamydospores and infectious cashew plants in the field while C_1, C_2, C_3 and are the positive weight of control related to its suggestion cost for controlling the disease with its transmission behaviour. The cost related to any control set-up is assumed to be non-linear and takes a quadratic form. The quadratic expressions $\frac{C_1 g_1^2}{2}$, $\frac{C_2 g_2^2}{2}$ and $\frac{C_3 g_3^2}{2}$ indicate the cost asociated with the application of organic soil amendments, advertisement and thermos-sanitation, respectively. The objective functional J introduced for the aim of minimizing the cost of implementing control measures and the number of diseased plants as

$J\left(g_1^*(t), (g_1^*(t), (g_1^*(t)\right) = \min\left(g_1(t), g_2(t), g_3(t) | g_1(t), g_2(t), g_3(t) \in U\right)$

where $U = (g_1(t), g_2(t), g_3(t))$ are measurable.

3.1. **Existence of Optimal Controls.**

The control strategies were evaluated for their existence following the approach applied by Bokil et al. [38]. The model (3) is transformed following the theorem.

Theorem 3.1

Assumed that $J(g_i)$ is subjected to system (3), then there existing optimal strategy $g_i^*, i = 1, 2, 3$ corresponding to fusarium wilt disease that minimizes $J(g_i)$ in U.

Proof:

The proof of existence evaluated at the following properties [39]:

1. The state variables and set of control measures are nonempty.
2. The computable control measures are closed and convex.

3. Each right-hand side of the state system is continuous, bounded above the sum of the bounded control measures and the state, and can be expressed as a linear function with values that depend on time and state.
4. The integrand $\ell(f, g)$ of the objective function is convex.
5. There exist constants M_1, $M_2 > 0$, and $\beta^* \geq 1$ such that the integrand of the objective functional satisfies $\ell \geq M_1 \left(\| g_1 \|^2 + \| g_2 \|^2 + \| g_3 \|^2\right)^{\frac{\beta^*}{2}} - M_2$

The result of the existence of the system confirms the first property [40]. Considering the meaning of a convex set, the control measures are closed and convex. Therefore, the second condition is satisfied. The state solutions of the linear state system in g_i are bounded, then the linear function bounds the right-hand side. Hence, M_1, $M_2 > 0$ and $\beta^* \geq 1$ satisfying

$Q_1X + Q_2F + \frac{C_1g_1^2}{2}(t) + \frac{C_2g_2^2}{2}(t) + \frac{C_3g_3^2}{2}(t) \geq M_1 \left(\| g_1 \|^2 + \| g_2 \|^2 + \| g_3 \|^2\right)^{\frac{\beta^*}{2}} - M_2$ because the state variable is bounded. Therefore, the optimal control is exists [39].

3.2. **Optimal Control Characterization.** This part represents the optimal control characteristic using Pontryagin's Maximum Principle, which is used to formulate the Lagrangian optimal problem for the aim of minimizing disease and intervention costs [41]. The Lagrangian optimal problem is created from the state, objective functional, and control variables. The objective functional in form of Lagrangian stated as:

$$\Gamma = Q_1X + Q_2F + \left(\frac{C_1g_1^2}{2} + \frac{C_2g_2^2}{2} + \frac{C_3g_3^2}{2}\right) dt. \tag{5}$$

In order to determine the minimum Lagrangian of optimal problem, Hamiltonian function (H) with respect to $g_1(t)$, $g_2(t)$ and $g_3(t)$ given as

$$H = Q_1X + Q_2F + \frac{C_1g_1^2}{2} + \frac{C_2g_2^2}{2} + \frac{C_3g_3^2}{2} + \sum_{j=1}^{8} P_j k_j. \tag{6}$$

where k_j represents the right-hand sides of the state variable jth equation and $j = 1, 2, ..., 8$ is the adjoint function set. The Hamiltonian contains the integrand of the objective function and the inner product of the right-hand side of state equations and the corresponding adjoint variable P_1, P_2, P_3, P_4, P_5, P_6, P_7 and P_8. Hamiltonian expanded form is written as:

