Asia Pac. J. Math. 2025 **12**:112

THE INFLUENCE OF AUTOMATION ON UNEMPLOYMENT IN THE JOB MARKET: A MATHEMATICAL MODEL WITH SIMULATIONS

ATHEER JAWAD KADHIM[1,*], ISRAA H. HASAN[2], NADIA M. G. AL-SAIDI[1], SUHA SHIHAB[1], AHMED SHAWKI JABER[3]

[1]College of Applied Sciences, University of Technology, Baghdad, Iraq

[2]College of Communication Engineering, University of Technology, Baghdad, Iraq

[3]Department of Mathematics Science, College of Science, Mustansiriyah University, Baghdad, Iraq

[*]Corresponding author: Atheer.J.Kadhim@uotechnology.edu.iq

Received Oct. 15, 2025

Abstract. Replacing human labor with technology, known as automation, impacts unemployment, a key global issue. A new mathematical model was developed to analyze unemployment in the context of automation, focusing on three variables: unemployed individuals, employees, and vacant positions. The model incorporates two factors: θ for jobs managed by automation and ψ for human competition. By formulating a system of nonlinear equations, the analysis reveals a single stable equilibrium point influenced by automation. Numerical simulations validate the theoretical results, showing that increased automation necessitates the creation of new jobs to balance machinery usage and reduce unemployment.

2020 Mathematics Subject Classification. 91B42.

Keywords and phrases: automation; unemployment; bifurcation; Lyapunov function.

1. Introduction

Automation (e.g., intelligent machines, robots, and artificial intelligence), which represents the head of the current civilization, poses a clear challenge to what we do to accomplish hard tasks and dangerous work instead of humans. It has added many features and benefits to humanity due to the accurate work it performs in various fields, including the medical field, where it has reached the level of precise, specialized surgeries to assist medical personnel. The expanded reliance on these smart technologies led to a decrease in the need for human labor, which has caused an imbalance between humans and them and thus increased unemployment, as they are competitors the humans in their work [1]- [3]. Many people around the world are impacted by the issue of unemployment. It is a sign of the state of the economy, and young people may have a harder time finding employment in underdeveloped nations.

DOI: 10.28924/APJM/12-112

The unemployment problem facing societies is due to several reasons, including the inefficiency of new graduates, where numerous institutions output of low academic qualifications; graduates cannot find jobs due to their lack of practical skills as they cannot integrate into the labor market, the use of smart machines instead of humans, or the use of artificial intelligence with its rapid development, which contributes to completing work in a typical time that humans are unable to achieve. Unemployment can lead to political, social, and economic issues for a nation. More common Psychological and mental problems in unemployed people can be heart disease and depression. As a result, maintaining social stability requires controlling unemployment. The process of controlling this society's major problem is done in several ways [4]- [5]. The challenge appears in identifying solutions to social dynamics that are dealt with in different disciplines, biological [6]- [8], physics [9], engineering [10], and many others. To comprehend the dynamics of issues such as poverty, social isolation, and injustice, interest in scientific research has increased. The essential elements for resolving these social issues are identified by presenting and analyzing nonlinear mathematical models. To comprehend the social and economic dynamics of the unemployment issue, numerous studies have been conducted recently. Misra and Singh in [11] introduced and examined a mathematical model for allocating housing to a population that has been rendered homeless by a natural disaster. Under certain conditions, the only equilibrium point is stable in their model. The unemployment problem requires immediate action, which stipulates stopping the current rise in unemployment rates before reaching a stage where it becomes difficult to find solutions to the problem. The previous ideas were expanded by proposing a model that simulates unemployment with the presence of the native population with immigrants [12]. Pathan and Bhathawala [13] via a nonlinear set of equations suggested and constructed a simulation model for unemployment that takes into account the competitive job market between newly arrived immigrants and unemployed natives. They supposed that there is job competition between unemployed people and recent immigrants, despite the fact that both the public and private sectors have worked quickly to create new positions. Likewise, the optimal control of the unemployment issue is by creating new jobs that fit the labor market and finding vacancies to be filled by the unemployed [14]. They designed a comprehensive model consisting of five equations describing those who have work, those who are temporarily unemployed, those who are regularly employed, and the number of permanent and temporary job opportunities. They found appropriate policies to reduce unemployment. Al-Malawi et. al [15] focused on one of the most important factors affecting the problem, which is the lack of government financial funding. In 2021 [16]- [17], they presented more modern models by dividing the unemployed into two categories: unemployed regular workers and unemployed skilled workers, and they focused on the extent of their impact on the labor market. El Yahyaoui and Amine [18] proposed a mathematical model to their system is divided into three categories: the primarily unemployed, employed persons, and periodically unemployed people. The effects of skills development on the governance of cyclical unemployment is an interesting work. The results prove that improving dexterity among the cyclically unemployed is associated with lowering unemployment. A fractional-order mathematical model was presented by Bansal and Mathur in [19] to examine the impacts of distinct skill-increase initiatives on young people. Their paper's primary goal is to investigate the effects of training initiatives that seek to development the skills of unemployment persons to reduce the unemployment rate as a whole. A phenomenon known as transcritical bifurcation occurs in nonlinear dynamical systems when a system parameter crosses a critical threshold, causing two equilibrium points to intersect and lose their stability. After the bifurcation point, one equilibrium that was initially stable turns unstable, while the other equilibrium concurrently becomes stable. It is observed in a number of applications, such as chemical reaction models, population dynamics, and even specific situations in image processing algorithms, where the system behavior suddenly changes [20].

This study presents a new mathematical model that takes into account the competition between unemployment and the workers to be used in analyzing the effects of automation on unemployment. It contributes by the following: Developing an automated nonlinear dynamical system for unemployment, in contrast to previous models that only consider conventional labor market dynamics, this work takes automation into account as a competing factor in the labor market. The relationship between human employment and automation-driven job displacement is captured by the three primary variables: open positions, employed individuals, and unemployed individuals. Global asymptotic stability is established by Lyapunov functions, which guarantee that the behavior of the system can be predicted under various employment scenarios. The impact of important variables on unemployment rates, including automation growth, job creation rates, and migration trends, is assessed using MATLAB as a numerical simulation. It assesses how automation influences employment trends and identifies policy interventions to mitigate technological unemployment. Moreover, a transition from the positive point to the other point, as a result of which we have obtained transcritical bifurcation at each parameter, is also accomplished. In addition to this introductory section, the remaining part of the paper is outlined as follows: The general mathematical model with the definition of the parameters and calculation of the equilibrium points is presented in Section 2. The stability analysis locally and globally is discussed in Section 3. Section 4 covers the numerical simulation of the proposed system, where the transcritical bifurcation at each parameter is performed in Section 5. Section 6 concludes the paper with possible avenues for further work.

