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PERFORMANCE LOSS IN DEPENDENT SEISMIC RISK MODELS

SHAFIRA FAUZIA UNTSA, NANANG SUSYANTO*, DANANG TEGUH QOYYIMI, DWI ERTININGSIH

Department of Mathematics, Universitas Gadjah Mada, Indonesia

*Corresponding author: nanang_susyanto@ugm.ac.id

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Abstract. Indonesia's vulnerability to seismic activity, stemming from its location at the convergence of four tectonic plates, highlights the critical need for accurate assessment of seismic losses. The seismic event of May 27, 2006, which led to substantial economic losses, underscores the importance of developing robust risk management strategies. A comprehensive understanding of dependencies between risks is essential for precise risk assessment, particularly given the diverse and interconnected nature of risk sources. The Collective Risk Model (CRM) provides insights into such collective risk events, emphasizing the need for effectively modeling dependency structures. This study examines the influence of the correlation coefficient on individual risk models and their associated risk measures, including variance, Value at Risk (VaR), and Tail Value at Risk (TVaR). Furthermore, the performance loss resulting from disregarding dependence is quantitatively analyzed. Specifically, analytic performance loss is derived under the assumption that losses follow a normal distribution, while numerical performance loss is explored for losses exhibiting a normal copula distribution.

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1. Introduction

Indonesia's geographic location at the convergence of four tectonic plates—the Eurasian Plate, the Indo-Australian Plate, the Philippine Sea Plate, and the Pacific Plate—results in a very high seismic risk [19]. Earthquakes originating from subduction zones across Indonesia have unpredictable return periods, contributing to uncertainties in assessing and managing earthquake disaster risks. The resulting seismic losses can impose significant physical and financial hardships on numerous households and businesses. A notable seismic event with substantial economic losses occurred on May 27, 2006, when

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a 6.3 Mw earthquake struck, severely affecting Bantul Regency and Yogyakarta City, as well as Klaten Regency and its surrounding areas. The direct economic impact was estimated at 3.1 billion USD, with 90% of the losses borne by the private sector and only 10% by the public sector [5]. Consequently, accurate measurement of seismic risk is essential for effective risk management and loss mitigation strategies.

In recent years, risk management has had to consider not only individual or isolated risks but also combinations of multiple sources of risk. A crucial question is whether these risks are interconnected. The 2004 Indian Ocean earthquake and tsunami are examples of such collective risk events. The Collective Risk Model (CRM) approach is commonly employed to assess earthquake disaster risks [24]. In actuarial science, CRM is used to describe the aggregate claim amount for a portfolio or non-life insurance contract, and it is based on three fundamental principles and assumptions. However, in practice, the independence assumptions underlying CRM are often violated [2]. This calls for a seismic risk assessment model that can account for dependencies between risks, emphasizing the need to model and describe their dependency structure.

The assumption of independence between policy classes in traditional actuarial models often does not reflect real-world situations. For instance, in a densely populated housing complex, a house located just 10 meters from a burning house is at a higher risk than one 100 meters away. This reality motivates the development of risk models that incorporate diverse dependency structures. In many practical situations, modeling the connection or dependence between variables influencing seismic losses is crucial. Several researchers have proposed seismic risk models that consider dependence, such as [9], [23], and [12]. Atkinson et al. [1] analyzed ground motion relationships in subduction zones via the maximum likelihood method, providing essential input for global seismic hazard analysis. Wan et al. [22] introduced a semi-parametric model based on elliptical copulas for better estimating the tail dependence function in financial markets. Hong and Goda [11] examined the use of attenuation relationships based on the pseudo-spectral acceleration (PSA) response. Klüppelberg et al. [14] studied the impact of inter-claim dependencies on insurance risk models across multiple business classes. Goda and Hong [8] investigated how spatial correlation and simultaneous seismic excitation affect seismic risk estimates in building clusters, finding that structural capacity uncertainties and local soil conditions play significant roles.

