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THE PROPOSED METHOD FOR SOLVING MULTI-OBJECTIVES FULLY FUZZY LINEAR PROGRAMMING PROBLEMS VIA RANKING FUNCTIONS WITH HEPTAGONAL MEMBERSHIP

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Abstract. An efficient technique for solving fully fuzzy multiple objectives linear programming (FFMOLP) problems is proposed, where all coefficients of the objective functions and constraint are represented as heptagonal fuzzy numbers. The aim of this study is to find the optimal solution (maximum or minimum) for the FFMOLP problem. The Heptagonal fuzzy numbers (HFN) have applications in various optimization problems requiring seven ambiguous parameters. Additionally, the ranking functions for linear and nonlinear heptagonal memberships were derived, and they can be used in several valuable applications in the future time. Numerical examples of fully fuzzy multi-objective functions are included to demonstrate the efficiency and accuracy of the proposed method. Moreover, tables are provided to clarify the results, and a comparison between the proposed methods linear and nonlinear heptagonal rankings is presented.

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1. Introduction

Optimization is a methodical and scientific approach that solves many administrative, scientific, industrial, economic, and military problems by providing systematic alternatives to achieve desired solutions [6]. Multi-objective optimization involves improving solutions across a specific set of equally critical objective functions and finding the maximum or minimum for all objectives to achieve the mathematical criterion. Moreover, multi-objective linear programming (MOLP) involves simultaneously optimizing multiple objectives (typically two or three) under linear constraints and then finding the best way to solve them. This approach is derived from the fuzzy multi-objective problem (FMOP), in

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which the parameters of the objective function and the constraints are all fuzzy numbers and include imprecision in improving the particular model. Therefore, stable and adaptable solutions can be found with the help of the ranking function of fuzzy numbers MOLP, which is an efficient method to increase efficiency and reduce vulnerability. The FMOP has various applications, such as production, storage, financial management, and more. Heptagonal fuzzy numbers (HFNs), which consist of seven vague parameters, have been widely utilized by researchers to solve various problems and find the best solution for them. Pattnaik M.(2013) studied sensitivity analysis for FMOLP problems in [7]. Many researchers have explored fuzzy ranking functions, K. Rathi and S.(2014) utilized ranking HFNs with vagueness numbers in [8]. Subsequently, Isra et al. [3] used decagonal fuzzy numbers with ranking functions for solving linear programming problems. Abdalqader O. (2017) explained FMOLP problems with triangular fuzzy numbers [2], also A. Mohamad in the same year demonstrated basic operations on HFN [9]. Tarabia M. et al. (2017) introdused a novel method for solving FMO nonlinear programming [11]. Rasha and Iden H. (2021) explained ranking methods for solving FMOP with trapezoidal fuzzy numbers [4]. Vandan et al. (2022) proposed a new technique to solve quadratic multi-objective functions with HFN [1]. Additionally, researchers in 2022 used project scheduling in [10], Natarajan et al. (2023) used HFNs with stroke disease in [5], and Eman et al. [12], studied ranking functions with fuzzy critical path problems for solving housing project. The suggested technique in this paper includes the membership of heptagonal fuzzy numbers, and then derives proposed ranking functions of heptagonal membership for linear and nonlinear. These functions are applied to solve FFMOLP problems to clarify that. The general structure of the research is as follows: Section 2 covers the primary definitions that will be needed. In section 3, the derivation of heptagonal fuzzy membership for linear and nonlinear functions will be presented. Section 4 presents the applications of the proposed novel technique through numerical examples to demonstrate the suggested method. Finally, the conclusion will be provided in section 5.

2. Some Basic Definitions:

This section includes some basic concepts as follows.