2. Dynamic construction of the mathematical model

Through mathematical simulation of a system in which an intelligent machine competes with the human race for its jobs, including unemployment, employees, and vacant jobs. The unemployment population is denoted by $A(t)$ at time t, the population of employees is denoted by $B(t)$, and a vacant position is denoted by $C(t)$ at time t, respectively, and $N = A + B$. The following presumptions are considered to subedit the dynamics of the system:

Fully qualified is the general condition of all these people competing to have a chance of a job in the unemployed category.

- It is permissible to employ a portion of the unemployed under conditions of war or depression or replace them by using automation.
- There is a relationship between the number of unemployed persons and the number of available vacancies; the average of transition from being unemployed to employed is jointly denoted by $\mathrm{A(t)}$ and $\mathrm{C(t)}$, respectively.
- The unemployed population may increase when either or both of the following cases occur, one of these is when the rate of employed people, denoted by $\mathrm{B(t)}$, could be influenced in case employers fire employees, or, on the contrary, some persons may resign.
- Migration of the peoples and death creates vacancies.
- The worker B in the model competes with A for job opportunities.

According to the above, the following governed nonlinear system of equations represents the model:

$$\begin{aligned} \frac{dA}{dt} &= N - \alpha\, AC + \beta\, B - \gamma\, A + \psi\, AB, \\ \frac{dB}{dt} &= \alpha\, AC - \beta\, B - \delta\, B - \psi\, AB, \\ \frac{dC}{dt} &= \delta\, B + \epsilon\, A - \theta\, C \end{aligned} \tag{1}$$

Table 1. The parameter description of the system (1).

Parameter	Description
N	*The total number of unemployed and employed people*
α	*The unemployment phenomenon ratio of unemployed persons*
β	*The number of workers who leave their jobs*
γ	*Annihilation or the end of life and the Rate of migration of unemployed people*
δ	*Death, rate of migration, and retirement of employed people*
ϵ	*The rate of the available vacancies created by the government and the private sectors*
θ	*The rate of jobs filled by automation in factories or any other type of job*
ψ	*The increased rate of competition between employment and unemployment*

TABLE 1. Description of parameters of system (1)

Theorem: Every solution of the system (1) is uniformly bounded when initiated in R_+^3 **Proof:** With the initial condition $(A(0), B(0), C(0)) \in R_+^3$, let it $(A(t), B(t), C(t))$ be any solution to the system (1). Via the following function $w(t) = A(t) + B(t) + C(t)$, one can get

$$\frac{dw}{dt} = N - \alpha\, AC + \beta\, B - \gamma\, A + \psi\, AB + \alpha\, AC - \beta\, B - \delta\, B - \psi\, AB + \delta\, B + \epsilon\, A - \theta\, C$$

So, acc, $\gamma > \epsilon$, we get, $\frac{dw}{dt} \le N - s\, W$, where $s = min(\gamma - \epsilon), \theta$, then
$w(t) \le M$ where $M = \frac{N}{s}$.
Hence, every solution of the system (1) is uniformly bounded.

2.1. Analyze and find the equilibrium points of the system.

The work here is devoted to a mathematical and economic analysis of all reasonable and mathematically acceptable equilibrium points that are compatible with the economic aspect of the proposed system, which is explained below.
From a purely mathematical point of view, all fixed points exist and can be found, but in our model, the following point $P_0 = (\bar{A}, 0, 0)$ where $\bar{A} = \frac{N}{\gamma}$ is considered mathematically acceptable, but it is not realistic to occur economically unless $(N = 0)$. This is not possible in reality, so it is unlikely that the number of workers and unemployed would be equal to zero, and it is contradictory to reach this point. OR $(\epsilon = 0)$, that is, the rate of available vacant jobs created by the government and the private sector and the number of unemployed people is equal to zero, because it is an unrealistic situation in life. The same thing is true about $P_1 = (\bar{\bar{A}}, \bar{\bar{B}}, 0)$, that is, it is illogical to have unemployed and employed people while no job opportunities are returning to the above-mentioned situation $(\epsilon = 0)$ in a state of complete stagnation.
Now, the only equilibrium point $P = (\tilde{A}, \tilde{B}, \tilde{C})$ where $\tilde{C} = \frac{\gamma B - \epsilon A}{\theta}$ substituted in the first and third equations in the model with some algebraic steps. So, $\hat{A}$, the following equation can be considered a positive solution.

$$R_1 A^2 + R_2 A + R_3 = 0, \tag{2}$$

Where $R_1 = \alpha\, \epsilon\, (\gamma + \delta)$, $R_2 = -[N\, \delta\, (\alpha - \psi\, \theta]$, $R_3 = N\, (\beta + \delta)\, \theta^2$. The solution of Equation (2) is $\tilde{A} = \frac{-R_2 + \sqrt{R_2^2 + 4R_1 R_2}}{2R_1} > 0$, exists provided that,

$$\alpha > \psi\, \theta \tag{3}$$

$$R_2^2 > 4R_1 R_2 \tag{4}$$

3. Detectors of the stability analysis

This section discusses the stability analysis of the system (1). Both the local and the global stability are considered.

3.1. **stability analysis of the system.** In this subsection, computing the Jacobian matrix of system (1) around each of the previous equilibrium points, the local stability analysis is discussed and illustrated. The mathematical form of the Jacobian matrix J for the system (1) with its details can be represented as:

$$æ = [b_i j]_{3\times 3} \tag{5}$$

with

$b_{11} = (\alpha\ C + \gamma - \psi\ B)$, $b_{12} = \beta + \psi\ A$, $b_{13} = -\alpha\ A$,
$b_{21} = \alpha\ C - \psi\ B$, $b_{22} = -(\beta + \delta) - \psi\ A$, $b_{23} = \alpha\ A$,
$b_{31} = \epsilon$, $b_{32} = \delta$, $b_{33} = -\theta$.

