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NOVEL MULTIPLE CRITERIA DECISION MAKING SORTING METHOD WITH APPLICATION

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Abstract. In this work, we consider multiple criteria sorting problems in which P alternatives from a set X have to be assigned to M predefined ordered classes. When eliciting most of the sorting models, Decision Makers (DMs) frequently have difficulty expressing precise values for the parameters (weights, category thresholds). Additionally, in particular contexts, the sorting model must maintain the homogeneity of groups of criteria expressing different dimensions (*e.g*, economic, governance, social and environmental) in the aggregation process. To overcome these two difficulties, this article proposes an extension to the KEMIRA multi-criteria choice method, namely KEMIRA-sort, which can handle both multi-criteria choice problems and multi-criteria sorting problems. This new method allows both to elicit weights and respect the homogeneity of criteria groups when assigning alternatives to predefined categories. An illustrative application drawn from a real case study on the sustainable management of small dams is provided to demonstrate the effectiveness of the proposed model.

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1. Introduction

In this paper, we are interested in the multiple criteria sorting problem which aims at assigning each alternative of a set $X = \{x^1, x^2, \ldots, x^P\}$ to M predefined ordered classes or categories $C_1 \prec C_2 \prec \cdots \prec C_M$, whre C_1 is the worst category and C_M is the best, taking into account the Q criteria and the preferences of the Decision Maker (DM). An aggregation model must be constructed to allow the assignment process to yield to a result in accordance with the DM's preferences. In this Multiple Criteria Decision Making (MCDM) context, two aggregation models are widely used: the outranking relation

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and the utility function [1,2]. Only multiple criteria sorting models based on a utility function are considered. More specifically, we are interested in building a sorting model using increasing functions and thresholds. This model could be considered an extension of the simple utility model for multiple criteria sorting presented in [2,3]. When an additive utility function is used for multiple criteria sorting, the DM is usually asked to determine the value of one or several parameters (weights, discrimination thresholds, etc.). These parameters are used to construct a preference model of the decision maker (DM). Generally it is not realistic to assume that the DM would easily provide the values of these parameters.

Therefore, methodologies for indirectly eliciting DMs' preferences have emerged. In the case of sorting problems, and precisely with respect to utility function-based models, few authors have proposed methodologies to infer preference parameters from assignment examples provided by DMs [4,5]. The literature is more prolific in the case of outranking relation based models for sorting problems [6–10]. These models propose to infer the preference parameters that best match the DMs' preference information by solving either a mathematical program [11] or by an evolutionary approach [8].

By heterogeneous criteria we mean those for which there is no natural compensation between strengths and weaknesses of the criteria (e.g, an economic criterion and an ecological criterion, or an economic criterion and a social criterion), in contrast to homogeneous criteria where there is a natural compensation between them (*e.g.* two economic, ecological or social criteria). This situation can occur for example when dealing with the sustainable management of resources where a strong productivity (economic criterion) can not naturally compensate for a loss of biodiversity (environmental criterion). In such a context, considering the criteria in homogeneous groups, during the process of aggregating the performance of the alternatives, could avoid undesirable compensations between the criteria and would contribute to legitimizing the results of an approach on the basis of the use of utility functions [12].

Recently, a new multiple criteria decision making method called KEmeny Median Indicator Ranks Accordance (KEMIRA) method, was presented, which considers the choice problems [13,14]. This approach takes into account the homogeneity of groups of criteria in their aggregation process. In this work we present a new sorting model that we call KEMIRA-sort, which is an extension of the former KEMIRA method, allowing us to address both multiple criteria choice problems and multiple criteria sorting problems.

The rest of the paper is structured as follows. Section 2 presents the procedure for computing the weights of the criteria and assigning the alternatives to predefined categories when the preferential information provided by the decision-maker makes it possible, within each group of criteria, to rank the criteria from the best to the worst and to set the minimum levels of performance to be met. Section 3 presents the original KEMIRA method. In section 4, we apply KEMIRA-sort method to an illustrative example. The last section provides conclusions and identifies further research ideas to be investigated.

2. Mathematical Formulation of the KEMIRA-Sort Model

We assume that the set of alternatives is $X = \{x^1, x^2, \ldots, x^P\}$; the Q criteria are partitioned into G groups indexed by $\{1, \ldots, G\}$ and the elements of each group are indexed in turn by $\{1, \ldots, n_i\}$ where $\forall i \in \{1, \ldots, G\}$, n_i is the size of group i, with of course $\sum_{i=1}^{G} n_i = G$ due to the partition of the Q criteria in G groups. Each criterion is thus indexed by a pair of natural numbers (i, j), the first one for the group, and the second one for the criterion within the group. For each criterion (i, j), we associate a weight $w_{i,j}$. On the basis of previous notation, for $k \in \{1, \ldots, P\}$, each alternative $x^k = (X^k_{1,1}, \ldots, X^k_{1,n_1}, x^k_{2,1}, \ldots, X^k_{2,n_2}, \ldots, X^k_{G,1}, \ldots, X^k_{G,n_G})$, is a real-valued vector of dimension Q, where $X^k_{i,j}$ represents the performance of alternative x^k on a specific criterion j belonging to group i.

