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UPPER AND LOWER SLIGHTLY $(\tau_1, \tau_2)\beta$-CONTINUOUS MULTIFUNCTIONS

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Abstract. This paper is concerned with the concepts of upper and lower slightly $(\tau_1, \tau_2)\beta$-continuous multifunctions. Furthermore, several characterizations of upper and lower slightly $(\tau_1, \tau_2)\beta$-continuous multifunctions are investigated. The relationships between slight $(\tau_1, \tau_2)\beta$-continuity and other forms of $(\tau_1, \tau_2)\beta$-continuity are also discussed.

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1. Introduction

In 2001, Noiri [37] introduced the notion of slightly β-continuous functions and studied the relationships between slight β-continuity, contra-β-continuity [30] and β-continuity [2]. Duangphui et al. [31] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, (Λ, sp)-continuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, pairwise M-continuous functions, (g, m)-continuous functions, almost (Λ, p)-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $(\Lambda, p(\star))$-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions, $\delta p(\Lambda, s)$-continuous functions, and slightly (m, μ)-continuous functions were presented in [45], [8], [49], [9], [23], [28], [29], [44], [41], [15], [14], [20], [5], [6], [7], [43] and [42], respectively.

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In 2005, Ekici [32] introduced and investigated slightly β-continuous multifunctions as a generalization of α-continuous multifunctions [39], γ-continuous multifunctions [1] and β-continuous multifunctions [38]. Laprom et al. [36] introduced and studied the notion of $\beta(\tau_1, \tau_2)$-continuous multifunctions. Viriyapong and Boonpok [50] introduced and investigated the concept of $(\tau_1, \tau_2)\alpha$-continuous multifunctions. Moreover, some characterizations of $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\imath^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, slightly (Λ, sp)-continuous multifunctions, (τ_1, τ_2)-continuous multifunctions, almost (τ_1, τ_2)-continuous multifunctions and slightly $(\tau_1, \tau_2)s$-continuous multifunctions were investigated in [24], [21], [26], [22], [48], [4], [10], [11], [13], [12], [18], [25], [16], [34], [19], [46], [17], [40], [35] and [33], respectively.

In this paper, we introduce the notions of upper and lower slightly $(\tau_1, \tau_2)\beta$-continuous multifunctions. We also investigate several characterizations of upper and lower slightly $(\tau_1, \tau_2)\beta$-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [27] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [27] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [27] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [27] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-*Cl*(A) *and* $\tau_1\tau_2$-*Cl*$(\tau_1\tau_2$-*Cl*$(A)) = \tau_1\tau_2$-*Cl*(A).

(2) *If* $A \subseteq B$, *then* $\tau_1\tau_2$-*Cl*$(A) \subseteq \tau_1\tau_2$-*Cl*$(B)$.

(3) $\tau_1\tau_2$-*Cl*(A) *is* $\tau_1\tau_2$-*closed.*

(4) A *is* $\tau_1\tau_2$-*closed if and only if* $A = \tau_1\tau_2$-*Cl*(A).

(5) $\tau_1\tau_2$-*Cl*$(X - A) = X - \tau_1\tau_2$-*Int*$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*clopen* [27] if A is both $\tau_1\tau_2$-open and $\tau_1\tau_2$-closed. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\beta$-*open* [24] (resp. $\alpha(\tau_1, \tau_2)$-*open* [47]) if $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$ (resp. $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A)))$). The complement of a $(\tau_1, \tau_2)\beta$-open (resp. $\alpha(\tau_1, \tau_2)$-open) set is called $(\tau_1, \tau_2)\beta$-*closed* (resp. $\alpha(\tau_1, \tau_2)$-*closed*). Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)\beta$-closed sets of X containing A is called the $(\tau_1, \tau_2)\beta$-*closure* of A and is denoted by $(\tau_1, \tau_2)\beta\text{-Cl}(A)$. The union of all $(\tau_1, \tau_2)\beta$-open sets of X contained in A is called the $(\tau_1, \tau_2)\beta$-*interior* of A and is denoted by $(\tau_1, \tau_2)\beta\text{-Int}(A)$.

Lemma 2. *For subsets A and B of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) $A \subseteq (\tau_1, \tau_2)\beta\text{-}Cl(A)$ *and* $(\tau_1, \tau_2)\beta\text{-}Cl((\tau_1, \tau_2)\beta\text{-}Cl(A)) = (\tau_1, \tau_2)\beta\text{-}Cl(A)$.

(2) *If* $A \subseteq B$, *then* $(\tau_1, \tau_2)\beta\text{-}Cl(A) \subseteq (\tau_1, \tau_2)\beta\text{-}Cl(B)$.