Now, the stability analysis at $P = (\hat{A}, \hat{B}, \hat{C})$ is discussed. The Jacobian matrix of the system (1) can be written as,

$$J = J(P) = \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \tilde{b}_{13} \\ \tilde{b}_{21} & \tilde{b}_{22} & \tilde{b}_{23} \\ \tilde{b}_{31} & \tilde{b}_{32} & \tilde{b}_{33} \end{bmatrix}$$

Then the characteristic equation of J is given by,

$$\lambda^3 + M_1\lambda^2 + M_2\lambda + M_3 = 0 \tag{6}$$

Where $M_1 = [\tilde{b}_{11} + \tilde{b}_{22} + \tilde{b}_{33}]$
$M_2 = \tilde{b}_{11}\ \tilde{b}_{22} + \tilde{b}_{33}\ (\tilde{b}_{11} + \tilde{b}_{22}) - \tilde{b}_{13}\ \tilde{b}_{31} - \tilde{b}_{23}\ \tilde{b}_{32} - \tilde{b}_{12}\ \tilde{b}_{21}$
$M_3 = \tilde{b}_{11}\ (\tilde{b}_{23}\ \tilde{b}_{32} - \tilde{b}_{22}\ \tilde{b}_{33}) + \tilde{b}_{12}\ (\tilde{b}_{21}\ \tilde{b}_{33} - \tilde{b}_{32}\ \tilde{b}_{31}) - \tilde{b}_{13}\ (\tilde{b}_{22}\ \tilde{b}_{31} - \tilde{b}_{21}\ \tilde{b}_{32})$

If and only if the following conditions are met, and with the help of the Routh-Horwitz criterion, equation (4) has the assertion of having eigenvalues with negative real parts, explain as,
$M_1 > 0, M_3 > 0$ and $\Delta = \frac{M_1 M_2 - M_3}{M_1} > 0$ Since $M_i > 0, i = 1, 3$. If these controls are applied, they will be met.

$$\tilde{b}_{11}\ (\tilde{b}_{23}\ \tilde{b}_{32} - \tilde{b}_{22}\ \tilde{b}_{33}) + \tilde{b}_{12}\ (\tilde{b}_{21}\ \tilde{b}_{33} - \tilde{b}_{32}\ \tilde{b}_{31}) - \tilde{b}_{13}\ (\tilde{b}_{22}\ \tilde{b}_{31} - \tilde{b}_{21}\ \tilde{b}_{32}) \tag{7}$$

$$\tilde{b}_{23}\ \tilde{b}_{32} < \tilde{b}_{22}\ \tilde{b}_{33} \tag{8}$$

$$\tilde{b}_{11}\ \tilde{b}_{22} + \tilde{b}_{33}\ (\tilde{b}_{11} + \tilde{b}_{22}) - \tilde{b}_{13}\ \tilde{b}_{31} - \tilde{b}_{23}\ \tilde{b}_{32} - \tilde{b}_{12}\ \tilde{b}_{21} \tag{9}$$

Since the three coefficients presented in (4) are positive. We just proved the following result.

$$M_1 M_2 - M_3 > 0 \tag{10}$$

So, $\Delta\ >\ 0$ presuming the conditions (7-9) are satisfied. Therefore, P is unstable unless the above conditions are met; otherwise, it is stable, see [21].

3.2. **Global stability analysis.** For the only point that is mathematically and economically acceptable, and whose existence and local stability have been discussed. Besides, Lyapunov's method is used to analyze the global stability of the system (1).

Theorem: The equilibrium point $P = (\hat{A}, \hat{B}, \hat{C})$ of the system (1) is globally asymptotically stable in the basin of attraction of Int.R^3_+ that satisfies the next condition.

$$\gamma\,\frac{(A-\tilde{A})^2}{A} - \frac{(\beta+\delta)}{B}(B-\tilde{B})^2(\frac{\theta}{C})(C-\tilde{C})^2 + \alpha\,(AC-\tilde{A}\,\tilde{C})(\frac{(A-\tilde{A})(B-\tilde{B})}{AB})$$
$$+\lambda(AB-\tilde{A}\,\tilde{B})[\frac{(B-\tilde{B})}{B} - \frac{(A-\tilde{A})}{A)}] > \frac{(C-\tilde{C})}{C}[\delta\,(B-\tilde{B}) + \epsilon\,(A-\tilde{A})] \tag{11}$$

with

$$\frac{(B-\tilde{B})}{B} > \frac{(A-\tilde{A})}{A} \tag{12}$$

$$A > \tilde{A}, B > \tilde{B} \tag{13}$$

$$C > \tilde{C} \tag{14}$$

Proof: A positive definite function is considered below as, $V : R^3_+ \to R$ is C^1,

$V = [A - \tilde{A} - \tilde{A}\,ln\,\frac{A}{\tilde{A}}] + [B - \tilde{B} - \tilde{B}\,ln\,\frac{B}{\tilde{B}}] + [C - \tilde{C} - \tilde{C}\,ln\,\frac{C}{\tilde{C}}]$

Some algebraic handling and concerning time t differentiation V getting,

$$\frac{dV}{dt} = -\gamma\frac{(A-\tilde{A})^2}{A} - (\frac{\beta+\delta}{B})(B-\tilde{B})^2 - \frac{\theta}{C}(C-\hat{C})^2 - \alpha\frac{AC-\tilde{A}\tilde{C}}{AB}[(A-\tilde{A})(B-\tilde{B})]$$
$$-\lambda(AB-\tilde{A}\tilde{B})[\frac{B-\tilde{B}}{B} - \frac{A-\tilde{A}}{A}] + \frac{C-\tilde{C}}{C}[\delta(B-\tilde{B}) + \epsilon(A-\tilde{A})]$$

Now, according to the conditions $(12) - (14)$, gives $\frac{dV}{dt}$ is negative definite. This confirms that the function V is a Lyapunov function.

Thus, it is verified that the single point of the system P is globally asymptotically stable.

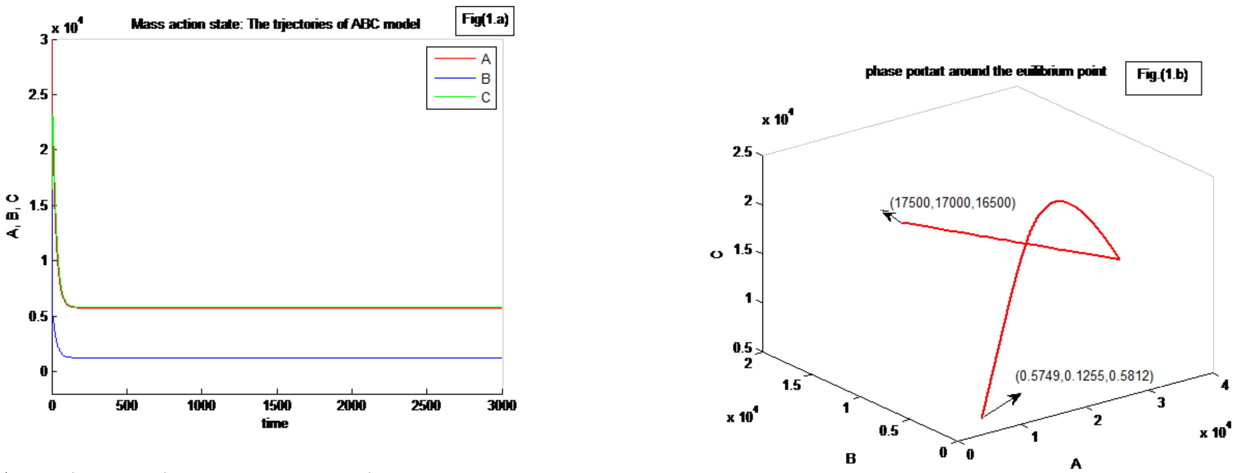
4. Practical numerical analysis and simulation of the system

Based on what has been discussed in previous sections, the problem in the system (1) is solved numerically using several MATLAB codes with the ode45 function, [22]. The solution visualizes the trajectories A, B, and C versus 300 days. Table 2 shows that fixing the system's parameters (1) ensures the previous theoretical calculations within four scenarios.

