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OPTIMAL ASSET ALLOCATION AND RISK MANAGEMENT ISSUES FOR INSURANCE COMPANIES

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Abstract. This paper studies the optimal asset allocation and risk management issues for insurance companies in a continuous-time model. The financial market consists of risk-free assets and risky assets. The surplus process of the insurer is approximated by a drifted Brownian motion. The insurer has to hedge its exposure to risky assets and insurance operations. Insurance company decision-makers are assumed to have loss aversion characteristics. Therefore, insurance company needs to obtain optimal investment and reinsurance strategies under loss aversion. We use the martingale method to derive the explicit solutions of optimal policy under this optimization criterion. Moreover, sensitivity analysis is presented in the end to show the economic behaviors of optimal strategies.

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1. Introduction

Mean-variance and expected utility maximization are by far the two predominant investment decision rules in financial portfolio selection. Portfolio theory in the dynamic setting has been established in the past thirty years, again centering around these two frameworks while employing heavily among others the martingale theory, convex duality and stochastic control; see Duffie [1], Karatzas and Shreve [2], and Föllmer and Schied [3] for systematic accounts on dynamic utility maximization, and Li and Ng [4], Zhou and Li [5], and Jin et al. [6] for recent studies on the mean-variance (including extensions to mean-risk) counterpart.

Expected utility theory (EUT), developed by von Neumann and Morgenstern [7] based on an axiomatic system, has an underlying assumption that decision makers are rational and risk averse when facing uncertainties. In the context of asset allocations, its basic tenets are: Investors evaluate wealth according to final asset positions; they are uniformly risk averse; and they are able to objectively

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evaluate probabilities. These, however, have long been criticized to be inconsistent with the way people do decision making in the real world. Substantial experimental evidences and market anomalies have suggested a systematic violation of the EUT principles, including the Allais paradox, Friedman and Savage puzzle, Ellesberg paradox, the return reversals and the equity premium puzzle.

Considerable attempts and efforts have been made to address the drawback of EUT, such as Lopes's SP/A model [8], Kahneman and Tversky's Prospect Theory [9], and Quiggin's Rank-Dependent Expected Utility [10]. The breakthrough in Kahneman and Tversky [9] has been a cornerstone of the prospect theory, in which Kahneman and Tversky proposed reference point and distortion of probability in portfolio theory. These ideas have been proven to be of great use and can result in lowing risk for an investor. Because the prospect theory describes human behavior better, more and more literatures study the loss aversion utility and distortion of probability in portfolio selection. Berkelaar et al. [11] firstly employed the martingale method to derive the optimal investment strategies with two utility functions under loss aversion in continuous case. Later, Gomes [12] considered the counterpart discrete model. Furthermore, Jarrow and Zhao [13] introduced a mean-variance framework under loss aversion. The above works only concerned the loss aversion in prospect theory. The distortion of probability in portfolio selection can refer to Jin and Zhou [14], Bernard and Ghossoub [15], He and Zhou [16] and references therein.

In addition, with permission of the insurance companies to invest, purchase reinsurance and acquire new business in financial markets in practice, the problem of optimal investment and reinsurance for a general insurance company has attracted more and more attention. Most works on the optimization problem for an insurer care about maximizing the expected utility of a smooth utility of terminal wealth. So the decision makers of the insurance company are often assumed to be strictly risk averse towards the terminal wealth. Browne [17] initiated the study of explicit solution for a firm to maximize the exponential utility of terminal wealth and minimize the probability of ruin with its surplus process given by the Lundberg risk model. Wang et al. [18] efficiently applied martingale method to study the optimal portfolio selection for insurer under the mean variance criterion as well as the expected constant absolute risk aversion (CARA) utility maximization. Under the constraint of no-shorting, Bai and Guo [19] studied the problem of optimal investment and reinsurance for an insurer under maximizing the expected exponential utility of terminal wealth as well as minimizing the probability of ruin. The readers are referred to, for example, Yang and Zhang [20], Xu et al. [21], Liang et al. [22], Guan and Liang [23], Liu et al. [24] and references therein. In the works mentioned above, the best strategy for avoiding market risk often includes a large allocation in risky assets (and a large percentage of insurance business). However, some people are reluctant to take risks from risky assets and insurance operations. In addition, some may seek out risk, thereby investing more in risky assets and retaining more of their insurance business. As a result, many of the optimal terminal wealth criteria in the literature may pose significant risks to insurers .

As far as we know, few works are devoted to the study of optimization problem for insurance companies under loss aversion, Guo [25] first investigated the optimal portfolio choice for an insurer under loss aversion, where a specific two-piece utility function is considered. Based on Guo [25], Chen and Yang [26] studied optimal reinsurance and investment strategies for an insurer in a stochastic market by considering the insurer's preference is represented by a two-piece utility function. Recently, Ma et al. [27] investigated optimal reinsurance and investment strategies with the assumption that the insurers can purchase proportional reinsurance contracts and invest their wealth in a financial market under an S-shaped utility. Since the existing works on the problem of optimal investment and reinsurance mainly care about the complexity in the market, we introduce here one different optimization criterion that is different from the smooth utility case. In this paper, we intend to investigate the optimal investment and reinsurance strategies for insurance companies under loss aversion. The financial market consists of both risk-free assets and risky assets whose price precess is modeled by a Geometric Brownian motion. The surplus process of the insurer is approximated by a drifted Brownian motion. As a result, the insurer has to manage risks from risky assets and insurance operations. The goal is to maximize the expected utility of terminal wealth. The utility function under loss aversion we adopt is firstly studied in Kahneman and Tversky [9]. The utility function is convex under a reference point while concave above the point. This leads to a risk-seeking attitude towards losses. Since the optimization problem is not a concave maximization problem, the optimal terminal wealth is a discontinuous function and so it seems that the stochastic programming method does not work here. We will apply the martingale method to derive the optimal investment and reinsurance strategies under loss aversion. Moreover, the sensitivity analysis in the end shows the economic behaviors of the optimal strategies. When the reference point is high, the loss-averse insurer becomes more concerned about volatilities that may cause the account of wealth to underperform the reference level, and thus, the lower wealth allocated in the risky asset and the less insurance business kept.

The organization of this paper is as follows. In section 2, the assumptions and model are described. Section 3 formulates the optimization problem we are going to consider under loss aversion. Section 4 solves the optimization problem and derives explicitly the corresponding optimal investment and proportional reinsurance strategies and the optimal wealth process by a martingale approach. Section 5 presents a sensitivity analysis to show the impact of the optimization criterion on the optimal strategies. Finally, Section 6 concludes this paper.

