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MULTIACTOR, MULTIDIMENSIONAL APPROACH INTEGRATING FUZZY LOGIC AND THE AHP METHOD FOR PRIORITIZING INTERVENTION AREAS IN A CRISIS CONTEXT (F-AHPD)

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Abstract. Burkina Faso is facing a growing terrorist threat, with major socio-economic and security impacts. This study proposes an innovative multi-actor, multidimensional classification approach to assess and rank affected areas. The objective is to provide a robust decision-support tool for policymakers, humanitarian organizations, and security forces to rigorously, transparently, and participatively prioritize interventions and the effective allocation of resources, thereby optimizing territory security. The new method, called Fuzzy-AHP for Decision-Making (F-AHPD), is based on two tools: fuzzy measure theory to integrate and assess the uncertainty and variability of data related to terrorism, and the AHP method to hierarchically structure the problem and verify the consistency of judgments provided by relevant stakeholders. The data used pertains to eight localities in the country and draws on certain information published on social media. The classification identified three categories of areas based on criticality level: critical, high risk, and moderate risk. The results show that 25% of the studied localities are classified as critical, 62.5% as high risk, and 12.5% as moderate risk. The method enabled the prioritization of criteria, with the frequency and severity of attacks being the most influential factors.

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1. Introduction

Like many countries, Burkina Faso is facing an escalating terrorist threat, resulting in significant socio-economic consequences [17]. Numerous localities are regularly affected by attacks, while response capacities, though existent, remain clearly insufficient to contain the crisis [1].

A structured approach to intervention prioritization is therefore essential. It should enable optimal allocation of resources to the most vulnerable areas and deliver a response that is both rapid and effective [20]. As these authors emphasize, decision-making in crisis contexts requires integrating multiple perspectives within complex scenarios.

This prioritization work benefits from being conducted collectively, involving experts from various disciplines [2, 10]. Indeed, the growing complexity of crises often exceeds the comprehension capacity of any single decision-maker [3]. Furthermore, available data is frequently incomplete or imprecise, necessitating models capable of formalizing uncertainty [13, 5]. Fuzzy logic, in particular, provides a suitable framework for representing expert judgments and reasoning from qualitative data.

To address these challenges, we propose a hybrid methodology combining fuzzy logic and Saaty's (1980) Analytic Hierarchy Process (AHP) [22]. This approach enables the integration of both information imprecision and diverse stakeholder perspectives, aiming to establish robust and transparent prioritization of intervention zones.

Following a literature review on the AHP method and fuzzy logic, we will present the proposed methodological framework. This will then be applied to a concrete case: terrorism risk assessment for localities in Burkina Faso. A critical analysis of F-AHPD will highlight its strengths, weaknesses, and innovations. Finally, a conclusion will summarize key findings, discuss their implications, and suggest future research directions.

2. Literature Review on the AHP Method and Fuzzy Logic

2.1. **The AHP Method: Fundamentals, Applications, and Integration with Other Methods.**

2.1.1. *Fundamentals and Applications.* The Analytic Hierarchy Process (AHP), developed by Thomas Saaty in the 1970s, is a structured method for complex multi-criteria decision-making. It enables the decomposition of a problem into a hierarchy, the evaluation of criteria through pairwise comparisons, and the synthesis of results to prioritize alternatives. Its strength lies in its ability to combine qualitative and quantitative judgments while checking the consistency of evaluations [15]. Today, AHP is applied in various fields such as public health for medical alliance resilience [4], management of competitive strategies in the Balkans [14], energy for prioritizing sustainable projects [12], and technology for software quality assessment [21].

Hierarchical Structure and Pairwise Comparisons

AHP is based on three fundamental principles:

- decomposition: The problem is divided into a hierarchy (goal > criteria > sub-criteria > alternatives);
- pairwise Comparisons: elements are evaluated two-by-two using Saaty's scale (1 to 9), where 1 indicates equality and 9 indicates absolute superiority [12];
- priority Synthesis: Eigenvectors of the comparison matrices generate local (per level) and global (aggregated) weights.

Judgment Consistency

A consistency ratio CR, calculated from the maximum eigenvalue($\lambda_{\max}$), validates the reliability of judgments. A $CR < 0.1$ is acceptable. Interactive tools using greedy algorithms adjust inconsistencies in real time, preserving the integrity of expert evaluations [7].

2.1.2. *Integration with Other Methods.*

- Fuzzy AHP: addresses judgment imprecision through membership functions, useful in risk assessment [21];
- SWOT-AHP: combines contextual analysis (Strengths/Weaknesses/Opportunities/Threats) with prioritization, as seen in the Korean study on health portals [9];
- AHP-ANP (Analytic Network Process): handles interdependent problems (e.g., public policies), overcoming the limitations of linear hierarchies [15].

2.2. **Fuzzy logic.**

2.2.1. *Concept of fuzzy logic.* It was in 1965 that Professor Lotfi A. Zadeh of the University of California (Berkeley) introduced fuzzy set theory in his seminal paper titled Fuzzy Sets, published in the journal Information and Control [13]. This revolutionary theory distinguishes itself from classical logic by enabling the mathematical modeling of imprecise and subjective notions, thereby aligning more closely with natural human reasoning [5]. Unlike the binary variables of Boolean logic, fuzzy sets manipulate linguistic variables (e.g., "tall," "small," "hot") that capture the gradation of properties in everyday language [8].

