

ALMOST (τ_1, τ_2) -CONTINUITY AND $(\tau_1, \tau_2)p$ -OPEN SETSCHALONGCHAI KLANARONG¹, SUPANNEE SOMPONG², CHAWALIT BOONPOK^{1,*}¹Mathematics and Applied Mathematics Research Unit, Department of Mathematics, Faculty of Science, Mahasarakham University, Maha Sarakham, 44150, Thailand²Department of Mathematics and Statistics, Faculty of Science and Technology, Sakon Nakhon Rajbhat University, Sakon Nakhon, 47000, Thailand

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ABSTRACT. This paper deals with the concept of almost (τ_1, τ_2) -continuous multifunctions. Furthermore, some characterizations of almost (τ_1, τ_2) -continuous multifunctions are investigated.

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1. INTRODUCTION

In 1968, Singal and Singal [35] introduced the concept of almost continuous functions as a generalization of continuity. Popa [34] defined almost quasi-continuous functions as a generalization of almost continuity [35] and quasi-continuity [20]. Munshi and Bassan [22] studied the notion of almost semi-continuous functions. Maheshwari et al. [19] introduced the concept of almost feebly continuous functions as a generalization of almost continuity [35]. Noiri [27] introduced and investigated the concept of almost α -continuous functions. Nasef and Noiri [23] introduced two classes of functions, namely almost precontinuous functions and almost β -continuous functions by utilizing the notions of preopen sets and β -open sets due to Mashhour et al [21] and Abd El-Monsef et al. [1], respectively. The class of almost precontinuity is a generalization of each of almost feeble continuity and almost α -continuity. The class of almost β -continuity is a generalization of almost quasi-continuity and almost semi-continuity. Keskin and Noiri [16] introduced the concept of almost b -continuous functions by utilizing the notion of b -open sets due to Andrijević [2]. The class of almost b -continuity is a generalization of almost precontinuity and almost semi-continuity. The class of almost β -continuity is a generalization of almost

b -continuity. In [6], the authors investigated some properties of (Λ, sp) -open sets. Viriyapong and Boonpok [38] studied several characterizations of (Λ, sp) -continuous functions by using (Λ, sp) -open sets and (Λ, sp) -closed sets. Moreover, several characterizations of strongly $\theta(\Lambda, p)$ -continuous functions, \star -continuous functions, θ - \mathcal{S} -continuous functions, pairwise almost M -continuous functions and almost $(\mu, \mu')^{(m,n)}$ -continuous functions were presented in [36], [5], [10], [13] and [14], respectively.

In 1982, Popa [33] introduced and studied the notion of almost continuous multifunctions. Popa and Noiri [31] introduced the notion of almost quasi-continuous multifunctions. Furthermore, several characterizations of almost quasi-continuous multifunctions were investigated in [26]. Popa et al. [29] introduced the concept of almost precontinuous multifunctions. Additionally, some characterizations of almost precontinuous multifunctions were studied in [32]. Popa and Noiri [30] introduced and investigated the notion of almost α -continuous multifunctions. Noiri and Popa [25] introduced the concept of almost β -continuous multifunctions. The further characterizations of almost β -continuous multifunctions were studied in [28]. Ekici and Park [15] introduced and studied almost γ -continuous multifunctions. Noiri and Popa [24] introduced and investigated the notion of almost m -continuous multifunctions as multifunctions from a set satisfying some minimal conditions into a topological space. Laprom et al. [18] introduced and studied the notion of almost $\beta(\tau_1, \tau_2)$ -continuous multifunctions. Viriyapong and Boonpok [39] introduced and investigated the concept of almost $(\tau_1, \tau_2)\alpha$ -continuous multifunctions. In particular, some characterizations of almost \star -continuous multifunctions, almost $\beta(\star)$ -continuous multifunctions, almost $(\tau_1, \tau_2)\delta$ -semicontinuous multifunctions and almost weakly (τ_1, τ_2) -continuous multifunctions were established in [11], [8], [7] and [9], respectively. In this paper, we investigate some characterizations of upper and lower almost (τ_1, τ_2) -continuous multifunctions.