Our objective is to compute the weight $w_{i,j}$ of each criterion (i, j) and to assign each alternative x^k to one of the M predefined ordered categories $C_1 \prec C_2 \prec \cdots \prec C_M$, by formulating and solving an optimization problem.

2.1. **Criteria priority and increasing functions.** For all $i \in \{1, \ldots, G\}$, we assume that the DM is able to provide a ranking of criteria from the most to the least preferred so that the relations (1) and the corresponding restrictions on the criteria weights (2) hold:

$$\begin{aligned}
&(1,1) \succsim (1,2) \succsim \ldots \succsim (1,n_1),\\
&(2,1) \succsim (2,2) \succsim \ldots \succsim (2,n_2),\\
&\qquad\vdots\\
&(G,1) \succsim (G,2) \succsim \ldots \succsim (G,n_G);
\end{aligned} \tag{1}$$

$$\begin{aligned}
&w_{1,1} \geq w_{1,2} \geq \ldots \geq w_{1,n_1},\\
&w_{2,1} \geq w_{2,2} \geq \ldots \geq w_{2,n_2},\\
&\qquad\vdots\\
&w_{G,1} \geq w_{G,2} \geq \ldots \geq w_{G,n_G}.
\end{aligned} \tag{2}$$

Therefore the performance $X^k_{i,j}$, $k \in \{1, 2, \ldots, P\}$, $i \in \{1, 2, \ldots, G\}$, $j \in \{1, 2, \ldots, n_i\}$, of alternatives w.r.t. the Q criteria, we normalize them. Here we choose an affine type of normalization and the normalized values are obtained by the relation

$$x^k_{i,j} = \tfrac{X^k_{i,j} - \min_i X^k_{i,j}}{\max_i X^k_{i,j} - \min_i X^k_{i,j}}. \tag{3}$$

In each group we assume that the weights are normalized and we verify

$$\sum_{j=1}^{n_i} w_{i,j} = 1,\ \forall i \in \{1, 2, \ldots, G\}. \tag{4}$$

Assuming that the lager values of variables $x^k_{i,j}$ represent better satisfaction with respect to the considered criterion, for an alternative x^k and for each group i, we compute its weighted average performance, denoted by W_i:

$$W_i(x^k) = \sum_{j=1}^{n_i} w_{i,j} \times x^k_{i,j}, \tag{5}$$

where weights $w_{i,j}$ satisfy requirements (2) and (4).

2.2. **Process of assignment to categories.** For each group i we introduce $M-1$ thresholds:

$$0 < \alpha_i^1 < \alpha_i^2 \ldots < \alpha_i^{M-1} < 1, \tag{6}$$

since we have M predefined ordered categories $C_1 \prec C_2 \prec \cdots \prec C_M$. Thus, for an alternative x^k, the assignment process is stated as follows :

$$\begin{aligned}
&\textbf{Step 1} : \text{if } \exists i \in \{1,2,\ldots,S\}, W_i(x^k) \leq \alpha_i^1 \text{ then } x^k \in C_1.\\
&\textbf{Step 2} : \text{if } \exists i \in \{1,2,\ldots,S\}, W_i(x^k) \leq \alpha_i^2 \text{ and } not(x^k \in C_1)\\
&\ \text{then } x^k \in C_2.\\
&\textbf{Step 3} : \text{if } \exists i \in \{1,2,\ldots,S\}, W_i(x^k) \leq \alpha_i^3 \text{ and } not(x^k \in C_1)\\
&\ \text{and } not(x^k \in C_2) \text{ then } x^k \in C_3.\\
&\vdots\\
&\textbf{Step M} : \text{if } x^k \text{ does not satisfy } \textit{Step1} \text{ to } \textit{Step M-1}, \text{ then } x^k \in C_M.
\end{aligned} \tag{7}$$

Note that this assignment process in (7) can equivalently be written as follows:

For each $i \in 1,\ldots,S$, introduce M thresholds $0 < \alpha_i^1 < \alpha_i^2 \ldots < \alpha_i^M = 1$. The assignment is then defined by

$$x^k \in C_{min(l\in\{1,\ldots,M\}: W_i(x^k)\leq \alpha_i^l \text{ for some } i\in\{1,\ldots,G\})}. \tag{8}$$

Considering the heterogenous nature of the criteria groups, the assignment rule (7), when applied, limits the compensation between poor performance on a given group of criteria and good performance on the other groups. Thus, poor performance on a single group of criteria logically determines the assignment. In the case where the compensation between poor performance on a given group of criteria and good performance on the other groups of criteria is allowed, the assignment procedure can be modified, while keeping the rest of the proposal, as follows:

$$x^k \in C_l \text{ if } \alpha^{l-1} \leq W(x^k) < \alpha^l, \tag{9}$$

$W(x^k)$ is the average performance of the alternative x^k on all criteria, α^l, $l \in \{2,3,...,M\}$ denote the performance levels set by the DM. Note also that a similar assignment procedure is implemented in the UTADIS method [15,16]

In this context, the partitioning of the set of criteria into groups is motivated by the fact that each group is supposed to contain homogeneous criteria (*e.g.*, group of economic criteria, group of environmental criteria, etc.) whose performance could be aggregated in the spirit of total compensation and in such a way that their number does not matter. If the partitioning of the set of criteria is motivated by other considerations, then the number of criteria per group might be relevant and integrated into the assignment rules. This aspect will be taken into account in future work.