Recent focus on individual risk models has highlighted their varied and complex dependence structures, which must be adequately accounted for since they can significantly influence the distribution of losses. Copulas offer an efficient solution to model such dependence in individual risk scenarios [3]. A copula captures the relationship between variables without being influenced by their marginal distributions [15]. Significant applications of copulas have emerged in various fields. Cossette et al. [4] recommended two construction methods to model the relationship between risks in an insurance portfolio within an individual risk framework: modeling individual and joint risk factors and the application of copulas. Goda and Ren [9] used copula functions to model the dependence between seismic losses of buildings and infrastructure, emphasizing the importance of correlated seismic effects on the probability distribution of aggregate losses from spatially distributed structures. Xu et al. [23] employed a copula-based model to investigate the joint probability of extreme rainfall and storm surge in Fuzhou City. Navarro and Sarabia [17] derived analytical expressions for the distribution of dependent random variables linked by a given copula, providing explicit models for specific copula and marginal distributions. Blier-Wong et al. [2] studied a copula-based CRM, using the Farlie-Gumbel-Morgenstern (FGM) copula to explore the dependency between claim frequency and severity.

To understand the total claim amount of a portfolio contract, it is necessary to sum a variable number of random variables (RVs) representing individual risks. The distribution of the sum of these RVs is key to defining aggregate risks [17]. This study focuses on understanding how the correlation coefficient affects individual risk models and evaluates its impact on various risk measures, including variance, Value at Risk (VaR), and Tail Value at Risk (TVaR). Additionally, we compare scenarios of ignoring dependence by assuming independence between random variables to those incorporating dependence. Performance loss analysis is then used to quantify the differences in risk measures between these scenarios, which serves as a main contribution to this paper. To illustrate this concept, we focus on the performance loss of assuming independence when aggregating risks that are actually dependent.

We explore two scenarios for the dependence of risks: losses modeled under a normal distribution and those under a normal copula distribution. The normal distribution is widely used in numerous applications, and performance loss under this assumption is derived analytically. However, normal distribution assumptions have limitations. To introduce more flexibility, we also consider a normal copula, where the marginal distributions are not necessarily normal. While more realistic, this scenario requires numerical, rather than analytical, computation of performance loss.

The remainder of the paper is structured as follows. Section 2 addresses the proposed performance loss theorem with the analytical performance loss for normally distributed losses, and investigates the numerical performance loss for losses modeled under a normal-copula distribution. Finally, Section 3 presents the conclusions.

2. Performance Loss

In catastrophe (CAT) modeling, the assumption of independence between events is often employed to simplify calculations and enhance the mathematical tractability of models. This assumption facilitates manageable computations when estimating aggregate losses across multiple regions or events, particularly in complex risk portfolios. However, the independence assumption can be problematic, as real-world events frequently exhibit dependencies, which can lead to underestimation or misrepresentation of risk. For instance, studies on competing risks highlight that assuming independence may allow for the use of common statistical methods but can result in incorrect risk estimates when events are actually dependent [20,21].

Moreover, Embrechts et al. [6] emphasize the challenges of understanding dependencies between risks. They caution that relying solely on simple linear correlation without accounting for other factors can lead to significant errors, particularly when risks do not adhere to normal distribution patterns. To address these limitations, we propose a performance loss metric to quantify the discrepancy between risk estimates obtained under the assumption of independence and those derived using models that incorporate dependency structures. This metric provides a more nuanced understanding of risk, supporting improved decision-making in risk management.

To quantify the effect of ignoring dependencies, we define the **performance loss (PL)** as:

$$\mathrm{PL}(\pi) = \pi(\mathrm{L}_{\mathrm{full}}) - \pi(\mathrm{L}_{\mathrm{ind}}), \tag{1}$$

where $\pi(\cdot)$ represents a risk measure, such as variance, Value at Risk (VaR), or Tail Value at Risk (TVaR). The term $\mathrm{L}_{\mathrm{full}}$ denotes the aggregate loss under the "full" or realistic model, where dependencies between events are explicitly considered. This model incorporates correlations or interactions between individual risks. Conversely, $\mathrm{L}_{\mathrm{ind}}$ represents the aggregate loss under the assumption of independence, wherein each event or risk is treated as if it occurs independently of others.