$$\begin{aligned} H = \quad & Q_1X + Q_2F + \frac{g_1^2C_1}{2} + \frac{g_2^2C_2}{2} + \frac{g_3^2C_3}{2} + P_1\left(aB\left(1 - \frac{B}{s}\right) + \tau Y - \left(\frac{\varepsilon_1F}{b+F} + \varepsilon_2X\right)g_1B\right) \\ & + P_2\left(\left(\frac{\varepsilon_1F}{b+F} + \varepsilon_2X\right)g_1B - (\sigma + \mu(1-g_3))L\right) + P_3\left(\sigma L - \left(\rho g_1 + \varphi + \mu(1-g_3) + \frac{eg_2W}{1+W}\right)X\right) \\ & + P_4\left(\rho g_1X - (\mu+\tau)Y\right) + P_5\left(\varphi X - \eta G(1-g_3)\right) + P_6\left(c_1L + c_2X + c_3G - xg_1A\right) \\ & + P_7\left(xg_1A + \frac{rg_2WF}{1+W} - \alpha F\right) + P_8\left(ng_2 + hX - yW\right). \end{aligned} \tag{7}$$

Let g_i^* be the optimal control set and x^* be the corresponding solution set of the state system (3) that minimize objective functional J , then there exist adjoint variables P such that

$$\begin{aligned} &\tfrac{dP}{dt} = -\frac{dH}{dx}, \text{ adjoint condition.} \\ &P_i(f) = 0, \text{ transversality condition.} \\ &\tfrac{dH}{dg} = 0 \text{ at } g_i^*, \text{ optimality conditions.} \end{aligned} \tag{8}$$

The adjoint system is obtained by taking the partial derivative of Lagrangian H , with respect to state variables as follows:

$$\begin{aligned} \frac{dH}{dB} &= P_1\left(a\left(\frac{B}{s}-1\right)+\frac{Ba}{s}+\left(\frac{\varepsilon_1 F}{b+F}+\varepsilon_2 X\right)g_1\right)-P_2 g_1\left(\frac{\varepsilon_1 F}{b+F}+\varepsilon_2 X\right) \\ \frac{dH}{dL} &= P_2\left(\sigma-\mu(g_3-1)\right)-P_3\sigma+P_6 c_1-P_5\upsilon, \\ \frac{dH}{dX} &= P_3\left(\upsilon+\rho g_1-\mu(g_3-1)\right)-P_6 c_2+P_5\upsilon+Q_1+P_4\rho g_1+BP_1\varepsilon_2 g_1+BP_2\varepsilon_2 g_1-P_8 h+\frac{rg_2 W}{W+1}, \\ \frac{dH}{dY} &= P_1\tau-P_4(\mu+\tau), \\ \frac{dH}{dG} &= P_6 c_3+P_5\eta(g_3-1), \\ \frac{dH}{dA} &= P_6 g_1 x-P_7 g_1 x, \\ \frac{dH}{dF} &= P_7\left(\alpha-\left(\frac{g_2 rW}{W+1}\right)\right)-Q_2-BP_1 g_1\left(\frac{\varepsilon_1}{b+F}-\frac{\varepsilon_1 F}{(b+F)^2}\right)+P_2 g_1 B\left(\frac{\varepsilon_1}{b+F}-\frac{\varepsilon_1 F}{(b+F)^2}\right), \\ \frac{dH}{dW} &= P_8 y-P_7\left(\frac{g_2 rF}{W+1}-\frac{g_2 rWF}{(W+1)^2}\right)+P_3 X\left(\frac{eg_2}{W+1}-\frac{g_2 eW}{(b+F)^2}\right). \end{aligned} \tag{9}$$

With consideration of transversality conditions $P(f) = 0$, then the optimality control problem is $\frac{\partial H}{\partial g_i} = 0$ wherever$i = 1, 2, 3$ is given as follows:

$$\begin{aligned} \frac{\partial H}{\partial g_1} &= C_1 g_1 - MP_6\eta + P_7 M\eta - P_3 X\rho + P_4 X\rho - P_1 B\left(\frac{\varepsilon_1 F}{b+F}+\varepsilon_2 X\right)+P_2 B\left(\frac{\varepsilon_1 F}{b+F}+\varepsilon_2 X\right)=0 \\ \frac{\partial H}{\partial g_2} &= C_2 g_2 + P_8 n + \frac{rP_7 WF}{W+1} - \frac{eP_3 GX}{W+1} = 0, \\ \frac{\partial H}{\partial g_3} &= C_3 g_3 + \eta P_5 G + xP_7 A + \mu P_2 L + \mu P_3 X = 0. \end{aligned} \tag{10}$$

