Asia Pac. J. Math. 2024 **11**:97

SEMI-ANALYTICAL SOLUTIONS OF FRACTIONAL TELEGRAPH EQUATIONS BY USING YANG DECOMPOSITION METHOD

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Received Jul. 5, 2024

Abstract. We provide new approximation solutions to telegraph equations with Caputo fractional operator using the Yang decomposition method in this paper. To confirm the suggested method's high accuracy, certain specific instances are given, and the resulting solutions are compared to the exact answer and analytical data. The findings show that, for lower degree of approximations, Yang decomposition method converge quickly to accurate solutions of the given problems.

2020 Mathematics Subject Classification. 35R11.

Key words and phrases. Yang transform; Adomian decomposition method; telegraph equation; Caputo fractional operator.

1. Introduction

During recent decades, researchers have been interested in studying fractional calculus and its applications, not only in mathematics but also in many other sciences, such as physics, thermodynamics, engineering, economics, etc. Fractional calculus has many applications in the field of electrical, electrochemistry, statistics, and probability. In addition, fractional differential equations can describe many cosmological phenomena that traditional differential equations cannot describe [1]- [8]. Differential equations of fractional order are particularly suited to describing critical aspects in finance, electromagnetic, acoustics, viscoelasticity, biochemistry, and material science [3]. Therefore, broad classes of semi-analytical and numerical techniques were used to solve these equations such as ADM, VIM, HPM, DTM, RDTM, LADM, LVIM, SADM, SVIM and other methods [8]- [60].

The Adomian decomposition technique is one of the important methods for finding the approximate solution to differential equations and it has been dealt with in many researches. The Yang transform is also one of the integral transforms used in solving differential equations. Therefore, when we merge the ADM with the Yang transform, we get a new method to find the approximate solution to differential

DOI: 10.28924/APJM/11-97

equations. In this paper, we apply the Yang decomposition technique to find solution of fractional differential equations with the fractional operator Caputo. The order of the paper is as follows: The basic definitions for calculus and fractional integration are presented in section 2, the method used is analyzed in section 3, many examples are given that explain the effectiveness of the method proposed in section 4, and finally, the conclusion is provided in section 5.

2. Preliminaries

This section [61]- [66] goes through some FC definitions and notation that will be used during this period of work.

Definition 2.1. *The fractional integral operator of order $v \geq 0$ Riemann Liouville, of $\varphi(\mu) \in C_\vartheta, \vartheta \geq -1$ is*

$$I^\alpha u(t) = \begin{cases} \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} u(t) d\tau, & \alpha > 0, \quad t > 0. \\ u(t), & \alpha = 0 \end{cases}$$

Properties of operator I^α :

1. $I^\alpha I^\sigma u(t) = I^{\alpha+\sigma} u(t)$.
2. $I^\alpha I^\sigma u(t) = I^\sigma I^\alpha u(t)$.
3. $I^\alpha t^m = \frac{\Gamma(m+1)}{\Gamma(\alpha+m+1)} t^{\alpha+m}$

Definition 2.2. *The Caputo fractional derivative of order α of $u(t)$ is*

$$\begin{aligned} D^\alpha u(t) &= I^{m-\alpha} D^m u(t) \\ &= \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} u^{(m)}(\tau) d\tau \end{aligned}$$

For $m-1 < \alpha < m, m \in N, t > 0$ and $u \in C_{-1}^m$.

The properties D^α are:

1. $D^\alpha k = 0$, *where* k *is a constant.*
2. $D^\alpha t^\sigma = \frac{\Gamma(\sigma+1)}{\Gamma(\sigma-\alpha+1)} t^{\sigma-\alpha}$,
3. $D^\alpha D^\sigma u(t) = D^{\alpha+\sigma} u(t)$
4. $I^\alpha D^\alpha u(t) = u(t) - \sum_{k=0}^{m-1} u^{(k)}(0) \frac{t^k}{k!}$.

Definition 2.3. *The MLF with $\alpha > 0$ is*

$$E_\alpha(z) = \sum_{m=0}^{\infty} \frac{z^\alpha}{\Gamma(m\alpha+1)}$$

3. Yang transform

Definition 2.4. *The Yang transform of the function is*

$$Y\{u(t)\} = \int_0^\infty e^{-\frac{t}{v}} u(t) dt, t > 0,$$

with v representing the transform variable.

Few properties of YT *is stated as.*

The YT $Y[f(t)]$ *of the caputo fractional derivative as defined by*

$$Y\left[{}_{0}^{c}D_{x}^{\alpha}f(t)\right]=\frac{Y[f(t)]}{v^{\alpha}}-\sum_{k=0}^{n-1}\frac{U^{(k)}\left(0^{+}\right)}{v^{(\alpha-k-1)}}$$

Where $\mathrm{n}-1<\alpha<n$

3. Formulation of Yang decomposition method for fractional telegraph equation

We now consider the following and hence illustrate the basic

$${}_{0}^{c}D_{x}^{\alpha}U(x,t)=A(x,t)\partial_{t}^{2}U(x,t)+B(x,t)\partial_{t}U(x,t)+C(x,t)U(x,t)+U^{r}(x,t)+g(x,t), \tag{3.1}$$

with the initial condition $U(0,t)$ and $U_{x}(o,t), 0<x<a, 0<\alpha\leq 2$ and $A(x,t), B(x,t), C(x,t)$ are continues functions and $U^{r}(x,t)$ is nonlinear function.