Note that the thresholds α_i^l are the performance levels that any alternative must meet; they are set by the DM. Therefore the DM is asked to express his preference on these thresholds in terms of the percentage of the best performance respectively in each group of criteria. Formally, for each group i and for each category C_l the DM is asked to set a number p strictly between 0 and 100 such that:

$$\begin{aligned} &\alpha_i^l = p\% \times \max_{k=1}^{P} W_i(x^k), \\ &0 < \alpha_i^1 < \alpha_i^2 \ldots < \alpha_i^{M-1} < 1. \end{aligned} \tag{10}$$

2.3. **Objective function.** Denote $|C_l|$ the number of elements in the set C_l and $fopt$ the function to be optimized. We define the objective function $fopt$ to be maximized by the formula

$$fopt = \sum_{l=1}^{M} l \times |C_l|. \tag{11}$$

Thus, the higher the value of the objective function $fopt$ is, the better the alternatives are globally assigned to their best categories. The motivation behind the choice of such an objective function is to give the alternatives the opportunity to be assigned to a better category in view of their respective performances.

2.4. **Optimization problem.** For a given set of weights satisfying relation (2), and associated performance levels as stated in inequations (6) expressing the preference information of the DM, corresponds an assignment of alternatives to categories via relation (6). Without further information from the decision-maker, we assume that, the alternatives will be assigned to the right categories if a corresponding set of weights, verifying relation (2), with the associated performance levels verifies relation (6) allowing them to be assigned as much as possible to better categories. We formalize this assumption through the optimization problem (12):

$$\begin{aligned} &\max_{w_{i,j}} fopt = \textstyle\sum_{l=1}^{M} l \times |C_l| \\ &w_{i,1} \geq w_{i,2} \geq \ldots \geq w_{i,n_i}, \forall i \in \{1, 2, \ldots, G\}, \\ &\textstyle\sum_{j=1}^{n_i} w_{i,j} = 1,\ \forall i \in \{1, 2, \ldots, G\}, \\ &0 < \alpha_i^1 < \alpha_i^2 < \ldots < \alpha_i^{M-1} < \alpha_i^M = 1, \\ &x^k \in C_{min(l \in \{1,\ldots,M\}: W_i(x^k) = \sum_{j=1}^{n_i} w_{i,j} \times x_{i,j}^k \leq \alpha_i^l \text{ for some } i \in \{1,\ldots,G\})}\ \forall\, k \in \{1, 2, \ldots, P\}. \end{aligned} \tag{12}$$

Note that, since the performance of an alternative is fixed, several sets of weights verifying the preferences of the DM can usually lead to the alternative being assigned to different categories. Therefore, taking an optimistic stance, we opt to favour sets of weights with corresponding performance levels verifying the preferences of the DM that would make it possible to globally assign the alternatives in the best possible categories.

Let us highlight two theoretical limit cases of our objective function. For each $i \in \{1, \dots, S\}$, define

$$\beta_i = \min_{k=1}^{P}(W_i(x^k) : W_i(x^k) > 0), \tag{13}$$

$$\gamma_i = \max_{k=1}^{P}(W_i(x^k) : W_i(x^k) > 0). \tag{14}$$

If we arbitrarily choose $0 < \alpha_i^1 < \alpha_i^2 \dots < \alpha_i^{M-1} < 1$ strictly between 0 and β_i; this guarantees that all the alternatives will be assigned to C_M, i.e., the objective function will be equal to $M \times P$, unless an alternative x^k has $W_i(x^k) = 0$ for some i. In that case, the objective function is equal to $M \times P$ minus the number of alternatives X^k so that $W_i(x^k) = 0$ for some i. In any case, the optimal value of the objective function is reached.

Note that the optimum is obtained with parameters α_i^l, which are chosen almost arbitrarily between 0 and β_i, irrespective of the weights. However in practice such a case is not realistic. Given that α_i^l are performance levels set by the DM, such a situation, if it occurs, would be tantamount to saying that the DM deliberately sets a very low level of performance that it is sure any alternative will pass.

Similarly, if we arbitrarily choose $0 < \alpha_i^1 < \alpha_i^2 \dots < \alpha_i^{M-1} < 1$ strictly between γ_i and 1, this guarantees that all the alternatives will be assigned to C_1, *i.e*, the objective function is equal to P which is its optimal value. This optimum is also obtained irrespective of the weights. However this situation is also not realistic because it would mean that the DM is deliberately setting a high level of performance that it is certain no alternative will meet.