To contextualize this innovation, a brief review of classical set theory is necessary. A classical set is characterized by sharp boundaries and binary membership: an element either fully belongs to the set or does not belong at all (true/false, 1/0) [16].

A classical set A is defined by a characteristic function $\varphi_A(x)$ that enforces a strict binary distinction between elements, allowing no representation of intermediate cases [16]. This function, also referred to as a binary membership function, assigns to every element x in the universe of discourse a value in

$\{0,1\}$:

$$\varphi : x \in A- > \varphi_A(x) \in 0,1 = \begin{cases} \varphi_A(x) = 1 & \text{if } x \in A \\ \varphi_A(x) = 0 & \text{if } x \notin A \end{cases} \tag{1}$$

"' This modeling, although fundamental in mathematics, has limitations in capturing the notion of gradation or partial membership, such as in the case of fuzzy attributes like "average height" [13].

To address this rigidity, fuzzy set theory introduced by Zadeh (1965) [13] generalizes this concept through a membership function $\mu_A(x)$ with values in the continuous interval $[0,1]$. This function quantifies the degree to which an element x belongs to the fuzzy set A:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \text{ fully belongs to } A \\ d \in (0,1) & \text{if } x \text{ partially belongs to } A \cdot \\ 0 & \text{if } x \text{ does not belong to } A \end{cases} \tag{2}$$

Unlike classical logic, which makes a sharp distinction, this modeling approach allows for the mathematical representation of imprecise concepts and categories with gradual boundaries, bringing it closer to natural human reasoning [5,8].

2.2.2. *Fuzzy number.*

Definition 2.1. *Trapezoidal fuzzy number.*

A trapezoidal fuzzy number $\widetilde{Z}$, denoted $\widetilde{Z} = (a,b,c,d)$, is a special case of a fuzzy number whose membership function $\mu_{\widetilde{Z}}(x)$ forms a trapezoid on the real axis $\mathbb{R}$ [6]. This parametric representation fully characterizes the fuzzy number using four critical values:

- *a and d define the support of $\widetilde{Z}$ (interval where $\mu_{\widetilde{Z}}(x) > 0$),*
- *b and c define its core (interval where $\mu_{\widetilde{Z}}(x) = 1$).*

Associated membership function:

$$\mu_{\widetilde{Z}}(x) = \begin{cases} 0 & \text{if } x \leq a \text{ or } x \geq d \\ \frac{x-a}{b-a} & \text{if } a < x < b \\ 1 & \text{if } b \leq x \leq c \\ \frac{d-x}{d-c} & \text{if } c < x < d \end{cases} . \tag{3}$$

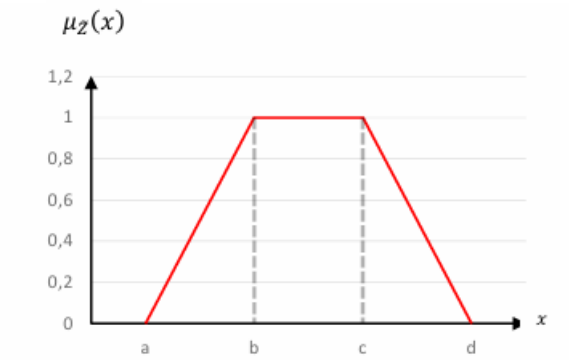

FIGURE 1. Membership function of a trapezoidal fuzzy number.

Definition 2.2. *: Triangular fuzzy number.*

A triangular fuzzy number $\widetilde{Z}$, denoted $\widetilde{Z} = (a, b, c)$, is a special case of a fuzzy number whose membership function $\mu_{\widetilde{Z}}(x)$ forms a triangle on the real axis $\mathbb{R}$ [6]. It is characterized by three parameters:

- *a: lower bound (start of the support),*
- *b: unique modal value (core where $\mu_{\widetilde{Z}}(b) = 1$),*
- *c: upper bound (end of the support).*

Associated membership function:

$$\mu_{\widetilde{Z}}(x) = \begin{cases} 0 & \text{if } x \leq a \text{ or } x \geq c \\ \frac{x-a}{b-a} & \text{if } a < x < b \\ 1 & \text{if } x = b \\ \frac{c-x}{c-b} & \text{if } b < x < c \end{cases} \tag{4}$$

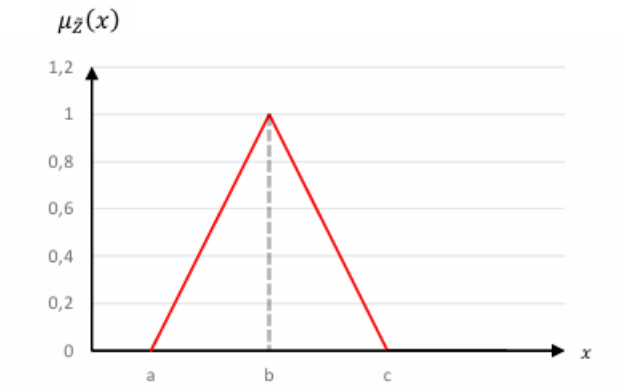

FIGURE 2. Membership function of a triangular fuzzy number.

