Asia Pac. J. Math. 2025 **12**:27

CHARACTERIZATIONS OF NEARLY (τ_1, τ_2)-CONTINUOUS FUNCTIONS

NIPAPORN CHUTIMAN[1], SUPUNNEE SOMPONG[2], CHAWALIT BOONPOK[1,*]

[1]Mathematics and Applied Mathematics Research Unit, Department of Mathematics, Faculty of Science, Mahasarakham University, Maha Sarakham, 44150, Thailand

[2]Department of Mathematics and Statistics, Faculty of Science and Technology, Sakon Nakhon Rajbhat University, Sakon Nakhon, 47000, Thailand

[*]Corresponding author: chawalit.b@msu.ac.th

Received Nov. 5, 2024

Abstract. This paper presents a new class of functions called nearly (τ_1, τ_2)-continuous functions. Furthermore, several characterizations and some properties concerning nearly (τ_1, τ_2)-continuous functions are investigated.

2020 Mathematics Subject Classification. 54C08; 54E55.

Key words and phrases. $(\tau_1, \tau_2)\theta$-open set; $(\tau_1, \tau_2)\theta$-closed set; nearly (τ_1, τ_2)-continuous function.

1. Introduction

In topology, there has been recently significant interest in characterizing and investigating the characterizations of some weak forms of continuity for functions. Weaker and stronger forms of open sets play an important role in the generalization of different forms of continuity. Using different forms of open sets, several authors have introduced and investigated various types of continuity. The concepts of (Λ, sp)-open sets, $s(\Lambda, sp)$-open sets, $p(\Lambda, sp)$-open sets, $\alpha(\Lambda, sp)$-open sets and $\beta(\Lambda, sp)$-open sets were studied in [8]. Viriyapong and Boonpok [34] investigated some characterizations of (Λ, sp)-continuous functions by using (Λ, sp)-open sets and (Λ, sp)-closed sets. Dungthaisong et al. [18] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [17] introduced and investigated the notion of almost $(\mu, \mu')^{(m,n)}$-continuous functions. Moreover, some characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions and pairwise M-continuous functions were presented in [30], [32], [1], [26], [6], [7], [5], [10], [12] and [13],

DOI: 10.28924/APJM/12-27

respectively. Carnahan [14] introduced the notion of N-closed sets in topological spaces. Noiri [23] studied several properties of N-closed sets and some separation axioms. The notion of N-continuous functions was introduced by Malghan and Hanchinamani [21]. Noiri and Ergun [22] investigated some characterizations of N-continuous functions. In [4], the present authors introduced and investigated the concept of (τ_1, τ_2)-continuous functions. Furthermore, several characterizations of almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions, slightly $(\tau_1, \tau_2)s$-continuous functions, slightly (τ_1, τ_2)-continuous functions, $\delta(\tau_1, \tau_2)$-continuous functions, faintly (τ_1, τ_2)-continuous functions, quasi $\theta(\tau_1, \tau_2)$-continuous functions and weakly quasi (τ_1, τ_2)-continuous functions were established in [2], [3], [25], [28], [24], [29], [27] and [16], respectively. Kong-ied et al. [20] introduced and investigated the concept of almost quasi (τ_1, τ_2)-continuous functions. Khampakdee et al. [19] introduced and studied the notion of almost weakly (τ_1, τ_2)-continuous functions. Quite recently, Kong-ied et al. [20] introduced and investigated the concept of almost nearly (τ_1, τ_2)-continuous functions. In this paper, we introduce the concept of nearly (τ_1, τ_2)-continuous functions. We also investigate several characterizations of nearly (τ_1, τ_2)-continuous functions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called *$\tau_1\tau_2$-closed* [11] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called *$\tau_1\tau_2$-open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the *$\tau_1\tau_2$-closure* [11] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the *$\tau_1\tau_2$-interior* [11] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [11] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-*Cl*(A) *and* $\tau_1\tau_2$-*Cl*$(\tau_1\tau_2$-*Cl*$(A)) = \tau_1\tau_2$-*Cl*(A).
(2) *If* $A \subseteq B$, *then* $\tau_1\tau_2$-*Cl*$(A) \subseteq \tau_1\tau_2$-*Cl*$(B)$.
(3) $\tau_1\tau_2$-*Cl*(A) *is* $\tau_1\tau_2$-*closed.*
(4) A *is* $\tau_1\tau_2$-*closed if and only if* $A = \tau_1\tau_2$-*Cl*(A).
(5) $\tau_1\tau_2$-*Cl*$(X - A) = X - \tau_1\tau_2$-*Int*$(A)$.