Applving the YT to both sides of (3.1), we have

$$\frac{Y[U(x,t)]}{v^{\alpha}}-\sum_{k=0}^{n-1}\frac{U^{(k)}\left(0^{+}\right)}{v^{(\alpha-k-1)}}=Y\left[\begin{array}{c}A(x,t)\partial_{t}^{2}U(x,t)+B(x,t)\partial_{t}U(x,t)\\+C(x,t)U(x,t)+U^{r}(x,t)+g(x,t)\end{array}\right], \tag{3.2}$$

or

$$\begin{aligned}Y[U(x,t)]=&v^{\alpha}\sum_{k=0}^{n-1}\frac{U^{(k)}\left(0^{+}\right)}{v^{(\alpha-k-1)}}+v^{\alpha}Y[g(x,t)]\\&+v^{\alpha}Y\left[A(x,t)\partial_{t}^{2}U(x,t)+B(x,t)\partial_{t}U(x,t)+C(x,t)U(x,t)+U^{r}(x,t)\right].\end{aligned} \tag{3.3}$$

Hence, applving the inverse YT to the both sides of (3.3), we conclude that.

$$\begin{aligned}U(x,t)=Y^{-1}&\left[v^{\alpha}\sum_{k=0}^{n-1}\frac{U^{(k)}\left(0^{+}\right)}{v^{(\alpha-k-1)}}+v^{\alpha}Y[g(x,t)]\right.\\&+v^{\alpha}Y\left[A(x,t)\partial_{t}^{2}U(x,t)+B(x,t)\partial_{t}U(x,t)+C(x,t)U(x,t)\right.\\&\left.+U^{r}(x,t)\right]\end{aligned} \tag{3.4}$$

So that

$$U(x,t)=\mu(x,t)+Y^{-1}\left[v^{\alpha}Y\left[A(x,t)\partial_{t}^{2}U(x,t)+B(x,t)\partial_{t}U(x,t)+C(x,t)U(x,t)+U^{r}(x,t)\right]\right], \tag{3.5}$$

Where

$$\mu(x,t)=Y^{-1}\left[v^{\alpha}\sum_{k=0}^{n-1}\frac{U^{(k)}\left(0^{+}\right)}{v^{(\alpha-k-1)}}+v^{\alpha}Y[g(x,t)]\right]. \tag{3.6}$$

Now, suppose that

$$U(x,t)=\sum_{n=0}^{\infty}U_n(x,t) \tag{3.7}$$

and

$$U^r(x,t)=\sum_{n=0}^{\infty}A_n(x,t).$$

Substituting series (3.7) in (3.5), we have

$$\begin{aligned}\sum_{n=0}^{\infty}U_n(x,t)=&\mu(x,t)\\&+Y^{-1}\left[v^{\alpha}Y\left[A(x,t)\partial_t^2U_n(x,t)+B(x,t)\partial_tU_n(x,t)+C(x,t)U_n(x,t)+A_n(x,t)\right]\right].\end{aligned} \tag{3.8}$$

For the recursive iteration system, by the computing of both side of (3.8), we get the components of the approximation as the of the following respectively.

$$U_0(x,t)=\mu(x,t).$$

$$U_1(x,t)=\left[Y^{-1}\left[v^{\alpha}Y\left[A(x,t)\partial_t^2U_0(x,t)+B(x,t)\partial_tU_0(x,t)+C(x,t)U_0(x,t)+A_0(x,t)\right]\right]\right] \tag{3.9}$$

$$U_2(x,t)=\left[Y^{-1}\left[v^{\alpha}Y\left[\begin{array}{c}A(x,t)\partial_t^2U_1(x,t)+B(x,t)\partial_tU_1(x,t)+\\C(x,t)U_1(x,t)+A_1(x,t)\end{array}\right]\right]\right] \tag{3.10}$$

$$U_3(x,t)=\left[Y^{-1}\left[v^{\alpha}Y\left[A(x,t)\partial_t^2U_2(x,t)+B(x,t)\partial_tU_2(x,t)+C(x,t)U_2(x,t)+A_2(x,t)\right]\right]\right]. \tag{3.11}$$

$$U_{n+1}(x,t)=\left[Y^{-1}\left[v^{\alpha}Y\left[A(x,t)\partial_t^2U_n(x,t)+B(x,t)\partial_tU_n(x,t)+C(x,t)U_n(x,t)+A_n(x,t)\right]\right]\right] \tag{3.12}$$

4. Convergence analysis

In this section, the sufficient condition that guarantees existence of a unique solution is introduced and we discuss the convergence of solution,

Theorem 4.1. *The equation*

$$U_{n+1}(x,t)=\mu(x,t)+\left[Y^{-1}\left[v^{\alpha}Y\left[\partial_t^2U_n(x,t)+\partial_tU_n(x,t)+U_n(x,t)+U_n^k\right]\right]\right],n\geq 0$$

hase unique solution whenever $0<\varepsilon<1$, *and*

$$\varepsilon=\frac{(L_1+L_2+L_3)\,t^{(\alpha+1)}}{(\alpha-1)!}$$

Proof. let $\mathrm{E}=(\mathrm{C}[I]_2\|\cdot\|)$ be a Banach space of all continuous function on $\boldsymbol{I}=[0,\ \mathrm{T}]$ with norm $\|\cdot\|$ we defin amapping $\mathrm{F}:\mathrm{E}\to\mathrm{E}$ where

$$U_{n+1}(x,t)=\mu(x,t)+\left[Y^{-1}\left[v^{\alpha}Y\left[L\left[U_n(x,t)\right]+M\left[U_n(x,t)\right]+U_n(x,t)+U_n^k\right]\right]\right]$$