N	α	β	γ	δ	ϵ	ψ	θ
300	1.08×10^{-2}	0.05	0.05	0.01	0.2	0.05	0.2

Table 2. The values of the parameters of system (1)

Scenario 1: With consideration of the values in Table 2, we designed a MATLAB code that visualized system (1)'s trajectories so that the unemployed, employed, and vacation segments reached an equilibrium point $P = (\tilde{A}, \tilde{B}, \tilde{C}) = (0.5749, 0.1255, 0.5812)$, see Figure (1-a). Additionally, we built another MATLAB code to confirm that P is globally asymptotically stable and to show its phase portrait. As shown in Figure (1-b).



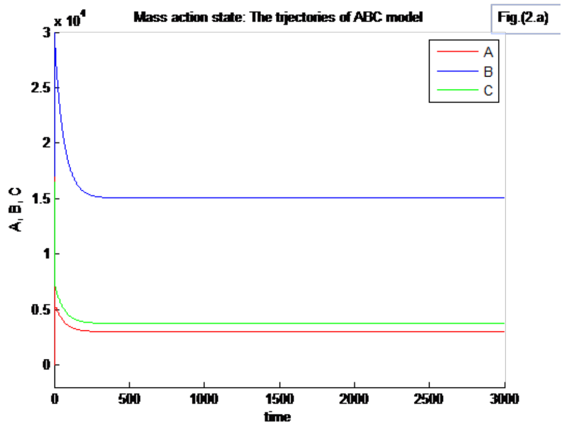
(A) The solution approaches asymptotically to the positive equilibrium point $P = (0.5749, 01255, 0.5812)$

(B) Phase portrait of the unique equilibrium points $P = (\tilde{A}, \tilde{B}, \tilde{C}) = [0.5749, 0.1255, 0.5812]$

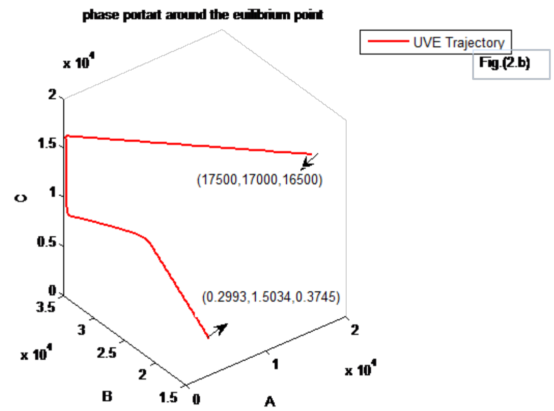
FIGURE 1. System (1) begins with different initial points $(17500, 17000, 16500)$, $(18500, 18000, 17500)$, $(19500, 19000, 18500)$, for the data given by Table 2.

Observe that according to the values of Table (2), as in Figure (1.a), there is a coinciding both unemployment and vacation, but the employee under this line as above explains.
Now, as in the following, a test of the system (1) via the numerical result of the parameter estimations, the information in Table 2 was settled for one transformed only each time, and fixed the rest.

Scenario 2: Parameter values difference α, (the account of unemployed persons according to the unemployment phenomenon) in range, $(1.0810^(-2) \leq \alpha \leq 4.0810^(-2))$ as in Figure $(1-a)$. Gradually, when the parameter's value increases, the number of workers increases, and the number of unemployed decreases, with job opportunities in between them. This is an excellent indicator that suggests a solution to the unemployment problem. We took one of the drawings to indicate this talk and an ideal value for the parameter $(\alpha = 20)$, as in Figures $(2.a)$ and $(2.b)$.



(A) The time series of the system (1) for the data given in Table 2, and ($\alpha = 20$) beginning at $(17500, 17000, 16500)$ approaches asymptotically to the point $P = (0.2293, 1.5034, 0.3745)$

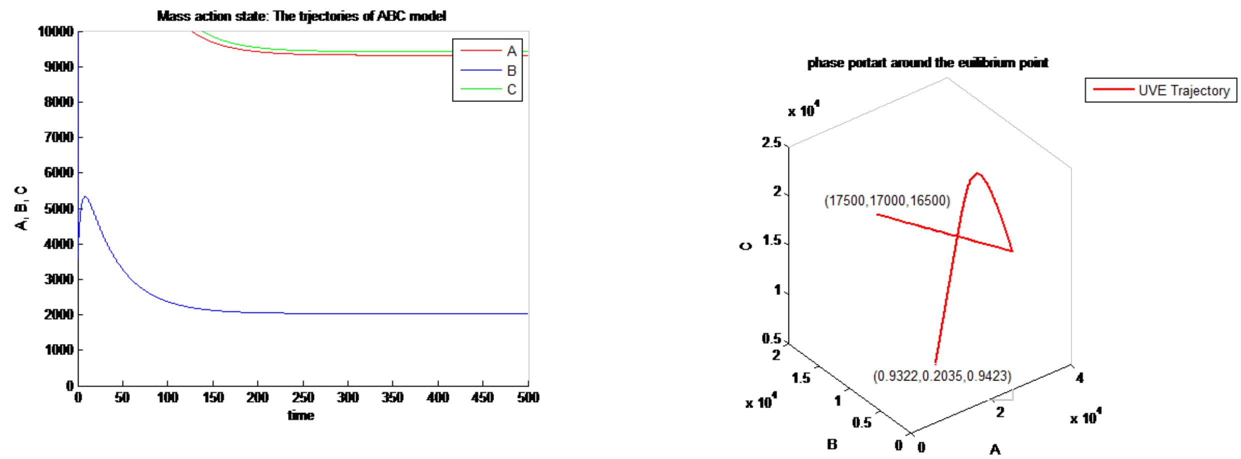


(B) The trajectory of the system (1), which approaches asymptotically the point $P = (0.2293, 1.5034, 0.3745)$.

FIGURE 2. System (1) with different values of α .

- By changing the parameter β (the rate of workers who leave their jobs due to problems of competing with the unemployed, which causes low wages and other disputes). Whatever the values of this parameter, it still does not affect drawing, and the results remain as shown in Figures $(1.a)$ and $(1.b)$.
- By differing γ (annihilation or the end of life and the rate of migration of unemployed people) in the range $(0.0001 \leq \gamma < 0.4)$, we get Figure $(3.a)$, opposite to Figure $(1 - a)$.