A subset A of a bitopological space (X, τ_1, τ_2) is said to be *$(\tau_1, \tau_2)r$-open* [35] (resp. *$(\tau_1, \tau_2)s$-open* [9], *$(\tau_1, \tau_2)p$-open* [9], *$(\tau_1, \tau_2)\beta$-open* [9]) if $A = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$, $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$, $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [33] if $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be $\alpha(\tau_1, \tau_2)$-*closed*. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\mathscr{N}(\tau_1, \tau_2)$-*closed* [31] if every cover of A by $(\tau_1, \tau_2)r$-open sets of X has a finite subcover. Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a $(\tau_1, \tau_2)\theta$-*cluster point* [35] of A if $\tau_1\tau_2$-Cl$(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the $(\tau_1, \tau_2)\theta$-*closure* [35] of A and is denoted by $(\tau_1, \tau_2)\theta$-Cl(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\theta$-*closed* [35] if $(\tau_1, \tau_2)\theta$-Cl$(A) = A$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be $(\tau_1, \tau_2)\theta$-*open*. The union of all $(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $(\tau_1, \tau_2)\theta$-*interior* [35] of A and is denoted by $(\tau_1, \tau_2)\theta$-Int(A).

Lemma 2. [35] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) *If A is $\tau_1\tau_2$-open in X, then $\tau_1\tau_2$-Cl$(A) = (\tau_1, \tau_2)\theta$-Cl$(A)$.*
(2) *$(\tau_1, \tau_2)\theta$-Cl(A) is $\tau_1\tau_2$-closed in X.*

3. Characterizations of nearly (τ_1, τ_2)-continuous functions

In this section, we introduce the notion of nearly (τ_1, τ_2)-continuous functions. Moreover, some characterizations of nearly (τ_1, τ_2)-continuous functions are discussed.

Definition 1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be nearly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be nearly (τ_1, τ_2)-continuous if f has this property at every point of X.*

Theorem 1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is nearly (τ_1, τ_2)-continuous at $x \in X$;*
(2) *$x \in \tau_1\tau_2$-Int$(f^{-1}(V))$ for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*
(3) *$x \in f^{-1}(\sigma_1\sigma_2$-Cl$(B))$ for each subset B of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure such that $x \in \tau_1\tau_2$-Cl$(f^{-1}(B))$;*
(4) *$x \in \tau_1\tau_2$-Int$(f^{-1}(B))$ for each subset B of Y such that*

$$Y - \sigma_1\sigma_2\text{-}Int(B)$$

is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $x \in f^{-1}(\sigma_1\sigma_2$-Int$(B))$.

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement and $x \in f^{-1}(V)$. By (1), there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$. Thus, $x \in U \subseteq f^{-1}(V)$. Since U is $\tau_1\tau_2$-open, we have $x \in \tau_1\tau_2\text{-Int}(f^{-1}(V))$.

(2) $\Rightarrow$ (3): Let B be any subset of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure. Then, $\sigma_1\sigma_2\text{-Cl}(B)$ is $\sigma_1\sigma_2$-closed and $Y - \sigma_1\sigma_2\text{-Cl}(B)$ is a $\sigma_1\sigma_2$-open set having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Suppose that

$$x \notin f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)).$$

Then, we have $x \in X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)) = f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(B))$ and hence $f(x) \in Y - \sigma_1\sigma_2\text{-Cl}(B)$. Since $Y - \sigma_1\sigma_2\text{-Cl}(B)$ is a $\sigma_1\sigma_2$-open set having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, by (2) we have

$$\begin{aligned} x \in \tau_1\tau_2\text{-Int}(f^{-1}(Y - \sigma_1\sigma_2\text{-Cl}(B))) &= \tau_1\tau_2\text{-Int}(X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))) \\ &= X - \tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))) \\ &\subseteq X - \tau_1\tau_2\text{-Cl}(f^{-1}(B)). \end{aligned}$$

Thus, $x \notin \tau_1\tau_2\text{-Cl}(f^{-1}(B))$.