(3) $(\tau_1, \tau_2)\beta\text{-}Cl(A)$ *is* $(\tau_1, \tau_2)\beta$*-closed.*

(4) A *is* $(\tau_1, \tau_2)\beta$*-closed if and only if* $A = (\tau_1, \tau_2)\beta\text{-}Cl(A)$.

(5) $(\tau_1, \tau_2)\beta\text{-}Cl(X - A) = X - (\tau_1, \tau_2)\beta\text{-}Int(A)$.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, following [3] we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and

$$F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}.$$

In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x\in A}F(x)$.

3. Upper and lower slightly $(\tau_1, \tau_2)\beta$-continuous multifunctions

In this section, we introduce the notions of upper and lower slightly $(\tau_1, \tau_2)\beta$-continuous multifunctions. Furthermore, some characterizations of upper and lower slightly $(\tau_1, \tau_2)\beta$-continuous multifunctions are discussed.

Definition 1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper slightly $(\tau_1, \tau_2)\beta$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-clopen set V of Y containing $F(x)$, there exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $F(U) \subseteq V$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper slightly $(\tau_1, \tau_2)\beta$-continuous if F has this property at every point of X.*

Theorem 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper slightly $(\tau_1, \tau_2)\beta$-continuous;*

(2) *$F^+(V)$ is $(\tau_1, \tau_2)\beta$-open in X for every $\sigma_1\sigma_2$-clopen set V of Y;*

(3) *$F^-(V)$ is $(\tau_1, \tau_2)\beta$-closed in X for every $\sigma_1\sigma_2$-clopen set V of Y.*

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-clopen set V of Y and $x \in F^+(V)$. Then, $F(x) \subseteq V$. Since F is upper slightly $(\tau_1, \tau_2)\beta$-continuous, there exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $F(U) \subseteq V$. Thus, $x \in U \subseteq F^+(V)$ and hence $x \in (\tau_1, \tau_2)\beta\text{-Int}(F^+(V))$. Therefore, we have $F^+(V) \subseteq (\tau_1, \tau_2)\beta\text{-Int}(F^+(V))$ and so $F^+(V)$ is $(\tau_1, \tau_2)\beta$-open in X.

(2) $\Leftrightarrow$ (3): This follows from the fact that $F^-(Y - B) = X - F^+(B)$ for every subset B of Y.

(2) $\Rightarrow$ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-clopen set V of Y containing $F(x)$. Then, $x \in F^+(V) = (\tau_1, \tau_2)\beta\text{-Int}(F^+(V))$. There exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $U \subseteq F^+(V)$. Thus, $F(U) \subseteq V$ and hence F is upper slightly $(\tau_1, \tau_2)\beta$-continuous at x. This shows that F is upper slightly $(\tau_1, \tau_2)\beta$-continuous. □

Definition 2. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower slightly $(\tau_1, \tau_2)\beta$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-clopen set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower slightly $(\tau_1, \tau_2)\beta$-continuous if F has this property at every point of X.*

Theorem 2. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is lower slightly $(\tau_1, \tau_2)\beta$-continuous;*
(2) *$F^-(V)$ is $(\tau_1, \tau_2)\beta$-open in X for every $\sigma_1\sigma_2$-clopen set V of Y;*
(3) *$F^+(V)$ is $(\tau_1, \tau_2)\beta$-closed in X for every $\sigma_1\sigma_2$-clopen set V of Y.*

Proof. The proof is similar to that of Theorem 1. □

Definition 3. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be slightly $(\tau_1, \tau_2)\beta$-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-clopen set V of Y containing $f(x)$, there exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $f(U) \subseteq V$.*

Corollary 1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is slightly $(\tau_1, \tau_2)\beta$-continuous;*
(2) *$f^{-1}(V)$ is $(\tau_1, \tau_2)\beta$-open in X for each $\sigma_1\sigma_2$-clopen set V of Y;*
(3) *$f^{-1}(V)$ is $(\tau_1, \tau_2)\beta$-closed in X for each $\sigma_1\sigma_2$-clopen set V of Y.*

Definition 4. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper $(\tau_1, \tau_2)\beta$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$, there exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $F(U) \subseteq V$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper $(\tau_1, \tau_2)\beta$-continuous if F has this property at each point of X.*

Theorem 3. *If a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper $(\tau_1, \tau_2)\beta$-continuous, then F is upper slightly $(\tau_1, \tau_2)\beta$-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-clopen set of Y containing $F(x)$. Since F is upper $(\tau_1, \tau_2)\beta$-continuous, there exists a $(\tau_1, \tau_2)\beta$-open set of X containing x such that $F(U) \subseteq V$. This shows that F is upper slightly $(\tau_1, \tau_2)\beta$-continuous. □

Definition 5. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower $(\tau_1, \tau_2)\beta$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for every $z \in U$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower $(\tau_1, \tau_2)\beta$-continuous if F has this property at each point of X.*

Theorem 4. *If a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower $(\tau_1, \tau_2)\beta$-continuous, then F is lower slightly $(\tau_1, \tau_2)\beta$-continuous.*

Proof. The proof is similar to that of Theorem 3. □

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-*extremally disconnected* [47] if the $\tau_1\tau_2$-closure of every $\tau_1\tau_2$-open set U of X is $\tau_1\tau_2$-open.

