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ALMOST QUASI (τ_1, τ_2)-CONTINUOUS FUNCTIONS

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Abstract. Our main purpose is to introduce the concept of almost quasi (τ_1, τ_2)-continuous functions. Moreover, several characterizations of almost quasi (τ_1, τ_2)-continuous functions are investigated.

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1. Introduction

In 1961, Marcus [19] introduced and studied the notion of quasi continuous functions. Popa [26] introduced and investigated the concept of almost quasi continuous functions. Neubrunnovaá [20] showed that quasi continuity is equivalent to semi-continuity due to Levine [17]. Popa and Stan [27] introduced and studied the notion of weakly quasi continuous functions. Weak quasi continuity is implied by quasi continuity and weak continuity [18] which are independent of each other. It is shown in [23] that weak quasi continuity is equivalent to weak semi-continuity due to Arya and Bhamini [1] and Kar and Bhattacharyya [15]. Duangphui et al. [14] introduced and investigated the concept of almost $(\mu, \mu')^{(m,n)}$-continuous functions. Moreover, some characterizations of strongly $\theta(\Lambda, p)$-continuous functions, (Λ, sp)-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, pairwise almost M-continuous functions and almost (g, m)-continuous functions were presented in [28], [31], [4], [7], [12] and [13], respectively. Bânzara and Crivăţ [2] introduced and studied the concept of quasi continuous multifunctions. Popa and Noiri [24] introduced the concept of almost quasi continuous multifunctions and investigated some characterizations of such multifunctions. Noiri and Popa [22] introduced and studied the notion of weakly quasi continuous multifunctions.

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Furthermore, several characterizations of weakly quasi continuous multifunctions have been obtained in [24].

In 1995, Popa and Noiri [25] introduced and investigated the concepts of upper and lower θ-quasi continuous multifunctions. Moreover, some characterizations of upper and lower θ-quasi continuous multifunctions were investigated in [21]. In [9], the author introduced and studied the notions of almost quasi $\star$-continuous multifunctions and weakly quasi $\star$-continuous multifunctions. In particular, some characterizations of almost $\star$-continuous multifunctions, almost $\beta(\star)$-continuous multifunctions and weakly quasi (Λ, sp)-continuous multifunctions were studied in [10], [6] and [30], respectively. Laprom et al. [16] introduced and investigated the concept of almost $\beta(\tau_1, \tau_2)$-continuous multifunctions. In [32], the present authors introduced and studied the notion of almost $(\tau_1, \tau_2)\alpha$-continuous multifunctions. Furthermore, some characterizations of almost $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions and almost (τ_1, τ_2)-continuous functions were established in [8], [5] and [3], respectively. In this paper, we introduce the notion of almost quasi (τ_1, τ_2)-continuous functions. We also investigate some characterizations of almost quasi (τ_1, τ_2)-continuous functions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [11] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [11] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [11] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [11] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-*Cl*(A) *and* $\tau_1\tau_2$-*Cl*$(\tau_1\tau_2$-*Cl*$(A)) = \tau_1\tau_2$-*Cl*(A).
(2) *If* $A \subseteq B$, *then* $\tau_1\tau_2$-*Cl*$(A) \subseteq \tau_1\tau_2$-*Cl*$(B)$.
(3) $\tau_1\tau_2$-*Cl*(A) *is* $\tau_1\tau_2$-*closed.*
(4) A *is* $\tau_1\tau_2$-*closed if and only if* $A = \tau_1\tau_2$-*Cl*(A).
(5) $\tau_1\tau_2$-*Cl*$(X - A) = X - \tau_1\tau_2$-*Int*$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [32] (resp. $(\tau_1, \tau_2)s$-*open* [8], $(\tau_1, \tau_2)p$-*open* [8], $(\tau_1, \tau_2)\beta$-*open* [8]) if $A = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$, $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$, $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp.

$(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is said to be $(\tau_1, \tau_2)r$-*closed*, $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [29] if $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is called $\alpha(\tau_1, \tau_2)$-*closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)s$-closed sets of X containing A is called the $(\tau_1, \tau_2)s$-*closure* [8] of A and is denoted by $(\tau_1, \tau_2)\text{-sCl}(A)$. The union of all $(\tau_1, \tau_2)s$-open sets of X contained in A is called the $(\tau_1, \tau_2)s$-*interior* [8] of A and is denoted by $(\tau_1, \tau_2)\text{-sInt}(A)$.