(3) $\Rightarrow$ (4): Let B be any subset of Y such that $Y - \sigma_1\sigma_2\text{-Int}(B)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. Suppose that $x \notin \tau_1\tau_2\text{-Int}(f^{-1}(B))$. Then, we have

$$\begin{aligned} x \in X - \tau_1\tau_2\text{-Int}(f^{-1}(B)) &= \tau_1\tau_2\text{-Cl}(X - f^{-1}(B)) \\ &= \tau_1\tau_2\text{-Cl}(f^{-1}(Y - B)) \end{aligned}$$

and by (3),

$$\begin{aligned} x \in f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - B)) &= f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(B)) \\ &= X - f^{-1}(\sigma_1\sigma_2\text{-Int}(B)). \end{aligned}$$

Thus, $x \notin f^{-1}(\sigma_1\sigma_2\text{-Int}(B))$.

(4) $\Rightarrow$ (1): Let V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - \sigma_1\sigma_2\text{-Int}(V) = Y - V$ which is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $x \in f^{-1}(\sigma_1\sigma_2\text{-Int}(V))$. By (4), we have $x \in \tau_1\tau_2\text{-Int}(f^{-1}(V))$. Therefore, there exists a $\tau_1\tau_2$-open set U of X containing x such that $x \in U \subseteq f^{-1}(V)$. Thus, $f(U) \subseteq V$. This shows that f is nearly (τ_1, τ_2)-continuous at x. □

Theorem 2. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is nearly (τ_1, τ_2)-continuous;*

(2) *$f^{-1}(V)$ is $\tau_1\tau_2$-open in X for each $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(3) *$f^{-1}(K)$ is $\tau_1\tau_2$-closed in X for every $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closed set K of Y;*

(4) *$\tau_1\tau_2$-Cl$(f^{-1}(B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ for every subset B of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure;*

(5) *$f^{-1}(\sigma_1\sigma_2\text{-}Int(B)) \subseteq \tau_1\tau_2\text{-}Int(f^{-1}(B))$ for every subset B of Y such that $Y - \sigma_1\sigma_2\text{-}Int(B)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed.*

Proof. $(1) \Rightarrow (2)$: Let V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement and $x \in f^{-1}(V)$. Since f is nearly (τ_1, τ_2)-continuous, by Theorem 1, $x \in \tau_1\tau_2\text{-Int}(f^{-1}(V))$. Thus, $\tau_1\tau_2\text{-Int}(f^{-1}(V)) \subseteq f^{-1}(V)$ and hence $f^{-1}(V)$ is $\tau_1\tau_2$-open in X.

$(2) \Rightarrow (3)$: The proof is obvious.

$(3) \Rightarrow (4)$: Let B be any subset of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure. Then, we have $\sigma_1\sigma_2\text{-Cl}(B)$ is a $\sigma_1\sigma_2$-closed set of Y and by (3), $f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ is $\tau_1\tau_2$-closed in X. Thus,

$$\begin{aligned} f^{-1}(B) &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)) \\ &= \tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))) \end{aligned}$$

and hence $\tau_1\tau_2\text{-Cl}(f^{-1}(B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$.