The delivered solution of g_1^*, g_2^* and g_3^* are

$$g_1^* = min\left(1, max\left(0, \frac{MP_6\eta + P_7M\eta - P_3X\rho + P_4X\rho - P_1B\left(\frac{\varepsilon_1 F}{b+F} + \varepsilon_2\right) + P_2B\left(\frac{\varepsilon_1 F}{b+F} + \varepsilon_2\right)}{C_1}\right)\right)$$

$$g_2^* = min\left(1, max\left(0, \frac{nP_8 + \frac{rP_7WF}{W+1} - \frac{eP_3GX}{W+1}}{C_2}\right)\right),$$

$$g_3^* = min\left(1, max\left(0, \frac{\eta P_5G + xP_7A + \mu P_2L + \mu P_3X}{C_3}\right)\right). \tag{11}$$

Since the second partial derivative of the Hamiltonian H with respect to controls g_1, g_2, and g_3 are positive. Such that $\frac{\partial^2 H}{\partial g_1^2} = C_1 > 0$, $\frac{\partial^2 H}{\partial g_2^2} = C_2 > 0$ and $\frac{\partial^2 H}{\partial g_3^2} = C_3 > 0$. Then, the optimal problem is minimum at control g_1, g_2 and g_3.

4. PARAMETER ESTIMATION OF FUSARIUM WILT DISEASE MODEL

Ordinary differential equations are frequently utilized in epidemiology to describe disease transmission dynamics. Still, systems of ODEs rarely yield quantitative solutions that are nearly identical to actual field observations due to inappropriate choice of parameters [42]. Hence, it is crucial to estimate model parameters for numerical analysis. This section presents data on dead cashew plants from seven districts in Mtwara and Lindi regions. The method used to estimate parameters is Maximum Likelihood. The actual data of dead cashew plants from Tandahimba, Masasi, Newala, Mtwara Rural, Liwale, Ruangwa, and Lindi Rural are used.

TABLE 2. Cashew plants death cases data due to fusarium wilt disease per village from 2018 to 2020.

Village code	Village Name	Death cases	Village Code	Village Name	Death Cases
1	Kitandi	15	14	Nakayaka	33
2	Chinongwe	45	15	Kitangali	50
3	Ruhokwe	46	16	Chikunja	30
4	Simana	23	17	Msikisi	16
5	Tuungane B	56	18	Makululu	15
6	Tuungane A	11	19	Mnavila	50
7	Ungonglo sokoni	24	20	Makong'onda	41
8	Majimaji	40	21	Likuledi	48
9	Legeza mwendo	15	22	Mchauru	7
10	Itete	50	23	Mpindimbi	9
11	Dihimba	39	24	Namyomyo	51
12	Tandika	31	25	Nangoo	190
13	Mwera sokoni	13	26	Nanganga	204

Source: Field data October, 2020

4.1. **Maximum Likelihood estimator.** The main goal of the Maximum Likelihood method is to maximaize the likelihood function. The likelihood function is defined as the sum of squares of residual, which is represented as

$$L(\theta) = \sum_{i=1}^{n} (x_i - x_i^{est})^2 \tag{12}$$

Where $\{x_i\}_{i=1}^{n}$ is actual data and $\{x_i^{est}\}_{i=1}^{n}$ is the model solution of equation (3) in a particular parameter values.The model was verified using the data of dead cashew plants collected from Lindi and Mtwara regions in Tanzania. The Maximum Likelihood estimator was used for parameter estimation. Table 3 summarizes estimated parameters for the fusarium wilt disease model.

TABLE 3. Estimated parameter values used in numerical simulation.

Parameter	**Definition**	**Value**	**Source**	**Estimated**
a	Cashew plant's growth rate	0.05	[43]	0.051
s	Carrying capacity of cashew plants	69	[44]	71
ε_1	Contact rate between susceptible plants with F	0.06	[45]	0.075
ε_2	Contact rate between susceptible with infected cashew plants	0.0018	[46]	0.029
b	Saturation constant rate of F in the ground	40	[47]	42.52
σ	Progression rate from L to X	0.05	[22]	0.58
μ	Natural death rate of cashew plants	0.01	Assumed	0.005
e	Death rate of X due to awareness	0.25	[11]	0.035
ρ	Plants recovery rate	0.3	[48,49]	0.359
τ	The rate of Y become susceptible	0.3	Assumed	0.28
x	Production of F from A	0.05	Assumed	0.052
α	Natural decay rate of F	0.048	[48]	0.054
r	Decay rate of F due to awareness	0.25	[11]	0.201
η	Dead plants left to decay on the farm	0.29	Assumed	0.35
c_1	Production rates of A from dead L	0.02	Assumed	0.029
c_2	Production rates of A from dead X	0.063	Assumed	0.083
c_3	Production rates of A from G	0.25	Assumed	0.458
n	Global awareness rate	0.003	[35]	0.004
h	Local awareness rate	0.025	[13]	0.028
y	Fading rate of farming awareness	0.015	[50]	0.01