2.1. **Fully Fuzzy Multi-Objective Linear Programming (FFMOLP).** A finite number of (n) objective functions represent an optimization problem maximum (minimum) to obtain a performance criterion. Mathematically, the problem can be expressed as below [7]: The FFMOLP problem with n objectives written as

$$Max(\tilde{z}_1, \tilde{z}_2, \cdots, \tilde{z}_n) = (\tilde{c}_1\tilde{x}, \tilde{c}_2\tilde{x}, \cdots, \tilde{c}_n\tilde{x}),$$

$$S.to\ \tilde{a}_m\tilde{x}(\cong)or(\leq)or(\geq)\tilde{b}_m, m = 1, 2, \cdots, k$$

$$\tilde{x} \geq 0$$

Where $\tilde{c}_n$ represents fuzzy parameters of MOF, $n = 1, 2, 3, \cdots$, $\tilde{a}_m$ and $\tilde{b}_m$ represent fuzzy parameters of constraints, and $\tilde{x}$ is the decision vector.

2.2. **Fuzzy and Heptagonal Fuzzy Numbers HFN.** Let the set the $\aleph \neq \Phi$. The fuzzy set A in $\aleph$ is a subset of the real membership function, $M_{\tilde{A}} : \aleph \to [0, 1]$ and descriptive by $\tilde{A}$ and $M_{\tilde{A}}(h)$ is explained the degree of the element h in fuzzy set A for every $h \in \aleph$ and $\tilde{A} = \{(h, M_{\tilde{A}}), h \in \aleph\}$.

3. Heptagonal Membership (HM) [2]

A fuzzy membership function for HFN $\tilde{A}_H = (p, q, r, s, t, e, v; w)$ where $p, q, r, s, t, e$and v are real numbers and $w \in [0, 1]$ weight function, we can define linear heptagonal membership as bellows.

3.1. **Linear Heptagonal Membership (LHM):.**

$$\mathcal{M}_{\tilde{A}_{LH}}(x) = \begin{cases} w\left(\frac{x-p}{q-p}\right) & p \leq x < q \\ w & q \leq x < r \\ w + (1-w)\left(\frac{x-r}{s-r}\right) & r \leq x < s \\ w + (1-w)\left(\frac{t-x}{t-s}\right) & s \leq x < t \\ w & t \leq x < e \\ w\left(\frac{v-x}{v-e}\right) & e \leq x < v \\ 0 & otherwise \end{cases} \tag{1}$$

By using $\sigma-$ cut and $0 \leq \sigma \leq 1$ for Eq.1 (see [2]).
The proposed membership in this project, where
$\inf \tilde{A}(\sigma) = \{x \in \tilde{A} \setminus \mathcal{M}_{HL}(x) \geq \sigma\}$ is the infimum of $\tilde{A}$, and
$\sup \tilde{A}(\sigma) = \{x \in \tilde{A} \setminus \mathcal{M}_{HL}(x) \leq \sigma\}$ is the greatest bound of $\tilde{A}$.
Defining $\sigma-$ cut of LHM as:

$$\tilde{A}_{LH}(\sigma) = \begin{cases} p + \left(\frac{q-p}{w}\right)\sigma = \inf_1\left(\tilde{A}(\sigma)\right) \\ r + \left(\frac{s-r}{1-w}\right)(\sigma - w) = \inf_2\left(\tilde{A}(\sigma)\right) \\ t + \left(\frac{s-t}{1-w}\right)(\sigma - w) = \sup_1\left(\tilde{A}(\sigma)\right) \\ v + \left(\frac{e-v}{w}\right)\sigma = \sup_2\left(\tilde{A}(\sigma)\right) \end{cases} \tag{2}$$

Whereas $0 < w < 1$, $\sigma \in [0, 1]$
Now using $\sigma-$ cut function for linear heptagonal (LH) fuzzy numbers for the Eq.2 formula then
$R_{LH}\left(\tilde{A}\right) = \frac{1}{2}\int_0^w \inf_1 \tilde{A}(\sigma)d\sigma + \frac{1}{2}\int_w^1 \inf_2 \tilde{A}(\sigma)d\sigma + \frac{1}{2}\int_w^1 \sup_1 \tilde{A}(\sigma)d\sigma + \frac{1}{2}\int_0^w \sup_2 \tilde{A}(\sigma)d\sigma$
Substituting Eq.2 in the above equation and solving it for σ, we deduce the following ranking function of LH fuzzy numbers

$$R_{LH}\left(\tilde{A}\right)=\frac{w}{4}\left[p+q+e+v\right]+\frac{(1-w)}{4}\left[r+2s+t\right] \tag{3}$$