2. PRELIMINARIES

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2) . The closure of A and the interior of A with respect to τ_i are denoted by $\tau_i\text{-Cl}(A)$ and $\tau_i\text{-Int}(A)$, respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ -closed [12] if $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$. The complement of a $\tau_1\tau_2$ -closed set is called $\tau_1\tau_2$ -open. Let A be a subset of a bitopological space (X, τ_1, τ_2) . The intersection of all $\tau_1\tau_2$ -closed sets of X containing A is called the $\tau_1\tau_2$ -closure [12] of A and is denoted by $\tau_1\tau_2\text{-Cl}(A)$. The union of all $\tau_1\tau_2$ -open sets of X contained in A is called the $\tau_1\tau_2$ -interior [12] of A and is denoted by $\tau_1\tau_2\text{-Int}(A)$.

Lemma 1. [12] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2) . For the $\tau_1\tau_2$ -closure, the following properties hold:*

- (1) $A \subseteq \tau_1\tau_2\text{-Cl}(A)$ and $\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Cl}(A)) = \tau_1\tau_2\text{-Cl}(A)$.
- (2) If $A \subseteq B$, then $\tau_1\tau_2\text{-Cl}(A) \subseteq \tau_1\tau_2\text{-Cl}(B)$.

- (3) $\tau_1\tau_2\text{-Cl}(A)$ is $\tau_1\tau_2\text{-closed}$.
- (4) A is $\tau_1\tau_2\text{-closed}$ if and only if $A = \tau_1\tau_2\text{-Cl}(A)$.
- (5) $\tau_1\tau_2\text{-Cl}(X - A) = X - \tau_1\tau_2\text{-Int}(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)p\text{-open}$ [9] (resp. $\alpha(\tau_1, \tau_2)\text{-open}$ [37]) if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$). The complement of a $(\tau_1, \tau_2)p\text{-open}$ (resp. $\alpha(\tau_1, \tau_2)\text{-open}$) set is called $(\tau_1, \tau_2)p\text{-closed}$ (resp. $\alpha(\tau_1, \tau_2)\text{-closed}$).

By a multifunction $F : X \rightarrow Y$, we mean a point-to-set correspondence from X into Y , and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \rightarrow Y$, following [3] we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and

$$F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}.$$

In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3. CHARACTERIZATIONS OF UPPER AND LOWER ALMOST (τ_1, τ_2) -CONTINUOUS MULTIFUNCTIONS

In this section, we investigate some characterizations of upper and lower almost (τ_1, τ_2) -continuous multifunctions.

Definition 1. [17] A multifunction $F : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be upper almost (τ_1, τ_2) -continuous at a point $x \in X$ if for each $\sigma_1\sigma_2\text{-open}$ set V of Y containing $F(x)$, there exists a $\tau_1\tau_2\text{-open}$ set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$. A multifunction $F : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be upper almost (τ_1, τ_2) -continuous if F has this property at each point of X .

Lemma 2. [17] For a multifunction $F : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) F is upper almost (τ_1, τ_2) -continuous;
- (2) $F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$ for every $\sigma_1\sigma_2\text{-open}$ set V of Y ;
- (3) $\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))) \subseteq F^-(K)$ for every $\sigma_1\sigma_2\text{-closed}$ set K of Y ;
- (4) $\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(B))$ for every subset B of Y ;
- (5) $F^+(\sigma_1\sigma_2\text{-Int}(B)) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B))))$ for every subset B of Y ;
- (6) $F^+(V)$ is $\tau_1\tau_2\text{-open}$ in X for every $(\sigma_1, \sigma_2)r\text{-open}$ set V of Y ;
- (7) $F^-(K)$ is $\tau_1\tau_2\text{-closed}$ in X for every $(\sigma_1, \sigma_2)r\text{-closed}$ set K of Y .

Theorem 1. For a multifunction $F : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) F is upper almost (τ_1, τ_2) -continuous;
- (2) $\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V))$ for every $(\sigma_1, \sigma_2)p\text{-open}$ set V of Y ;

- (3) $\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V))$ for every $(\sigma_1, \sigma_2)p$ -open set V of Y ;
 (4) $F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$ for every $(\sigma_1, \sigma_2)p$ -open set V of Y .