2. Assumptions and Model

Let $(\Omega, \mathcal{F}, P)$ be a given complete probability space with a filtration $(\mathcal{F}_t)$, $t \in [0, T]$ satisfying the usual conditions, i.e. the filtration contains all P-null sets and is right continuous, where $T \in (0, +\infty)$ is a finite constant and represents the time horizon; $(\mathcal{F}_t)$ stands for the information available up to time t and any decision made at time t is based on this information. All stochastic processes in this paper are assumed to be well defined and adapted processes in this probability space.

2.1. **Financial market.**

We assume that the insurer can invest in the capital market where two types of assets are traded continuously on a finite horizon $[0, T]$. For simplicity, we assume that the financial market consists of a risk-free asset and a risky asset. The price of risk-free asset $P_0(t)$ is given by

$$dP_0(t) = P_0(t) r dt\,,\ \ P_0(0) = 1, \tag{1}$$

and the price of risky asset $P_1(t)$ satisfying

$$dP_1(t) = P_1(t)\left[\mu dt + \sigma_1 dW_1(t)\right]\ ,\ \ P_1(0) > 0, \tag{2}$$

where r is the risk-free interest rate, μ is the appreciation rate. $\sigma_1 > 0$ is the volatility, and $W_1(t)$ is a 1-dimensional standard Brownian motion on the filtered complete probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$. In general, we assume that $\mu > r \geq 0$.

2.2. **Surplus process.**

We consider an insurer whose surplus process is modeled by a diffusion approximation model. To understand the diffusion approximation model better, it is advantageous to start from the classical Cramér-Lundberg model. In the Cramér-Lundberg model the claims arrive according to a homogeneous Poisson process $\{K(t)\}$ with intensity λ; the individual claim sizes are $Y_i, i = 1, 2, 3...$, which are assumed to be independent of $\{K(t)\}$ and be independent and identically distributed (i.i.d.) positive random variables with finite first and second-order moments given by $\mu_0 = EY = \int_0^\infty y dF(y) < \infty$ and $\sigma_0^2 = E(Y^2) = \int_0^\infty y^2 dF(y) < \infty$, respectively. Then the surplus process of the insurer without reinsurance and investment follows

$$U(t) = x_0 + c_0 t - R(t) = x + c_0 t - \sum\nolimits_{i=1}^{K(t)} Y_i, \tag{3}$$

where $x_0 > 0$ is the initial reserve of an insurance company; c_0 is the premium rate which is assumed to be calculated according to the expected value principle, i.e., $c_0 = (1+\theta)\lambda\mu_0$, where θ is the safety loading of the insurer. $R(t) = \sum_{i=1}^{K(t)} Y_i$ is a compound Poisson process defined on $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$, which represents the cumulative amount of claims in time interval $[0, t]$.

By Grandll [28], the Cramér-Lundberg model can be approximated by the following diffusion model

$$dU(t) = \theta\lambda\mu_0 dt + \sigma_2 dW_2(t), \tag{4}$$

where $\theta\lambda\mu_0$ can be regarded as the premium return rate of the insurer, ${\sigma_2}^2 = \lambda{\sigma_0}^2$ measures the volatility of the insurer's surplus, $W_2(t)$ is a standard Brownian motion, which is independent of $W_1(t)$. It is worth pointing out that the diffusion approximation model (4) works well for the large insurance portfolios, where an individual claim is relatively small compared to the size of surplus. The diffusion approximation model has been used in much existing literature, for example, Browne [17], Promislow and Young [29], Gerber and Shiu [30], Bai and Guo [19], Cao and Wan [31], Chen et al. [32], Zeng and Li [33], and so on.

In addition, the insurer is allowed to purchase proportional reinsurance or acquire new business (for example, acting as a reinsurer of other insurers, see B$\ddot{a}$uerle [34]) at each moment in order to control insurance business risk. The proportional reinsurance or new business level is associated with the value of risk exposure $q(t) \in [0, +\infty)$ at any time $t \in [0, T]$. $q(t) \in [0, 1]$ corresponds to a proportional reinsurance cover and shows that the cedent should divert part of the premium to the reinsurer at the rate of $(1 - q(t))(1 + \eta)\lambda\mu_0$, where η is the safety loading of the reinsurer satisfying $\eta \geq \theta > 0$. In return, for each claim occurring at time t, the reinsurer pays $100(1 - q(t))\%$ of the claim, and the cedent pays the rest. $q(t) \in (1, +\infty)$ corresponds to acquiring new business (acting as a reinsurer for other insurers). When a reinsurance policy $\{q(t) : t \in [0, T]\}$ is adopted, the corresponding diffusion approximation dynamics for the surplus process becomes

$$dU(t) = [\theta\lambda\mu_0 - (1 - q(t))(1 + \eta)\lambda\mu_0]\, dt + \sigma_2 q(t) dW_2(t). \tag{5}$$

2.3. **Wealth process.**

Assume that the insurer can dynamically purchase proportional reinsurance, acquire new business and invest in the financial market over the time interval $[0, T]$ and there is no transaction coat in the financial market and the insurance market. A trading policy is donated by a pair of stochastic processes $h = \{\pi(t), q(t)\}_{t\in[0,T]}$, where $q(t)$ and $\pi(t)$ are the value of the risk exposure and the dollar amount invested in the risky asset at time t, respectively. The dollar amount invested in the risk-free asset at time t is $X(t) - \pi(t)$, where $X(t)$ is the wealth process associated with strategy h. Then the evolution of $X(t)$ can be described as

$$\begin{cases} dX(t) = [X(t)r + \pi(t)(\mu - r) + \theta\lambda\mu_0 - (1 - q(t))(1 + \eta)\lambda\mu_0]\, dt + \sigma_1\pi(t) dW_1(t) \\ \qquad\qquad + \sigma_2 q(t) dW_2(t) \\ X(0) = x_0. \end{cases} \tag{6}$$

Definition 2.1. (Admissible strategy) Let $\vartheta := [0, T] \times R$. For any fixed $t \in [0, T]$, a trading policy $h = \{\pi(t), q(t)\}_{t\in[0,T]}$ is said to be admissible if it satisfies that

(1) $\pi(t)$ and $q(t)$ are predictable mappings with respect to $\mathcal{F}_t$;

(2) for all $t \in [0,T]$, $q(t) \geq 0$ and $E\left[\int_t^T (\pi(s)^2 + q(s)^2)ds\right] < +\infty$;

(3) $(X(t), h)$ is the unique solution to the stochastic differential equation (6).

In addition, let $\prod(t,x)$ denote the set of all admissible strategies with respect to initial condition $(t,x) \in \vartheta$.