3.2. **Nonlinear Heptagonal Membership (NLHM):.** Assume that $\tilde{A}_H=(p,q,r,s,t,e,v;w)$ be NL-HFN, with weight function $0<w<1$and $k>1$

$$\mathcal{M}_{\tilde{A}_{NLH}}(x)=\begin{cases} w\left(\frac{x-p}{q-p}\right)^k & p\le x<q \\ w & q\le x<r \\ w+(1-w)\left(\frac{x-r}{s-r}\right)^k & r\le x<s \\ w+(1-w)\left(\frac{t-x}{t-s}\right)^k & s\le x<t \\ w & t\le x<e \\ w\left(\frac{v-x}{v-e}\right)^k & e\le x<v \\ 0 & otherwise \end{cases} \tag{4}$$

Now by using $\sigma-$cut with $0\le\sigma\le1$, for Eq.4 (see [9]). Then the proposed nonlinear membership in this section, where the $\sigma-$cut (NLH) function as:

$$\tilde{A}_{NLH}(\sigma)=\begin{cases} p+\left(\frac{q-p}{w^{1/k}}\right)\sigma^{1/k}=\inf_1\left(\tilde{A}(\sigma)\right) \\ r+\left(\frac{s-r}{(1-w)^{1/k}}\right)(\sigma-w)^{1/k}=\inf_2\left(\tilde{A}(\sigma)\right) \\ t+\left(\frac{s-t}{(1-w)^{1/k}}\right)(\sigma-w)^{1/k}=\sup_1\left(\tilde{A}(\sigma)\right) \\ v+\left(\frac{e-v}{w^{1/k}}\right)\sigma^{1/k}=\sup_2\left(\tilde{A}(\sigma)\right) \end{cases} \tag{5}$$

Whereas $0<w<1$, $\sigma\in[0,1]$

Now applying $\sigma-$cut function for NLHFN to find ranking functions as follows

$R_{NLH}\left(\tilde{A}\right)=\frac{1}{2}\int_0^w \inf_1\tilde{A}(\sigma)d\sigma+\frac{1}{2}\int_w^1 \inf_2\tilde{A}(\sigma)d\sigma+\frac{1}{2}\int_w^1 \sup_1\tilde{A}(\sigma)d\sigma+\frac{1}{2}\int_0^w \sup_2\tilde{A}(\sigma)d\sigma$

Substituting Eq.5 in the above equation and solving it for σ, we can obtain the following formula with $k>1$ which represents the ranking of NLH function

$$R_{NLH}\left(\tilde{A}\right)=\frac{w}{2(k+1)}\left(p+k(q+e)+v\right)+\frac{(1-w)}{2(k+1)}\left(r+k(2s)+t\right) \tag{6}$$

The following section will present an application to illustrate the novel mechanism.

4. Numerical Examples:

In this section we will provide the following example FFMOLP to illustrate the proposed technique.

Example 4.1.

$$\begin{aligned}
Max\tilde{z}_1 &= (2.3, 2.5, 2.8, 3, 3.3, 3.7, 4)\tilde{x}_1 + (0.3, 0.5, 0.82, 1, 1.3, 1.7, 2)\tilde{x}_2 \\
&+(1.3, 1.5, 1.8, 2, 2.3, 2.7, 3)\tilde{x}_3 + (0.3, 0.5, 0.8, 1, 1.3, 1.7, 2)\tilde{x}_4 \\
Max\tilde{z}_2 &= (0.3, 0.5, 0.8, 1, 1.3, 1.7, 2)\tilde{x}_1 - (0.3, 0.5, 0.9, 1, 0.5, 0, -0.5)\tilde{x}_2 \\
&+(1.3, 1.5, 1.8, 2, 2.3, 2.7, 3)\tilde{x}_3 + (3.3, 3.5, 3.82, 4, 4.3, 4.7, 5)\tilde{x}_4 \\
Max\tilde{z}_3 &= (-0.3, -0.5, -0.9, -1, -0.5, 0, 0.5)\tilde{x}_1 + (4.3, 4.5, 4.82, 5, 5.3, 5.7, 6)\tilde{x}_2 \\
&+(0.3, 0.5, 0.8, 1, 1.3, 1.7, 2)\tilde{x}_3 + (1.3, 1.5, 1.8, 2, 2.3, 2.7, 3)\tilde{x}_4
\end{aligned}$$