$(4) \Rightarrow (5)$: Let B be any subset of Y such that $Y - \sigma_1\sigma_2\text{-Int}(B)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. Then by (4), we have

$$\begin{aligned} X - \tau_1\tau_2\text{-Int}(f^{-1}(B)) &= \tau_1\tau_2\text{-Cl}(X - f^{-1}(B)) \\ &= \tau_1\tau_2\text{-Cl}(f^{-1}(Y - B)) \\ &\subseteq \tau_1\tau_2\text{-Cl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(B))) \\ &\subseteq f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(B)) \\ &= X - f^{-1}(\sigma_1\sigma_2\text{-Int}(B)). \end{aligned}$$

Thus, $f^{-1}(\sigma_1\sigma_2\text{-Int}(B)) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(B))$.

$(5) \Rightarrow (1)$: Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Thus by (5),

$$x \in f^{-1}(V) = f^{-1}(\sigma_1\sigma_2\text{-Int}(V)) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(V)).$$

By Theorem 1, f is nearly (τ_1, τ_2)-continuous at x. This shows that f is nearly (τ_1, τ_2)-continuous. □

Corollary 1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is nearly (τ_1, τ_2)-continuous if $f^{-1}(K)$ is $\tau_1\tau_2$-closed in X for every $\mathscr{N}(\sigma_1, \sigma_2)$-closed set K of Y.*

Proof. Let V be any $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - V$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. By the hypothesis,

$$\begin{aligned} X - f^{-1}(V) &= f^{-1}(Y - V) \\ &= \tau_1\tau_2\text{-Cl}(X - f^{-1}(V)) \end{aligned}$$

$$= X - \tau_1\tau_2\text{-Int}(f^{-1}(V))$$

and hence $f^{-1}(V) = \tau_1\tau_2\text{-Int}(f^{-1}(V))$. It follows from Theorem 2 that f is nearly (τ_1, τ_2)-continuous. □

Recall that a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-*regular* [15] if for each $\tau_1\tau_2$-closed set F and each point $x \in X - F$, there exist disjoint $\tau_1\tau_2$-open sets U and V such that $x \in U$ and $F \subseteq V$.

Theorem 3. *Let (Y, σ_1, σ_2) be a (σ_1, σ_2)-regular space. For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is nearly (τ_1, τ_2)-continuous;*
(2) *$f^{-1}((\sigma_1, \sigma_2)\theta\text{-Cl}(B))$ is $\tau_1\tau_2$-closed in X for every subset B of Y such that $(\sigma_1, \sigma_2)\theta\text{-Cl}(B)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed;*
(3) *$f^{-1}(K)$ is $\tau_1\tau_2$-closed in X for every $\mathscr{N}(\sigma_1, \sigma_2)$-closed $(\sigma_1, \sigma_2)\theta$-closed set K of Y;*
(4) *$f^{-1}(V)$ is $\tau_1\tau_2$-open in X for each $(\sigma_1, \sigma_2)\theta$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y such that $(\sigma_1, \sigma_2)\theta\text{-Cl}(B)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. Then, $(\sigma_1, \sigma_2)\theta\text{-Cl}(B)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $\sigma_1\sigma_2$-closed. Thus by Theorem 2, $f^{-1}((\sigma_1, \sigma_2)\theta\text{-Cl}(B))$ is $\tau_1\tau_2$-closed in X.

(2) $\Rightarrow$ (3): Let K be any $\mathscr{N}(\sigma_1, \sigma_2)$-closed $(\sigma_1, \sigma_2)\theta$-closed set of Y. Then, we have $K = (\sigma_1, \sigma_2)\theta\text{-Cl}(K)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and by (2), $f^{-1}(K)$ is $\tau_1\tau_2$-closed in X.

(3) $\Rightarrow$ (4): Let V be any $(\sigma_1, \sigma_2)\theta$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - V$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $(\sigma_1, \sigma_2)\theta$-closed. By (3), we have

$$\begin{aligned} X - f^{-1}(V) = f^{-1}(Y - V) &= \tau_1\tau_2\text{-Cl}(f^{-1}(Y - V)) \\ &= \tau_1\tau_2\text{-Cl}(X - f^{-1}(V)) \\ &= X - \tau_1\tau_2\text{-Int}(f^{-1}(V)) \end{aligned}$$

and hence $f^{-1}(V)$ is $\tau_1\tau_2$-open in X.