Where $L\left[U_n(x,t)\right]\equiv\partial_t^2U_n(x,t)]$ and $M\left[U_n(x,t)\right]\equiv\partial_tU_n(x,t)$

Now suppose that $L\left[U_n(x,t)\right]$ and $M\left[U_n(x,t)\right]$ is also lipschitzian with

$$\text{and } |LU - L\widetilde{U}| \le L_2|U - \widetilde{U}||MU - M\widetilde{U}| \le L_1|U - \widetilde{U}|$$

Where L_1 and L_2 is lipschitz constant respectively and $U, \widetilde{U}$ is defferent values of the function

$$\|\mathrm{F} - \widetilde{\mathrm{F}}\| = \max_{t\in I} | \left[Y^{-1}\left[v^{\alpha}Y[LU(x,t)] + M[U(x,t)] + U(x,t) + U^k(x,t)\right]\right]\right]$$
$$- \left[Y^{-1}\left[v^{\alpha}Y\left[L[\widetilde{U}(x,t)] + M[\widetilde{U}(x,t)] + \widetilde{U}(x,t) + \widetilde{U^k}\right]\right]\right]$$

$$\le \max_{t\in I} | \left[Y^{-1}\left[v^{\alpha}Y\left[L[U(x,t) - L[\widetilde{U}(x,t)]\right] + Y^{-1}\left[v^{\alpha}Y[M[U(x,t)] - M[\widetilde{U}(x,t)]\right.\right.\right.$$
$$+Y^{-1}\left[v^{\alpha}Y\left[U^k(x,t)\right] - \widetilde{U^k}(x,t)\right]\right]\right] |$$

$$\le \max_{t\in I} | L_1Y^{-1}\left[v^{\alpha}Y[U(x,t) - \widetilde{U}(x,t)]\right.$$
$$\left.+L_2Y^{-1}\left[v^{\alpha}Y[U(x,t) - \widetilde{U}(x,t)] + L_3Y^{-1}\left[v^{\alpha}Y[U(x,t) - \widetilde{U}(x,t)]\right]\right]\right] |$$

$$\le \max_{t\in I}(L_1 + L_2 + L_3)\left[Y^{-1}\left[v^{\alpha}Y[U(x,t) - \widetilde{U}(x,t)]\right]\right]$$

$$\le (L_1 + L_2 + L_3)\left[Y^{-1}\left[v^{\alpha}Y[U(x,t) - \widetilde{U}(x,t)]\right]\right]$$
$$= \frac{(L_1 + L_2 + L_3)\,t^{\alpha-1}}{(\alpha-1)!}\left\|\left[Y^{-1}\left[v^{\alpha}Y[U(x,t) - \widetilde{U}(x,t)]\right]\right]\right\|,$$

under the condition $0 < \varepsilon < 1$, the mapping contraction. Therefore, by Banach fixed point theorem for contraction, there exists a unique solution to the equation. □

Theorem 4.2. *The solution of equation 3.1 and with initial condition* $\sum_{k=0}^{n-1}\frac{U_0^{(k)}}{v^{n-k-1}}$ *will be convergence.*

Proof. Let s_n be the one partial sum., I.e, $s_n = \sum_{i=0}^{n} U_i(x,t)$. We shall prove that $\{\mathrm{S_n}\}$ is a Cauchy sequence in a Banach space E. By using a new formulation of Adomian polynomials, we get

$$R(s_n) = \widetilde{A_n} + \sum_{r=0}^{n-1}\widetilde{A_r}$$

$$U^k(s_n) = \widetilde{A_n} + \sum_{r=0}^{n-1}\widetilde{A_c}$$

$$\|s_n - s_m\| = \max_{t\in I}|s_n - s_m| = \max_{t\in I}\left|\sum_{i=m+1}^{n}\widetilde{U}_l(x,t)\right|$$

$$\le \max_{t\in I}\left|\begin{array}{c} Y^{-1}\left[v^{\alpha}Y\left[\sum_{i=m+1}^{n} L\left[U_{n-1}^{n}(x,t)\right]\right]\right] \\ +Y^{-1}\left[v^{\alpha}Y\left[\sum_{i=m+1}^{n} M\left[U_{n-1}^{n}(x,t)\right]\right]\right] \\ +Y^{-1}\left[v^{\alpha}Y\left[\sum_{i=m+1}^{n}\left[A_{n-1}(x,t)\right]\right]\right] \end{array}\right|$$

$$\leq \max_{t\in I}\left|\begin{array}{l} Y^{-1}\left[v^{\alpha}Y\left[\sum_{i=m}^{n-1}L\left[U_n(x,t)\right]\right]\right] \\ +Y^{-1}Y\left[\sum_{i=m}^{n-1}M\left[U_n(x,t)\right]\right] \\ +Y^{-1}\left[v^{\alpha}Y\left[\sum_{i=m+1}^{n}\left[A_{n-1}(x,t)\right]\right]\right] \end{array}\right|$$

$$\leq \max_{t\in I}\left|\begin{array}{c} Y^{-1}\left[v^{\alpha}Y\left[L\left(s_{n-1}\right)-L\left(s_{m-1}\right)\right]\right] \\ +Y^{-1}\left[v^{\alpha}Y\left[M\left(s_{n-1}\right)-M\left(s_{m-1}\right)\right]\right] \\ v^{\alpha}Y\left[\sum_{i=m+1}^{n}\left[U^{k}\left(s_{n-1}\right)-U^{k}\left(s_{m-1}\right)\right]\right] \end{array}\right|$$

$$\leq L_1\max_{t\in I}Y^{-1}\left|\left[v^{\alpha}Y\left[\left(s_{n-1}\right)-\left(s_{m-1}\right)\right]\right]\right|$$

$$+L_2\max_{t\in I}Y^{-1}\left|\left[v^{\alpha}Y\left[\left(s_{n-1}\right)-\left(s_{m-1}\right)\right]\right]\right|$$

$$+L_3\max_{t\in I}Y^{-1}\left|\left[v^{\alpha}Y\left[\left(s_{n-1}\right)-\left(s_{m-1}\right)\right]\right]\right|$$

$$\leq \frac{\left(L_1+L_2+L_3\right)t^{(\alpha-1)}}{(\alpha-1)!}\left\|s_{n-1}+s_{m-1}\right\|.$$