s.to

$$\begin{aligned}
&(1.3, 1.5, 1.8, 2, 2.3, 2.7, 3)\tilde{x}_1 + (0.3, 0.5, 0.8, 1, 1.3, 1.7, 2)\tilde{x}_2 \\
&+(3.3, 3.5, 3.8, 4, 4.3, 4.7, 5)\tilde{x}_3 + (2.3, 2.5, 2.8, 3, 3.3, 3.7, 4)\tilde{x}_4 \\
&\leq (59.3, 59.5, 59.8, 60, 60.3, 60.7, 61) \\
&(2.3, 2.5, 2.8, 3, 3.3, 3.7, 4)\tilde{x}_1 + (3.3, 3.5, 3.8, 4, 4.3, 4.7, 5)\tilde{x}_2 \\
&+(0.3, 0.5, 0.8, 1, 1.3, 1.7, 2)\tilde{x}_3 + (1.3, 1.5, 1.8, 2, 2.3, 2.7, 3)\tilde{x}_4 \\
&\leq (59.3, 59.5, 59.8, 60, 60.3, 60.7, 61) \\
&\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0
\end{aligned}$$

with the optimal crisp solution of MOF, is as follows

$$\begin{aligned}
Max\ z_1 &= 66, x_1 = 18, x_2 = 0, x_3 = 6, x_4 = 0. \\
Max\ z_2 &= 80, x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 20. \\
Max\ z_3 &= 75, x_1 = 0, x_2 = 15, x_3 = 0, x_4 = 0.
\end{aligned}$$

After solving previous systems by finding the values of parameters for the first objective function $(Max\tilde{z}_1)$ with the constraints to find the values of parameters for $\tilde{z}_1$ then solve the second objective function $(Max\tilde{z}_2)$ with constraints by finding the values of parameters for $Max\tilde{z}_2$, and also solve the third objective function $(Max\tilde{z}_3)$ with the same constraints to find the values of parameters for $\tilde{z}_3$. The results for parameters $\tilde{A}_H = (p, q, r, s, t, e, v)$ of $\tilde{z}_i (i = 1, 2, 3)$ for MOF heptagonal fuzzy numbers for above example, as follows:

$$\begin{aligned}
Max\tilde{z}_1 &= (67.536, 66.937, 66.3, 66, 65.683, 65.442, 65.357), \\
&\tilde{x}_1 = (24.708, 22.3, 19.5, 18, 16.15, 14.22, 13.07), \tilde{x}_2 = (0, 0, 0, 0, 0, 0, 0), \\
&\tilde{x}_3 = (8.236, 7.437, 6.5, 6, 5.383, 4.742, 4.357), \tilde{x}_4 = (0, 0, 0, 0, 0, 0, 0). \\
Max\tilde{z}_2 &= (85.082, 83.3, 81.584, 80, 78.57, 77.105, 76.52),
\end{aligned}$$

$$\tilde{x}_1 = (0,0,0,0,0,0,0), \tilde{x}_2 = (17.176,0,0,0,0,0,0),$$

$$\tilde{x}_3 = (0,0,0,0,0,0,0), \tilde{x}_4 = (25.782,23.8,21.375,20,18.27,16.405,13.7).$$

$$Max\tilde{z}_3 = (77.269,76.5,75.851,75,74.323,73.619,73),$$

$$\tilde{x}_1 = (0,0,0,0,0,0,0), \tilde{x}_2 = (17.969,17,15.736,15,14.02,12.914,12.2),$$

$$\tilde{x}_3 = (0,0,0,0,0,0,0), \tilde{x}_4 = (0,0,0,0,0,0,0).$$

By using Eq.3 to compute ranking function of LHM and Eq.6 to compute ranking function of NLHM for $Max\tilde{z}_1$, $Max\tilde{z}_2$ and $Max\tilde{z}_3$ from the above data. We obtained the following results which display in table 1 for $Max\tilde{z}_i$, $i = 1,2,3$ when using ranking functions LHM and NLHM for example 4.1 when $0 \leq w \leq 1$ and $k > 2$, suppose that $k = 2$ in Eq.6.