3. Formulation of the Problem

Most works on the optimization problem for insurer care about maximizing the expectation of a smooth utility of terminal wealth, in order to find the optimal strategies within $[0,T]$. However, in the real world, some individuals are unwilling to take the risks from risky assets and insurance business. They may be more interested to allocate money in the risk-free asset and keep less insurance business. Besides, others may be risk seeking and invest more in risky assets and keep more insurance business. The general optimization problem only characterizes a risk averse investor and cannot reflect others' behavior towards risk. In this section, we formulate one different optimization problem, which better manage the risks for the insurer.

This section formulates the optimization problem under loss aversion. Kahneman and Tversky (1979) [9] firstly established the theory of prospect theory. They stated that people always make decisions relative to some reference levels. The reference levels may be different for different people. They judge the account of the wealth over (under) the reference as gains (losses). People often act differently towards gains and losses. In fact, people are more sensitive to losses than gains. They also demonstrated their idea based on the following utility function:

$$U(X(T)) = \begin{cases} A(X(T)-\xi)^{\gamma_1}, & X(T) > \xi; \\ -B(\xi - X(T))^{\gamma_2}, & X(T) \leq \xi. \end{cases} \tag{7}$$

where A and B are positive constants, $0 < \gamma_1 \leq 1$, $0 < \gamma_2 \leq 1$. Statistics are showed in Kahneman and Tversky (1979) [9] to support the above utility function. The investor is risk-averse towards gains while risk-seeking towards losses. The reference point ξ is chosen in advance. For the insurer, the reference point ξ can be chosen to be connected with the premium rate and initial wealth. The utility function is convex when the wealth is less than ξ and concave when the wealth is bigger than ξ.

Following the utility maximization criterion, the problem of optimal investment and reinsurance strategies for an insurer can be formulated as follows:

$$\begin{cases} \max\limits_{h \in \Pi} E\{U[X(T)]\} \\ s.t.\ X(t) \text{ satisfies } (6) \\ X(t) \geq 0.\ \forall t \in [0,T]. \end{cases} \tag{8}$$

where $X(t) \geq 0$, $\forall t \in [0,T]$ reflects that the insurance company is not bankrupt throughout the investment period $[0,T]$.

4. Solution to the Optimization Problem

In this section, we use martingale algorithm to solve (8). The previous section allows us to change dynamic maximization problem (8) with the mean constraint into a static problem. We are thus led to a constrained optimization problem which is solved by standard Lagrange multipliers methods.

Define

$$H(t) = \exp\left\{-rt + \int_0^t \theta_1(s)dW_1(s) + \int_0^t \theta_2(s)dW_2(s) - \frac{1}{2}\int_0^t [\theta_1(s)^2 + \theta_2(s)^2]ds\right\}, \tag{9}$$

where $\theta_1(t) = -\frac{\mu - r}{\sigma_1}$ and $\theta_2(t) = -\frac{(1+\eta)\lambda\mu_0}{\sigma_2}$. Then, We have the following conclusion.

Proposition 4.1. If $H(t)$ is defined by (9) for $t \in [0,T]$, then $H(t)X(t) + \int_0^t H(s)c_2 ds$ is a martingale under the probability measure P, where $c_2 = (1 + \eta - \theta)\lambda\mu_0$.

Proof. We write $H(t)$ in the following differential form:

$$dH(t) = H(t)[-rdt + \theta_1(t)dW_1(t) + \theta_2(t)dW_2(t)]. \tag{10}$$

By Ito's formula, we have

$$\begin{aligned} d\left[H(t)X(t)\right] &= H(t)dX(t) + X(t)dH(t) + d\left[H(t), X(t)\right] \\ &= -H(t)c_2dt + H(t)\left[X(t)\theta_1(t) + \sigma_1\pi(t)\right]dW_1(t) \\ &\quad + H(t)\left[X(t)\theta_2(t) + \sigma_2 q(t)\right]dW_2(t) \end{aligned} \tag{11}$$

where $[H(t), X(t)]$ denotes the quadratic co-variation of $H(t)$ and $X(t)$. We pray for integration on both sides of (11) and get

$$\begin{aligned} H(t)X(t) + \int_0^t H(s)c_2ds = x_0 &+ \int_0^t H(s)\left[X(s)\theta_1(s) + \sigma_1\pi(s)\right]dW_1(t) \\ &+ \int_0^t H(s)\left[X(s)\theta_2(s) + \sigma_2 q(s)\right]dW_2(t). \end{aligned} \tag{12}$$

This shows that $H(t)X(t) + \int_0^t H(s)c_2 ds$ can be represented as an Ito integral with respect to the Brownian motions $W_1(t)$ and $W_2(t)$, and therefore is a Martingale under P. □

Of course, a martingale must be super-martingale under P. The super-martingale property applied to (12) implies the following constraint for the optimization problem:

$$E[H(T)X(T) + \int_0^T H(s)c_2ds] \leq x_0. \tag{13}$$

As in Guo [25], we now show that this constraint plays a decisive role in the optimization problem.

Theorem 4.1. *Let $\psi \geq 0$ be an $\mathcal{F}_t$-measurable random variable, then for a given initial wealth x_0 satisfying $E[H(T)\psi + \int_0^T H(s)c_2 ds] = x_0$, there exists a policy $h = [\pi(t), q(t)]$, such that $h = [\pi(t), q(t)] \in \Pi$, $t \in [0, T]$, and $X^h(T) = \psi$.*

Proof. Define a martingale

$$M(t) = E[H(T)\psi + \int_0^T H(s)c_2 ds|\mathcal{F}_t].$$

According to the Martingale representation theorem (e.g., Cont and Tankov [35], Proposition 9.4), there exist two predictable processes $\varphi_1 : \Omega \times [0, T] \mapsto R$ and $\varphi_2 : \Omega \times [0, T] \mapsto R$ satisfying

$$\int_0^T \varphi_1(s)^2 ds < \infty, \quad a.s., \qquad \int_0^T \varphi_2(s)^2 ds < \infty, \quad a.s.$$

such that

$$\begin{aligned} M(t) &= E\left[H(T)\psi + \int_0^T H(s)c_2 ds\right] + \int_0^t \varphi_1(s)dW_1(s) + \int_0^t \varphi_2(s)dW_2(s) \\ &= x_0 + \int_0^t \varphi_1(s)dW_1(s) + \int_0^t \varphi_2(s)dW_2(s). \end{aligned} \tag{14}$$

Thus, it is easy to see that

$$H(T)\psi + \int_0^T H(s)c_2 ds = x_0 + \int_0^T \varphi_1(s)dW_1(s) + \int_0^T \varphi_2(s)dW_2(s). \tag{15}$$

Comparing $dW_1(t)$-term and $dW_2(t)$-term respectively in (15) with those in (12) taking $t = T$, it is reasonable to conjecture

$$\begin{cases} \pi(t) = \dfrac{1}{\sigma_1 H(t)} \left[\varphi_1(t) - H(t)X(t)\theta_1(t)\right], \\ q(t) = \dfrac{1}{\sigma_2 H(t)} \left[\varphi_2(t) - H(t)X(t)\theta_2(t)\right]. \end{cases} \tag{16}$$

Then we need to check whether the policy defined in (16) is admissible.