5. NUMERICAL SIMULATION

This section assesses the numerical outcomes of the optimal control methods, utilizing the forward-backwards sweep and fourth-order Runge-Kutta method in simulations. We analyse the effectiveness of control strategies, such as organic soil amendments using aqueous extracts, advertisements to increase farming awareness, and thermos-sanitation for burnig exposed, infected, and diseased-induced plants. Each control strategy is evaluated based on the estimated parameter values listed in Table 3.

Control Scenarios

Strategy S_1: Effect of organic soil amendments and advertisement $(g_1 \neq 0, g_2 \neq 0, g_3 = 0)$.

Strategy S_2: Effect of organic soil amendments and thermos-sanitation $(g_1 \neq 0, g_2 = 0, g_3 \neq 0)$.

Strategy S_3: Effect of thermos-sanitation and advertisement $(g_1 = 0, g_2 \neq 0, g_3 \neq 0)$.

Strategy S_4: Effect of organic soil amendments, advertisement and thermos-sanitation $(g_1 \neq 0, g_2 \neq 0, g_3 \neq 0)$.

5.1. **Effect of Organic Soil Amendments and Advertisement.** The effects of organic soil amendment $g_1(t)$ and advertisement $g_2(t)$ are implemented. Figures 1 (a) and (b) show that, the infected plants slowly decrease as the population of chlamydospores drops. This method effectively reduces the population of chlamydospores in the field, as indicated in Figure 1 (b). The control profiles in Figure 1(c) show that the control strategies should be fully implemented to minimise disease. Farmers must continuously apply organic soil amendment and advertisement to eliminate the disease.