In this case, there is an inverse proportion, meaning that the more the value of the parameter decreases, the more unemployment and job opportunities increase, while the number of workers decreases, and vice versa. The more the value of the parameter increases, the more the number of unemployed increases, the more job opportunities decreases, and the more the number of workers increases.



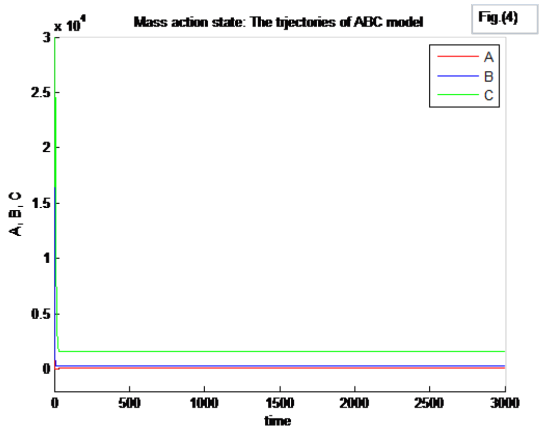
(A) System (1) with ($\gamma = 0.89$) for the data given in Table 2, beginning at $(17500, 17000, 16500)$ time series, approaches asymptotically to the point $P = (0.9322, 0.2035, 0.9423)$

(B) System (1) has a trajectory that approaches asymptotically to the point $P = (0.9322, 0.2035, 0.9423)$.

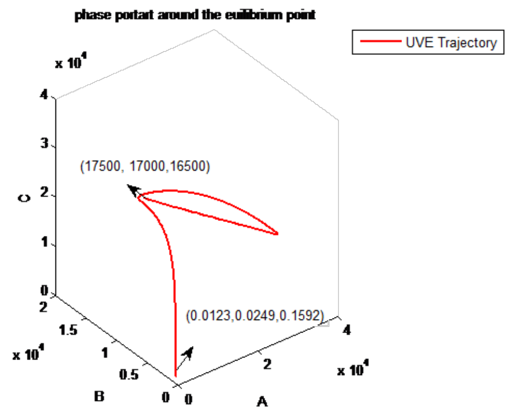
FIGURE 3. System (1) with different values of γ .

Therefore, in the range $(0.4 \leq \gamma \leq 0.99)$, it gives Figure $(3.a)$ in comparison to Figure $(1-a)$. With the increase in the value of the parameter, the job opportunities available to the unemployed begin to match the job opportunities available to them, and in contrast, the number of workers increases significantly, which is the opposite of Figure $(3.a)$, until it reaches Figure $(1.a)$.

Scenario 3: By varying δ (with respect of employed people it can represent death, rate of migration, and retirement), when $(0.0001 \leq \delta \leq 0.1)$. it has no effect, and the drawing and results remain as shown in Figures $(1-a)$ and $(1.b)$, while, for $(0.2 \leq \delta \leq 1)$, with the increase in the parameter value, the job opportunities available to the unemployed increased with a significant decrease in the number of unemployed and employed people, as shown in Figure 4, typically at $(\delta = 1)$.



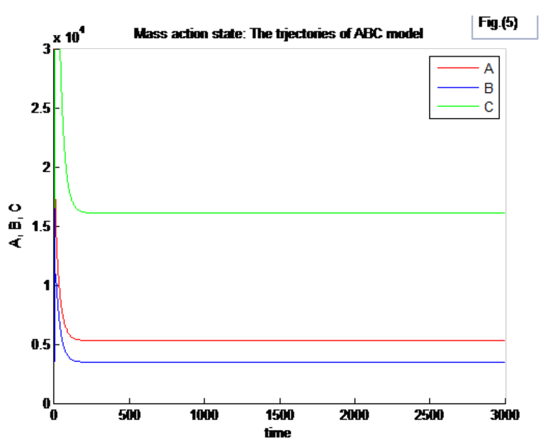
(A) System(1) with ($\delta = 1$) for the data given in Table 2, beginning at $(17500, 17000, 16500)$ time series, approaches asymptotically to the point $P = (0.0123, 0.2035, 0.1592)$.



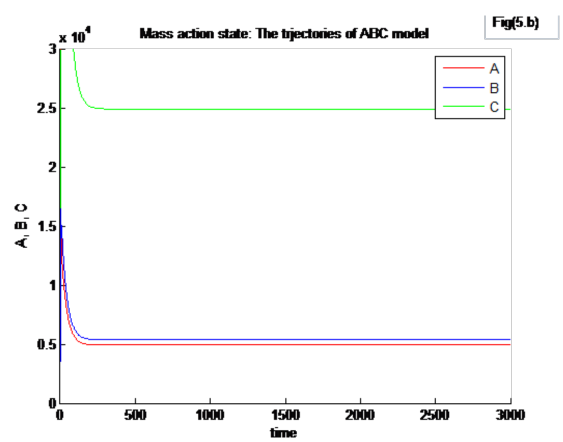
(B) With ($\delta = 1$) system(1) has a trajectory that approaches asymptotically to the point $P = (0.0123, 0.0249, 0.1592)$.

FIGURE 4. System (1) with different values of δ .

By assorted ϵ, at the interval $(0.1 \leq \epsilon \leq 0.99)$, slowly increasing the value of the rate of government and private job opportunities, distinguish a significant increase in the job opportunities plan, at contract by a low values in the number of unemployed due to the availability of new job opportunities. as shown in the Figure (5.a), fit at ($\epsilon = 0.6$), reaching at the number of unemployed almost matches the number of workers, nearly at ($\epsilon = 0.99$) as illustrated in Figure 5.b



(A) With ($\epsilon = 0.6$) System(1) for the data given in Table 2, beginning at $(17500, 17000, 16500)$ approaches asymptotically to the point $P = (0.5305, 0.3474, 1.6089)$.

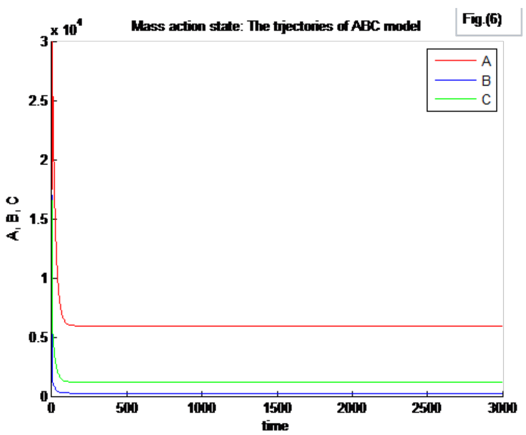


(B) With ($\epsilon = 0.99$) system(1) has a trajectory that approaches asymptotically to the point $P = (0.4925, 0.5376, 2.4893)$..