3. Proposed Methodological Framework: Fuzzy–AHP for Decision-Making (F-AHPD)

F-AHPD is a fuzzy multi-criteria decision-making process structured into 6 steps:

3.1. Description of the F-AHPD method.

This study deviates from the conventional framework of the Analytic Hierarchy Process (AHP) by adopting a fuzzy modeling of evaluations. In line with the foundational work of Buckley (1985) [11] on the fuzzy extension of AHP, we represent the performances of alternatives and the weights of criteria using triangular fuzzy numbers:

$$\widetilde{X_{ijk}} = \left(x_{ijk}^{l}, x_{ijk}^{m}, x_{ijk}^{r}\right) \; et \; \widetilde{W_{jk}} = \left(\widetilde{W_{jk}^{l}}, \widetilde{W_{jk}^{m}}, \ldots, \widetilde{W_{jk}^{r}}\right).$$

where i=1,2, …, n; j=1,2, …, m et k=1,2, …, s .

Step 1:Aggregation of expert evaluations and weights: Fusion of fuzzy evaluations and weights via min/average/max.

- The vector of aggregated fuzzy evaluations for an alternative a_i (i=1,2, …, n). The aggregated fuzzy evaluations are calculated based on equation (5).

$$x_{ij}^{l} = \min_{k}\left\{x_{ijk}^{l}\right\};\; x_{ij}^{m} = \frac{1}{s}\sum_{k=1}^{s} x_{ijk}^{m};\; x_{ij}^{r} = \max_{k}\left\{x_{ijk}^{r}\right\}. \tag{5}$$

  This yields the following vector: $\widetilde{X_{ij}} = \left(x_{ij}^{l}, x_{ij}^{m}, x_{ij}^{r}\right)$.

- The vector of aggregated fuzzy weights for a criterion c_j (j=1,2, …, m). The aggregated fuzzy weights are calculated based on equation (6).

$$w_{j}^{l} = \min_{k}\left\{w_{jk}^{l}\right\};\; w_{j}^{m} = \frac{1}{s}\sum_{k=1}^{s} x_{jk}^{m};\; w_{j}^{r} = \max_{k}\left\{w_{jk}^{r}\right\}. \tag{6}$$

  This yields the following vector:$\widetilde{w_j} = \left(w_{j}^{l}, w_{j}^{m}, w_{j}^{r}\right)$.

- The matrices of aggregated fuzzy evaluations and aggregated fuzzy weights are respectively (7):

$$\widetilde{X} = \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{13} & \ldots & \tilde{x}_{1m} \\ \tilde{x}_{21} & \tilde{x}_{22} & \tilde{x}_{23} & \ldots & \tilde{x}_{2m} \\ \tilde{x}_{31} & \tilde{x}_{32} & \tilde{x}_{33} & \ldots & \tilde{x}_{3m} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \tilde{x}_{n1} & \tilde{x}_{n2} & \tilde{x}_{n3} & \ldots & \tilde{x}_{nm} \end{pmatrix} \; et \; \widetilde{W} = \left(\widetilde{W}_1, \widetilde{W}_2, \ldots, \widetilde{W}_m\right) \tag{7}$$

Step 2: Defuzzification: conversion of fuzzy numbers into crisp values using Yager's index.

In accordance with the defuzzification method established by Yager (1981) [18], the triangular fuzzy numbers representing evaluations and weights are converted into crisp numerical values by calculating the ranking index, followed by standard normalization to obtain a consistent weight distribution

summing to unity. The crisp value, denoted as I, is obtained using equation (8):

$$I(x_{ij}^l, x_{ij}^m, x_{ij}^r) = \frac{1}{4}(x_{ij}^l + 2x_{ij}^m + x_{ij}^r) \text{ et } I(w_j^l, w_j^m, w_j^r) = \frac{1}{4}(w_j^l + 2w_j^m + w_j^r) \tag{8}$$

This yields the following decision matrices:

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \dots & x_{1m} \\ x_{21} & x_{22} & x_{23} & \dots & x_{2m} \\ x_{31} & x_{32} & x_{33} & \dots & x_{3m} \\ \dots & \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & x_{n3} & \dots & x_{nm} \end{pmatrix} et\ W = (W_1, W_2, \dots, W_m)\,. \tag{9}$$

The weights are then normalized using the equation (10):

$$W_j^N = \frac{W_j}{\sum_{j=1}^{m} W_j}. \tag{10}$$

And the matrix of normalized weights is given by (11):

$$W^N = \left(W_1^N, W_2^N, \dots, W_m^N\right) \tag{11}$$

Step 3:Pairwise comparison matrices: Construction of judgment matrices based on relative deviations from evaluations, normalized for homogeneity.