TABLE 1. The results for $Max\tilde{z}_i$, $i = 1,2,3$ when using ranking of linear and nonlinear heptagonal fuzzy numbers for example 4.1

w	$Max\tilde{z}_1$ (**LHF**)	$Max\tilde{z}_1$ $k = 2$(**NLHF**)	$Max\tilde{z}_2$ (**LHF**)	$Max\tilde{z}_2$ $k = 2$(**NLHF**)	$Max\tilde{z}_3$ (**LHF**)	$Max\tilde{z}_3$ $k = 2$(**NLHF**)
0	66.021	66.014	80.037	80.025	75.043	75.029
0.1	66.050	66.040	80.084	80.066	75.049	75.034
0.2	66.080	66.066	80.130	80.107	75.054	75.040
0.3	66.110	66.092	80.177	80.148	75.059	75.045
0.4	66.139	66.118	80.224	80.189	75.065	75.051
0.5	66.169	66.144	80.271	80.231	75.070	75.056
0.6	66.199	66.1707	80.317	80.272	75.075	75.062
0.7	66.228	66.1968	80.364	80.313	75.081	75.068
0.8	66.258	66.222	80.411	80.354	75.086	75.073
0.9	66.288	66.249	80.457	80.396	75.091	75.079
1	66.318	66.275	80.504	80.4373	75.097	75.084

The results indicate that they were all MOF variables are good when we used a fully fuzzy heptagonal numbers for FFMOF from $w = 0$ and the results improve when w closer to 1 for linear and nonlinear heptagonal functions and the results of the linear was the best from nonlinear for all $Max\tilde{z}_i$, $i = 1,2,3$ also when $k = 2$ in Eq.6.

Example 4.2.

$$Min\tilde{z}_1 = (5.1,5.3,5.5,6,6.5,6.7,6.9)\tilde{x}_1 + (1.1,1.3,1.5,2,2.5,2.7,2.9)\tilde{x}_2$$
$$+(2.1,2.3,2.5,3,3.5,3.7,3.9)\tilde{x}_3$$

$$\begin{aligned} Min\tilde{z}_2 &= (4.1, 4.3, 4.5, 5, 5.5, 5.7, 5.9)\tilde{x}_1 + (0.1, 0.3, 0.5, 1, 1.5, 1.7, 1.9)\tilde{x}_2 \\ &\quad +(1.1, 1.3, 1.5, 2, 2.5, 2.7, 2.9)\tilde{x}_3 \end{aligned}$$

$$\begin{aligned} \text{s.to} \quad & (1.1, 1.3, 1.5, 2, 2.5, 2.7, 2.9)\tilde{x}_1 - (0.1, 0.3, 0.5, 1, 1.5, 1.7, 1.9)\tilde{x}_2 \\ & +(1.1, 1.3, 1.5, 2, 2.5, 2.7, 2.9)\tilde{x}_3 \geq (3.1, 3.3, 3.5, 4, 4.5, 4.7, 4.9) \\ & (0.1, 0.3, 0.5, 1, 1.5, 1.7, 1.9)\tilde{x}_1 + (-0.1, -0.3, -0.5, 0, 0.5, 0.7, 0.9)\tilde{x}_2 \\ & +(3.1, 3.3, 3.5, 4, 4.5, 4.7, 4.9)\tilde{x}_3 \geq (3.1, 3.3, 3.5, 4, 4.5, 4.7, 4.9) \\ & (0.1, 0.3, 0.5, 1, 1.5, 1.7, 1.9)\tilde{x}_1 + (2.1, 2.3, 2.5, 3, 3.5, 3.7, 3.9)\tilde{x}_2 \\ & +(1.1, 1.3, 1.5, 2, 2.5, 2.7, 2.9)\tilde{x}_3 \geq (6.1, 6.3, 6.5, 7, 7.5, 7.7, 7.9) \\ & \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq 0 \end{aligned}$$