We suppose that the DM is able to set the value of α_i^l with the constraints:

$$0 < \beta_i \leq \alpha_i^1 < \alpha_i^2 \dots < \alpha_i^{M-1} < \gamma_i < 1. \tag{15}$$

2.5. **Algorithm to solve the optimization problem.** To solve the optimization problem (12), we propose and implement the Algorithm 1. Let us make some comments on this algorithm.

- Points 1 to 6 of Algorithm 1 refer to the random choice of an initial vector of weights w^0, satisfying conditions (2) and (4), from which the assignment process (7) is computed and the corresponding value the objective function $fopt^0$ is calculated. Afterward a perturbation of this vector w^0 in a randomly direction Δw is done and we obtain the vector $w^1 = w_0 + \epsilon \times \Delta w$. We call w^0 the current weight and the corresponding assignment is called the current assignment.
- Points 7 to 18 ensure that the w^1 vector satisfies the conditions (2) and (4) of weights decreasing in each group and their normalization.
- Points 19 to 31 run the new assignment with the vector of weights w^1 and compare the resulting objective function $fopt^1$ to its previous value $fopt^0$. If one has an improvement in the value of the objective function,*i.e.*,$fopt^1 > fopt^0$, then the assignment issued from w^1 is better than that issued from w^0. Thus, we keep w^1 and the corresponding assignment which become the current weights vector and the current assignment and we must back to point 5 of the algorithm.

If not we must return to point 2 of the algorithm and initialize a new current weight vector w^0. This approach allows us to avoid any local minimums that could prevent a better solution from being obtained. Therefore we repeat these scenarios until we reach the maximum of the authorized iteration $\max_{iter}$ and the resulting current weight vector becomes the solution of the optimization problem. The corresponding assignment is selected as the best assignment of the alternatives to the predefined categories.

- The value of $\max_{iter}$ must be chosen to reach at least the limit point where the assignment to categories no longer changes and becomes stable. Therefore, by running Algorithm 1, we assume that for any set of input data, there exists a value of $\max_{iter}$, $N_{\max}$, from which the assignment to categories will no longer change. The proof of this assumption is one of the perspectives of the current work. In practice, the value of the $\max_{iter}$ must be initialized at a sufficiently high level to ensure that the $\max_{iter} \geq N_{\max}$.
- Recalling that only the ranking of criteria from the best to the worst in each group of criteria is asked to the DM. Generally, this type of information can be provided without any great difficulty by the DM. Owing to this weak precision of information on the DM's preferences w.r.t. criteria, the space of feasible weights to explore is infinite. Additional information obtained from the decision-maker could be integrated into problem 12 and then reduce the set of feasible solutions. This question is beyond the scope of this work and will be examined in our future work.
- In decision-making, we know a rank reversal as a change in the rank ordering of the preferability of alternative possible decisions when, for example, the method of choosing changes or the set of other available alternatives changes [17, 18]. The study of a rank reversal problem in the application of the KEMIRA-sort method consisted of observing the possible undesirable assignment of alternatives to ordered categories when the set of available alternatives changeg. For instance, if after a change in the set of alternatives, an alternative x^i assigned to a lower category than an alternative x^j is now assigned to a better category than x^j, such a change is an undesirable assignment. This investigation is important because for a given group of criteria, the preference thresholds which specify the desired levels of performance, are provided by the DM in terms of the percentage of the best average performance of alternatives w.r.t. criteria belonging to the concerned group.

Algorithm 1 KEMIRA Sort assignment

1: Fixed algorithm parameters: $10^{-3} \leq \epsilon \leq 10^{-1}$, $\max_{iter}$ the maximum number of iterations, Thresholds: $0 < \beta_i \leq \alpha_i^1 < \alpha_i^2 \ldots < \alpha_i^{M-1} < \gamma_i < 1$,
2: Choose an initial vector of weights satisfying conditions (2) and (4): $w^0 = (w_{1,1}^0, w_{1,2}^0, \ldots, w_{1,n_1}^0; w_{2,1}^0, w_{2,2}^0, \ldots, w_{2,n_2}^0; \ldots; w_{S,1}^0, w_{S,2}^0, \ldots, w_{S,n_S}^0)$;
3: Run the condition (7) of the assignment process;
4: Compute the value of the objective function $fopt^0$ as shown in relation (11);
5: Randomly selected direction vector Δw: $\Delta w = (\Delta_{1,1}, \ldots, \Delta_{1,n_1}; \Delta_{2,1}, \ldots, \Delta_{2,n_2}; \ldots; \Delta_{G,1}, \ldots, \Delta_{G,n_G})$, $-1 \leq \Delta_{i,j} \leq 1$;
6: The vector $w^1 = w^0 + \epsilon \times \Delta w$ is calculated.
7: **if** w^1 does not satisfy restrictions (2) and (4) **then**
8:     the corrections proposed in [14] are carried out:
9:     **if** $w_{i,j}^1 < 0$ **then**
10:         change: $w_{i,j}^1 = 0$;
11:     **end if**
12:     **if** $w_{i,j}^1 < w_{i,j+1}^1$ **then**
13:         change: $w_{i,j}^1 = w_{i,j+1}^1$;
14:     **end if**
15:     **if** $t_i = \sum_{j=1}^{n_i} w_{i,j}^1 \neq 1$ **then**
16:         change: $w_{i,l}^1 = \frac{w_{i,l}^1}{t_i}$;
17:     **end if**
18: **end if**
19: the number of iterations $iter$ is calculated
20: **if** $iter > \max_{iter}$ **then**
21:     stop the algorithm.
22: **else**
23:     Run the condition (7) of the assignment process with the current weight vector w^1 satisfying condition (2);
24:     Compute the value of the current objective function $fopt^1$ as shown in relation (11)
25:     **if** $fopt^1 > fopt^0$ **then**
26:         change: $fopt^0 = fopt^1$; $w^0 = w^1$;
27:         go to point 5 of the algorithm.
28:     **else**
29:         go to point 2 of the algorithm.
30:     **end if**
31: **end if**