Let $n=m+1$, then

$$\left\|s_{m+1}+s_m\right\|\leq\varepsilon\left\|s_m+s_{m-1}\right\|\leq\varepsilon^2\left\|s_{m-1}+s_{m-2}\quad\right\|\leq\cdots\leq\varepsilon^m\left\|s_1+s_0\right\|$$

Where $\varepsilon=\frac{(L_1+L_2+L_2)t^{(\alpha-1)}}{(\alpha-1)!}$.

Similarly, we have, from the triangle inequality we have

$$\left\|s_{m+1}-s_m\right\|\leq\left\|s_{m+2}-s_{m+1}\right\|\leq\cdots\leq\left\|s_n-s_{n-1}\right\|\leq\left[\varepsilon^m+\varepsilon^{m+1}+\cdots+\varepsilon^{n-1}\right]\leq$$
$$\left\|s_1-s_0\right\|\leq\varepsilon^m\frac{1-\varepsilon^{n-m}}{\varepsilon}\left\|U_1\right\|,$$

since $0<\varepsilon<1$ we have

$$\left(1-\varepsilon^{n-m}\right)<1;\text{ then }\left\|s_n-s_m\right\|\frac{\varepsilon^m}{1-\varepsilon}\max_{t\in I}\left\|U_1\right\|$$

However, $|U_1|<\infty$ (since $U(x,t)$ is bounded),

So , as $m\to\infty$ then $\|s_n-s_m\|\to$ hence $\{s_n\}$ is a cauchy sequance in E, therefore the series $\sum_{n=0}^{\infty}U_n$ convergence and the prove is complete. □

5. Illustrative Examples

Example 5.1. *Consider the one-dimensional space FTE*

$$ {}_0^cD_x^{\alpha}U(x,t)=D_t^2U(x,t)+D_tU(x,t)+U(x,t),\quad 0<x<1,\quad 0<\alpha\leq 2, \tag{5.1}$$

with the initial and boundary conditions

$$U(0,t) = e^{-t},$$
$$U_x(0,t) = e^{-t},$$
$$U(x,0) = e^{x}$$
$$U_t(x,0) = 0.$$

Applying the YT on the both side of (5.1), *we have*

$$Y\left[{}_0^cD_x^\alpha U(x,t)\right] - Y\left[D_t^2U(x,t) + D_tU(x,t) + U(x,t)\right] = 0 \tag{5.2}$$

or

$$\frac{Y[U(x,t)]}{v^\alpha} - \sum_{k=0}^{m-1}\frac{U^{(k)}}{v^{\alpha-k-1}} = Y\left[D_t^2U(x,t) + D_tU(x,t) + U(x,t)\right] \tag{5.3}$$

$$\frac{Y[U(x,t)]}{v^\alpha} - \frac{U_{(0)}^{(0)}}{v^{\alpha-0-1}} - \frac{U_0^{(1)}}{v^{\alpha-1-1}} = Y\left[D_t^2U(x,t) + D_tU(x,t) + U(x,t)\right]$$

$$Y[U(x,t)] = v^\alpha\left[\frac{e^{-t}}{v^{\alpha-1}} + \frac{e^{-t}}{v^{\alpha-2}}\right] + v^\alpha Y\left[D_t^2U(x,t) + D_tU(x,t) + U(x,t)\right]$$

$$Y[U(x,t)] = (1-v)e^{-t} + v^\alpha Y\left[D_t^2U(x,t) + D_tU(x,t) + U(x,t)\right]. \tag{5.4}$$

Applying the invers YT to the both side of (5.4), *we get*

$$U(x,t) = (1-x)e^{-t} + Y^{-1}\left[\left[v^\alpha Y\left[D_t^2U(x,t) + D_tU(x,t) + U(x,t)\right]\right]\right].$$

Then, we have

$$U_0(x,t) = e^{-t}(1-x)$$

Next, when we use $U_0(x,t)$ *to calculate* $U_1(x,t)$

$$\begin{aligned}
U_1(x,t) &= Y^{-1}\left[v^\alpha Y\left[D_t^2U_0(x,t) + D_tU_0(x,t) + U_0(x,t)\right]\right] \\
&= Y^{-1}\left[v^\alpha Y\left[D_t^2\left[e^{-t}(1-x)\right] + D_t\left[e^{-t}(1-x)\right] + e^{-t}(1-x)\right]\right] \\
&= Y^{-1}\left[v^\alpha Y\left[(1-x)D_t^2\left(e^{-t}\right) + (1-x)D_t\left(e^{-t}\right) + e^{-t}(1-x)\right]\right] \\
&= Y^{-1}\left[v^\alpha Y\left[(1-x)\left(e^{-t}\right) - (1-x)\left(e^{-t}\right) + e^{-t}(1-x)\right]\right] \\
&= Y^{-1}\left[v^\alpha Y\left[e^{-t}(1-x)\right]\right] \\
&= Y^{-1}\left[\left[e^{-t}\left(v - v^2\right)v^\alpha\right]\right] \\
&= Y^{-1}\left[\left[e^{-t}\left(v^{\alpha+1} - v^{\alpha+2}\right)\right]\right] \\
&= \left[e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha+1}{\Gamma(\alpha+2)}\right]\right]
\end{aligned}$$