(4) $\Rightarrow$ (1): Let V be any $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Since (Y, σ_1, σ_2) is (σ_1, σ_2)-regular, V is $(\sigma_1, \sigma_2)\theta$-open in Y and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then by (4), we have $f^{-1}(V)$ is $\tau_1\tau_2$-open in X. By Theorem 2, f is nearly (τ_1, τ_2)-continuous. □

Acknowledgements. This research project was financially supported by Mahasarakham University.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] C. Boonpok, J. Khampakdee, Almost Strong $\theta(\Lambda, p)$-Continuity for Functions, Eur. J. Pure Appl. Math. 17 (2024), 300–309. https://doi.org/10.29020/nybg.ejpam.v17i1.4975.

[2] C. Boonpok, P. Pue-On, Characterizations of Almost (τ_1, τ_2)-Continuous Functions, Int. J. Anal. Appl. 22 (2024), 33. https://doi.org/10.28924/2291-8639-22-2024-33.

[3] C. Boonpok, C. Klanarong, On Weakly (τ_1, τ_2)-Continuous Functions, Eur. J. Pure Appl. Math. 17 (2024), 416–425. https://doi.org/10.29020/nybg.ejpam.v17i1.4976.

[4] C. Boonpok, N. Srisarakham, (τ_1, τ_2)-Continuity for Functions, Asia Pac. J. Math. 11 (2024), 21. https://doi.org/10.28924/APJM/11-21.

[5] C. Boonpok, on Some Spaces via Topological Ideals, Open Math. 21 (2023), 20230118. https://doi.org/10.1515/math-2023-0118.

[6] C. Boonpok, N. Srisarakham, Weak Forms of (Λ, b)-Open Sets and Weak (Λ, b)-Continuity, Eur. J. Pure Appl. Math. 16 (2023), 29–43. https://doi.org/10.29020/nybg.ejpam.v16i1.4571.

[7] C. Boonpok, $\theta(\star)$-Precontinuity, Mathematica, 65 (2023), 31–42. https://doi.org/10.24193/mathcluj.2023.1.04.

[8] C. Boonpok, J. Khampakdee, (Λ, sp)-Open Sets in Topological Spaces, Eur. J. Pure Appl. Math. 15 (2022), 572–588. https://doi.org/10.29020/nybg.ejpam.v15i2.4276.

[9] C. Boonpok, $(\tau_1, \tau_2)\delta$-Semicontinuous Multifunctions, Heliyon, 6 (2020), e05367. https://doi.org/10.1016/j.heliyon.2020.e05367.

[10] C. Boonpok, on Characterizations of $\star$-Hyperconnected Ideal Topological Spaces, J. Math. 2020 (2020), 9387601. https://doi.org/10.1155/2020/9387601.

[11] C. Boonpok, C. Viriyapong, M. Thongmoon, on Upper and Lower (τ_1, τ_2)-Precontinuous Multifunctions, J. Math. Computer Sci. 18 (2018), 282–293. https://doi.org/10.22436/jmcs.018.03.04.

[12] C. Boonpok, Almost (G, m)-Continuous Functions, Int. J. Math. Anal. 4 (2010), 1957–1964.

[13] C. Boonpok, M-Continuous Functions in Biminimal Structure Spaces, Far East J. Math. Sci. 43 (2010), 41–58.

[14] D. Carnahan, Locally Nearly Compact Spaces, Boll. Un. Mat. Ital. (4), 6 (1972), 146–153.

[15] M. Chiangpradit, S. Sompong, C. Boonpok, On Characterizations of (τ_1, τ_2)-Regular Spaces, Int. J. Math. Comput. Sci. 19 (2024), 1329–1334.

[16] M. Chiangpradit, S. Sompong, C. Boonpok, Weakly Quasi (τ_1, τ_2)-Continuous Functions, Int. J. Anal. Appl. 22 (2024), 125. https://doi.org/10.28924/2291-8639-22-2024-125.

[17] T. Duangphui, C. Boonpok and C. Viriyapong, Continuous Functions on Bigeneralized Topological Spaces, Int. J. Math. Anal. 5 (2011), 1165–1174.