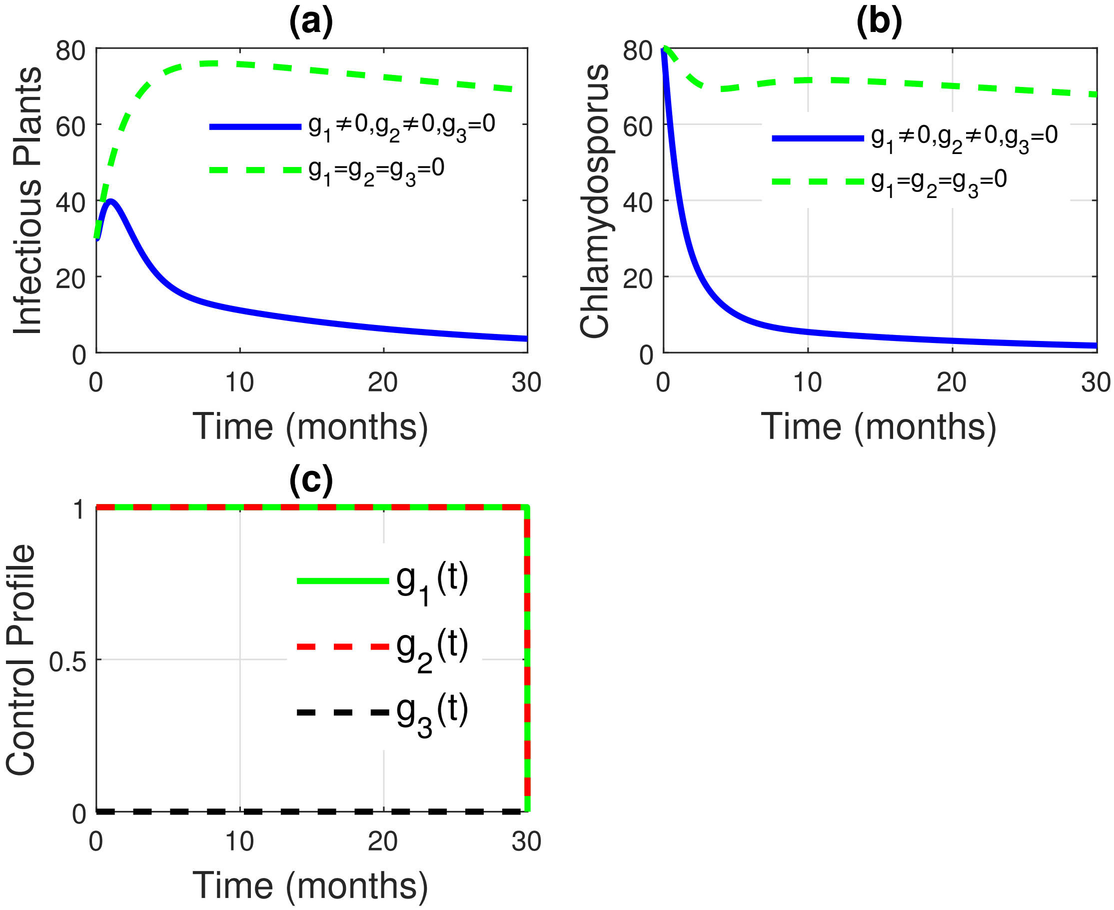

FIGURE 1. Effect of using the organic soil amendments and advertisement.

5.2. **Effect of thermos-sanitation and organic soil amendments.** This strategy investigates the effect of organic soil amendments g_1 and thermos-sanitation g_3. Figures 2 (a) and (b) show that the infected plants and chlamydospores decrease slowly and reach disease-free at 12 and 23 months, respectively. There is a possibility of controlling the disease due to the decrease of chlamydospores. The control profiles in Figure 2 (c) indicate that applying organic soil amendments and thermos-sanitation can minimize the disease after fully applying the control strategies for 26 months. The application of control strategies can be reduced as the infection is reduced to zero. This method indicates the possibility of eliminating the disease within 30 months.

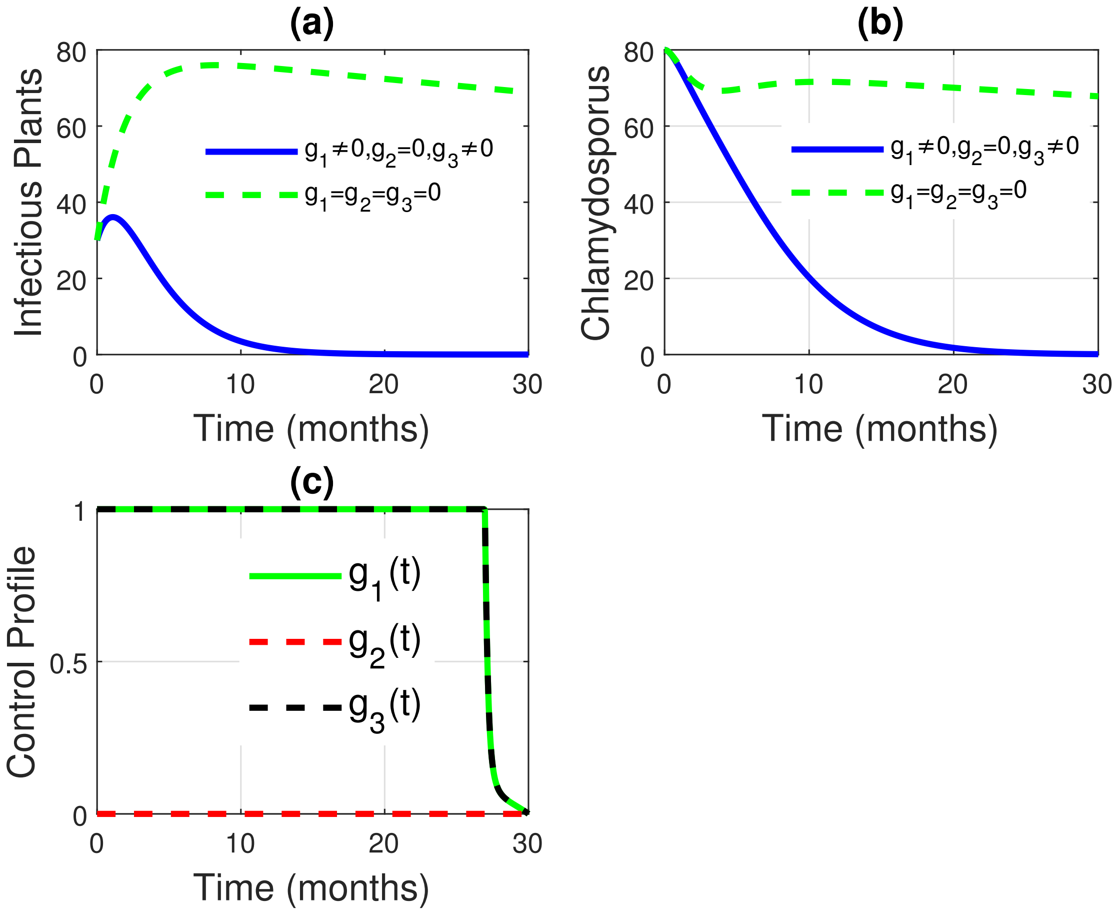

FIGURE 2. Impact of implementing organic soil amendments and thermos-sanitation.