- For each criterion c_j, the mean absolute deviatione $e(c_j)$ of the aggregated fuzzy evaluations related to it is calculated as a measure of the dispersion of expert judgments according to equation (12). This metric quantifies the inter-decision-maker variability for a given criterion:

$$e(c_j) = \frac{1}{n}\sum_{i=1}^{n} |x_{ij} - x_e(j)|. \tag{12}$$

  where x_{ij} and $x_e(j)$ are the aggregated evaluations and their arithmetic mean, respectively.

- Pairwise comparison judgments are determined using equations (13) and (14).

  If the criterion c_j is to be maximized, then:

$$J_{ilj} = \begin{cases} \text{round}\left(\frac{|x_{ij}-x_{lj}|}{e(c_j)} + 1\right) & \text{if} \quad x_{ij} > x_{lj} \\ \frac{1}{\text{round}\left(\frac{|x_{ij}-x_{lj}|}{e(c_j)}+1\right)} & \text{else} \end{cases} \tag{13}$$

  If the criterion c_j is to be minimized, then:

$$J_{ilj} = \begin{cases} \text{round}\left(\frac{|x_{ij}-x_{ij}|}{e(c_j)} + 1\right) & \text{if} \quad x_{ij} < x_{lj} \\ \frac{1}{\text{round}\left(\frac{|x_{ij}-x_{lj}|}{e(c_j)}+1\right)} & \text{else} \end{cases} \tag{14}$$

  where "round(x)" denotes the integer closest to x.

- The judgment matrix is thus given by (15):

$$J_j = \begin{pmatrix} J_{11j} & J_{12j} & \cdots & J_{1nj} \\ J_{21j} & J_{22j} & \cdots & J_{2nj} \\ \vdots & \vdots & \ddots & \vdots \\ J_{n1j} & J_{n2j} & \cdots & J_{nnj} \end{pmatrix}. \tag{15}$$

- Normalization of the judgment matrix.

  Normalization of judgments via linear transformation to homogenize measurement scales. Each normalized judgment is determined by equation (16):

$$N_{ilj} = \frac{J_{ilj}}{\sum_{i=1}^{n} J_{ilj}}. \tag{16}$$

The matrix of normalized judgments is thus represented as follows:

$$N_j = \begin{pmatrix} N_{11j} & N_{12j} & \cdots & N_{1nj} \\ N_{21j} & N_{22j} & \cdots & N_{2nj} \\ \vdots & \vdots & \ddots & \vdots \\ N_{n1j} & N_{n2j} & \cdots & N_{nnj} \end{pmatrix}. \tag{17}$$

Step 4: Priority of alternatives: Calculation of priority scores by averaging normalized judgments. The priority P_{ij} of an alternative a_i relative to a criterion c_j is given by equation (18):

$$P_{ij} = \frac{1}{n} \sum_{l=1}^{n} N_{ilj} \tag{18}$$

and we construct the matrix P_j of priorities as follows:

$$P_j = \begin{pmatrix} P_{1j} \\ P_{2j} \\ \vdots \\ P_{nj} \end{pmatrix} \tag{19}$$

Step 5: Consistency control: Verification of judgment consistency via the consistency ratio (inspired by AHP).

- Calculate the eigenvalues of the judgment matrix using the equation: (20):

$$J_j * P_j = \lambda_j * P_j \tag{20}$$

with $\lambda_j = \begin{pmatrix} \lambda_{1j} \\ \lambda_{2j} \\ \vdots \\ \lambda_{nj} \end{pmatrix}$

- Calculate the maximum eigenvalue using the equation: (21).

$$\lambda_{j\,\max} = \frac{\sum_{i=1}^{n} \lambda_{ij}}{n} \tag{21}$$

- Calculate the consistency index using the equation: (22):

$$CI_j = \frac{\lambda_{j\,\max} - n}{n-1} \tag{22}$$

- Identify the randomized index RI from the following table:

TABLE 1. Random Indices [22].

taille de la matrice	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

- Calculate the fuzzy consistency ratio using the equation: (23)

$$CR_j = \frac{CI_j}{RI_j} \tag{23}$$

If: $\begin{cases} \mathrm{CR_j} < 0,1 \text{ then the consistency of the judgments is acceptable} \\ CR_j \geq 0,1 \text{ then the consistency of the judgments is not acceptable.} \end{cases}$

Step 6: Terminal aggregation: Calculation of global scores through linear combination of priorities and weights. After validating the consistency of judgments (steps 1-4), the global score S_i of each alternative a_i is calculated by aggregating the priorities using the equation: (24).

$$S_i = \sum_{j=1}^{m} W_j^N * P_{ij} \tag{24}$$

3.2. **Computational study on F-AHPD.** Complexity analysis, particularly time complexity analysis, is a fundamental step in evaluating an algorithmic method. Indeed, it enables:

Understanding the scalability of the method's execution time with the size of input data,

Allocating the necessary computational resources (processor, memory) to execute the method,

Identifying the most computationally expensive steps of the method,

Comparing the method with other approaches.

Step 1: Aggregation of fuzzy matrices.