Lemma 2. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $(\tau_1, \tau_2)\text{-sCl}(A) = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)) \cup A$ [5];

(2) $(\tau_1, \tau_2)\text{-sInt}(A) = \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)) \cap A$.

3. Almost quasi (τ_1, τ_2)-continuous functions

We begin this section by introducing the notion of almost quasi (τ_1, τ_2)-continuous functions.

Definition 1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost quasi (τ_1, τ_2)-continuous at a point $x \in X$ if for every $\sigma_1\sigma_2$-open set V of Y containing $f(x)$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a nonempty $\tau_1\tau_2$-open set G such that $G \subseteq U$ and $f(G) \subseteq (\sigma_1, \sigma_2)\text{-sCl}(V)$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost quasi (τ_1, τ_2)-continuous if f is almost quasi (τ_1, τ_2)-continuous at each point of X.*

Theorem 1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is almost quasi (τ_1, τ_2)-continuous at $x \in X$;*

(2) *for every $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq (\sigma_1, \sigma_2)\text{-sCl}(V)$;*

(3) *$x \in (\tau_1, \tau_2)\text{-sInt}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V)))$ for every $\sigma_1\sigma_2$-open set V of Y containing $f(x)$;*

(4) *$x \in \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))))$ for every $\sigma_1\sigma_2$-open set V of Y containing $f(x)$.*

Proof. (1) $\Rightarrow$ (2): Let $\mathscr{U}(x)$ the family of all $\tau_1\tau_2$-open sets of X containing x. Let V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. For each $H \in \mathscr{U}(x)$, there exists a nonempty $\tau_1\tau_2$-open set G_H such that $G_H \subseteq H$, $f(G_H) \subseteq (\sigma_1, \sigma_2)\text{-sCl}(V)$. Let $W = \cup\{G_H \mid H \in \mathscr{U}(x)\}$. Then W is $\tau_1\tau_2$-open in X and $x \in \tau_1\tau_2\text{-Cl}(W)$. Put $U = W \cup \{x\}$, then U is a $(\tau_1, \tau_2)s$-open set of X containing x and $f(U) \subseteq (\sigma_1, \sigma_2)\text{-sCl}(V)$.

(2) $\Rightarrow$ (3): Let V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Then, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq (\sigma_1, \sigma_2)\text{-sCl}(V)$. Thus, $x \in U \subseteq f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))$ and hence

$$x \in U \subseteq (\tau_1, \tau_2)\text{-sInt}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))).$$

(3) ⇒ (4): Let V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. By (3), $x \in (\tau_1, \tau_2)$-sInt$(f^{-1}((\sigma_1, \sigma_2)$-sCl$(V)))$. Now, put

$$U = (\tau_1, \tau_2)\text{-sInt}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))).$$

Then, we have U is $(\tau_1, \tau_2)s$-open in X and by Lemma 2,

$$\begin{aligned} x \in U &\subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(U)) \\ &\subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V)))). \end{aligned}$$

(4) ⇒ (1): Let U be any $\tau_1\tau_2$-open set of X containing x and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. Then, we have

$$x \in \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))))$$

and hence $U \cap \tau_1\tau_2$-Int$(f^{-1}((\sigma_1, \sigma_2)$-sCl$(V))) \neq \emptyset$. Put

$$W = U \cap \tau_1\tau_2\text{-Int}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))).$$

Then, we have W is a nonempty $\tau_1\tau_2$-open set of X such that $W \subseteq U$, $f(W) \subseteq (\sigma_1, \sigma_2)$-sCl$(V)$. This shows that f is almost quasi (τ_1, τ_2)-continuous at x. □

Theorem 2. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is almost quasi (τ_1, τ_2)-continuous;*

(2) *for each $x \in X$ and every $\sigma_1\sigma_2$-open set V of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq (\sigma_1, \sigma_2)$-sCl$(V)$;*

(3) *$f^{-1}(V)$ is $(\tau_1, \tau_2)s$-open in X for every $(\sigma_1, \sigma_2)r$-open set V of Y;*

(4) *$f^{-1}(V) \subseteq (\tau_1, \tau_2)$-sInt$(f^{-1}((\sigma_1, \sigma_2)$-sCl$(V)))$ for every $\sigma_1\sigma_2$-open sets V of Y;*

(5) *(τ_1, τ_2)sCl$(f^{-1}(\sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(B))))) \subseteq f^{-1}(\sigma_1\sigma_2$-Cl$(B))$ for every subset B of Y;*

(6) *$f^{-1}(V) \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(f^{-1}((\sigma_1, \sigma_2)$-sCl$(V))))$ for every $\sigma_1\sigma_2$-open set V of Y.*

Proof. (1) ⇒ (2): The proof follows from Theorem 1.