5.3. **Impact of thermos-sanitation and advertisement.** This method implements the effects of thermos-sanitation and advertisement. Figures 3 (a) and (b) detected that infected plants and chlamydospores declined but not to the extent of a disease-free situation in the soil. Slowly reducing infected plants and chlamydospores shows there is no possibility of eliminating the disease by implementing sanitation and advertisement control strategies. The control profiles in Figure 3 (c) show that the intervention are completely applied for 30 months. The application of thermos-sanitation and advertisement shows there is no possibility of making soil free from fungus. Thus, an alternative approach can be proposed to eradicate the disease beyond relying solely on thermos-sanitation and advertising.

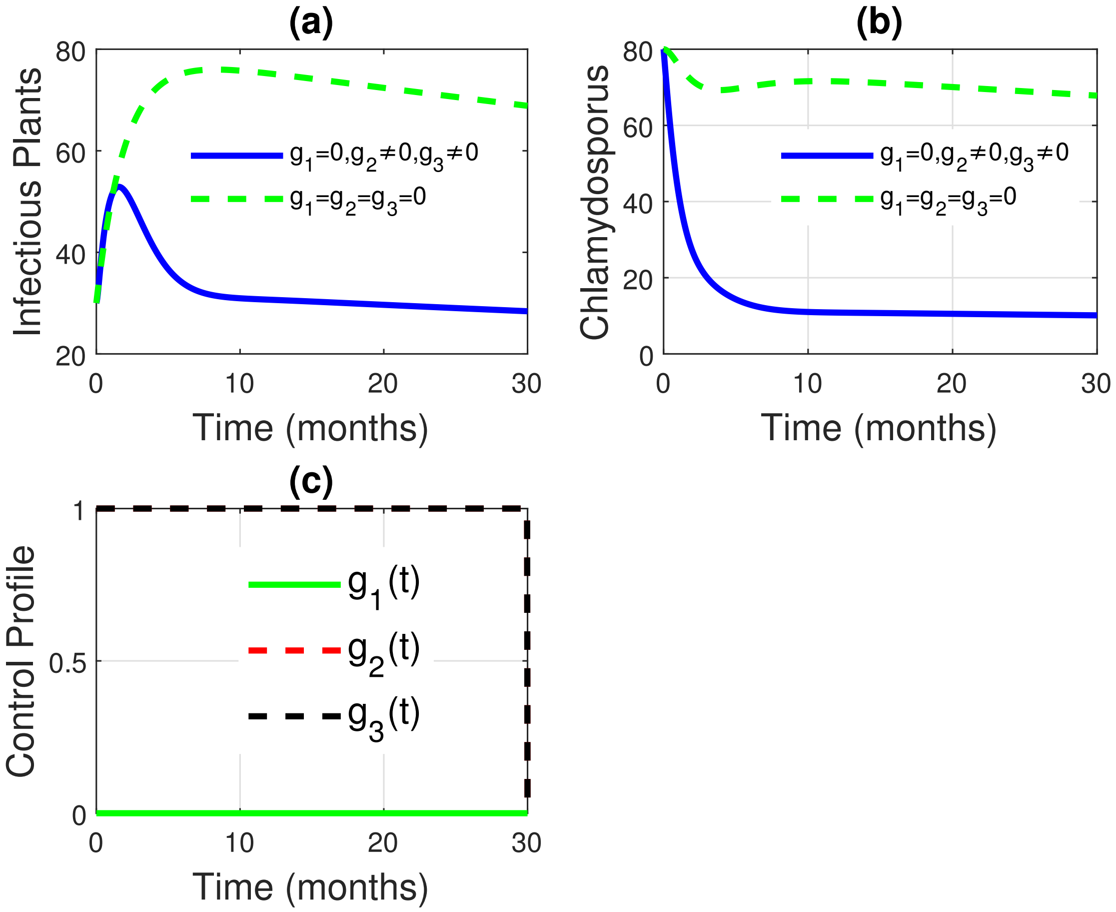

FIGURE 3. . Effect of using thermos-sanitation and advertisement.