- Element-wise operations.
  - For each fuzzy evaluation

    $\tilde{x}_{ij}$ (for n alternatives and m criteria):
    - Calculation of x_{ij}^{l} : min over s experts $\rightarrow O(s)$
    - Calculation of x_{ij}^{m} : average over s experts $\rightarrow O(s)$
    - Calculation of x_{ij}^{r} : max over s experts $\rightarrow O(s)$

  - For each fuzzy weight $\tilde{w}_j$ (for m criteria):
    - Same operations $\rightarrow O(s)$ per criterion.
- Total complexity:
  - Evaluations: $n \times m \times 3 \times s \rightarrow O(n \cdot m \cdot s)$
  - Weights: $m \times 3 \times s \rightarrow O(m \cdot s)$
  - Total: $O(n \cdot m \cdot s)$

Step 2: Defuzzification and Normalization

- Défuzzification:
  - Apply Yager's formula to each evaluation $(n \times m)$ and each weight (m) $\rightarrow O(n \cdot m)$
- Normalization of weights:
  - Sum of the m weights $\rightarrow O(m)$
  - Division of each weight by the sum $\rightarrow O(m)$
- Total complexity: $O(n \cdot m)$

Step 3: Construction of pairwise comparison matrices

- Calculation of the mean absolute deviation $e\left(c_j\right)$:
  - For each criterion j:
    - Average of x_{ij} over n alternatives $\rightarrow$ O(n)
    - Calculation of $|x_{ij} - x_e(j)|$ for n alternatives $\rightarrow$ O(n)
    - Average of these deviations $\rightarrow$ O(n)
  - For m criteria $\rightarrow O(m \cdot n)$
- Construction of judgments J_{ilj}:
  - For each criterion j , and each pair of actions (i, l) (with n^2 pairs):
    - Calculation of $|x_{ij} - x_{lj}|$, division by $e\left(c_j\right)$, addition, and rounding $\rightarrow O(1)$.
  - For m m criteria $\rightarrow O\left(m \cdot n^2\right)$
- Normalization of judgments N_{ilj}:
  - For each criterion j and each column l of the matrix J_j :
    - Sum of the n n elements of the column $\rightarrow O(n)$
    - Division of each element by the sum. $\rightarrow O(n)$
  - For m criteria and n columns $\rightarrow O\left(m \cdot n^2\right)$
- Total complexity: $O\left(m \cdot n^2\right)$

Step 4: Determination of priorities. P_{ij}

- For each criterion j and each alternative i:
  - Average of the n elements in row i of $N_j \rightarrow O(n)$
- For m criteria $\rightarrow O\left(m \cdot n^2\right)$

Step 5: Consistency control.

- Calculation of eigenvalues:
  - For each criterion j:
    - Matrix-vector product $J_j \cdot P_j \to O\left(n^2\right)$
    - Calculation of $\lambda_{ij} = \left(J_j \cdot P_j\right)_i / P_{ij} \to O(n)$
    - Average of the $\lambda_{ij} \to O(n)$
  - For m criteria $\to O\left(m \cdot n^2\right)$
- Calculation of $CI_j, CR_j : O(1)$ per criterion $\to O(m)$
- Total complexity: $O\left(m \cdot n^2\right)$

Step 6: Final aggregation

- For each alternative i:
  - Weighted sum over m criteria $\to O(m)$
- For n n alternatives: $\to O(n \cdot m)$

Conclusion: The complexity analysis reveals that the proposed method has an overall algorithmic complexity dominated by $\mathcal{O}\left(\max\left(n \cdot m \cdot s, m \cdot n^2\right)\right)$.

Table 2. Multiple criteria methods and their complexities. [19].

Method	Reference	Complexity	Multi-decision-maker
TOPSIS	Hwang et Yoon (1981)	$\mathcal{O}(m \cdot n)$	No
ELECTRE (I, II & III)	Roy (1968, 1978a, 1978b)	$\mathcal{O}\left(m \cdot n^2\right)$	No
PROMETHEE (I, II et III)	Brans & Vincke (1985)Brans Mareschal (1986, 1994)	&	No
AHP	Saaty (1980)	$\mathcal{O}\left(m^3 + m \cdot n^3\right)$	No
AHP de groupe	Ossadnik et al. (2016)	$\mathcal{O}\left(\right.$ l.m. $\left.n^2\right)$	Yes
DS/AHP	Beynon et al. (2000)	$\mathcal{O}\left(\mathrm{l}m \cdot n^2\right)$	Yes
AHP/VAHP	Soltanifar et Kamyabi (2024).	$\mathcal{O}\left(\mathrm{l}m \cdot n^2\right)$	Yes
AHP-SWARA	Zolfani et Saparauskas (2013)	$\mathcal{O}\left(m \cdot n + m^2\right)$	Yes
SWARA-VAHP	Keršulienė et al. (2010)	$\mathcal{O}\left(l.n^2.m\right)$	Yes
AHP-BWM	Rezaei (2015)	$\mathcal{O}\left(m \cdot n + m^2\right)$	No
BWM-VAHP	Mi et al. (2019)	$\mathcal{O}\left(l.mn + m^2\right)$	Yes
WM-AHP	Dong et al. (2010)	$\mathcal{O}\left(\mathrm{l}m \cdot n^2\right)$	Yes
BM-AHP	Ishizaka et Labib (2011)	$\mathcal{O}\left(m \cdot n^2\right)$	No
CAHP	Ngoie et al. (2022)	$\mathcal{O}(\max(n.m.l, m.n^3))$	Yes

4. Application of F-AHPD to a Concrete Case: Terrorism Risk Assessment for Localities in Burkina Faso

In this section, we present a case study in which F-AHPD was applied step by step.