3. KEMIRA Method

For each group i, we introduce one threshold $0 < \alpha_i < 1$ and denote $B = \{x^k \in X : W_i(x^k) = \sum_{j=1}^{n_i} w_{i,j} \times x_{i,j}^k > \alpha_i \, \forall i \in \{1, \ldots, G\}\}$ the set of the best alternative and $|B|$ the number of elements in B. Using the same notations above, when running the original KEMIRA method [14], the objective is to compute the weight $w_{i,j}$ of each criterion (i,j) and to choose the best alternatives by solving the optimization problem (16).

$$\begin{array}{l} \max_{w_{i,j}} fopt_2 = |B| \\ w_{i,1} \geq w_{i,2} \geq \ldots \geq w_{i,n_i}, \forall i \in \{1, 2, \ldots, G\}, \\ \sum_{j=1}^{n_i} w_{i,j} = 1, \ \forall i \in \{1, 2, \ldots, G\}, \\ x^k \in B \forall \ k \in \{1, 2, \ldots, P\}. \end{array} \tag{16}$$

If we consider the KEMIRA-sort method in the case where we have only two assignment categories, then category C_2 and set B have exactly the same elements, *i.e.*, $C_2 = B$. Additionally, for all $w_{i,j}$ the value $fopt_2 = |C_2|$ is maximum if the value $fopt = 2 \times |C_2| + |C_1|$ is maximal and vice versa. As a result, the solutions of the mathematical program (16) are exactly the same as those of mathematical program (12) when considering only two categories and vice versa. The use of the KEMIRA method means to use the KEMIRA-sort method with only two assignment categories. The new MCDM method proposed, KEMIRA-sort, is therefore an extension of the KEMIRA method.

4. A real world application

4.1. **Presentation of the case study.** To illustrate the methodology, we consider a real case study concerning the prioritization of the best sustainable management methods for small dams (water reservoirs) in the city of Ouagadougou (West Africa).

The fight against the degradation of small dams (water reservoirs) in the city of Ouagadougou has often been approached in a thematic and not holistic way, hence poor results obtained. Therefore, we believe that the policy to fight against the degradation of water reservoirs must be the result of a participatory approach.

We identified the target parties, which included the local populations (local residents and operators) using the banks of the dams and the communal authorities. A group of municipal authorities and an environmental expert acted as decision-maker (DM).

During our interviews with the local residents, operators of the banks and the municipal authorities, twelve (12) actions were identified as being able to slow this degradation. From these twelve (12) identified actions, seven (7) scenarios (alternatives) or dam management methods have been constructed. It was a grouping of the twelve (12) isolated actions identified in three major management topics; those topics range from the rehabilitation of water reservoirs to rationalization of water uses and protection of water reservoirs. In addition to these scenarios, three others scenarios result from the development of water reservoirs by the Burkinabe government. These are the prestige development, the development of tourism and the socioecological planning of reservoirs. Finally, a last scenario concerning the possible privatization of water reservoirs was proposed.

let us set $X = \{x^1, x^2, x^3, x^4, x^5, x^6, x^7\}$:

- x^1 : Rehabilitation of water reservoirs;
- x^2 : Rationalization of water use;
- x^3 : Protection;
- x^4 : Prestige development;
- x^5 : Tourism development;
- x^6 : Socioecological planning;
- x^7 : Privatization of water reservoirs.