After that using $U_1(x,t)$, we get

$$U_2(x,t) = Y^{-1}\left[v^\alpha Y\left[D_t^2 U_1(x,t) + D_t U_1(x,t) + U_1(x,t)\right]\right]$$

$$U_2(x,t) = Y^{-1}\left(v^\alpha Y\left[D_t^2\left[e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha+1}{\Gamma(\alpha+2)}\right] + D_t\left[e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha+1}{\Gamma(\alpha+2)}\right] + \right.\right.\right.$$

$$\left.\left.\left. e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha+1}{\Gamma(\alpha+2)}\right]\right]\right]\right)$$

$$= Y^{-1}\left[v^\alpha Y\left[e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha+1}{\Gamma(\alpha+2)}\right] - \left[e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha+1}{\Gamma(\alpha+2)}\right] + e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha+1}{\Gamma(\alpha+2)}\right]\right]\right.\right.$$

$$= Y^{-1}\left[v^\alpha Y\left[e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha+1}{\Gamma(\alpha+2)}\right]\right.\right.$$

$$= e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right]$$

Now Use $U_2(x,t)$ colculus $U_3(x,t)$

$$U_3(x,t) = Y^{-1}\left[v^\alpha Y\left[D_t^2\left[e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right]\right] + D_t\left[e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right]\right] + \right.\right.$$

$$\left.\left.\left[e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right]\right]\right]\right]$$

$$U_3(x,t) = Y^{-1}\left[v^\alpha Y\left[\left[e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right]\right] - \left[e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right]\right] + \right.\right.$$

$$\left.\left.\left[e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right]\right]\right.\right.$$

$$U_3(x,t) = Y^{-1}\left[v^\alpha Y\left[e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right]\right]\right]$$

$$U_3(x,t) = e^{-t}\left[\frac{x^{4\alpha}}{\Gamma(4\alpha+1)} + \frac{x^{4\alpha}+1}{\Gamma(4\alpha+2)}\right]$$

So that, the approximate solution is

$$U(x,t) = e^{-t}(1-x) + e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha+1}{\Gamma(\alpha+2)}\right] + e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right]$$
$$+ e^{-t}\left[\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha}+1}{\Gamma(2\alpha+2)}\right] + \cdots$$

$$U(x,t) = \sum_{m=0}^{\infty} e^{-t}\left[\frac{x^{m\alpha}}{\Gamma(x^{m\alpha}+1)} + \frac{x^{m\alpha+1}}{\Gamma(x^{m\alpha}+2)}\right]$$

When $\alpha = 2$, we have

$$U(x,t) = e^{x-t}$$

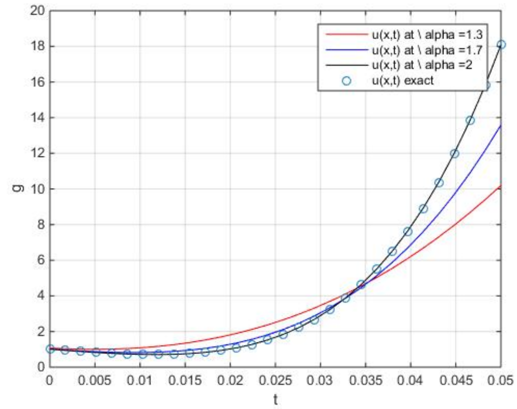

FIGURE 1. Plot of approximate solutions $U(x,t)$ at different values of α at $t = 0.5$ and compression with exact solution e^{x-t}.

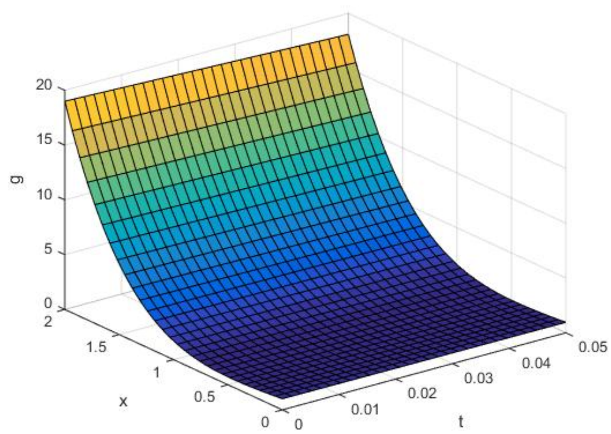

FIGURE 2. The surface shows the YADM solution $U(x,t)$ for example 1, when $\alpha = 2$.

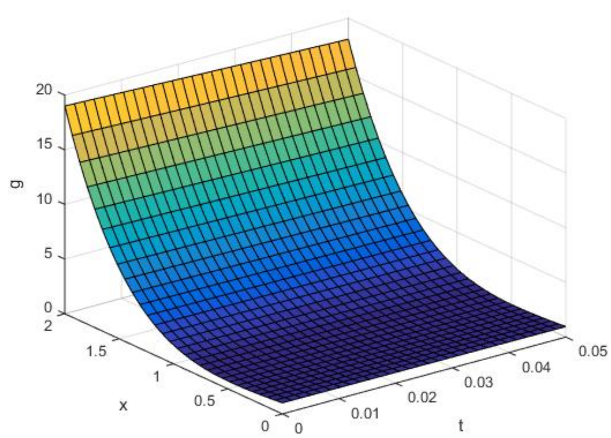

FIGURE 3. The surface shows the YADM solution $U(x,t)$ for example 1, when $\alpha = 1.7$.