Proof. (1) \Rightarrow (2): Let V be any $(\sigma_1, \sigma_2)p$ -open set of Y . Then, $\sigma_1\sigma_2\text{-Cl}(V)$ is $\sigma_1\sigma_2$ -closed in Y and by Lemma 2, we have

$$\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)).$$

(2) \Rightarrow (3): Let V be any $(\sigma_1, \sigma_2)p$ -open set of Y . By (2),

$$\begin{aligned} & \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(V)))) \\ & \subseteq \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ & \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)). \end{aligned}$$

(3) \Rightarrow (4): Let V be any $(\sigma_1, \sigma_2)p$ -open set of Y . Thus by (3), we have

$$\begin{aligned} & X - \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ & = \tau_1\tau_2\text{-Cl}(X - F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ & = \tau_1\tau_2\text{-Cl}(F^-(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ & = \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V)))) \\ & = \tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(Y - \sigma_1\sigma_2\text{-Cl}(V)))) \\ & \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ & = F^-(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ & \subseteq F^-(Y - V) \\ & = X - F^+(V) \end{aligned}$$

and hence $F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$.

(4) \Rightarrow (1): Let V be any $(\sigma_1, \sigma_2)r$ -open set of Y . Then, V is $(\sigma_1, \sigma_2)p$ -open in Y and by (4),

$$F^+(V) \subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) = \tau_1\tau_2\text{-Int}(F^+(V)).$$

Thus, $F^+(V)$ is $\tau_1\tau_2$ -open in X . It follows from Lemma 2 that F is upper almost (τ_1, τ_2) -continuous. \square

Definition 2. [17] A multifunction $F : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be lower almost (τ_1, τ_2) -continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$ -open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $\tau_1\tau_2$ -open set U of X containing x such that $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)) \cap F(z) \neq \emptyset$ for each $z \in U$. A multifunction $F : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be lower almost (τ_1, τ_2) -continuous if F has this property at each point of X .

Lemma 3. [17] For a multifunction $F : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) F is lower almost (τ_1, τ_2) -continuous;
- (2) $F^-(V) \subseteq \tau_1\tau_2\text{-Int}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$ for every $\sigma_1\sigma_2$ -open set V of Y ;
- (3) $\tau_1\tau_2\text{-Cl}(F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))) \subseteq F^+(K)$ for every $\sigma_1\sigma_2$ -closed set K of Y ;
- (4) $\tau_1\tau_2\text{-Cl}(F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(B))$ for every subset B of Y ;
- (5) $F^-(\sigma_1\sigma_2\text{-Int}(B)) \subseteq \tau_1\tau_2\text{-Int}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B))))$ for every subset B of Y ;
- (6) $F^-(V)$ is $\tau_1\tau_2$ -open in X for every $(\sigma_1, \sigma_2)r$ -open set V of Y ;
- (7) $F^+(K)$ is $\tau_1\tau_2$ -closed in X for every $(\sigma_1, \sigma_2)r$ -closed set K of Y .

Theorem 2. For a multifunction $F : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) F is lower almost (τ_1, τ_2) -continuous;
- (2) $\tau_1\tau_2\text{-Cl}(F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$ for every $(\sigma_1, \sigma_2)p$ -open set V of Y ;
- (3) $\tau_1\tau_2\text{-Cl}(F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$ for every $(\sigma_1, \sigma_2)p$ -open set V of Y ;
- (4) $F^-(V) \subseteq \tau_1\tau_2\text{-Int}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$ for every $(\sigma_1, \sigma_2)p$ -open set V of Y .

Proof. The proof is similar to that of Theorem 1. □

Definition 3. [4] A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be almost (τ_1, τ_2) -continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$ -open set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$ -open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$. A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be almost (τ_1, τ_2) -continuous if f has this property at each point of X .

Corollary 1. For a function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:

- (1) f is almost (τ_1, τ_2) -continuous;
- (2) $\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$ for every $(\sigma_1, \sigma_2)p$ -open set V of Y ;
- (3) $\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(V)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(V))$ for every $(\sigma_1, \sigma_2)p$ -open set V of Y ;
- (4) $f^{-1}(V) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$ for every $(\sigma_1, \sigma_2)p$ -open set V of Y .

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CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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