A literature review on the dimensions of sustainable development identified a series of issues related to the sustainable management of water reservoirs in the city of Ouagadougou. The issues identified are grouped according to the dimensions of sustainable development (economic, ecological, social and governance). On this basis twelve (12) criteria and their relevant measurement indicators have been developed to evaluate the seven alternatives. The set of twelve criteria has been subdivided in four (4) groups $G_4 = \{(4,1),(4,2)\}$, $G_3 = \{(3,1),(3,2)\}$, $G_2 = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\}$, $G_1 = \{(1,1),(1,2)\}$:

- G_1 : Economic group
  - $(1,1)$ importance of income generating activities;
  - $(1,2)$ financial cost.
- G_2 : Social group
  - $(2,1)$ Importance of voluntary displacement
  - $(2,2)$ Degree of acceptability
  - $(2,3)$ Importance of potential exposure
  - $(2,4)$ Water withdrawal
  - $(2,5)$ Flood risk
  - $(2,6)$ Development of recreational activities around dams
- G_3 : Ecological group
  - $(3,1)$ Ecological function
  - $(3,2)$ Importance of biodiversity
- G_4 : Governance group
  - $(4,1)$ Importance of conflicts of use
  - $(4,2)$ Involvement of all stakeholders in dam management

Ranking of the criteria in each group i as well as the performance levels α_i^l has been provided by the DM. Note that while it was easy for the DM to rank the criteria within each subgroup, the determination of thresholds or performance levels α_i^l was not obvious.

The evaluations of the different scenarios w.r.t. the criteria were carried out through field observations and surveys of local residents and operators of the bank.

The computed evaluation matrix is given in Table 1. This problem has been solved with a multiple criteria choice method, namely the KEMIRA method [19] to select the best sustainable management method for small dams. The scenarios x^2, x^3 and x^6 are selected as the best ones. For the reader interested, more details on this structuring phase of the application can be found in [19].

TABLE 1. Evaluating Matrix

	(1,1)	(1,2)	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)	(3,1)	(3,2)	(4,1)	(4,2)
x^1	0.81	0	0.87	0	0.5	1	0	0	0	0.5	0	1
x^2	1	0	0.67	1	1	0	0	0	1	1	1	1
x^3	0.1	1	1	1	1	0	1	0	1	1	1	0
x^4	0	1	0.16	1	0	1	1	1	0	0.5	0	0
x^5	0.51	0	0.03	0.5	0	1	1	1	0	0.5	0	0
x^6	0.2	1	0.1	0.5	0.5	0	1	0	0.5	0.5	0	1
x^7	0.07	1	0	0	0.5	0.5	0	0	0	0.5	1	0

4.2. **Application of KEMIRA-sort to the case study.** To apply KEMIRA-sort in this problem we have to change the statement of the problem. Rather than selecting the best sustainable management methods for small dams, we will now search to prioritize the best sustainable management methods for small dams in terms of their level of sustainability. therefore, we decide to assign the scenarios or alternatives to four ordered categories $C_1 \prec C_2 \prec C_3 \prec C_4$ which represent four levels of sustainability so that the best category C_4 corresponds to the set of the best alternatives selected via the KEMIRA method. We recall that an alternative x^k is selected as the best one by the KEMIRA method if the following relation is satisfied:

$$\forall i \in \{1, 2, ..., G\}, W_i(x^k) > \alpha_i. \tag{17}$$

and the algorithm of eliciting the corresponding weights is stopped when the cardinal of the set of the best alternatives representing the objective function is maximal.

Only one level of performance specified by the α_i value must be satisfied in each of the G groups by an alternative x^k to be selected as best. Among the alternatives that do not satisfy this selection rule (17), it could be necessary to make the differences between those that satisfy the rule on only one or two or three groups. The KEMIRA-sort method, with different performance levels and corresponding assignment categories, takes this need into account.

In our sorting case, an alternative that satisfies rule (17) is equivalently assigned to the category C_M, *i.e.*, the best category, according to the last *Step* of the condition (7) which is therefore satisfied when the number of categories M is equal to 4 and α_i^3 equal to α_i.

Note that theKEMIRA-sort method offers the possibility to select the second or third best category of alternatives that have acceptable performance. This is an interesting advantage. Indeed, in our case study of dam management in Burkina Faso, if for some reason none of the best category alternatives could be implemented, a survey of the second best category alternatives identified by KEMIRA-sort method could be conducted and an alternative from this category, with fewer constraints (and of course less efficient) than those in the first category, could be selected and implemented.

In fact, to applying KEMIRA-sort with only two categories of assignment is equivalent to using the original KEMIRA method for the choice problem.

4.3. **Results and discussion.** In point 1 of our algorithm proposal, the parameters were set as follows:

$$\begin{gathered}\epsilon = 10^{-2}, \max_{iter} = 4\\ \alpha_i^1 = 10\% \times \max_{k=1}^{P} W_i(x^k); \forall i \in \{1,2,3,4\};\\ \alpha_i^2 = 20\% \times \max_{k=1}^{P} W_i(x^k); \forall i \in \{1,2,3,4\};\\ \alpha_i^3 = 30\% \times \max_{k=1}^{P} W_i(x^k); \forall i \in \{1,2,3,4\}.\end{gathered} \tag{18}$$

The results of four successive iterations of our proposed algorithm applied to the problem are given in Tables 2,3,4,5.