The optimal crisp solutions of MOF for example 4.2 as follows:

$$\begin{aligned} Min\ z_1 &= 8.62, x_1 = 0, x_2 = 0.75, x_3 = 2.375. \\ Min\ z_2 &= 5.5, x_1 = 0, x_2 = 0.75, x_3 = 2.375. \end{aligned}$$

By using the same procedure for example 4.1 we obtained the results for parameters of $Min\tilde{z}_i, i = 1, 2$ which are shown as follows:

$$\begin{aligned} Min\tilde{z}_1 &= (7.678, 7.950, 8.166, 8.625, 8.700, 9.235, 9.411), \\ &\quad \tilde{x}_1 = (0, 0, 0, 0, 0, 0, 0), \tilde{x}_2 = (1.363, 1.153, 1, 0.75, 0.75, 0.556, 0.517), \\ &\quad \tilde{x}_3 = (2.942, 2.804, 2.667, 2.375, 1.950, 2.090, 2.028). \\ Min\tilde{z}_1 &= (3.372, 3.992, 4.5, 5.5, 6, 6.588, 6.856), \\ &\quad \tilde{x}_1 = (0, 0, 0, 0, 0, 0, 0), \tilde{x}_2 = (1.363, 1.153, 1, 0.75, 0.75, 0.556, 0.517), \\ &\quad \tilde{x}_3 = (2.942, 2.804, 2.667, 2.375, 1.950, 2.090, 2.028). \end{aligned}$$

By using Eq.3 to compute ranking function of linear heptagonal membership and Eq.6 to compute ranking function of nonlinear heptagonal membership for $Min\tilde{z}_1$, and $Min\tilde{z}_2$ from the above data that will be demonstrated above. Obtaining the following results which display in table 2 for $Min\tilde{z}_i$,$i = 1, 2$ when using ranking LHM and NLHM to example 4.2 where $0 \leq w \leq 1$, also when $k = 2$. The results are indicating that closer to crisp solutions when we used a fully fuzzy numbers for FFMOF from $w = 0$ and the results are improved and becomes well when w closer to 1 in both linear and nonlinear.

TABLE 2. The results for $Min\tilde{z}_i$, $i = 1, 2$when using ranking of linear and nonlinear heptagonal fuzzy numbers for example 4.2

w	$Min\tilde{z}_1$ (LHF)	$Min\tilde{z}_1$ $k=2$(NLHF)	$Min\tilde{z}_2$ (LHF)	$Min\tilde{z}_2$ $k=2$(NLHF)
0	8.529	8.561	5.375	5.416
0.1	8.525	8.562	5.357	5.398
0.2	8.521	8.564	5.340	5.379
0.3	8.517	8.565	5.323	5.361
0.4	8.513	8.567	5.307	5.343
0.5	8.509	8.568	5.289	5.324
0.6	8.505	8.570	5.272	5.306
0.7	8.501	8.571	5.255	5.288
0.8	8.496	8.573	5.238	5.269
0.9	8.493	8.574	5.221	5.251
1	8.489	8.576	5.204	5.238

5. CONCLUSION

This paper studied a fuzzy function for heptagonal fuzzy number with ranking membership linear and nonlinear. In addition to derive laws of ranking heptagonal membership linear and nonlinear then used them in the optimal solution for FFMOLP problems. The novel technique has proven its ability to find the accuracy solutions. It can be noted that the optimal values of the multiple objective functions $Max\tilde{z}_i$, $i = 1, 2, 3$ were better when ranking linear heptagonal function used than nonlinear that scalar in example 4.1, also can notice that the results improve from $w = 0$ and become better when w approach to 1. It's possible to say the profit in the solution is harmonious with the values of each one multi-objective functions so obtaining compatible solution results for all values of $Max\tilde{z}_i$, $i = 1, 2, 3$ see table 1. Also for example 4.2 we can notice that the optimal values of FFMOLP for $Min\tilde{z}_i$, $i = 1, 2$ are well when used heptagonal fuzzy numbers and the numerical results improve from the beginning of zero and continue to improve until reaching one, and Table 2 show that. From here decision makers can control all steps of the solution to make our approach useful and can be applied in many life problems which the information is uncertain but the results are fairly acceptable.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

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