To check $\pi(t)$ is admissible, we only need to check that $\int_0^T |\pi(t)|dt < \infty$, a.s. We define some notations:

$$||f(t)||_\infty = \max_{0 \leq t \leq T} |f(t)|, \quad ||f(t)||_2 = [\int_0^T |f(t)|^2 dt]^{\frac{1}{2}}.$$

According to (16), we have

$$\begin{aligned} \int_0^T |\pi(t)|dt &= \int_0^T |{\sigma_1}^{-1} H(t)^{-1}\varphi_1(t) + X(t)(\mu - r){\sigma_1}^{-2}|dt \\ &\leq \int_0^T |{\sigma_1}^{-1}||H(t)^{-1}||_\infty \varphi_1(t) + ||X(t)||_\infty (\mu - r){\sigma_1}^{-2}|dt \\ &\leq {\sigma_1}^{-1}||H(t)^{-1}||_\infty \int_0^T \varphi_1(t)dt + ||X(t)||_\infty (\mu - r){\sigma_1}^{-2}T \\ &\leq {\sigma_1}^{-1}||H(t)^{-1}||_\infty ||\varphi_1(t)||_2 + ||X(t)||_\infty (\mu - r){\sigma_1}^{-2}T \\ &< \infty, \text{a.s.} \end{aligned}$$

The last inequality follows from the uniformly bounded conditions.

Due to the non-negativity constraints of admissible reinsurance strategy, we define two regions:

$$D_1 := \left\{(t,x) \in [0,T] \times R^+ | x < A_1(t)\right\},$$

$$D_2 := \left\{(t,x) \in [0,T] \times R^+ | x \geq A_1(t)\right\}.$$

where $A_1(t) = \frac{\varphi_2(t)}{H(t)\theta_2}$.

Firstly, we consider region D_1. It is obvious that $[\varphi_2(t) - H(t)X(t)\theta_2(t)] < 0$. Hence, we take the value $q(t) \equiv 0$, which satisfies the admissibility. In a word, in region D_1, we conjecture the form of trading strategy as follows:

$$\begin{cases} \pi(t) = \dfrac{1}{\sigma_1 H(t)} \left[\varphi_1(t) - H(t)X(t)\theta_1(t)\right], \\ q(t) = 0. \end{cases} \tag{17}$$

Then, we consider region D_2. In this region, it is easy to find that the policy defined in (16) is admissible. Hence, in region D_2, the conjecture of trading strategy is given by (16). □

According to Theorem 4.1, any $\mathcal{F}_t$-measurable random variable $\psi \geq 0$ with $E[H(T)\psi + \int_0^T H(s)c_2 ds] = x_0$ can be financed via trading an admissible policy h such that $X^h(T) = \psi$. So to determine the optimal policy h^* in the dynamic maximization problem (8), which depends on the time variable t, it is sufficient to maximize over all possible random variable ψ's . That is to say, the dynamic maximization problem (8) is equivalent to the following static optimization problem:

$$\begin{cases} \max\limits_{\psi \geq 0} E[U(\psi)] \\ s.t.\ E\left[H(T)\psi + \displaystyle\int_0^T H(s)c_2 ds\right] \leq x_0. \end{cases} \tag{18}$$

Theorem 4.2 characterizes the optimal solutions of the optimization problem (18).

Theorem 4.2. *The optimal terminal wealth of a loss averse insurer with* $0 < \gamma_1 < 1$ *and* $0 < \gamma_2 < 1$ *is given by*

$$\psi^* = \begin{cases} \xi + \left\{x_0 - E[H(T)\xi + \int_0^T H(s)c_2 ds]\right\} \frac{H(T)^{\frac{1}{\gamma_1 - 1}}}{E(H(T)^{\frac{\gamma_1}{\gamma_1 - 1}})}, & \xi \leq x_0 e^{rT} - c_2 \frac{e^{rT}-1}{r}; \\ 0, & \xi > x_0 e^{rT} - c_2 \frac{e^{rT}-1}{r}. \end{cases} \tag{19}$$

Proof. Denote $u_1(x) = A(x-\xi)^{\gamma_1}$, $u_2(x) = -B(\xi - x)^{\gamma_2}$. To solve the problem (18), firstly, we assume that

$$E\left[H(T)\xi + \int_0^T H(s)c_2 ds\right] \leq x_0.$$

If $\psi > \xi$, the Lagrangian function $L(\psi, y)$ of problem (18) can be written as

$$L(\psi, y) = E\left\{u_1(\psi) + y[x_0 - H(T)\psi - \int_0^T H(s)c_2 ds]\right\}. \tag{20}$$

where y is the Lagrangian multiplier. Equating the derivatives of Lagrangian function (20) with respect to ψ and y respectively to zero, we obtain

$$\begin{cases} \dfrac{\partial L}{\partial \psi} = E\left[{u_1}'(\psi) - yH(T)\right] = 0, \\ \dfrac{\partial L}{\partial y} = x_0 - H(T)\psi - \displaystyle\int_0^T H(s)c_2 ds = 0. \end{cases} \tag{21}$$

Solving equation (21), we have

$$\psi_1^* = \xi + \left[\frac{A\gamma_1}{yH(T)}\right]^{\frac{1}{1-\gamma_1}}. \tag{22}$$

While, the Lagrangian multiplier y is determined by the constraint

$$\begin{aligned} E[H(T)\psi_1^* + \int_0^T H(s)c_2 ds] =& \ E[H(T)\xi + (\alpha\gamma_1)^{\frac{1}{1-\gamma_1}} y^{\frac{1}{\gamma_1-1}} H(T)^{\frac{\gamma_1}{\gamma_1-1}} + \int_0^T H(s)c_2 ds] \\ =& \ x_0 \end{aligned}$$

which is satisfied by setting

$$y^{\frac{1}{\gamma_1-1}} = \frac{x_0 - E[H(T)\xi + \int_0^T H(s)c_2 ds]}{(A\gamma_1)^{\frac{1}{1-\gamma_1}} E[H(T)^{\frac{\gamma_1}{\gamma_1-1}}]}$$