TABLE 2. First iteration of KEMIRA-sort

	W_1	W_2	W_3	W_4	C_1	C_2	C_3	C_4
x^1	0.81	0.87	0	0	1	0	0	0
x^2	1	0.67	1	1	0	0	0	1
x^3	0.1	1	1	1	1	0	0	0
x^4	0	0.16	0	0	1	0	0	0
x^5	0.51	0.03	0	0	1	0	0	0
x^6	0.2	0.1	0.5	0	1	0	0	0
x^7	0.07	0	0	1	1	0	0	0
α_k^1	0.1	0.1	0.1	0.1				
α_k^2	0.2	0.2	0.2	0.2				
α_k^3	0.3	0.3	0.3	0.3				

$w^0 = (1,0;1,0,0,0,0,0;1,0;1,0)$;
$fopt^0 = 10$; $fopt^1 = 10$.
$fopt^1 \leq fopt^0$;update the value of w^0 by generating randomly a new w^0:
$w^0 = (0.507, 0.507; 0.332, 0.204, 0.205, 0.212, 0.022, 0.027; 0.507, 0.5; 0.493, 0.494)$.
apply (i) to (iv) of point 6 of the algorithm to correct w^0:
$w^0 = (0.5, 0.5; 0.325, 0.207, 0.207, 0.207, 0.027, 0.27; 0.503, 0.497; 0.5, 0.5)$;

TABLE 3. Second iteration of KEMIRA-sort

	W_1	W_2	W_3	W_4	C_1	C_2	C_3	C_4
x^1	0.405	0.594	0.248	0.5	0	0	1	0
x^2	0.5	0.632	1	1	0	0	0	1
x^3	0.55	0.766	1	0.5	0	0	0	1
x^4	0.5	0.52	0.248	0	1	0	0	0
x^5	0.255	0.374	0.248	0	1	0	0	0
x^6	0.6	0.266	0.5	0.5	0	0	0	1
x^7	0.535	0.207	0.248	0.5	0	0	1	0
α_k^1	0.06	0.076	0.1	0.1				
α_k^2	0.12	0.153	0.2	0.2				
α_k^3	0.18	0.229	0.3	0.3				

$fopt^0 = 10$; $fopt^1 = 20$.

$fopt^1 > fopt^0$; update the value of w^0:

$\Delta = (0.82, -0.44; -0.62, -0.14, -0.22, 0.84, 0.84, 0.3; -0.68, 0.02; -0.24, 0.44),\ \epsilon = 10^{-2}$

$w^1 = w^0 + \epsilon\Delta = (0.508, 0.496; 0.319, 0.206, 0.205, 0.216, 0.035, 0.03; 0.496, 0.497; 0.498, 0.504)$

Apply (i) to (iv) of point 6 of the algorithm to correct w^1:

$w^1 = (0.506, 0.494; 0.309, 0.209, 0.209, 0.209, 0.034, 0.029; 0.5, 0.5; 0.5, 0.5)$.

Change $w^0 = w^1$

TABLE 4. Third iteration of KEMIRA-sort

	W_1	W_2	W_3	W_4	C_1	C_2	C_3	C_4
x^1	0.410	0.583	0.25	0.5	0	0	1	0
x^2	0.506	0.626	1	1	0	0	0	1
x^3	0.544	0.762	1	0.5	0	0	0	1
x^4	0.494	0.531	0.25	0	1	0	0	0
x^5	0.258	0.386	0.25	0	1	0	0	0
x^6	0.595	0.274	0.5	0.5	0	0	0	1
x^7	0.529	0.209	0.25	0.5	0	0	1	0
α_k^1	0.059	0.076	0.1	0.1				
α_k^2	0.119	0.152	0.2	0.2				
α_k^3	0.158	0.228	0.3	0.3				

$fopt^0 = 20$; $fopt^1 = 20$.

$fopt^1 \leq fopt^0$;update the value of w^0 by randomly generating a new w^0:

$w^0 = (0.772, 0.227; 0.172, 0.183, 0.189, 0.172, 0.179, 0.099; 0.867, 0.128; 0.498, 0.492)$;

Apply (i) to (iv) of point 6 of the algorithm to correct w^0:

$w^0 = (0.773, 0.227; 0.184, 0.184, 0.184, 0.175, 0.175, 0.097; 0.871, 0.129; 0.503, 0.497)$

TABLE 5. Fourth iteration of KEMIRA-sort

	W_1	W_2	W_3	W_4	C_1	C_2	C_3	C_4
x^1	0.626	0.428	0.064	0.497	0	1	0	0
x^2	0.773	0.492	1	1	0	0	0	1
x^3	0.305	0.728	1	0.503	0	0	0	1
x^4	0.227	0.661	0.064	0	1	0	0	0
x^5	0.394	0.545	0.064	0	1	0	0	0
x^6	0.382	0.378	0.5	0.497	0	0	0	1
x^7	0.281	0.18	0.064	0.503	0	1	0	0
α_i^1	0.077	0.072	0.1	0.1				
α_i^2	0.154	0.145	0.2	0.2				
α_i^3	0.231	0.218	0.3	0.3				
$fopt^0 = 20$; $fopt^1 = 18$.								
$fopt^1 \leq fopt^0$;update the value of w^0 by randomly generating a new w^0								

We carried out the proposed algorithm in the C program and solved it via Code::Block17.12 in less than 0.58 seconds with the setting parameter. The first iteration leads to an assignment of alternatives to categories with the objective function $fopt^0 = 10$ (see Table 2) whereas the second and third iteration lead to the same assignment of alternatives to categories with objective function $fopt^1 = 20$ (see Tables 3 and 4). The fourth iteration leads to an assignment with an objective function $fopt^1 = 20$ (see Table 5). At this stage of our program the weights and assignments to be considered as solutions of our multicriteria sorting problem are those stemming from Tables 3 and 4 which have the highest objective function value $fopt^1 = 20$. Several sets of weights can lead to the same optimal assignment since the weight determination is not unique.