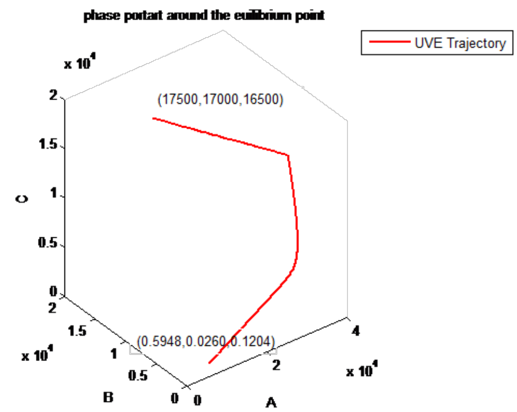
FIGURE 5. System (1) with different values of δ .

Scenario 4: Varying the parameter θ (the rate of jobs filled by automation robots and artificial intelligence machines in factories or any other type of job). In the range $(0.001 \leq \theta \leq 0.99)$. It gives the results shown in Figure 6.

As the rate of jobs occupied by automation robots and artificial intelligence machines gradually increases in factories or any other type of jobs there will be a significant increase in unemployment, accompanied by a decrease in the number of workers and in the job opportunities plan, as shown in Figure 6, usually at $(\theta = 0.99)$.



(A) With $(\theta = 0.99)$ the system(1) has time series approaches asymptotically to the unique point $P = (0.5948, 0.0260, 0.1204)$.

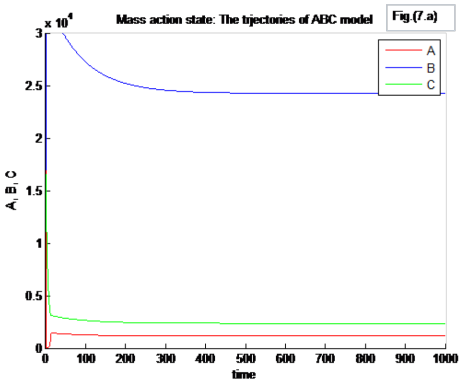


(B) With $(\theta = 0.99)$ system(1) has a trajectory that approaches asymptotically to the unique point $P = (0.5948, 0.0260, 0.1204)$.

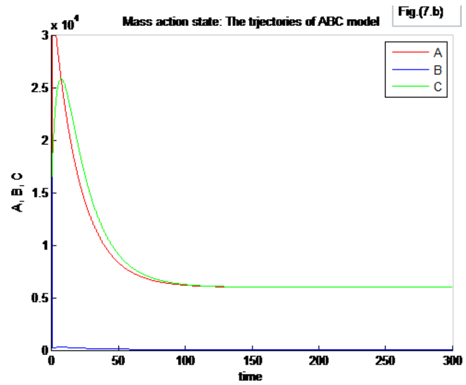
FIGURE 6. System (1) with different values of θ .

By varying the parameter ψ (the increased rate of competition between employment and unemployment) in the range $(0.001 \leq \psi \leq 0.99)$. It gives the results shown in Figure(7-a).

The parameter variation stage goes through two stages. When it is decreasing, this leads to a noticeable increase in the number of workers and a noticeable decline and decrease in the number of unemployed and job opportunities together typically at $(\psi = 0.001)$. As presented in Figure(7-a) coincide with Figure (2-b), while in the case of an increase in the value of the parameter. The previous scenario is completely reversed, such that the number of workers declines and diminishes, while it closely matches the increase in both the number of unemployed and job opportunities typically at $(\psi = 0.99)$ as shown in Figure (7-b).



(A) with ($\psi = 0.001$) the system(1) has time series approaches asymptotically to the unique point $P = (0.1161, 2.4555, 0.2391)$.



(B) With ($\psi = 0.99$) system(1) has a trajectory that approaches asymptotically to the unique point $P = (0.5987, 0.0065, 0.5990)$.

FIGURE 7. System (1) with different values of θ .

5. Bifurcation Analysis

After studying the numerical behavior of the system and choosing a set of parameters contributing to the study of the proposed model, we study the bifurcation in the presence of a single fixed point according to our proposed model, and the following cases appear.

- For the parameter θ: the rate of jobs occupied by automation (robots and artificial intelligence machines in factories or any other type of jobs) plays an effective role in influencing the stability of the solution for the system by choosing a specific threshold for this parameter ($\theta \geq 1$) in which the system switches from a stable state to an unstable state at the positive point $P = (\hat{A}, \hat{B}, \hat{C})$. Otherwise, the positive point remains stable, and this phenomenon of transition from stability to instability is called transcritical bifurcation, as shown in Figure 8 with a typical value ($\theta = 1.2$) in comparison to Figure 6.

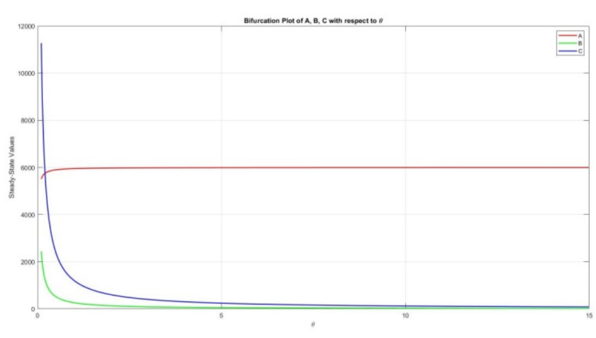


FIGURE 8. The solution of the system(1) for the data given by Table 2, which shows that local bifurcation (Transcritical) for $\theta = 1.2$.

Figure 8 shows the threshold of the parameter θ it reaches the state of mass unemployment and the collapse of the labor market with a complete loss of workers and a complete disappearance of job opportunities. This is a state that portends a catastrophe on the human level and calls for economic explanations that warn of the need for governmental and political intervention to reconsider the size and method of automation's intervention in the labor market and to take the correct economic measures to avoid this catastrophe.

- For the parameter ψ: the increased rate of competition between employment and unemployment plays an effective role in influencing the stability of the solution for the system by choosing the specific threshold for this parameter $(\psi \geq 0.99)$ at which the system switches from a stable state to an unstable state at the positive point $P = (\hat{A}, \hat{B}, \hat{B})$. Otherwise, the positive point remains stable, and this phenomenon of transition from stability to instability is called transcritical bifurcation, as shown in Figure 9 with a typical value $(\psi = 3)$.