Lemma 3. [47] *For a bitopological space (X, τ_1, τ_2), the following properties are equivalent:*

(1) *(X, τ_1, τ_2) is (τ_1, τ_2)-extremally disconnected.*
(2) *Every $(\tau_1, \tau_2)r$-open set of X is $\tau_1\tau_2$-closed.*
(3) *Every $(\tau_1, \tau_2)r$-closed set of X is $\tau_1\tau_2$-open.*

Definition 6. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper almost $(\tau_1, \tau_2)\beta$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$, there exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper almost $(\tau_1, \tau_2)\beta$-continuous if F has this property at each point of X.*

Lemma 4. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper almost $(\tau_1, \tau_2)\beta$-continuous;*
(2) *for each $x \in X$ and each $(\sigma_1, \sigma_2)r$-open set V of Y containing $F(x)$, there exists a $(\tau_1, \tau_2)\beta$-open set of X containing x such that $F(U) \subseteq V$.*

Theorem 5. *If a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper slightly $(\tau_1, \tau_2)\beta$-continuous and (Y, σ_1, σ_2) is (τ_1, τ_2)-extremally disconnected, then F is upper almost $(\tau_1, \tau_2)\beta$-continuous.*

Proof. Let $x \in X$ and V be any $(\sigma_1, \sigma_2)r$-open set of Y containing $F(x)$. Then, by Lemma 3 we have V is $\sigma_1\sigma_2$-clopen in Y. Since F is upper slightly $(\tau_1, \tau_2)\beta$-continuous, there exists a $(\tau_1, \tau_2)\beta$-open set of X containing x such that $F(U) \subseteq V$. By Lemma 4, F is upper almost $(\tau_1, \tau_2)\beta$-continuous. □

Definition 7. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower almost $(\tau_1, \tau_2)\beta$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $(\tau_1, \tau_2)\beta$-open* *set U of X containing x such that $\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V)) \cap F(z) \neq \emptyset$ for every $z \in U$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower almost $(\tau_1, \tau_2)\beta$-continuous if F has this property at each point of X.*

Lemma 5. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is lower almost $(\tau_1, \tau_2)\beta$-continuous;*

(2) *for each $x \in X$ and each $(\sigma_1, \sigma_2)r$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $(\tau_1, \tau_2)\beta$-open set of X containing x such that $U \subseteq F^-(V)$.*

Theorem 6. *If a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower slightly $(\tau_1, \tau_2)\beta$-continuous and (Y, σ_1, σ_2) is (τ_1, τ_2)-extremally disconnected, then F is lower almost $(\tau_1, \tau_2)\beta$-continuous.*

Proof. By utilizing Lemma 5, this can be proved similarly to that of Theorem 5. □

Definition 8. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper weakly $(\tau_1, \tau_2)\beta$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$, there exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper weakly $(\tau_1, \tau_2)\beta$-continuous if F has this property at each point of X.*

Theorem 7. *If a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper weakly $(\tau_1, \tau_2)\beta$-continuous, then F is upper slightly $(\tau_1, \tau_2)\beta$-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-clopen set of Y containing $F(x)$. Since F is upper weakly $(\tau_1, \tau_2)\beta$-continuous, there exists a $(\tau_1, \tau_2)\beta$-open set of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V) = V$. This shows that F is upper slightly $(\tau_1, \tau_2)\beta$-continuous. □

Definition 9. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower weakly $(\tau_1, \tau_2)\beta$-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $(\tau_1, \tau_2)\beta$-open set U of X containing x such that $\sigma_1\sigma_2$-Cl$(V) \cap F(z) \neq \emptyset$ for every $z \in U$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower weakly $(\tau_1, \tau_2)\beta$-continuous if F has this property at each point of X.*

Theorem 8. *If a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower weakly $(\tau_1, \tau_2)\beta$-continuous, then F is lower slightly $(\tau_1, \tau_2)\beta$-continuous.*

Proof. The proof is similar to that of Theorem 7. □

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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