Asia Pac. J. Math. 2025 **12**:50

DIFFERENT TYPES OF MICRO OPEN SETS IN MICRO IDEAL TOPOLOGICAL SPACES (MITS)

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Received Mar. 20, 2025

Abstract. This paper presents Micro $\beth_{Id}$-open sets in micro ideal topological spaces, analyzes their fundamental properties, and uses this set to present and learn the various kinds of sets known as Micro $\beth\beth_{Id}$-open and Micro $\beth\beth\beth_{Id}$-open sets. Numerous theorems and characterizations pertaining to these sets have been proven.

2020 Mathematics Subject Classification. 54A05; 54D10; 54D80.

Key words and phrases. micro $\beth_{Id}$-open sets; micro $\beth\beth_{Id}$-open sets; micro $\beth\beth\beth_{Id}$-open sets.

1. Introduction

Kuratowski [1] studied ideals in topological spaces. Jankovic and Hamlett [2] explored additional properties of Ideal Space. AL-Omeri Wadei et al. [3] examine additional e-I-open set properties, derive multiple e-I-continuous function characterizations, and look into how these relate to other kinds of functions. Lellis Thivagar [4] first proposed Nanotopology in 2013. The lower approximation, upper approximation, and boundary region concepts form the foundation of this topology. It has minimum of 3 open sets, including universal and empty sets, also maximum of 5. There are a lot of works around Nanotopology with ideal [5,6]. All Nanotopologies are Microtopologies. In 2019, Chandrasekar [7] introduced micro topology, which was defined as Micro closed, Micro open, Micro interior, and Micro closure. Nano topology was extended into Micro topology. Additionally, there are a minimum of four open sets and a maximum of nine. A new concept of spaces known as Micro ideal topological spaces (MITS) is introduced by S. GANESAN et al. [8], who also look into the relationship between Micro topological space and Micro ideal topological spaces. Micro $\beth_{Id}$-open sets, Micro $\beth\beth_{Id}$-open sets, and Micro $\beth\beth\beth_{Id}$-open sets are the new classes of sets that we are primarily interested in studying. Numerous theorems and characterizations pertaining to these sets have been proven.

DOI: 10.28924/APJM/12-50

2. Preliminary

Definition 2.1. [7] $(\aleph, \tau\mathcal{Z}(\mathcal{X}))$ *is Nanotopological space now* $\mu_{ic}\mathcal{Z}(\mathcal{X}) = \{N \cup (N_0 \cap \mu_{ic})\} : N, N_0 \in \tau\mathcal{Z}(\mathcal{X})$ *and named it Microtopology of* $\tau\mathcal{Z}(\mathcal{X})$ *by* μ_{ic} *where* $\mu_{ic} \notin \tau\mathcal{Z}(\mathcal{X})$.

Definition 2.2. [7] *The Microtopology* $\mu_{ic}\mathcal{Z}(\mathcal{X})$, *adheres to these axioms:*

(1) $\aleph, \emptyset \in \mu_{ic}\mathcal{Z}(\mathcal{X})$.
(2) *Any sub-collection of* $\mu_{ic}\mathcal{Z}(\mathcal{X})$ *has a union of its elements.*
(3) *Any finite sub-collection of* $\mu_{ic}\mathcal{Z}(\mathcal{X})$ *has an intersection of its elements in* $\mu_{ic}\mathcal{Z}(\mathcal{X})$.

The Microtopology on $\aleph$ regarding the $\mathcal{X}$ is then denoted by $\mu_{ic}\mathcal{Z}(\mathcal{X})$. The triplet is known as Micro topological spaces and consists of $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}))$ Micro open sets are the elements of $\mu_{ic}\mathcal{Z}(\mathcal{X})$, and a Micro closed set is the complement of a Micro open set.

Definition 2.3. [7] *The Microinterior of a set* $\mathcal{K}$ *is* $\mu_{ic}\ Int(\mathcal{K}) = \cup\{\chi : \chi$ *is Micro open and* $\chi \subseteq \mathcal{K}\}$. *The Microclosure of a set* $\mathcal{K}$ *is* $\mu_{ic}Cl(\mathcal{K}) = \cap\{\chi : \chi$ *is Micro closed and* $\mathcal{K} \subseteq \chi\}$.