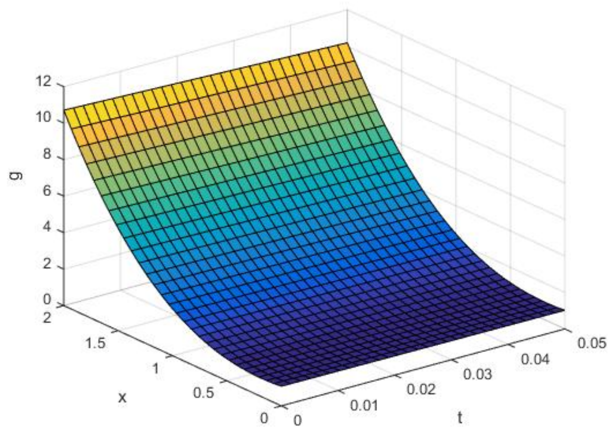

FIGURE 4. The surface shows the YADM solution $U(x,t)$ for example 1, when $\alpha = 1.3$.

Example 5.2. *Consider the following space-fractional nonlinear telegraph equation.*

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} - u^2(x,t) + xu(x,t)u_x(x,t), \quad x,t \geq 0 \quad , \quad 0 < \alpha \leq 2 \tag{5.5}$$

with the initial conditions

$$u(x,0) = x, u_t(x,0) = x,$$

By taking Yang transform for (5.5), *we have*

$$\mathrm{Y}\left[\frac{\partial^\alpha u}{\partial t^\alpha}\right] = \mathrm{Y}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} - u^2(x,t) + xu(x,t)u_x(x,t)\right]$$
$$\frac{Y[u(x,t)]}{v^\alpha} - \frac{u(x,0)}{v^{\alpha-1}} - \frac{u_t(x,o)}{v^{\alpha-2}} = \mathrm{Y}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} - u^2(x,t) + xu(x,t)u_x(x,t)\right]$$

Arrangement and substitute the initial condition, we get

$$Y[u(x,t)] = vx + v^2 x + v^\alpha Y\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} - u^2(x,t) + xu(x,t)u_x(x,t)\right]$$

Applying the invers Yang transform

$$u(x,t) = Y^{-1}\left[vx + v^2 x\right] + Y^{-1}\left[v^\alpha Y\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} - u^2(x,t) + xu(x,t)u_x(x,t)\right]\right]$$
$$u(x,t) = x + xt + Y^{-1}\left[v^\alpha Y\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} - u^2(x,t) + xu(x,t)u_x(x,t)\right]\right]$$

Hence

$$u_{n+1}(x,t) = (x+xt) + Y^{-1}\left(v^{\alpha} Y\left[\sum_{n=0}^{\infty}\frac{\partial^2 u_n(x,t)}{\partial x^2} + \sum_{n=0}^{\infty}\frac{\partial u_n(x,t)}{\partial x} - \sum_{n=0}^{\infty} A_n(x,t) + \sum_{n=0}^{\infty} B_n x u(x,t) u_x(x,t)\right]\right)$$

The initial term

$$u_0(x,t) = x + xt$$
$$u_1(x,t) = \left(\frac{t^{\alpha}}{\Gamma(\alpha+1)}x\right),$$
$$u_2(x,t) = \left(\frac{t^{\alpha+1}}{\Gamma(\alpha+2)}x\right),$$
$$u_3(x,t) = \left(\frac{t^{\alpha+2}}{\Gamma(\alpha+3)}x\right)$$

Therefore, the approximate is

$$u_n(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots = x + xt + \frac{t^{\alpha}}{\Gamma(\alpha+1)}x + \frac{t^{\alpha+1}}{\Gamma(\alpha+2)}x + \frac{t^{\alpha+2}}{\Gamma(\alpha+3)}x + \cdots$$

substituting $\alpha = 2$, *We obtain the exact solution of standard Telegraph Equation in the following from*

$$u(x,t) = xe^{t}$$

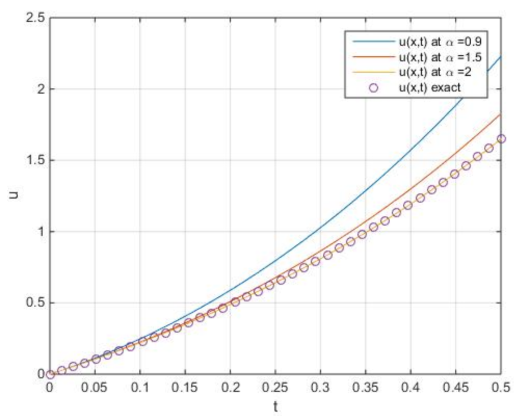

Figure 5. Plot of approximate solutions $U(x,t)$ at different values of α at $t = 0.5$ and compression with exact solution e^{x-t}.