5.4. **Effect of organic soil amendments, thermos-sanitation and advertisement.** The effects of implementing all three control strategies are indicated in Figures 4 (a) and (b), which show that the population of infected plants decrease slowly, even if not to zero. The chlamydospores population approach to zero in 30 months shows the possibility of disease elimination as control strategies are continuously implemented. The control profiles in Figure 4 (c) show that the control variables g_1, g_2 and g_3 is at their highest after 30 months and then drop to zero. Applying organic soil amendments, thermos-sanitation, and advertisement shows the ability to reduce disease after full application for 30 months.

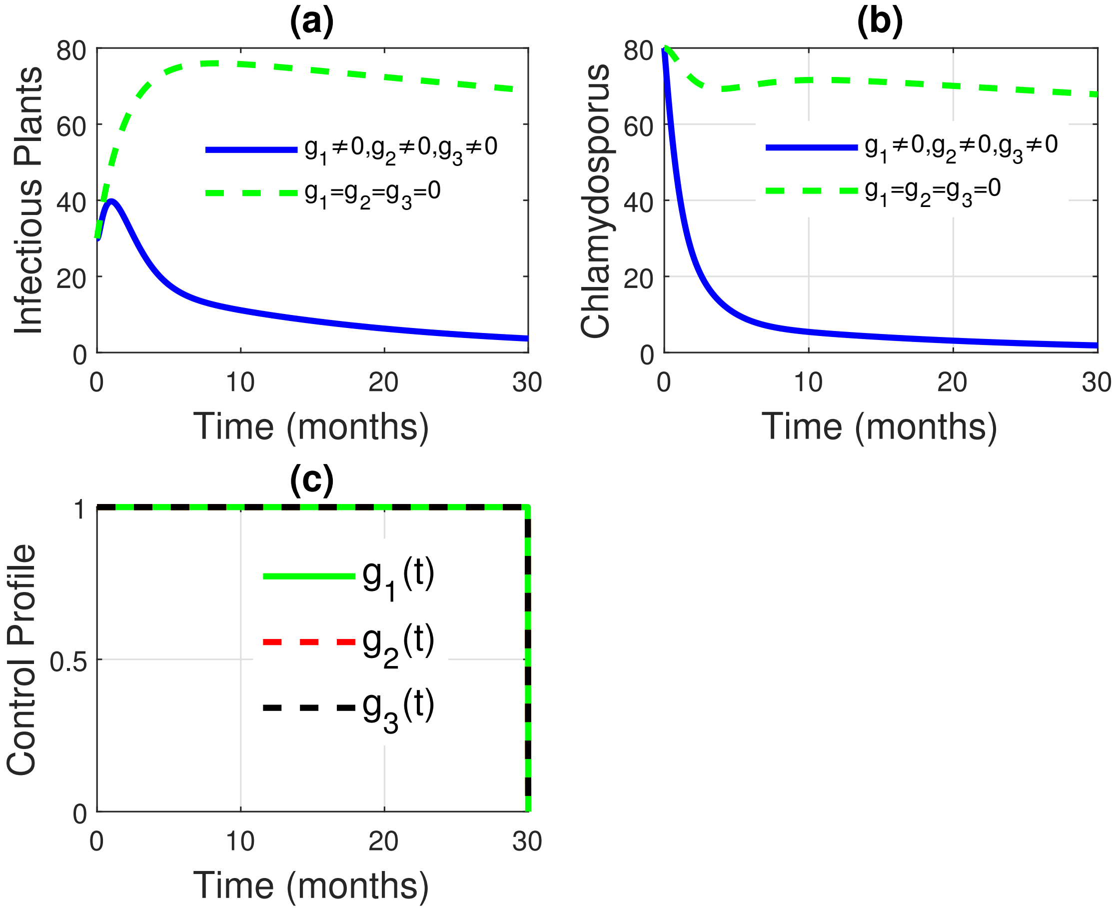

Figure 4. Impact of implementing organic soil amendments, thermos-sanitation and advertisement .