Definition 2.4. [7] *Any two Micro sets* $\mathcal{K}$ *and* χ *in* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}))$,

(1) $\mathcal{K}$ *is a Micro closed set iff* $\mu_{ic}Cl(\mathcal{K}) = \mathcal{K}$.
(2) $\mathcal{K}$ *is Micro open set iff* $\mu_{ic}Int(\mathcal{K}) = \mathcal{K}$.
(3) $\mathcal{K} \subseteq \chi \rightarrow \mu_{ic}Int(\mathcal{K}) \subseteq \mu_{ic}Int(\chi)$ *and* $\mu_{ic}Cl(\mathcal{K}) \subseteq \mu_{ic}Cl(\chi)$.
(4) $\mu_{ic}Cl(\mu_{ic}Cl(\mathcal{K})) = \mu_{ic}Cl(\mathcal{K})$ *and* $\mu_{ic}Int(\mu_{ic}Int(\mathcal{K})) = \mu_{ic}Int(\mathcal{K})$.
(5) $\mu_{ic}Cl(\mathcal{K} \cup \chi) \supseteq \mu_{ic}Cl(\mathcal{K}) \cup \mu_{ic}Cl(\chi)$.
(6) $\mu_{ic}Cl(\mathcal{K} \cap \chi) \subseteq \mu_{ic}Cl(\mathcal{K}) \cap \mu_{ic}Cl(\chi)$.
(7) $\mu_{ic}Int(\mathcal{K} \cup \chi) \supseteq \mu_{ic}Int(\mathcal{K}) \cup \mu_{ic}Int(\chi)$.
(8) $\mu_{ic}Int(\mathcal{K} \cap \chi) \subseteq \mu_{ic}Int(\mathcal{K}) \cap \mu_{ic}Int(\chi)$.
(9) $\mu_{ic}Cl(\mathcal{K}^c) = [\mu_{ic}Int(\mathcal{K})]^c$.
(10) $\mu_{ic}Int(\mathcal{K}^c) = [\mu_{ic}Cl(\mathcal{K})]^c$.

Definition 2.5. [8] *Let* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *that is a (MITS) and Suppose* $p(\aleph)$ *is the collection of all subsets of* $\aleph$. *A set operator* $(.)^*_{\mu_{ic}} : p(\aleph) \rightarrow p(\aleph)$. *For a subset* $\mathcal{K} \subset \aleph, \mathcal{K}^*_{\mu_{ic}}(\mu_{ic}, Id) = \{\mathcal{K} \in: Sm \cap \mathcal{K} \notin Id$, *for every* $S_m \in S_m(\aleph)\}$ *is named the Microlocal function (briefly,* μ_{ic}*-local function) of* $\mathcal{K}$ *regarding the Id and* μ_{ic}. *Let's just write* $\mathcal{K}^*_{\mu_{ic}}$ *for* $\mathcal{K}^*_{\mu_{ic}}(Id, \mu_{ic})$.

Example 2.6. [8] *Let* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *that is a (MITS) and for every* $\mathcal{K} \subseteq \aleph$.

(1) *If* $Id = \{\emptyset\}$ *then* $p(\aleph) = \mu_{ic}Cl(\mathcal{K})$,
(2) *If* $Id = P(\aleph)$, *then* $\mathcal{K}^*_{\mu_{ic}} = \emptyset$.

Theorem 2.7. [8] *Let* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *that is a (MITS) and* $\mathcal{K}, \chi$ *be subsets of* $\aleph$. *Then*

(1) $\mathcal{K} \subseteq \chi \to \mathcal{K}^*_{\mu_{ic}} \subseteq \chi^*_{\mu_{ic}}$,

(2) $Id \subseteq Id' \to \mathcal{K}^*_{\mu_{ic}}(Id') \subseteq \mathcal{K}^*_{\mu_{ic}}(Id)$,

(3) $\mathcal{K}^*_{\mu_{ic}} = \mu_{ic}Cl(\mathcal{K}^*_{\mu_{ic}}) \subseteq \mu_{ic}Cl(\mathcal{K})(\mathcal{K}^*_{\mu_{ic}}$ *is a Micro closed subset of* $\mu_{ic}Cl(\mathcal{K}))$,

(4) $(\mathcal{K}^*_{\mu_{ic}})^*_{\mu_{ic}} \subseteq \mathcal{K}^*_{\mu_{ic}}$,

(5) $\mathcal{K}^*_{\mu_{ic}} \cup \chi^*_{\mu_{ic}} = (\mathcal{K} \cup \chi)^*_{\mu_{ic}}$,

(6) $\mathcal{K}^*_{\mu_{ic}} - \chi^*_{\mu_{ic}} = (\mathcal{K} - \chi)^*_{\mu_{ic}} - \chi^*_{\mu_{ic}} \subseteq (\mathcal{K} - \chi)^*_{\mu_{ic}}$,