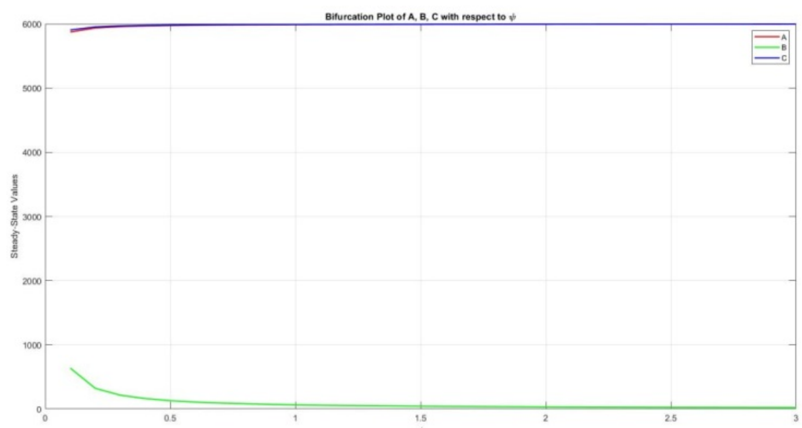


FIGURE 9. solution of system(1) for the data given by Table 2, which shows that local bifurcation (Transcritical) for $(\psi = 3)$.

Figure 9 shows that we have obtained a transition from the positive point to a point where the unemployed match the job opportunities with a complete absence of the number of workers, when the coefficient (ψ) is an unrealistic situation in the labor market.

- For the parameter ϵ: the rate of the available vacancies created by the government with the private sector and the number of unemployed people play an effective role in influencing the stability of the solution for the system by choosing a specific threshold for this parameter $(\epsilon = 0.001)$ at which the system switches from a stable state to an unstable state at the positive point $P = (\tilde{A}, \tilde{B}, \tilde{C})$. Otherwise, the positive point remains stable, and this phenomenon of transition from stability to instability is called transcritical bifurcation, as shown in Figure 10 with a typical value $(\epsilon = 0.001)$.

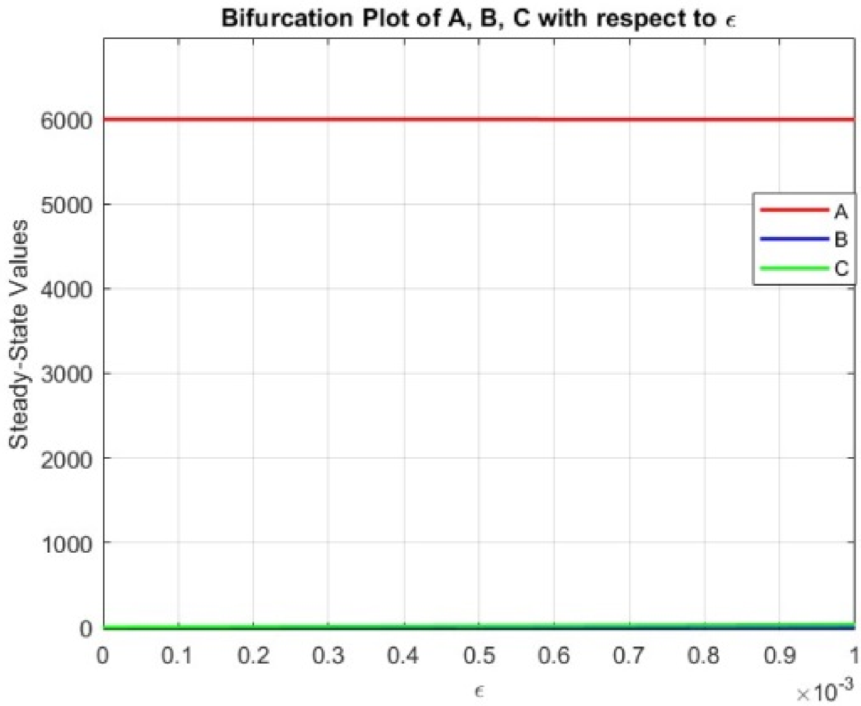


FIGURE 10. The solution of system (1) for the data given by Table 2, which shows the local bifurcation (Transcritical) for $\epsilon = 0.001$.

From Figure 10, it is clear that we have obtained a transition from the positive point to a point of complete absence in the number of workers and job opportunities when the parameter ϵ is equal to zero, while only the number of unemployed remains, which is an unrealistic situation in the labor market.

- For the parameter γ: Annihilation or the end of life, and the rate of migration of unemployed people) plays an effective role in influencing the stability of the solution for the system by choosing a specific threshold for this parameter $(\gamma = 10)$ at which the system switches from a stable state to an unstable state at the positive point $P = (\tilde{A}, \tilde{B}, \tilde{C})$. Otherwise, the positive point remains stable, and this phenomenon of transition from stability to instability is called transcritical bifurcation, as shown in Figure 11 with a typical value $(\gamma = 10)$.

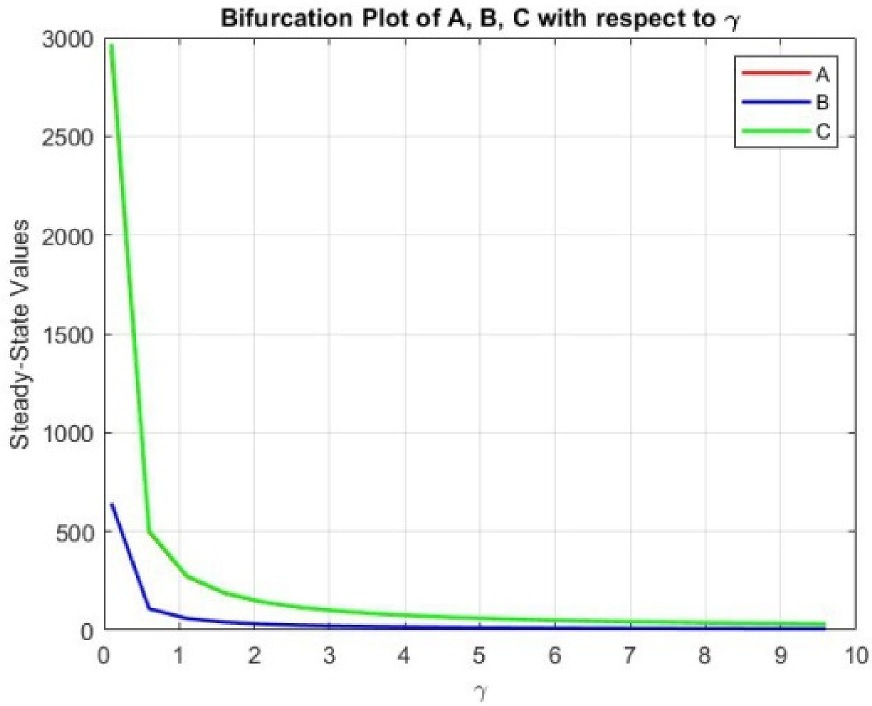


FIGURE 11. The solution of system(1) for the data given by Table 2, which shows the local bifurcation (Transcritical) for $\gamma = 10$.