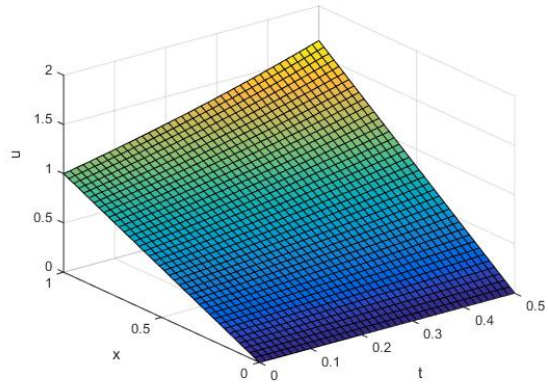

FIGURE 6. The surface shows the YADM solution $U(x,t)$ for example 2, when α

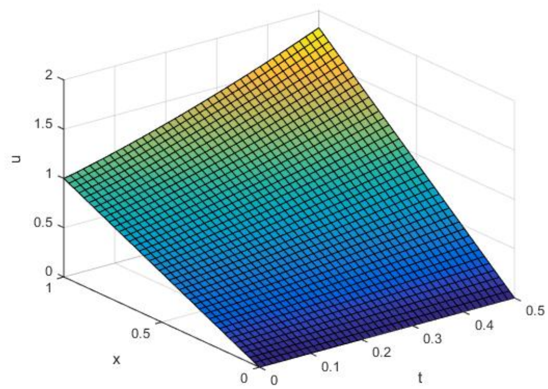

FIGURE 7. The surface shows the YADM solution $U(x,t)$ for example 2, when $\alpha = 1.5$

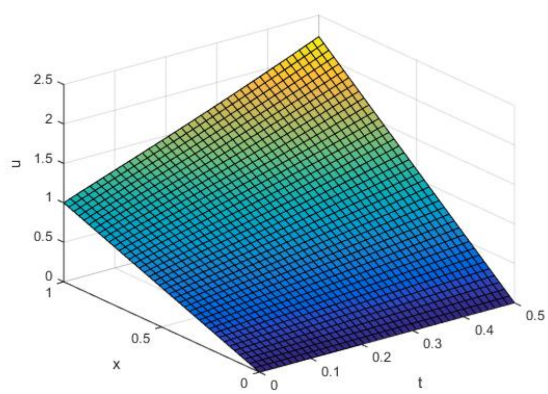

FIGURE 8. The surface shows the YADM solution $U(x,t)$ for example 1, when $\alpha = 1.5$.

Example 5.3. *Consider the space-fractional linear telegraph equation of nonhomogeneous type*

$$\frac{\partial^\alpha u}{\partial x^\alpha} = \frac{\partial^2 u}{\partial^2 t} + \frac{\partial u}{\partial t} + u - x^2 - t + 1 \quad ,0 < \alpha < 2, t \geq 0 \tag{5.6}$$

with initial condition

$$u(0,t) = t, \quad u_x(0,t) = 0$$

Taking the yang transform of the equation

$$Y\left[\frac{\partial^\alpha u}{\partial x^\alpha}\right] = Y\left[\frac{\partial^2 u}{\partial^2 t} + \frac{\partial u}{\partial t} + u\right] + Y\left[-x^2 - t + 1\right]$$

$$Y\left[\frac{u(x,t)}{v^\alpha}\right] - \frac{u(0,t)}{v^{\alpha-1}} - \frac{u_x(0,t)}{v^{\alpha-2}} = Y\left[\frac{\partial^2 u}{\partial^2 t} + \frac{\partial u}{\partial t} + u\right] + \left[-2v^{\alpha+2} - v^\alpha t + v^\alpha\right]$$

$$Y[u(x,t)] = \left[v^\alpha t - 2v^{2\alpha+2} - v^{2\alpha} t + v^{2\alpha}\right] + v^\alpha Y\left[\frac{\partial^2 u}{\partial^2 t} + \frac{\partial u}{\partial t} + u\right]$$

Appling invers yang transform it give

$$u(x,t) = Y^{-1}\left[v^\alpha t - 2v^{2\alpha+2} - v^{2\alpha} t + v^{2\alpha}\right] + Y^{-1}\left[v^\alpha Y\left[\frac{\partial^2 u}{\partial^2 t} + \frac{\partial u}{\partial t} + u\right]\right]$$

$$+ Y^{-1}\left[v^\alpha Y\left[\frac{\partial^2 u}{\partial^2 t} + \frac{\partial u}{\partial t} + u\right]\right] = t - \frac{2x^{\alpha+2}}{\Gamma(\alpha+3)} - \frac{tx^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha}{\Gamma(\alpha+1)}$$

$$u_0(x,t) = t - \frac{2x^{\alpha+2}}{\Gamma(\alpha+3)} - \frac{tx^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha}{\Gamma(\alpha+1)}$$

$$u_1(x,t) = Y^{-1}\left[v^\alpha Y\left[\frac{\partial^2 u_0}{\partial^2 t} + \frac{\partial u_0}{\partial t} + u_0\right]\right]$$

$$= Y^{-1}\left[v^\alpha Y\left[0 + 1 - \frac{x^\alpha}{\Gamma(\alpha+1)} + t - \frac{2x^{\alpha+2}}{\Gamma(\alpha+3)} - \frac{tx^\alpha}{\Gamma(\alpha+1)} - \frac{tx^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha}{\Gamma(\alpha+1)}\right]\right]$$

$$= \frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{tx^\alpha}{\Gamma(\alpha+1)} - \frac{2x^{\alpha+2}}{\Gamma(\alpha+3)} - \frac{tx^{2\alpha}}{\Gamma(\alpha+1)}$$

$$u_2(x,t) = Y^{-1}\left[v^\alpha Y\left[\frac{\partial^2 u_1}{\partial^2 t} + \frac{\partial u_1}{\partial t} + u_1\right]\right]$$

$$= \frac{2x^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{x^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{tx^{2\alpha}}{\gamma(2\alpha+1)} - \frac{2x^{3\alpha+2}}{\gamma(3\alpha+3)} - \frac{tx^{3\alpha}}{\gamma(3\alpha+1)}$$