(7) $V \in \mu_{ic} \to V \cap \mathcal{K}^*_{\mu_{ic}} = V \cap (V \cap \mathcal{K})^*_{\mu_{ic}} \subseteq (V \cap \mathcal{K})^*_{\mu_{ic}}$,

Definition 2.8. [8] *Let* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id))$ *that is a* $(MITS)$. *The set operator* $\mu_{ic}\ Cl^*_{Id}$ *is called a Micro* $*$*-closure and is defined as* $\mu_{ic}\ Cl^*_{Id}\ (\mathcal{K}) = \mathcal{K} \cup \mathcal{K}^*_{\mu_{ic}} for \mathcal{K} \subseteq \aleph$.

Theorem 2.9. [8] *The following requirements are met by the set operator* $\mu_{ic}\ Cl^*_{Id}$:

(1) $\mathcal{K} \subseteq \mu_{ic}Cl^*_{Id}(\mathcal{K})$,

(2) $\mu_{ic}Cl^*_{Id}(\emptyset) = \emptyset$ *and* $\mu_{ic}Cl^*_{Id}(\aleph) = \aleph$,

(3) *If* $\mathcal{K} \subseteq \chi$, *then* $\mu_{ic}\ Cl^*_{Id}(\mathcal{K}) \subseteq \mu_{ic}Cl^*_{Id}(\chi)$,

(4) $\mu_{ic}Cl^*_{Id}(\mathcal{K}) \cup \mu_{ic}Cl^*_{Id}(\chi) = \mu_{ic}Cl^*_{Id}(\mathcal{K} \cup \chi)$,

(5) $\mu_{ic}Cl^*_{Id}(\mu_{ic}Cl^*_{Id}(\mathcal{K})) = \mu_{ic}Cl^*_{Id}(\mathcal{K})$.

Theorem 2.10. [8] *Let* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id))$ *that is a* $(MITS)$. *For every* $\mathcal{K} \subseteq \aleph$ *If* $\mathcal{K} \subseteq \mathcal{K}^*_{\mu_{ic}}$, *then* $\mathcal{K}^*_{\mu_{ic}} = \mu_{ic}Cl(\mathcal{K}^*_{\mu_{ic}}) = \mu_{ic}Cl(\mathcal{K}) = \mu_{ic}Cl^*_{Id}(\mathcal{K})$.

Definition 2.11. [9] *Consider the space* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}))$ *as a Microtopological also* $\mathcal{K} \subseteq \aleph$. *Then* $\mathcal{K}$ *is named Micro* α*-open if* $\mathcal{K} \subseteq \mu_{ic}Int(\mu_{ic}Cl(\mu_{ic}Int(\mathcal{K})))$.

3. Micro $\daleth_{\mathbf{Id}}$-open sets

Definition 3.1. *Let* $\mathcal{K}$ *be a subset of* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *that is a* $(MITS)$, *then* $\mathcal{K}$ *is*

(1) *Micro Id - open set* [8] *if* $\mathcal{K} \subset \mu_{ic}Int(\mathcal{K}^*_{\mu_{ic}})$

(2) *Micro* $\daleth$*-open* [10] *if* $\mathcal{K} \subset \mu_{ic}Int(\mu_{ic}Cl(\mathcal{K})) \cup \mu_{ic}Cl(\mu_{ic}Int(\mathcal{K}))$.

(3) *Micro* $\daleth_{Id}$*-open if* $\mathcal{K} \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$.

The family members of all Micro $\daleth$ *- open* (*Micro* $\daleth_{Id}$*-open*) *of the space* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *will be denoted by Micro* $\daleth O(\aleph)$ (*Micro* $\daleth_{Id}O(\aleph)$).

A sub set $\mathcal{K}$ *of* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *that is a* $(MITS)$. *This is referred to as Micro* ð *-closed* (*Micro* $\daleth_{Id}$*-closed*) *if its complement is Micro* $\daleth$*-open* (*Micro* $\daleth_{Id}$*-open*).

Theorem 3.2. *Let* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *that is a* $(MITS)$, *then every*

(1) *Micro open is Micro* $\daleth_{Id}$*-open.*

(2) *Micro Id -open is Micro* $\daleth_{Id}$*-open.*

Proof.

(1) Let $\mathcal{K} \subset \aleph$ if $\mathcal{K}$ is open in $\mu_{ic}\mathcal{Z}(\mathcal{X})$ we have $\mathcal{K} = \mu_{ic}Int(\mathcal{K}) \subset \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$. Then $\mathcal{K}$ is Micro $\sqsupset_{Id}$- open.

