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ALMOST NEARLY (τ_1, τ_2)-CONTINUOUS MULTIFUNCTIONS

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Abstract. This paper is concerned with the concept of almost nearly (τ_1, τ_2)-continuous multifunctions. Furthermore, some characterizations of almost nearly (τ_1, τ_2)-continuous multifunctions are considered.

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1. Introduction

The concept of almost continuous functions was introduced by Singal and Singal [52]. Popa [47] defined almost quasi-continuous functions as a generalization of almost continuity [52] and quasi-continuity [42]. Munshi and Bassan [43] studied the notion of almost semi-continuous functions. Maheshwari et al. [40] introduced the concept of almost feebly continuous functions as a generalization of almost continuity [52]. Dungthaisong et al. [31] introduced and investigated the concept of $g_{(m,n)}$-continuous functions. In [14], the present authors studied some properties of (Λ, sp)-open sets and (Λ, sp)-closed sets. Viriyapong and Boonpok [62] investigated several characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets. Duangphui et al. [30] introduced and studied the notion of almost $(\mu, \mu')^{(m,n)}$-continuous functions. Moreover, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions and weakly (τ_1, τ_2)-continuous functions were presented

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in [53], [56], [1], [48], [13], [9], [10], [21], [25], [26], [2], [3] and [4], respectively. Kong-ied et al. [38] introduced and studied the concept of almost quasi (τ_1, τ_2)-continuous functions. Chiangpradit et al. [28] introduced and investigated the notion of weakly quasi (τ_1, τ_2)-continuous functions. Thongmon et al. [55] introduced and studied the concept of rarely (τ_1, τ_2)-continuous functions. Malghan and Hanchinamani [41] introduced the concept of N-continuous functions. Noiri and Ergun [45] investigated some characterizations of N-continuous functions. Ekici [33] introduced and studied the concept of nearly continuous multifunctions as a generalization of semi-continuous multifunctions and N-continuous functions.

In 2004, Ekici [32] introduced and investigated the notion of almost nearly continuous multifunctions as a generalization of nearly continuous multifunctions and almost continuous multifunctions [46]. Laprom et al. [39] introduced and investigated the concept of $\beta(\tau_1, \tau_2)$-continuous multifunctions. Furthermore, several characterizations of $(\tau_1, \tau_2)\alpha$-continuous multifunctions, $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, almost quasi $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\imath^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, (τ_1, τ_2)-continuous multifunctions, almost (τ_1, τ_2)-continuous multifunctions, weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (τ_1, τ_2)-continuous multifunctions and s-$(\tau_1, \tau_2)p$-continuous multifunctions were established in [63], [20], [18], [23], [17], [61], [5], [8], [22], [11], [6], [7], [15], [19], [12], [35], [16], [58], [51], [36], [57], [49] and [59], respectively. Khampakdee et al. [34] introduced and investigated the concept of c-(τ_1, τ_2)-continuous multifunctions. Pue-on et al. [50] introduced and studied the notion of almost quasi (τ_1, τ_2)-continuous multifunctions. Noiri and Popa [44] introduced and investigated the notion of almost nearly m-continuous multifunctions as multifunctions from a set satisfying some minimal conditions into a topological spaces. Carpintero et al. [27] introduced and studied the notion of nearly ω-continuous multifunctions as a weaker form of nearly continuous multifunctions. In this paper, we introduce the concept of almost nearly (τ_1, τ_2)-continuous multifunctions. We also investigate several characterizations of almost nearly (τ_1, τ_2)-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i

are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [24] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [24] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [24] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [24] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) *$A \subseteq \tau_1\tau_2$-Cl(A) and $\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Cl$(A)) = \tau_1\tau_2$-Cl(A).*
(2) *If $A \subseteq B$, then $\tau_1\tau_2$-Cl$(A) \subseteq \tau_1\tau_2$-Cl$(B)$.*
(3) *$\tau_1\tau_2$-Cl(A) is $\tau_1\tau_2$-closed.*
(4) *A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2$-Cl(A).*
(5) *$\tau_1\tau_2$-Cl$(X - A) = X - \tau_1\tau_2$-Int$(A)$.*

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [63] (resp. $(\tau_1, \tau_2)s$-*open* [20], $(\tau_1, \tau_2)p$-*open* [20], $(\tau_1, \tau_2)\beta$-*open* [20]) if $A = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$, $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$, $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [60] if $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be $\alpha(\tau_1, \tau_2)$-*closed*. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\mathscr{N}(\tau_1, \tau_2)$-*closed* [54] if every cover of A by $(\tau_1, \tau_2)r$-open sets of X has a finite subcover.

Lemma 2. [29] *Let (X, τ_1, τ_2) be a bitopological space. If V is a $\tau_1\tau_2$-open set of X having $\mathscr{N}(\tau_1, \tau_2)$-closed complement, then*

$$\tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(V))$$

is a $(\tau_1, \tau_2)r$-open set having $\mathscr{N}(\tau_1, \tau_2)$-closed complement.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3. Almost nearly (τ_1, τ_2)-continuous multifunctions

In this section, we introduce the notion of almost nearly (τ_1, τ_2)-continuous multifunctions. Moreover, several characterizations of almost nearly (τ_1, τ_2)-continuous multifunctions are discussed.