4.1. **Problem.** Faced with the increasing number of terrorist attacks and limited resources, the Ministry of Defense of Burkina Faso must urgently and systematically prioritize intervention zones to effectively address population needs and optimize the use of security and humanitarian resources.

A multidisciplinary group comprising security experts, humanitarian representatives, and local delegates was assembled to assess and rank eight localities severely affected by terrorism:

Mouhoun Loop (BM), Cascades (CA), Center-East (CE), Center-North (CN), East (ES), Upper Basins (HB), North (NO), Sahel (SA).

The prioritization must be based on a multi-criteria analysis including:

(1) Frequency of Attacks (FA): Measurement: number of attacks recorded in the last month.
(2) Severity of Attacks (GA): Measurement: number of deaths per attack (in tens) over the last month.
(3) Population Displacement (DP): Measurement: number of displaced persons (in hundreds) per month.
(4) Psychological Impact (IP): Scale:
    - Mild: temporary, low-intensity disruption.
    - Moderate: perceptible distress, but manageable.
    - High: significant impairment of daily functioning.
    - Chronic: long-term, entrenched disorders.
(5) Impact on Development (ID): Measurement: reduction in foreign and local investments (qualitative or quantitative).

Expert assessments are synthesized into decision matrices to systematically analyze the criticality of each zone according to the selected criteria.

Table 3. Fuzzy decision matrix of decision-maker 1

Criteria →	FA	GA	DP	IP	ID
Minimum scale →	0	0	0	0	0
Maximum scale →	10	50	300	5	100
Alternatives ↓ \Weights →	(3, 5, 6)	(5, 7, 8)	(2, 4, 5)	(1, 3, 4)	(2, 3, 5)
CA	(3, 5, 7)	(10, 20, 30)	(70, 80, 100)	(2, 3, 4)	(60, 80, 85)
SA	(1, 2, 3)	(4, 7, 9)	(10, 20, 30)	(1, 2, 3)	(15, 20, 30)
CN	(2, 4, 5)	(8, 25, 35)	(60, 100, 110)	(1, 3, 4)	(60, 75, 85)
CE	(2, 5, 6)	(10, 15, 20)	(100, 110, 150)	(1, 3, 4)	(50, 70, 75)
HB	(3, 7, 8)	(25, 30, 40)	(80, 150, 200)	(2, 3, 4)	(20, 30, 35)
ES	(1, 2, 3)	(5, 8, 12)	(15, 25, 30)	(1, 2, 3)	(10, 15, 20)
BM	(2, 5, 6)	(15, 20, 30)	(70, 120, 150)	(2, 3, 4)	(40, 60, 70)
NO	(3, 5, 6)	(10, 15, 25)	(80, 100, 120)	(1, 3, 4)	(50, 70, 80)

Table 4. Fuzzy decision matrix of decision-maker 2

Criteria →	FA	GA	DP	IP	ID
Minimum scale →	0	0	0	0	0
Maximum scale →	10	50	300	5	100
Alternatives ↓ \Weights →	(3, 4, 7)	(5, 6, 7)	(2, 3, 6)	(2, 3, 5)	(2, 3, 6)
CA	(3, 4, 6)	(15, 20, 25)	(60, 70, 75)	(1, 2, 4)	(40, 70, 80)
SA	(2, 3, 4)	(6, 8, 15)	(5, 15, 20)	(1, 2, 3)	(10, 15, 20)
CN	(1, 3, 4)	(10, 20, 30)	(40, 50, 55)	(2, 3, 4)	(40, 60, 70)
CE	(1, 5, 6)	(15, 20, 40)	(60, 70, 80)	(1, 3, 4)	(30, 40, 45)
HB	(4, 8, 9)	(20, 35, 50)	(80, 100, 120)	(2, 3, 4)	(70, 90, 95)
ES	(2, 3, 4)	(3, 10, 12)	(7, 10, 15)	(1, 2, 3)	(5, 10, 15)
BM	(2, 4, 6)	(10, 15, 20)	(60, 70, 100)	(1, 2, 4)	(40, 50, 60)
NO	(3, 4, 5)	(15, 20, 30)	(70, 90, 110)	(2, 3, 4)	(45, 60, 70)