Substitution of $y^{\frac{1}{\gamma_1-1}}$ in (22) gives us the optimal solution of (20) via the following formula:

$$\psi_1^* = \xi + \left\{x_0 - E[H(T)\xi + \int_0^T H(s)c_2 ds]\right\} \frac{H(T)^{\frac{1}{\gamma_1-1}}}{E[H(T)^{\frac{\gamma_1}{\gamma_1-1}}]}. \tag{23}$$

If $\psi \le \xi$, the utility function $u_2(\psi)$ is continuous and convex in the closed interval $[0, \xi]$. Therefore the local optimal solution ψ_2^* is located at one of the boundaries $\psi_2^* = 0$ or $\psi_2^* = \xi$. Furthermore it is easy to check $\psi_2^* = 0$ and $\psi_2^* = \xi$ satisfy the constraint condition

$$E\left[H(T)\psi + \int_0^T H(s)c_2 ds\right] \le x_0.$$

Since $U(\cdot)$ is not concave, we need to compare the local maxima ψ_1^* and ψ_2^* to determine the global maximum. Firstly we compare ψ_1^* to $\psi_2^* = \xi$:

$$\begin{aligned} U[\psi_1^*] - U[\xi] =& \ u_1(\psi_1^*) - u_2(\xi) \\ =& \ A[\psi_1^* - \xi]^{\gamma_1} \\ =& \ A\left[\frac{A\gamma_1}{yH(T)}\right]^{\frac{\gamma_1}{1-\gamma_1}} > 0. \end{aligned}$$

Hence $\psi_2^* = \xi$ is never the optimal level of terminal wealth. Similarly by comparing ψ_1^* to $\psi_2^* = 0$, we find

$$\begin{aligned} U[\psi_1^*] - U[\xi] &= u_1(\psi_1^*) - u_2(0) \\ &= A[\psi_1^* - \xi]^{\gamma_1} + B\xi^{\gamma_2} \\ &= A\left[\frac{A\gamma_1}{yH(T)}\right]^{\frac{\gamma_1}{1-\gamma_1}} + B\xi^{\gamma_2} > 0. \end{aligned}$$

So $\psi_2^* = 0$ is not the optimal level of terminal wealth too. We conclude that ψ_1^* is the optimal solution of the static problem (18), when

$$E\left[H(T)\xi + \int_0^T H(s)c_2 ds\right] \leq x_0.$$

Then, we assume that

$$E\left[H(T)\xi + \int_0^T H(s)c_2 ds\right] > x_0.$$

If $\psi > \xi$, the Lagrangian function (18) has no optimal solution; If $\psi \leq \xi$, similarly according to the continuity and convexity of the utility function $u_2(\psi)$, the local optimal solution ψ_2^* is located at one of the boundaries $\psi_2^* = 0$ or $\psi_2^* = \xi$. But $\psi_2^* = \xi$ does not satisfy the constraint

$$E\left[H(T)\psi + \int_0^T H(s)c_2 ds\right] \leq x_0.$$

So we conclude that $\psi_2^* = 0$ is the optimal solution of the static problem (20) when

$$E\left[H(T)\xi + \int_0^T H(s)c_2 ds\right] > x_0.$$

It is easy to calculate

$$E\left[H(T)\xi + \int_0^T H(s)c_2 ds\right] = \xi e^{-rT} + c_2\frac{1-e^{-rT}}{r}.$$

Let ψ^* be the optimal solution of the problem (18). Then ψ^* can be written as

$$\psi^* = \begin{cases} \xi + \left\{x_0 - E[H(T)\xi + \int_0^T H(s)c_2 ds]\right\} \frac{H(T)^{\frac{1}{\gamma_1 - 1}}}{E(H(T)^{\frac{\gamma_1}{\gamma_1 - 1}})}, & \xi \leq x_0 e^{rT} - c_2\frac{e^{rT}-1}{r}; \\ 0, & \xi > x_0 e^{rT} - c_2\frac{e^{rT}-1}{r}. \end{cases} \tag{24}$$

□

Note that the optimal terminal wealth is a discontinuous function. In good states ($\xi \leq x_0 e^{rT} - c_2\frac{e^{rT}-1}{r}$) the loss-averse agent behaves like the CRRA agent and obtains wealth above the reference level; In bad states ($\xi > x_0 e^{rT} - c_2\frac{e^{rT}-1}{r}$), the insurer ends up with zero wealth. Since the insurer is mostly concerned with small changes in wealth relative to the threshold the gambling behavior below the threshold causes the insurer to incur large losses in these bad states.

In the previous section, we characterized the optimal terminal wealth of a loss-averse insurer. In what follows, we derive closed-form solutions for the optimal policies when the price process of risky asset follows a geometric Brownian motion. When applying the martingale methodology the optimal strategies are derived not given in feedback form as with stochastic dynamic programming. Instead, the optimal strategies are derived as a function of the wealth process. Theorem 4.3 presents closed-form expressions of the optimal policy, the optimal wealth process and the optimal expected utility of terminal wealth.

Theorem 4.3. *Consider the optimal investment and reinsurance problem for an insurance company and the decision makers are assumed to be loss averse. Then:*

(i) *The optimal trading policy* $h^* = [\pi^*(t), q^*(t)]$ *is given by*

$$\begin{cases} \pi^*(t) = \dfrac{1}{(1-\gamma_1){\sigma_1}^2}\left[X^*(t) - (\xi - \dfrac{1}{r}c_2)e^{-r(T-t)} - \dfrac{1}{r}c_2\right](\mu - r), \\ q^*(t) = \dfrac{1}{(1-\gamma_1){\sigma_2}^2}\left[X^*(t) - (\xi - \dfrac{1}{r}c_2)e^{-r(T-t)} - \dfrac{1}{r}c_2\right](1+\eta)\lambda\mu_0. \end{cases} \tag{25}$$

where $\pi^*(t)$ *and* $q^*(t)$ *denote the optimal investment strategy and the optimal reinsurance strategy respectively.*

(ii) *The corresponding optimal wealth process* $X^*(t), t \in [0, T]$ *is given by*

$$\begin{aligned} X^*(t) &= (\xi - \frac{1}{r}c_2)e^{-r(T-t)} + \frac{1}{r}c_2 + (x_0 - \xi e^{-rT} - c_2\frac{1-e^{-rT}}{r})\frac{Z(t)}{H(t)} \\ &= (\xi - \frac{1}{r}c_2)e^{-r(T-t)} + \frac{1}{r}c_2 + (x_0 - \xi e^{-rT} - c_2\frac{1-e^{-rT}}{r}) \\ &\quad \times \exp\left\{rt + \frac{1}{\gamma_1 - 1}\int_0^t \theta_1(s)dW_1(s) + \frac{1}{\gamma_1 - 1}\int_0^t \theta_2(s)dW_2(s)\right. \\ &\quad \left. + \frac{1-2\gamma_1}{2(\gamma_1-1)^2}\int_0^t [\theta_1(s)^2 + \theta_2(s)^2]ds\right\}. \end{aligned} \tag{26}$$