Definition 1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost nearly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open sets V, V' of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements such that $x \in F^+(V) \cap F^-(V')$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V')))$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost nearly (τ_1, τ_2)-continuous if F is almost nearly (τ_1, τ_2)-continuous at each point x of X.*

Theorem 1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is almost nearly (τ_1, τ_2)-continuous;*

(2) *for each $x \in X$ and for every $(\sigma_1, \sigma_2)r$-open sets V, V' of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements such that $F(x) \subseteq V$ and $F(x) \cap V' \neq \emptyset$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(z) \subseteq V$ and $F(z) \cap V' \neq \emptyset$ for every $z \in U$;*

(3) *for each $x \in X$ and for every $\sigma_1\sigma_2$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed sets K, K' of Y such that $x \in F^+(Y - K) \cap F^-(Y - K')$, there exists a $\tau_1\tau_2$-closed set $H \neq X$ such that $x \in X - H$ and $F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \cup F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K'))) \subseteq H$;*

(4) *$F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V')))$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-open sets V, V' of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements;*

(5) *$F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \cup F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K')))$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed sets K, K' of Y;*

(6) *$F^+(V) \cup F^-(V')$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)r$-open sets V, V' of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements;*

(7) *$F^-(K) \cap F^+(K')$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)r$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed sets K, K' of Y.*

Proof. (1) ⇒ (2): Let $x \in X$ and V, V' be any $(\sigma_1, \sigma_2)r$-open sets of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements such that $F(x) \subseteq V$ and $F(x) \cap V' \neq \emptyset$. By (1), there exists a $\tau_1\tau_2$-open set U of X containing x such that

$$\begin{aligned} U &\subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V'))) \\ &= F^+(V) \cap F^-(V'). \end{aligned}$$

(2) ⇒ (1): It is sufficient to observe that for any $(\sigma_1, \sigma_2)r$-open sets V, V' of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements the sets $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ and $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V'))$ are $(\sigma_1, \sigma_2)r$-open in Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements.

(1) ⇒ (3): Let $x \in X$ and K, K' be any $\sigma_1\sigma_2$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed sets of Y such that $x \in F^+(Y - K) \cap F^-(Y - K')$. Then, $Y - K$ and $Y - K'$ are $\sigma_1\sigma_2$-open sets having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements. Since F is almost nearly (τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that

$$U \subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - K))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - K')))$$

$$= X - [F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \cup F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K')))].$$

It is clear that $H = X - U$ is $\tau_1\tau_2$-closed in X and the inclusion $F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \cup F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K'))) \subseteq H$ is satisfied.

(3) $\Rightarrow$ (1): The proof is similar to the proof (1) $\Rightarrow$ (3).

(1) $\Rightarrow$ (4): Let V, V' be any $\sigma_1\sigma_2$-open sets of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements. Let

$$x \in F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V'))).$$

Then, we have $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ and $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V'))$ are $\sigma_1\sigma_2$-open sets of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements. By the definition of almost nearly (τ_1, τ_2)-continuous at a point x, there exists a $\tau_1\tau_2$-open set U of X containing x such that

$$U \subseteq F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V'))).$$

Since U is $\tau_1\tau_2$-open, we have

$$x \in \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V'))))$$

and hence

$$\begin{aligned}&F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V')))\\ &\subseteq \tau_1\tau_2\text{-Int}(F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V')))).\end{aligned}$$

Thus, $F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V')))$ is $\tau_1\tau_2$-open in X.

(4) $\Rightarrow$ (1): The proof is clear.

(4) $\Rightarrow$ (5): Let K, K' be any $\sigma_1\sigma_2$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed sets of Y. Then, we have $Y - K$ and $Y - K'$ are $\sigma_1\sigma_2$-open sets having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements. By (4),

$$\begin{aligned}&F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - K))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - K')))\\ &= F^+(Y - \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \cap F^-(Y - \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K')))\\ &= [X - F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))] \cap [X - F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K')))]\\ &= X - [F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \cup F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K')))]\end{aligned}$$

is $\tau_1\tau_2$-open in X. Thus,

$$F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \cup F^+(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K')))$$

is $\tau_1\tau_2$-closed in X.

(5) $\Rightarrow$ (4): It can be obtained similarly as (4) $\Rightarrow$ (5).

(4) $\Rightarrow$ (6): It is easily seen that the set

$$F^+(V) \cap F^+(V') = F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \cap F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V')))$$

for every $(\sigma_1, \sigma_2)r$-open sets V, V' of Y.

$(6) \Rightarrow (4)$: The proof is a consequence of Lemma 2.

$(6) \Rightarrow (7)$: Let K, K' be any $(\sigma_1, \sigma_2)r$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed sets of Y. Then, we have $Y - K$ and $Y - K'$ are $(\sigma_1, \sigma_2)r$-open sets having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complements. By (6), we have

$$F^+(Y - K) \cup F^-(Y - K') = X - (F^-(K) \cap F^+(K'))$$

is $\tau_1\tau_2$-open in X. Therefore, $F^-(K) \cap F^+(K')$ is $\tau_1\tau_2$-closed in X.

$(7) \Rightarrow (6)$: It can be obtained similarly as $(6) \Rightarrow (7)$. □

Definition 2. [37] *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost nearly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost nearly (τ_1, τ_2)-continuous if f has this property at every point of X.*

Corollary 1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is almost nearly (τ_1, τ_2)-continuous;*

(2) *for each $x \in X$ and for every $(\sigma_1, \sigma_2)r$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$- closed complement containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$;*

(3) *for each $x \in X$ and for every $\sigma_1\sigma_2$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed set K of Y such that $x \in f^{-1}(Y - K)$, there exists a $\tau_1\tau_2$-closed set $H \neq X$ such that $x \in X - H$ and $f^{-1}(\sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(K))) \subseteq H$;*

(4) *$f^{-1}(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V)))$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(5) *$f^{-1}(\sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(K)))$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed set K of Y;*

(6) *$f^{-1}(V)$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)r$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(7) *$f^{-1}(K)$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)r$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed set K of Y.*

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