The distinction between $\mathrm{L}_{\mathrm{full}}$ and $\mathrm{L}_{\mathrm{ind}}$ forms the basis for understanding performance loss in various risk estimation scenarios. To illustrate this concept, we first examine a specific case where joint risks follow a normal distribution. This approach enables us to analyze how dependencies between risks influence aggregate losses and how risk measures can be derived explicitly from $\mathrm{L}_{\mathrm{full}}$ and $\mathrm{L}_{\mathrm{ind}}$. In this case, the explicit form of the performance loss is provided.

For the second case, to capture non-linear dependencies, we consider individual risks with specific marginal distributions while modeling their dependencies using a copula. This approach accommodates complex dependency structures, providing a more flexible representation of aggregate risk. The performance loss in this scenario is evaluated numerically to address the challenges posed by the non-linear nature of the dependency structure. However, due to the lack of an analytical expression for the performance loss, it is computed using numerical methods.

The dependence for both cases is determined by a parameter called "correlation." In this study, we focus exclusively on positive values of correlation, as real-world risk modeling consistently observes positive dependencies between locations. When one risk increases, others tend to follow suit due to spatial dependence. This positive correlation aligns with scenarios such as portfolio risk, where interrelated risks often rise or fall together.

2.1. **Normal Distributed Losses.** Let's assume that the true joint distribution of individual risks X and Y follow a normal distribution, with mean $(\mu_X, \mu_Y)^T$ and covariance matrix $\begin{pmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix}$. Here, ρ represents the correlation coefficient between X and Y. The aggregate claim amount is $\mathrm{L}_{\mathrm{full}} = X + Y$, is normally distributed with mean $\mu_X + \mu_Y$ and variance $\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y$. As mentioned in the main aim of this paper, we will compute the performance loss defined in (1). Under independence assumption, the loss is also normally distributed with mean $\mu_X + \mu_Y$ and variance $\sigma_X^2 + \sigma_Y^2$ (note that under normal model, independence equivalent with zero correlation). From these distributions, we can calculate the PL under different risk measures: variance, Value at Risk (VaR), and Tail Value at Risk (TVaR), which are essential in understanding the behavior of aggregate risks in the portfolio. The performance loss under all risk measures will be given analytically as provided in the following theorem.

Theorem 2.1. *Given two losses X and Y that are jointly normal with mean $(\mu_X, \mu_Y)^T$ and covariance matrix $\begin{pmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix}$. Suppose L_{full} is a model considered dependencies and L_{ind} is a model that assumes independence (ignore dependence). The performance loss risk measure variance (var), Value at Risk (Var), and Tail Value at Risk (TVaR) is given as follows:*

$$PL(var) = 2\rho\sigma_X\sigma_Y, \tag{2}$$

$$PL(VaR) = \left(\sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y} - \sqrt{(\sigma_X^2 + \sigma_Y^2)}\right)\Phi^{-1}(p), \quad \forall p \in (0,1), \tag{3}$$

and

$$PL(TVaR) = \left(\sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y} - \sqrt{(\sigma_X^2 + \sigma_Y^2)}\right)\frac{\phi[\Phi^{-1}(p)]}{1-p} \quad \forall p \in (0,1), \tag{4}$$

respectively. Here, the notations ϕ and Φ^{-1} represent the probability density function (pdf) and the quantile function (inverse cumulative distribution function) of the standard normal distribution, respectively.