[18] W. Dungthaisong, C. Boonpok, C. Viriyapong, Generalized Closed Sets in Bigeneralized Topological Spaces, Int. J. Math. Anal. 5 (2011), 1175–1184.

[19] C. Boonpok, S. Sompong, J. Khampakdee, Almost Weakly (τ_1, τ_2)-Continuous Functions, Eur. J. Pure Appl. Math. 18 (2025), 5721. https://doi.org/10.29020/nybg.ejpam.v18i1.5721.

[20] B. Kong-ied, A. Sama-Ae and C. Boonpok, Almost Nearly (τ_1, τ_2)-Continuous Functions, Int. J. Anal. Appl. 23 (2025), 14. https://doi.org/10.28924/2291-8639-23-2025-14.

[21] S.R. Malghan, V.V. Hanchinamani, N-Continuous Functions, Ann. Soc. Sci. Bruxelles, 98 (1984), 69–79.

[22] T. Noiri, N. Ergun, Notes on N-Continuous Functions, Res. Rep. Yatsushiro Coll. Tech. 11 (1989), 65–68.

[23] T. Noiri, N-Closed Sets and Some Separation Axioms, Ann. Soc. Sci. Bruxelles, 88 (1974), 195–199.

[24] C. Prachanpol, C. Boonpok, C. Viriyapong, $\delta(\tau_1, \tau_2)$-Continuous Functions, Eur. J. Pure Appl. Math. 17 (2024), 3730–3742. https://doi.org/10.29020/nybg.ejpam.v17i4.5335.

[25] P. Pue-on, S. Sompong, C. Boonpok, Slightly $(\tau_1, \tau_2)s$-Continuous Functions, Int. J. Math. Comput. Sci. 20 (2025), 217–221.

[26] P. Pue-on, C. Boonpok, $\theta(\Lambda, p)$-Continuity for Functions, Int. J. Math. Comput. Sci. 19 (2024), 491–495.

[27] N. Srisarakham, S. Sompong, C. Boonpok, Quasi $\theta(\tau_1, \tau_2)$-Continuous Functions, Eur. J. Pure Appl. Math. 18 (2025), 5722. https://doi.org/10.29020/nybg.ejpam.v18i1.5722.

[28] N. Srisarakham, S. Sompong, C. Boonpok, Slight (τ_1, τ_2)-Continuity for Functions, Int. J. Math. Comput. Sci. 20 (2025), 211–215.

[29] N. Srisarakham, A. Sama-Ae, C. Boonpok, Characterizations of Faintly (τ_1, τ_2)-continuous functions, Eur. J. Pure Appl. Math. 17 (2024), 2753–2762.

[30] N. Srisarakham, C. Boonpok, Almost (Λ, p)-Continuous Functions, Int. J. Math. Comput. Sci. 18 (2023), 255–259.

[31] M. Thongmoon, A. Sama-Ae, C. Boonpok, Upper and Lower Near (τ_1, τ_2)-Continuity, Eur. J. Pure Appl. Math. 18 (2025), 5633. https://doi.org/10.29020/nybg.ejpam.v18i1.5633.

[32] M. Thongmoon, C. Boonpok, Strongly $\theta(\Lambda, p)$-Continuous Functions, Int. J. Math. Comput. Sci. 19 (2024), 475–479.

[33] N. Viriyapong, S. Sompong, C. Boonpok, (τ_1, τ_2)-Extremal Disconnectedness in Bitopological Spaces, Int. J. Math. Comput. Sci. 19 (2024), 855–860.

[34] C. Viriyapong, C. Boonpok, (Λ, sp)-Continuous Functions, WSEAS Trans. Math. 21 (2022), 380–385. https://doi.org/10.37394/23206.2022.21.45.

[35] C. Viriyapong, C. Boonpok, $(\tau_1, \tau_2)\alpha$-Continuity for Multifunctions, J. Math. 2020 (2020), 6285763. https://doi.org/10.1155/2020/6285763.