Table 5. Fuzzy decision matrix of decision-maker 3

Criteria →	FA	GA	DP	IP	ID
Minimum scale →	0	0	0	0	0
Maximum scale →	10	50	300	5	100
Alternatives ↓ \Weights →	(4, 5, 6)	(5, 6, 8)	(3, 4, 6)	(2, 3, 4)	(2, 3, 5)
CA	(2, 3, 6)	(15, 20, 25)	(60, 65, 70)	(1, 3, 4)	(40, 50, 60)
SA	(1, 3, 4)	(5, 10, 15)	(10, 20, 25)	(1, 2, 3)	(15, 20, 25)
CN	(3, 4, 5)	(10, 20, 30)	(50, 80, 85)	(2, 3, 4)	(20, 30, 35)
CE	(3, 4, 5)	(12, 15, 30)	(40, 70, 80)	(1, 2, 4)	(35, 40, 50)
HB	(4, 5, 7)	(20, 25, 30)	(80, 120, 150)	(2, 3, 4)	(10, 20, 25)
ES	(1, 2, 4)	(2, 5, 8)	(5, 10, 20)	(1, 2, 3)	(5, 10, 15)
BM	(3, 5, 7)	(10, 12, 15)	(60, 70, 85)	(2, 3, 4)	(15, 25, 30)
NO	(4, 5, 6)	(10, 15, 17)	(40, 50, 60)	(2, 3, 4)	(25, 40, 60)

4.2. **Resolution. Step 1**: The following table summarizes the aggregated fuzzy evaluations and weights, calculated using the equations (5) et (6).

Table 6. Fuzzy synthesis matrix

Criteria →	FA	GA	DP	IP	ID
Minimum scale →	0	0	0	0	0
Maximum scale →	10	50	300	5	100
Alternatives ↓ \Weights →	(3, 4.66, 7)	(5, 6.33, 8)	(2, 3.66, 6)	(1, 3, 5)	(2, 3, 6)
CA	(2, 4, 7)	(10, 20, 30)	(60, 71.66, 100)	(1, 2.66, 4)	(40, 66.66, 85)
SA	(1, 2.66, 4)	(4, 8.33, 15)	(5, 18.33, 30)	(1, 2, 3)	(10, 18.33, 30)
CN	(1, 3.66, 5)	(8, 21.66, 35)	(40, 76.66, 110)	(1, 3, 4)	(20, 55, 85)
CE	(1, 4.66, 6)	(10, 16.66, 40)	(40, 83.33, 150)	(1, 2.66, 4)	(30, 50, 75)
HB	(3, 6.66, 9)	(20, 30, 50)	(80, 123.33, 200)	(2, 3, 4)	(10, 23.33, 35)
ES	(1, 2.33, 4)	(2, 7.66, 12)	(5, 15, 30)	(1, 2, 3)	(5, 11.66, 20)
BM	(2, 4.66, 7)	(10, 15.66, 30)	(60, 86.66, 150)	(1, 2.66, 4)	(15, 45, 70)
NO	(3, 4.66, 6)	(10, 16.66, 30)	(40, 80, 120)	(1, 3, 4)	(25, 56.66, 80)

Step 2: Défuzzification

The following table presents the precise (crisp) values calculated using the equation: (8).

Table 7. Fuzzy synthesis matrix in crisp values

Criteria →	FA	GA	DP	IP	ID
Minimum scale →	0	0	0	0	0
Maximum scale →	10	50	300	5	100
Alternatives ↓ \Weights →	4.83	6.415	3.83	3	3.5
CA	4.25	20	75.83	2.58	64.58
SA	2.58	8.915	17.915	2	19.165
CN	3.33	21.58	75.83	2.75	53.75
CE	4.08	20.83	89.165	2.58	51.25
HB	6.33	32.5	131.665	3	22.915
ES	2.415	7.33	16.25	2	12.08
BM	4.58	17.83	95.83	2.58	43.75
NO	4.58	18.33	80	2.75	54.58

Step 3: Pairwise Comparison Matrices.

For each criterion c_j, pairwise comparisons between alternatives are calculated using equations (13) and (14), then synthesized in matrix form. Example with the criterion "Frequency of Attacks":

Table 8. Judgment Matrix According to Criterion FA

Alternatives ↓ \ **Alternatives** →	CA	SA	CN	CE	HB	ES	BM	NO
CA	1	$\frac{1}{3}$	$\frac{1}{2}$	1	3	$\frac{1}{3}$	1	1
SA	3	1	2	3	5	1	$\frac{1}{3}$	$\frac{1}{3}$
CN	2	$\frac{1}{2}$	1	2	4	$\frac{1}{2}$	2	2
CE	1	$\frac{1}{3}$	$\frac{1}{2}$	1	3	$\frac{1}{3}$	2	2
HB	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	1	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{3}$
ES	3	1	2	3	5	1	3	3
BM	1	3	$\frac{1}{2}$	$\frac{1}{2}$	3	$\frac{1}{3}$	1	1
NO	1	3	$\frac{1}{2}$	$\frac{1}{2}$	3	$\frac{1}{3}$	1	1

Step 4 and 5: Priorities of actions and consistency ratios.

For each criterion c_j, the priorities of the alternatives are calculated using the equation (18), and the consistency ratios are determined via the equation (23). The following table presents these priorities by criterion along with their corresponding consistency ratios.

Table 9. Priority matrix and consistency ratios.