Proof. For **variance** as the risk measure, as previously mentioned, the variance of the aggregate loss under the $\mathrm{L}_{\mathrm{full}}$ model is given by

$$\mathrm{var}(\mathrm{L}_{\mathrm{full}}) = \sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y,$$

whereas under the $\mathrm{L}_{\mathrm{ind}}$ model, it is

$$\mathrm{var}(\mathrm{L}_{\mathrm{ind}}) = \sigma_X^2 + \sigma_Y^2.$$

Consequently, the performance loss under the risk measure variance is:

$$PL(\mathrm{var}) = \mathrm{var}(\mathrm{L}_{\mathrm{full}}) - \mathrm{var}(\mathrm{L}_{\mathrm{ind}}) = 2\rho\sigma_X\sigma_Y,$$

as required.

For the **Value at Risk (VaR)** as the risk measure, for any confidence level $p \in (0,1)$, we have:

$$\begin{aligned}\mathrm{VaR}_p(\mathrm{L_{full}}) &= \mu_{\mathrm{L_{full}}} + \sigma_{\mathrm{L_{full}}}\Phi^{-1}(p) \\ &= (\mu_X + \mu_Y) + \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}\,\Phi^{-1}(p),\end{aligned} \tag{5}$$

and

$$\begin{aligned}\mathrm{VaR}_p(\mathrm{L_{ind}}) &= \mu_{\mathrm{L_{ind}}} + \sigma_{\mathrm{L_{ind}}}\Phi^{-1}(p) \\ &= (\mu_X + \mu_Y) + \sqrt{\sigma_X^2 + \sigma_Y^2}\,\Phi^{-1}(p).\end{aligned} \tag{6}$$

Therefore, the performance loss under the risk measure VaR is:

$$PL(\mathrm{VaR}) = \mathrm{VaR}_p(\mathrm{L_{full}}) - \mathrm{VaR}_p(\mathrm{L_{ind}}) = \left(\sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y} - \sqrt{\sigma_X^2 + \sigma_Y^2}\right)\Phi^{-1}(p),$$

as claimed.

For the **Tail Value at Risk (TVaR)** as the risk measure, for any confidence level $p \in (0,1)$, we have:

$$\begin{aligned}\mathrm{TVaR}_p(\mathrm{L_{full}}) &= \mu_{\mathrm{L_{full}}} + \sigma_{\mathrm{L_{full}}}\frac{\phi[\Phi^{-1}(p)]}{1-p} \\ &= (\mu_X + \mu_Y) + \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}\,\frac{\phi[\Phi^{-1}(p)]}{1-p},\end{aligned} \tag{7}$$

and

$$\begin{aligned}\mathrm{TVaR}_p(\mathrm{L_{ind}}) &= \mu_{\mathrm{L_{ind}}} + \sigma_{\mathrm{L_{ind}}}\frac{\phi[\Phi^{-1}(p)]}{1-p} \\ &= (\mu_X + \mu_Y) + \sqrt{\sigma_X^2 + \sigma_Y^2}\,\frac{\phi[\Phi^{-1}(p)]}{1-p}.\end{aligned} \tag{8}$$

Therefore, the performance loss under the risk measure TVaR is:

$$PL(\mathrm{TVaR}) = \mathrm{TVaR}_p(\mathrm{L_{full}}) - \mathrm{TVaR}_p(\mathrm{L_{ind}}) = \left(\sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y} - \sqrt{\sigma_X^2 + \sigma_Y^2}\right)\frac{\phi[\Phi^{-1}(p)]}{1-p},$$

as we want to prove. □

As expected, the performance losses for all three risk measures increase as the correlation, representing the dependence levels, also increases. This indicates that when aggregate risk is computed under the assumption of independence between individual risks that are actually dependent, the computation will be undervalued. Theorem 2.1 provides the exact value of this undervaluation compared to the true value of the aggregate risk. To illustrate this numerically, Figure 1 shows the performance losses for all three risk measures when we set $\sigma_X = 1$, $\sigma_Y = 2$, and $p = 0.95$. It is evident that higher dependence between risks leads to greater performance loss. Among the risk measures used, variance yields the largest performance loss, while VaR and TVaR exhibit almost similar performance losses. The higher performance loss associated with variance may be attributed to the fact that variance is calculated "quadratically."