(iii) *The insurer's optimal expected utility of terminal wealth is given by*

$$\begin{aligned} E[U(X^*(T))] = A[x_0 - \xi e^{-rT} + c_2\frac{1-e^{-rT}}{r}]^{\gamma_1} \\ \times \exp\left\{\gamma_1 rT + \frac{1}{2}\frac{\gamma_1}{1-\gamma_1}\int_0^T [\theta_1(s)^2 + \theta_2(s)^2]ds\right\}. \end{aligned} \tag{27}$$

Proof. We derive the optimal policy $h^* = \{\pi^*(t), q^*(t)\}_{t\in[0,T]}$ in the dynamic problem (8) with the corresponding optimal terminal wealth ψ_1^* satisfying

$$X^*(T) = X^{h^*}(T) = \psi_1^*.$$

Multiplying by $H(T)$ and then taking conditional expectation on both sides gives

$$E\left[H(T)X^*(T) + \int_0^T H(s)c_2 ds|\mathcal{F}_t\right] = E\left[H(T)\psi_1^* + \int_0^T H(s)c_2 ds|\mathcal{F}_t\right]. \tag{28}$$

According to Proportion 4.1, (28) can be rewritten as

$$H(t)X^*(t)+\int_0^t H(s)c_2ds=H(t)\xi e^{-r(T-t)}+\int_0^t H(s)c_2ds-\frac{1}{r}c_2(e^{-r(T-t)}-1)H(t)$$
$$+\left[x_0-E(H(T)\xi+\int_0^T H(s)c_2ds)\right]\frac{E(H(T)^{\frac{\gamma_1}{\gamma_1-1}}|\mathcal{F}_t)}{E(H(T)^{\frac{\gamma_1}{\gamma_1-1}})}. \tag{29}$$

Then we obtain

$$H(t)X^*(t)=H(t)[\xi e^{-r(T-t)}-\frac{1}{r}c_2(e^{-r(T-t)}-1)]$$
$$+\left[x_0-E(H(T)\xi+\int_0^T H(s)c_2ds)\right]\frac{E(H(T)^{\frac{\gamma_1}{\gamma_1-1}}|F_t)}{E(H(T)^{\frac{\gamma_1}{\gamma_1-1}})}. \tag{30}$$

Introduce an exponential martingale

$$Z(t)=\exp\left\{\frac{\gamma_1}{\gamma_1-1}\int_0^t\theta_1(s)dW_1(s)+\frac{\gamma_1}{\gamma_1-1}\int_0^t\theta_2(s)dW_2(s)\right.$$
$$\left.-\frac{1}{2}\frac{(\gamma_1)^2}{(\gamma_1-1)^2}\int_0^t[\theta_1(s)^2+\theta_2(s)^2]ds\right\}. \tag{31}$$

According to $Z(t)$, $H(t)^{\frac{\gamma_1}{\gamma_1-1}}$ can be rewritten as

$$H(t)^{\frac{\gamma_1}{\gamma_1-1}}=Z(t)\exp\left\{-\frac{\gamma_1}{\gamma_1-1}rt+\frac{1}{2}\frac{\gamma_1}{(\gamma_1-1)^2}\int_0^t[\theta_1(s)^2+\theta_2(s)^2]ds\right\}. \tag{32}$$

Denote

$$f(t)=\exp\left\{-\frac{\gamma_1}{\gamma_1-1}rt+\frac{1}{2}\frac{\gamma_1}{(\gamma_1-1)^2}\int_0^t[\theta_1(s)^2+\theta_2(s)^2]ds\right\},$$

then the fraction of (29) on the right-hand side can be rewritten as

$$\frac{E[H(T)^{\frac{\gamma_1}{\gamma_1-1}}|\mathcal{F}_t]}{E[H(T)^{\frac{\gamma_1}{\gamma_1-1}}]}=\frac{E[f(T)Z(T)|\mathcal{F}_t]}{E[f(T)Z(T)]}=\frac{f(T)E[Z(T)|\mathcal{F}_t]}{f(T)E[Z(T)]}=\frac{Z(t)}{Z(0)}=Z(t).$$

The last equality holds because $Z(0)=1$. Substituting back into (29), and since

$$E[H(T)\xi+\int_0^T H(s)c_2ds]=\xi e^{-rT}+c_2\frac{1-e^{-rT}}{r},$$

we obtain

$$H(t)X^*(t)=H(t)[(\xi-\frac{1}{r}c_2)e^{-r(T-t)}+\frac{1}{r}c_2]+(x_0-\xi e^{-rT}-c_2\frac{1-e^{-rT}}{r})Z(t). \tag{33}$$

Taking differential on both sides of (33), we get

$$\begin{aligned} d[H(t)X^*(t)] &= [(\xi - \frac{1}{r}c_2)e^{-r(T-t)} + \frac{1}{r}c_2]dH(t) \\ &\quad + H(t)d[(\xi - \frac{1}{r}c_2)e^{-r(T-t)} + \frac{1}{r}c_2] + [x_0 - \xi e^{-rT} - c_2\frac{1-e^{-rT}}{r}]dZ(t) \\ &= H(t)[(\xi - \frac{1}{r}c_2)e^{-r(T-t)}r]dt \\ &\quad + H(t)[(\xi - \frac{1}{r}c_2)e^{-r(T-t)} + \frac{1}{r}c_2][-rdt + \theta_1(t)dW_1(t) + \theta_2(t)dW_2(t)] \\ &\quad + \frac{\gamma_1}{\gamma_1 - 1}H(t)[X^*(t) - (\xi - \frac{1}{r}c_2)e^{-r(T-t)} - \frac{1}{r}c_2][\theta_1(t)dW_1(t) + \theta_2(t)dW_2(t)] \\ &= -c_2H(t)dt + \frac{1}{\gamma_1 - 1}H(t)\theta_1(t)[X^*(t)\gamma_1 - (\xi - \frac{1}{r}c_2)e^{-r(T-t)} - \frac{1}{r}c_2]dW_1(t) \\ &\quad + \frac{1}{\gamma_1 - 1}H(t)\theta_2(t)[X^*(t)\gamma_1 - (\xi - \frac{1}{r}c_2)e^{-r(T-t)} - \frac{1}{r}c_2]dW_2(t). \end{aligned} \tag{34}$$