From Figure 11, it is clear that we have obtained a transition from the positive point to the point of complete absence in the number of unemployed workers and job opportunities when the coefficient $\gamma = 10$, which is an unrealistic and impossible situation in the labor market.

6. Conclusions

Using three nonlinear equations in a system represented by three dynamic variables unemployed, employed, and vacancies. This mathematical model of unemployment examines the entry of automation into the labor market and its influence on the proposed model. Only one positive equilibrium is derived to describe this phenomenon, plus certain conditions assert that the positive equilibrium point can be both stable locally and globally. Via Lyapunov, the dynamics of the model are studied. Observations show that the positive equilibrium point is asymptotically stable everywhere, as shown in Figure 1.

Authors' Contributions. All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] N. Masoud, Artificial Intelligence and Unemployment Dynamics: An Econometric Analysis in High-Income Economies, Technol. Sustain. 4 (2024), 30–50. https://doi.org/10.1108/techs-04-2024-0033.

[2] C. Magazzino, M. Mele, M. Mutascu, An Artificial Neural Network Experiment on the Prediction of the Unemployment Rate, J. Polic. Model. 47 (2025), 471–491. https://doi.org/10.1016/j.jpolmod.2024.10.004.

[3] M.J. Idrisi, D. Geteye, P. Shanmugasundaram, Modeling the Complex Interplay: Dynamics of Job Displacement and Evolution of Artificial Intelligence in a Socio-Economic Landscape, Int. J. Netw. Distrib. Comput. 12 (2024), 185–194. https://doi.org/10.1007/s44227-024-00025-0.

[4] M. Badiuzzaman, M. Rafiquzzaman, Automation and Robotics: A Review of Potential Threat on Unskilled and Lower Skilled Labour Unemployment in Highly Populated Countries, Int. Bus. Manag. 14 (2020), 16–24.

[5] A.K. Misra, M. Kumari, Dynamic Relationship Between Informal Sector and Unemployment: A Mathematical Model, Int. J. Bifurc. Chaos 34 (2024), 2450018. https://doi.org/10.1142/s0218127424500184.

[6] A.J. Kadhima, A.A. Majeed, Epidemiological model Involving Two Diseases in Predator Population with Holling Type-II Functional Response, Int. J. Nonlinear Anal. Appl. 12 (2021), 2085–2107.

[7] K. AL-Jubouri, R. Hussien, N. Al-Saidi, Modeling of an Eco-Epidemiological System Involving Various Epidemic Diseases with Optimal Harvesting, Eurasian J. Math. Comput. Appl. 8 (2020), 4–27. https://doi.org/10.32523/2306-6172-2020-8-2-4-27.

[8] Z.K. Alabacy, S.H. Salih, S. Khlil, N.M.G. Al-Saidi, Stabilizing Ecosystem Dynamics: The Toxicity Effect on an Ecosystem in the Presence of Self-Defence, Commun. Math. Biol. Neurosci. 2024 (2024), 62. https://doi.org/10.28919/cmbn/8575.

[9] M. Rasheed, O. Alabdali, S. Shihab, A. Rashid, T. Rashid, On the Solution of Nonlinear Equation for Photovoltaic Cell Using New Iterative Algorithms, J. Phys.: Conf. Ser. 1999 (2021), 012078. https://doi.org/10.1088/1742-6596/1999/1/012078.

[10] S.H. Salih, N.M. Al-Saidi, A. Malik, S.J. Obaiys, Analyzing Stability and Controlling Chaos in a Unique Host Parasitoid Discrete Dynamical System with Applications in Artificial Intelligence, Math. Model. Eng. Probl. 11 (2024), 2152–2162. https://doi.org/10.18280/mmep.110817.

[11] A. Misra, A.K. Singh, A Mathematical Model for Unemployment, Nonlinear Anal.: Real World Appl. 12 (2011), 128–136. https://doi.org/10.1016/j.nonrwa.2010.06.002.

[12] L. Harding, M. Neamţu, A Dynamic Model of Unemployment with Migration and Delayed Policy Intervention, Comput. Econ. 51 (2016), 427–462. https://doi.org/10.1007/s10614-016-9610-3.

[13] G.N. Pathan, P.H. Bhathawala, Unemployment Discussion With a Mathematical Model, Int. J. Bus. Manag. Econ. Res. 3 (2016), 19–27.

[14] S.B. Munoli, S. Gani, S.R. Gani, A Mathematical Approach to Employment Policies: An Optimal Control Analysis, Int. J. Stat. Syst. 12 (2017), 549–565.

[15] R.M. Al-Maalwi, H.A. Ashi, S. Al-sheikh, Unemployment Model, Appl. Math. Sci. 12 (2018), 989–1006. https://doi.org/10.12988/ams.2018.87102.

[16] R. Al-Maalwi, S. Al-Sheikh, H. Ashi, S. Asiri, Mathematical Modeling and Parameter Estimation of Unemployment with the Impact of Training Programs, Math. Comput. Simul. 182 (2021), 705–720. https://doi.org/10.1016/j.matcom.2020.11.018.

[17] S. Al-Sheikh, R. Al-Maalwi, H.A. Ashi, A Mathematical Model of Unemployment with the Effect of Limited Jobs, Comptes Rendus. Mathématique 359 (2021), 283–290. https://doi.org/10.5802/crmath.164.

[18] M. El Yahyaoui, S. Amine, Mathematical Modeling of Unemployment Dynamics with Skills Development and Cyclical Effects, Partial. Differ. Equ. Appl. Math. 11 (2024), 100800. https://doi.org/10.1016/j.padiff.2024.100800.

[19] K. Bansal, T. Mathur, Impact of Skills Development on Youth Unemployment: A Fractional-order Mathematical Model, Math. Methods Appl. Sci. 47 (2024), 14286–14303. https://doi.org/10.1002/mma.10272.

[20] Y.A. Kuznetsov, Elements of Applied Bifurcation Theory, Springer, Cham, 2023. https://doi.org/10.1007/978-3-031-22007-4.

[21] Z.K. Alabacy, A.A. Majeed, The Local Bifurcation Analysis of Two Preys Stage-Structured Predator Model with Anti-Predator Behavior, J. Phys.: Conf. Ser. 2322 (2022), 012061. https://doi.org/10.1088/1742-6596/2322/1/012061.

[22] A.A. Majeed, A.J. Kadhim, The Bifurcation Analysis and Persistence of the Food Chain Ecological Model with Toxicant, J. Phys.: Conf. Ser. 1818 (2021), 012191. https://doi.org/10.1088/1742-6596/1818/1/012191.