(2) ⇒ (3): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y and $x \in f^{-1}(V)$. Then, we have $f(x) \in V$ and there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $f(U) \subseteq V$. Thus, $x \in U \subseteq f^{-1}(V)$ and hence

$$x \in (\tau_1, \tau_2)\text{-sInt}(f^{-1}(V)).$$

Therefore, $f^{-1}(V) \subseteq (\tau_1, \tau_2)$-sInt$(f^{-1}(V))$. This shows that $f^{-1}(V)$ is $(\tau_1, \tau_2)s$-open in X.

(3) ⇒ (4): Let V be any $\sigma_1\sigma_2$-open set of Y and $x \in f^{-1}(V)$. Then, we have $f(x) \in V \subseteq (\sigma_1, \sigma_2)$-sCl$(V)$. Thus, $x \in f^{-1}((\sigma_1, \sigma_2)$-sCl$(V))$. By Lemma 2, (σ_1, σ_2)-sCl(V) is $(\sigma_1, \sigma_2)r$-open in Y. Then by (3), $f^{-1}((\sigma_1, \sigma_2)$-sCl$(V))$ is $(\tau_1, \tau_2)s$-open in X and

$$x \in (\tau_1, \tau_2)\text{-sInt}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))).$$

Thus, $f^{-1}(V) \subseteq (\tau_1, \tau_2)$-sInt$(f^{-1}((\sigma_1, \sigma_2)$-sCl$(V)))$.

$(4) \Rightarrow (5)$: Let B be any subset of Y. Then, we have $Y - \sigma_1\sigma_2$-Cl(B) is $\sigma_1\sigma_2$-open in Y. By (4) and Lemma 2,

$$\begin{aligned}
&X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))\\
&= f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(B))\\
&\subseteq (\tau_1, \tau_2)\text{-sInt}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(Y - \sigma_1\sigma_2\text{-Cl}(B))))\\
&= (\tau_1, \tau_2)\text{-sInt}(X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))))\\
&= X - (\tau_1, \tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B)))))
\end{aligned}$$

and hence

$$(\tau_1, \tau_2)\text{-sCl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)).$$

$(5) \Rightarrow (6)$: Let V be any $\sigma_1\sigma_2$-open set of Y. Then $Y - V$ is $\sigma_1\sigma_2$-closed in Y. By (5) and Lemma 2,

$$\begin{aligned}
\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(Y - V))))) &\subseteq f^{-1}(Y - V)\\
&= X - f^{-1}(V).
\end{aligned}$$

Moreover, we have

$$\begin{aligned}
&\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(Y - V)))))\\
&= \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))))\\
&= \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(X - f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))))\\
&= X - \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V)))).
\end{aligned}$$

Thus, $f^{-1}(V) \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(f^{-1}((\sigma_1, \sigma_2)$-sCl$(V))))$.

$(6) \Rightarrow (1)$: Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$. By (6), we have

$$x \in f^{-1}(V) \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}((\sigma_1, \sigma_2)\text{-sCl}(V))))$$

and by Lemma 2, $x \in f^{-1}(V) \subseteq (\tau_1, \tau_2)$-sInt$(f^{-1}((\sigma_1, \sigma_2)$-sCl$(V)))$. Put $U = (\tau_1, \tau_2)$-sInt$(f^{-1}((\sigma_1, \sigma_2)$-sCl$(V)))$, then U is $(\tau_1, \tau_2)s$-open set of X containing x such that $f(U) \subseteq (\sigma_1, \sigma_2)$-sCl$(V)$. This shows that f is almost quasi (τ_1, τ_2)-continuous. □

Theorem 3. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is almost quasi (τ_1, τ_2)-continuous;*

(2) *(τ_1, τ_2)-sCl$(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2$-Cl$(V))$ for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;*

(3) *(τ_1, τ_2)-sCl$(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2$-Cl$(V))$ for every $(\sigma_1, \sigma_2)s$-open set V of Y;*

(4) *$f^{-1}(V) \subseteq (\tau_1, \tau_2)$-sInt$(f^{-1}(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y.*

Proof. The proof follows from Theorem 2. □

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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