We then iterate the algorithm a thousand times and the highest value of the objective function obtained was $fopt^1 = 20$. The first assignment and the corresponding weights obtained with the highest value of the objective function $fopt^1 = 20$ are shown in table 6. Less than 9.69 seconds were necessary to compute the thousand iterations. Considering solution of our multicriteria sorting problem as given in Tables 3, 4 or 5 we can see that the alternatives X^2, X^3 and X^6 which were the best selected by the KEMIRA method are the only ones belonging to the C_4 category.

TABLE 6. Thousandth iteration of KEMIRA-sort

	W_1	W_2	W_3	W_4	C_1	C_2	C_3	C_4
x^1	0.407	0.403	0.249	0.369	0	0	1	0
x^2	0.503	0.498	1	1	0	0	0	1
x^3	0.548	0.708	1	0.631	0	0	0	1
x^4	0.497	0.661	0.249	0	1	0	0	0
x^5	0.256	0.540	0.249	0	1	0	0	0
x^6	0.598	0.350	0.5	0.369	0	0	0	1
x^7	0.532	0.163	0.249	0.631	0	0	1	0
α_k^1	0.059	0.07	0.1	0.1				
α_k^2	0.119	0.141	0.2	0.2				
α_k^3	0.179	0.212	0.3	0.3				

$w^1 = (0.503, 0.497; 0.192, 0.192, 0.179, 0.147, 0.146, 0.146; 0.503, 0.497; 0.631, 0.369)$

$fopt^0 = 20$; $fopt^1 = 20$.

Some remarks need to be pointed out to obtain better results when solving sorting problem with the new algorithm KEMIRA-sort.

remark 1: The algorithm must be run for many iterations to ensure that the highest value of the objective function $fopt$ will be reached.

remark 2: The DM sets the values of thresholds α_i^l while respecting the condition $0 < \beta_i \leq \alpha_i^1 < \alpha_i^2 \ldots < \alpha_i^{M-1} < \gamma_i < 1$. This operation which is not easy, and sometimes boring for the DM, must be carried out in interaction with the DM in order to properly translate his preferential information through the said thresholds. The operation could be carried out indirectly through the expression of the DM's preferential information in the form of examples of assignment to predefined categories [20].

4.4. **On rank reversal in KEMIRA-sort.** On the basis of the results of applying the KEMIRA-sort method to the case study, as shown in Table 6, we noted the following:

a) when we removed one alternative from the set of alternatives and run the KEMIRA-sort algorithm, the six other alternatives kept their same assignment categories;

b) when we removed two or more alternatives from the set of alternatives, a stability or a shift to a best category was noted for some alternatives when the removed alternatives belonged to the best category;

c) an alternative assigned to a category at least as good as another remained assigned to a category at least as good as that other one if a category change occurred.

These partial experimental results indicate that, on the basis of a reference situation, a rank reversal problem did not appear when we applied the KEMIRA-sort method by changing the set of alternatives. As a matter of fact, only changes in the best alternatives (*i.e.*, belonging to the best category) could lead to a shift to the best or worst category; and there are intrinsic performances of alternatives w.r.t. those of these best alternatives which could allow them to shift from one category to another. Of course, there are more hypotheses than robust results that need to be verified empirically.

5. Conclusion

In this paper, we presented an algorithm for the multiple criteria sorting problem, namely KEMIRA-sort, which is based on gradient-descent and increasing functions with DM's preference thresholds allowing both to elicit weights and respect the homogeneity of criteria groups when assigning alternatives to predefined categories. The algorithm uses the preference information given by the DM on the priorities between criteria belonging to the same group to elicit the weights of all the criteria and assign alternatives to predefined categories by solving an optimization problem. The objective function to be maximized is an overall measure of the quality of assignment of alternatives to the best categories. This algorithm appears as an extension of the original KEMIRA method, for choosing problem, that we recover when using the KEMIRA-sort method with only two categories. The algorithm is proven to be efficient in terms of the stability of the results and computing times when it is applied to a real case study problem. Further work can be pursued to investigate the convergence of the algorithm by establishing an axiomatization of this new sorting method. An indirect elicitation of the DM's preference thresholds for increasing functions via preference information stemming from assignment examples given by the DM should also be investigated and integrated into the algorithm. In addition the behavior of this new algorithm for multiple criteria sorting problems should be tested empirically by applying it to many other sorting problems and the results should be compared with those obtained with other sorting methods such as ELECTRE TRI [1,21] and MR-Sort [10].

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