6. Cost-effective Analysis

The cost-effectiveness analysis helps make a proper decision on choosing the intervention that minimises the spread of fusarium wilt disease at a possible minimum cost. It determines and proposes the best cost-effective strategy to apply with limited resources. This is accomplished by establishing a correlation between the costs and outcomes of each intervention, as determined by evaluating the incremental cost-effective ratio (ICER), which is defined as the additional cost for each additional intervention impact. Incrementally, when analysing two or more competing intervention strategies, one intervention is related to the next less operative option [51,52]. The total number of diseased plants averted is the ICER denominator, while the ICER numerator is the total difference in intervention costs.

$$ICER = \frac{\text{Difference in total cost between control strategies}}{\text{Difference in total number of infectious plants averted by control strategies}} \tag{13}$$

The infection averted plant is determined by subtracting the total number of infected plants under control from those without control. The status que is the total number of infections without applying a control strategy. The ICER is determined by considering the combination of organic soil amendments and advertisement as S_1, the combination of organic soil amendments and thermos-sanitation as S_2, the combination of thermos-sanitation and advertisement as S_3, and the combination of organic soil amendments, advertisement and thermos-sanitation as S_4. The total cost and infections averted were estimated using the parameter values from Table 3, as shown in Table 4 below.

TABLE 4. The individual strategy total cost and the number of diseases prevented

Total Infected	**Total Infected**	**Total Infected averted**	**Total cost**
Status quo	141630	–	0
S_1	19686	121944	700.00
S_2	25863	115767	497.00
S_3	46829	94801	650.00
S_4	19715	121915	950.00

The quantitative outcomes broken down by control effects for the control strategies are shown in Table 4, then we reverse the order of the control strategies and putting them in ascending order of increasing efficacy as shown in Table 5.

TABLE 5. Control measures in ascending order of effectiveness

Control measures	**Total Infected**	**Total cost** ($)
S_3	94801	650.00
S_2	115767	497.00
S_4	121915	950.00
S_1	121944	700.00

We compare the strategy S_3 and strategy S_2 using the calculated cost-effective ratio (ICER) as follows:

$$ICER(S_3) = \frac{650.00}{94801} = 0.006856 \tag{14}$$

$$ICER(S_2) = \frac{497.00 - 650.00}{115767 - 94801} = -0.0073 \tag{15}$$

TABLE 6. Shows the ICER for strategy S_3 and strategy S_2

Control measures	**Total Infected averted**	**Total cost** ($)	**ICER**
S_3	94801	650.00	0.006856
S_2	115767	497.00	−0.0073

The ICER of strategy S_2 is lower than that of strategy S_3 when compared the two methods as shown in Table 6. The option S_3 is more expensive and ineffectual than the tactic S_2 . The strategy S_3 is eliminated from the list of potential choices, and the ICER is recalculated and compared for the strategy S_2 and strategy S_4.

Table 7. Shows the ICER for strategy S_2 and strategy S_4

Control measures	**Total Infected averted**	**Total cost** ($)	**ICER**
S_2	115767	497.00	0.004294
S_4	121915	950.00	0.073682

Table 7 indicates that method S_2 is less expensive than strategy S_4 as ICER(S_2) is less than the ICER(S_4). Then, the strategy S_4 is omitted. The ICER is recalculated and compared for the strategy S_2 and strategy S_1.

Table 8. Shows the ICER for strategy S_1 and strategy S_2

Control measures	**Total Infected averted**	**Total cost** ($)	**ICER**
S_2	115767	497.00	0.004294
S_1	121944	700.00	0.032864

Strategies S_2 and S_1 are contrasted and shows the strategy S_1 is ineffective and more costly than the approach S_2 as shown in Table 8. The control S_2 is less expensive and more efficient since it has a lower ICER. The strategy S_1 must be dropped from the list of alternatives. The strategy S_2 with the control variable based on organic soil amendments and thermos-sanitation has the lowest ICER and is the most economically advantageous strategy.

7. Discussion and Conclusions

This study introduced the fusarium wilt disease model with optimal control to minimize disease transmission. The control variables such as thermos-sanitation to reduce the production and transmission of chlamydospores in the field by burning debris and sanitising farming tools, advertisements to communicate accurate and pertinent farming knowledge, and organic soil amendments for suppressing the growth and activity of pathogenic fusarium fungus incorporated in the model to combat fusarium wilt disease. The study focuses on finding effective and affordable control techniques. The Pontryagin maximum principle is used to investigate the optimal strategies for disease control. Numerical results suggest that thermos-sanitation and organic soil amendment significantly reduce the disease. The cost-effectiveness analysis indicates that employing thermos-sanitation and organic soil amendments is the best cost-effective method for eliminating the disease epidemic under limited resources. Based on the numerical results, we recommended that the Tanzania government invest in organic soil amendments and thermos-sanitation to minimise fusarium wilt disease and reduce its economic impact on families and the nation.

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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