Since $H(t)X^*(t)$ also satisfies (11), we have

$$\begin{aligned} d\left[H(t)X^*(t)\right] &= H(t)dX^*(t) + X^*(t)dH(t) + d\left[H(t), X^*(t)\right] \\ &= -H(t)c_2dt + H(t)\left[X^*(t)\theta_1(t) + \sigma_1\pi(t)\right]dW_1(t) \\ &\quad + H(t)\left[X^*(t)\theta_2(t) + \sigma_2 q(t)\right]dW_2(t). \end{aligned} \tag{35}$$

Comparing $dW_1(t)$-term and $dW_2(t)$-term of equation (34) with those of equation (35), the optimal policy is given by

$$\begin{cases} \pi^*(t) = \dfrac{1}{(1-\gamma_1){\sigma_1}^2}\left[X^*(t) - (\xi - \frac{1}{r}c_2)e^{-r(T-t)} - \frac{1}{r}c_2\right](\mu - r), \\ q^*(t) = \dfrac{1}{(1-\gamma_1){\sigma_2}^2}\left[X^*(t) - (\xi - \frac{1}{r}c_2)e^{-r(T-t)} - \frac{1}{r}c_2\right](1+\eta)\lambda\mu_0. \end{cases} \tag{36}$$

Finally, it is easy to prove that the policy in Equation (36) is admissible. So, it is the optimal policy of the optimization problem (8). From (33), we easily derive the optimal wealth process:

$$\begin{aligned} X^*(t) &= (\xi - \frac{1}{r}c_2)e^{-r(T-t)} + \frac{1}{r}c_2 + (x_0 - \xi e^{-rT} - c_2\frac{1-e^{-rT}}{r})\frac{Z(t)}{H(t)} \\ &= (\xi - \frac{1}{r}c_2)e^{-r(T-t)} + \frac{1}{r}c_2 + (x_0 - \xi e^{-rT} - c_2\frac{1-e^{-rT}}{r}) \\ &\quad \times \exp\left\{rt + \frac{1}{\gamma_1 - 1}\int_0^t \theta_1(s)dW_1(s) + \frac{1}{\gamma_1 - 1}\int_0^t \theta_2(s)dW_2(s)\right. \\ &\quad \left. + \frac{1-2\gamma_1}{2(\gamma_1 - 1)^2}\int_0^t [\theta_1(s)^2 + \theta_2(s)^2]ds\right\}. \end{aligned} \tag{37}$$

Substituting ψ_1^* into the value function in the maximization problem (18), we can derive the optimal expected utility:

$$\begin{aligned} E[U(X^*(T))] &= E[U(\psi_1^*)] \\ &= E[u_1(\psi_1^*)] \\ &= A[x_0 - \xi e^{-rT} + c_2 \frac{1-e^{-rT}}{r}]^{\gamma_1} E\left(\frac{H(T)^{\frac{1}{\gamma_1-1}}}{E[H(T)^{\frac{\gamma_1}{\gamma_1-1}}]}\right)^{\gamma_1} \\ &= A[x_0 - \xi e^{-rT} + c_2 \frac{1-e^{-rT}}{r}]^{\gamma_1} E\left(\frac{H(T)^{\frac{1}{\gamma_1-1}}}{f(T)E[Z(T)]}\right)^{\gamma_1} \\ &= A[x_0 - \xi e^{-rT} + c_2 \frac{1-e^{-rT}}{r}]^{\gamma_1} f(T)^{1-\gamma_1}. \end{aligned}$$

The last equality holds because $E[Z(T)] = 1$. Substituting the expression of $f(t)$ into the above formula, the insurer's optimal expected utility of terminal wealth is given by

$$\begin{aligned} E[U(X^*(T))] = A[x_0 - \xi e^{-rT} + c_2 \frac{1-e^{-rT}}{r}]^{\gamma_1} \\ \times \exp\left\{\gamma_1 rT + \frac{1}{2}\frac{\gamma_1}{1-\gamma_1}\int_0^T [\theta_1(s)^2 + \theta_2(s)^2]ds\right\}. \end{aligned} \tag{38}$$

□

Note that (1) the optimal policy depends on the wealth process, which is a realistic and important conclusion, however, a lot of literature could not have such results; the parameters of the capital market and the insurance market have impact on the optimal policy; the reference level of the insurer has impact on the optimal policy; from the above results, we find that the optimal dollar amount invested in the risky asset and the optimal reinsurance proportion both decrease with respect to the reference level, that is to say, the higher aspiration level to determine gains and losses, the less amount the insurer invests in the risky asset and the less insurance business the insurer keeps. (2) The optimal expected utility of terminal wealth decreases with respect to the reference level, which implies that the higher reference level of the insurer, the smaller the optimal utilities.

5. Numerical Examples

In this section, we present a sensitivity analysis to explore the economic behavior of the optimal investment and reinsurance strategies. Since the optimal strategies are stochastic, we apply the Monte Carlo Methods (MCM) to show the impacts of economic parameters on the optimal strategies. Throughout the sensitivity analysis, unless otherwise stated, the basic parameters are given by: $\mu = 0.2, r = 0.05$, $\sigma_0 = 1$, $\sigma_1 = 2$, $\eta = 1.5$, $\mu_0 = 0.1$, $x_0 = 10$, $\lambda = 0.2$, $\theta = 1$, $T = 10$, $\xi = 5$, $\gamma_1 = 0.2$, $\gamma_2 = 0.3$, $A = 1$, $B = 2.25$. Since the impacts of the economic parameters on the optimal investment and reinsurance strategies have been studied in many literatures, see Zeng and Li [33], Liang et al. [22] and etc., we mainly investigate the influence of the loss aversion on the optimal strategies. Different loss aversion functions are corresponding to different people. This section in fact describes the optimal strategies for different people.

Figures 1-4 show that the optimal dollar amount invested in the risky asset and the optimal reinsurance proportion both increase with respect to time t, namely, as time escapes, the insurer should invest more in the risky asset and keep more insurance business.