Similarly, with find out the components ahead as authors have found out the previous components

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots$$

$$= t - \frac{2x^{\alpha+2}}{\Gamma(\alpha+3)} - \frac{tx^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{tx^\alpha}{\Gamma(\alpha+1)}$$

$$- \frac{2x^{\alpha+2}}{\Gamma(\alpha+3)} - \frac{tx^{2\alpha}}{\Gamma(\alpha+1)} + \frac{2x^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{x^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{tx^{2\alpha}}{\Gamma(2\alpha+1)}$$

$$- \frac{2x^{3\alpha+2}}{\Gamma(3\alpha+3)} - \frac{tx^{3\alpha}}{\Gamma(3\alpha+1)}$$

When $\alpha = 2$, we get YADM solution

$$u(x,t) = t - \frac{2x^4}{\Gamma(5)} - \frac{tx^2}{\Gamma(3)} + \frac{x^2}{\Gamma(3)} + \frac{x^2}{\Gamma(3)} + \frac{tx^2}{\Gamma(3)} - \frac{2x^6}{\Gamma(7)} - \frac{tx^4}{\Gamma(5)} + \frac{2x^4}{\gamma\Gamma(5)} - \frac{x^6}{\Gamma(7)} + \frac{tx^4}{\Gamma(5)} - \frac{2x^8}{\Gamma(9)} - \frac{tx^6}{\Gamma(7)}.$$

This solution is equivalent to exact solution

$$u(x,t) = t + x^2.$$

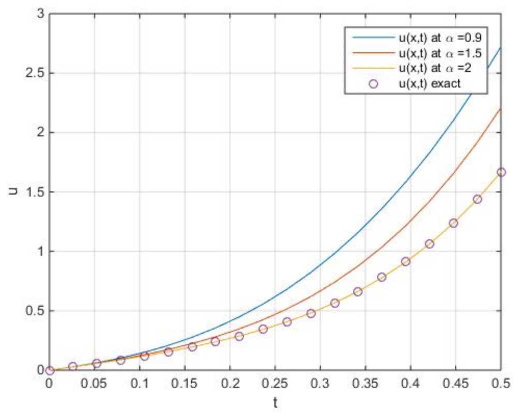

FIGURE 9. Plot of approximate solutions $U(x,t)$ at different values of α at $t = 0.5$ and compression with exact solution.

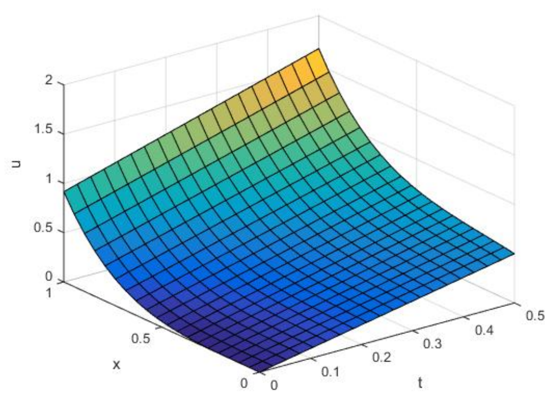

FIGURE 10. The surface shows the YADM solution $U(x,t)$ for example 2, when $\alpha = 2$

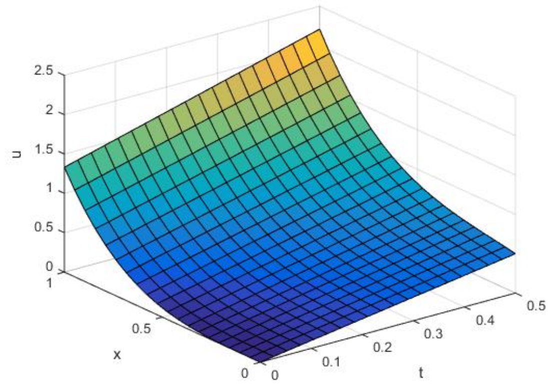

FIGURE 11. The surface shows the YADM solution $U(x,t)$ for example 2, when $\alpha = 1.5$

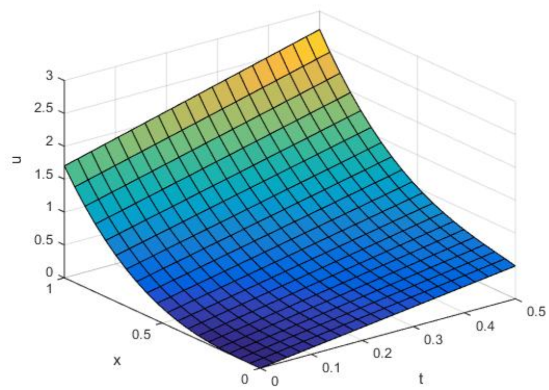

FIGURE 12. The surface shows the YADM solution $U(x,t)$ for example 2, when $\alpha = 0.9$

6. Conclusion

In conclusion, this article investigates the use of YDM to obtain approximate analytical solutions of telegraph equation. Through a careful comparative analysis between these approximate solutions and exact solutions, supported by 2D and 3D graphs generated using the Maple platform, the analysis sheds light on the accuracy and confidence of the YDM in solving fractional differential equations.

Authors' Contributions

All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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