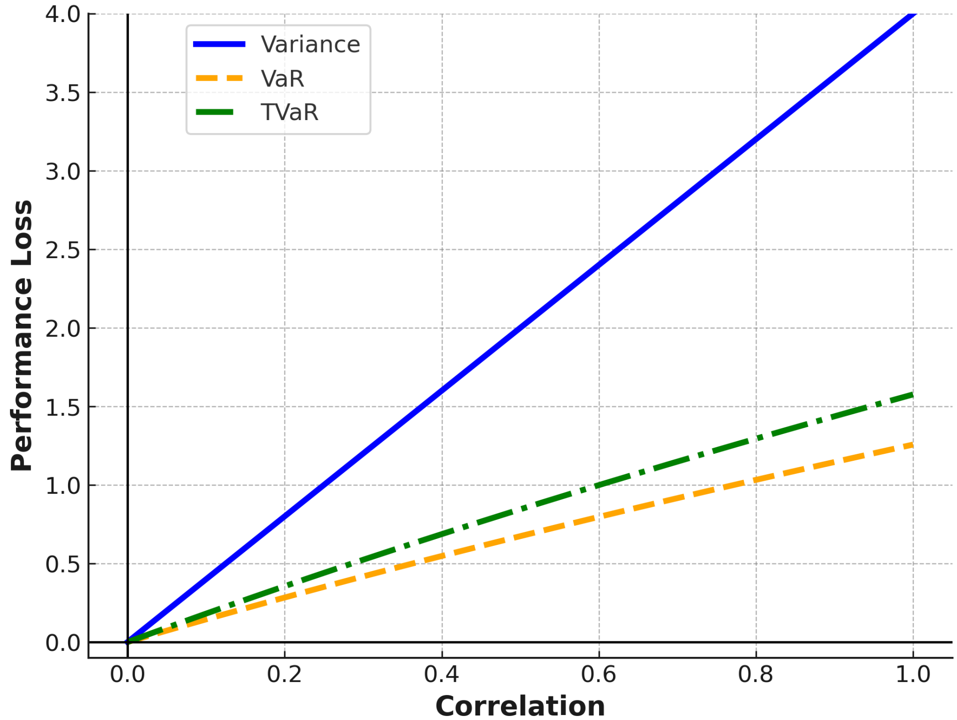

FIGURE 1. Performance loss for jointly normal risks

2.2. **Normal-Copula Distributed Losses.** This subsection discusses the performance loss of the Copula-based individual risk model, employing a numerical approach due to the absence of an analytic formula. In particular, we investigate the performance loss numerically for each previously discussed risk measure using the Normal copula approach. We assume that the losses follow a log-Normal distribution. This choice is particularly relevant for loss data, as it reflects the multiplicative nature of losses, ensuring all values are positive and skewed—features commonly observed in risk modeling [16]. The dependence between the two risks is modeled by a Normal copula characterized by the correlation value $\rho \in (0,1)$.

Let X and Y denote the individual risks coupled by a Normal copula with correlation ρ. As mentioned earlier, X and Y are assumed to follow log-Normal distributions with parameters (μ_X, σ_X) and (μ_Y, σ_Y), respectively. Under the Normal copula with correlation ρ, the joint cumulative distribution function (cdf) of $(X,Y)^\top$ is given by:

$$F(x,y) = C(F_X(x), F_Y(y); \rho) = \Phi_\rho\left(\Phi^{-1}(F_X(x)), \Phi^{-1}(F_Y(y))\right), \tag{9}$$

where F_X and F_Y represent the cdfs of X and Y, respectively. Here, Φ_ρ denotes the joint cdf of bivariate normal with correlation ρ.

Due to the absence of an analytic formula for the three risk measures for $X+Y$ under the Normal copula, we calculate the performance loss numerically. The method is straightforward: for a fixed correlation value ρ, we generate samples from a Normal copula distribution with correlation ρ. Next, we compute the aggregate losses for all three risk measures—variance, VaR, and TVaR—numerically. Finally, the aggregate loss under independence is subtracted from the aggregate loss under the Normal copula to obtain the performance loss.