The impact of the risk averse level on the optimal investment policy and the optimal reinsurance policy with respect to time t are shown in Figure 1 and Figure 2 respectively. Three different risk averse levels are: $\gamma_1 = 0.2$, $\gamma_1 = 0.4$, $\gamma_1 = 0.6$. Specially, Figure 1 illustrates that the optimal investment policy $\pi^*(t)$ is increasing with respect to the coefficient of risk aversion γ_1, i.e., the more the insurer dislikes risk, the less amount the insurer invests in the risky asset; Figure 2 tells us that the optimal reinsurance policy $q^*(t)$ is also increasing with respect to the coefficient of risk aversion γ_1, that is to say, the less risk averse the insurer, the more insurance business the insurer keeps;

The impact of the reference level on the optimal investment policy and the optimal reinsurance policy with respect to time t are shown in Figure 3 and Figure 4 respectively. Three different reference levels are: $\xi = 1$, $\xi = 5$, $\xi = 10$. Specially, Figure 3 and Figure 4 show that both the optimal investment policy $\pi^*(t)$ and the optimal reinsurance policy $q^*(t)$ are decreasing with respect to the reference level ξ. When the reference level is increased, the insurer tends to adopt a lower allocation in the risky asset and keep less insurance business, since the loss aversion insurer becomes more concerned about volatilities that may cause the account of wealth to underperform the reference level.

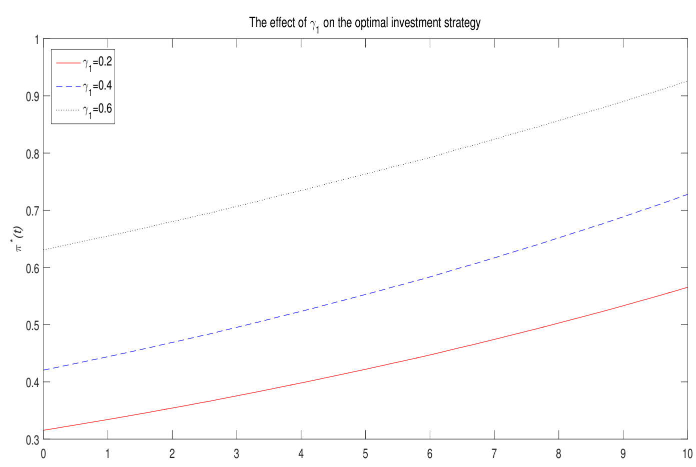

Figure 1. The effect of γ_1 on the optimal investment policy.

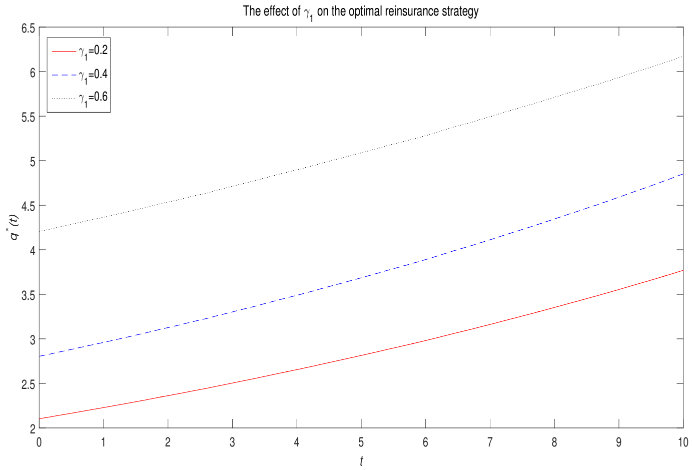

FIGURE 2. The effect of γ_1 on the optimal reinsurance policy.

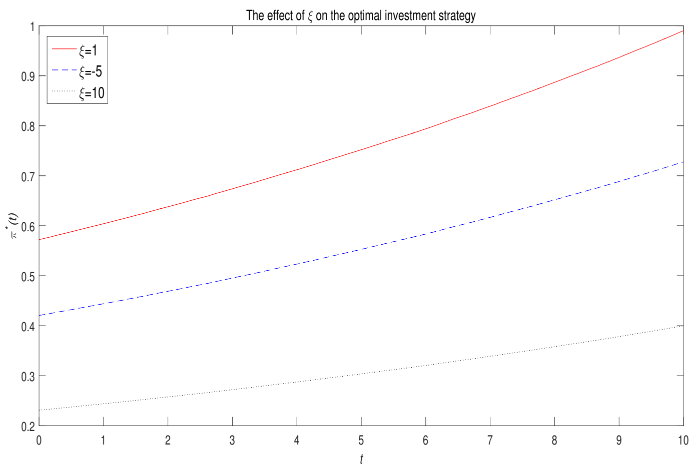

FIGURE 3. The effect of ξ on the optimal investment policy.

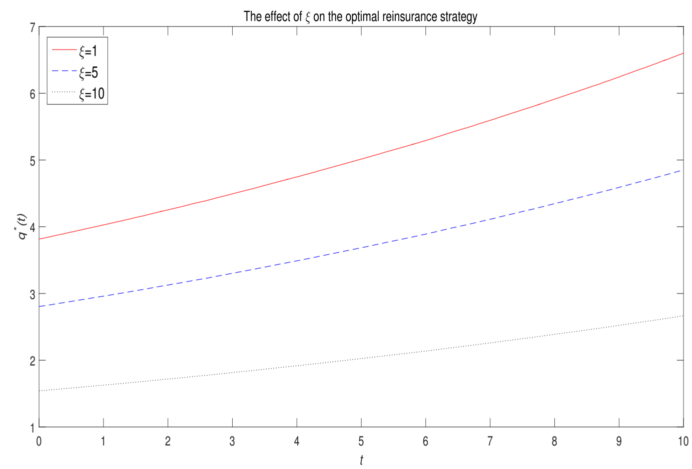

FIGURE 4. The effect of ξ on the optimal reinsurance policy.

6. Conclusions

In this paper, we study the optimal asset allocation and risk management issues for insurance companies in a continuous-time model. The insurer is allowed to invest in a financial market and purchase proportional reinsurance and acquire new business. The surplus process of the insurer is assumed to follow a diffusion approximation model and the financial market consists of risk-free assets and risky assets. With the help of martingale approach, we change the dynamic maximization problem into a static optimization problem. The closed-form expressions for the optimal policies and the optimal wealth process are derived. In the end, we present a sensitivity analysis to show the impact of parameters on the optimal investment and reinsurance policies.

This paper considers the continuous-time optimal investment and reinsurance problem for an insurer under loss aversion. Our work is just a basic framework. There are still many works needed to be investigated in this direction. For example, (1) this paper assumes that the price process of risky asset and the surplus process are both driven by diffusion processes in order to derive closed-form solutions, it is noteworthy to extend this work to a jump-diffusion case because the real financial markets are often of such cases. (2) In our problems, we do not consider the probability distortion, which is one of the major ingredients of the Prospect Theory, it is also worth investigating the optimal investment and reinsurance problem under loss aversion and probability distortion. (3) As the insurer updates its policy continuously in our optimization problem, it may be more interesting to consider a time coefficient of reference point.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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