Alternatives ↓ \ **Criteria** →	FA	GA	DP	IP	ID
Normalized weights →	0.22	0.3	0.18	0.14	0.16
CA	0.08	0.08	0.10	0.10	0.04
SA	0.17	0.25	0.26	0.26	0.2
CN	0.15	0.07	0.1	0.07	0.07
CE	0.11	0.07	0.08	0.11	0 .07
HB	0.03	0.03	0.04	0.03	0.19
ES	0.24	0.27	0.26	0.27	0.26
BM	0.11	0.12	0.06	0.10	0.10
NO	0.11	0.11	0.10	0.06	0.07
Consistency Ratio CR →	0.09	0.03	0.01	0.03	0.01

Step 6: Final aggregation.

With the consistency of judgments verified, the table summarizes the global scores (calculated via the equation (24)), the criticality levels and the ranking of localities.

TABLE 10. Final Decision Matrix

Alternatives	Risk Score (S_i)	Criticality Level	Rank
Cascades (CA)	0.080	High	7^{th}
Sahel (SA)	0.227	Critical	2^{nd}
Center-North (CN)	0.093	High	5^{th}
Center-East (CE)	0.086	High	6^{th}
Upper-Basins (HB)	0.057	Moderate	8^{th}
East (ES)	0.260	Critical	1^{st}
Mouhoun Loop (BM)	0.101	High	3^{rd}
North (NO)	0.094	High	4^{th}

4.2.1. *Results and Recommendations.*

- Distribution of Criticality Levels (Critical threshold: > 0.200, high threshold: $0.075 - 0.200$, moderate threshold: < 0.075)
  - Critical zones (25%): 2 out of 8 regions (East and Sahel)
  - High-risk zones (62.5%): 5 out of 8 regions
  - Moderate-risk zones (12.5%): 1 out of 8 regions.

  Observations:

  The predominance of the Frequency and Severity of attacks criteria (0.22 and 0.3 , respectively) in the score calculations explains the significant elevation in the criticality level of the affected zones. 87.5% of the zones exhibit a high to critical risk level.
- Targeted resource allocation:
  - allocate 50% of resources to the 2 critical zones;
  - 40 % to the 5 high-risk zones;
  - 10 % to the moderate-risk zone.

  An urgent mitigation of the frequency and severity criteria of attacks through the implementation of a targeted strategy could constitute an absolute operational priority.

5. CRITICAL ANALYSIS OF F-AHPD

5.1. **Strengths of F-AHPD.** The new method stands out for its robust handling of uncertainty through fuzzy numbers and expert aggregation, coupled with a built-in consistency control inspired by AHP

(with a CR threshold < 0.1). Relying on adaptive matrices using the mean deviation as a scaling factor, it enables fine discrimination of options through pairwise comparisons, while offering a unique ability to merge divergent expert opinions in a structured and reliable manner.

5.2. **Weaknesses of F-AHPD.** The method has certain limitations, including an algorithmic complexity of $\mathcal{O}\left(\max\left(n \cdot m \cdot s, m \cdot n^2\right)\right)$ which restricts its application to large datasets, a sensitivity to aggregation operators (min / average / max) potentially introducing bias, persistent subjectivity in the selection of weights and initial evaluations, and finally, an inability to capture the temporal dynamics of crises.

5.3. **Innovations of F-AHPD.** The proposed method builds upon several key innovations: firstly, the use of the mean absolute deviation $e\left(c_j\right)$ as a scaling factor in pairwise judgments enables dynamic adaptation to data dispersion; secondly, the use of Yager's index for defuzzification ensures internal consistency by applying the same formula to both scores and weights; finally, the approach originally combines fuzzy logic and AHP's consistency control, thereby providing a flexible yet rigorous foundation for decision support in uncertain contexts.

6. Conclusion

The F-AHPD (Fuzzy-AHP for Decision-making) method, which combines fuzzy logic and the AHP method, constitutes a relevant tool for prioritizing intervention zones in security crisis contexts. It enables structured and transparent decision-making, effectively integrating data imprecision and the diversity of expert perspectives. Its balance between mathematical rigor and operationality makes it a suitable approach for critical decisions involving a moderate number of options.

While the results generally align with the perceptions of local stakeholders (particularly in Burkina Faso), the method has certain limitations, including its algorithmic complexity and sensitivity to initial choices. These constraints do not undermine its validity in similar contexts (humanitarian crises, risk management), where it contributes to more efficient resource allocation.

To strengthen the method, several areas for improvement are considered: a comparative study with other multi-criteria methods (TOPSIS, ELECTRE) on concrete cases; the integration of machine learning (neural networks, Bayesian methods) to automatically calibrate criterion weights from historical data; the exploration of alternative defuzzification techniques (centroid, bisector) to improve output accuracy; and finally, the development of a specific consistency index for fuzzy matrices, going beyond the current framework borrowed from AHP.

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- Frédéric Nikièma: Conception of the work, methodology, writing.
- Zoïnabo Savadogo: Implementation of the models, writing.

- Mifiamba Soma: Investigation, Review, writing.
- Somdouda Sawadogo: validation, writing.

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