For our experiment, we set $\mu_X = 0, \sigma_X = 0.3, \mu_Y = 0.3$, and $\sigma_Y = 0.25$. We generate 100,000 samples and set the significance level $p = 0.95$. We then compute the performance loss for each risk measure using the procedure explained before.

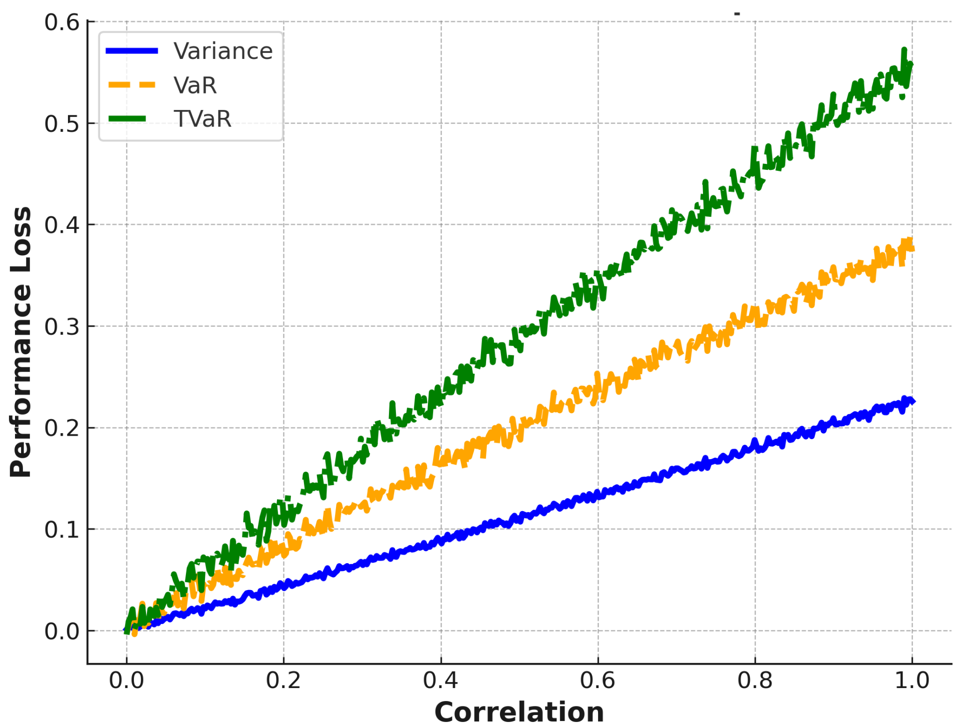

Figure 2. Graphic of performance loss Variance Copula Approach

Based on Figure 2, we can conclude the influence of the correlation coefficient value (ρ) on performance loss. The analysis reveals the following points:

(i) As the correlation coefficient (ρ) increases, there is a corresponding increase in performance loss, indicating that the overall risk also increases. This highlights that greater interdependence between risks can amplify the vulnerability of a portfolio.

(ii) Conversely, as the correlation coefficient (ρ) decreases, performance loss tends to decrease, indicating a reduction in the overall risk. This suggests that lower correlation coefficients among risks promote better diversification and reduce the likelihood of significant losses.

3. Conclusion

This study highlights the critical role of incorporating dependency structures in risk aggregation models. Using a simple normal assumption and a normal copula approach, we analyzed the impact of correlation on risk measures—variance, Value at Risk (VaR), and Tail Value at Risk (TVaR)—and demonstrated how dependencies significantly influence portfolio losses. Our results show that higher correlation increases both overall risk and performance loss, underscoring the importance of accounting for interdependencies in risk models. Assuming independence underestimates potential losses, leading to overly optimistic risk assessments. Performance loss, defined as the difference between dependent and independent scenarios, rises with correlation, indicating greater vulnerability in portfolios with stronger dependencies. In conclusion, integrating correlation structures into risk models is crucial for accurate risk estimation.

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