(2) Let $\mathcal{K}$ is Micro Id -open $\rightarrow$ $\mathcal{K} \cup \mu_{ic}Int(\mathcal{K}^*_{\mu_{ic}}) \subseteq \mu_{ic}Int(\mu_{ic}Cl^*(\mathcal{K}) \subseteq \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$ which proves that $\mathcal{K}$ is Micro $\sqsupset_{Id}$-open.

□

Remark 3.3. *However, the opposite does not have to be true.*

Example 3.4.

(1) *Let* $\aleph = \{P_1, P_2, P_3\}$ *with* $\aleph/\mathcal{Z} = \{\{P_1\}, \{P_3\}, \{P_2\}\}$ *and* $\mathcal{X} = \{P_1, P_2\}$. *Then* $\tau\mathcal{Z}(\mathcal{X}) = \{\emptyset, X, \{P_1\}, \{P_1, P_2\}\}$ *and* $Id = \{\mathcal{O}, \{P_2\}\}, \mu_{ic} = \{P_2\}, \mu_{ic}\mathcal{Z}(\mathcal{X}) = \{\emptyset, X, \{P_1\}, \{P_2\}, \{P_1, P_2\}\}$. *Then* $\mathcal{K} = P_1, P_3$ *is Micro* $\sqsupset_{Id}$*-open. However, it's not Micro open.*

(2) *Let* $\aleph = \{P_1, P_2, P_3\}$ *with* $\aleph/\mathcal{Z} = \{\{P_1\}, \{P_3\}, \{P_2\}\}$ *and* $\mathcal{X} = \{P_1, P_2\}$. *Then* $\tau\mathcal{Z}(\mathcal{X}) = \{\emptyset, X, \{P_1\}, \{P_2\}, \{P_1, P_2\}\}$ *and* $Id = \{\mathcal{O}, \{P_3\}\}, \mu_{ic} = \{P_2, P_3\}, \mu_{ic}\mathcal{Z}(\mathcal{X}) = \{\emptyset, X, \{P_1\}, \{P_2\}, \{P_1, P_2\}, \{P_2, P_3\}\}$. *Then* $\mathcal{K} = \{P_1, P_3\}$ *is Micro* Id *–open. However, it's not satisfy Micro* $\sqsupset_{Id}$*-open.*

Theorem 3.5. *Let* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *that is a (MITS), and* $\mathcal{K} \subset \aleph$ *we have:*

(1) $Id = \emptyset$*, then* $\mathcal{K}$ *is Micro* $\sqsupset_{Id}$*-open* $\rightarrow$ $\mathcal{K}$ *is Micro* $\sqsupset$*-open.*

(2) $Id = \mathbb{P}(X)$*, then* $\mathcal{K}$ *is Micro* $\sqsupset_{Id}$*-open* $\rightarrow$ $\mathcal{K}$ *Micro* α*-open.*

Proof.

(1) Let $Id = \emptyset, \mathcal{K}$ is Micro $\sqsupset_{Id}$-open and $\mathcal{K} \subset \aleph$. Therefore $\mathcal{K} \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$. We have $\mu_{ic}Cl^*_{Id}(\mathcal{K}) = \mu_{ic}Cl(\mathcal{K})), \mu_{ic}Int^*_{Id}(\mathcal{K})) = \mu_{ic}Int(\mathcal{K}))$ and $\mathcal{K}^* = \mu_{ic}Cl(\mathcal{K})$ Hence $\mathcal{K} \subset \mu_{ic}Int(\mu_{ic}Cl(\mathcal{K})) \cup \mu_{ic}Cl(\mu_{ic}Int(\mathcal{K}))$.

(2) Let $Id = P(X)$ then $\mathcal{K}^*_{\mu_{ic}} = \emptyset$, for any $\mathcal{K} \subset \aleph$. Since $\mathcal{K}$ is Micro $\sqsupset_{Id}$-open, we have $\mathcal{K} \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \rightarrow \mathcal{K} \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})) = \mu_{ic}Int[\mu_{ic}Int(\mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K}))) \cup \mu_{ic}\ Cl^*_{Id}(\mathcal{K})] \subset \mu_{ic}Int[\mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Cl^*_{Id} (\mathcal{K})] \subset \mu_{ic}Int[\mu_{ic}Cl^*_{Id}(\mu_{ic}Int^*_{Id}(\mathcal{K} \cup \mathcal{K}))] \subset \mu_{ic}Int[\mu_{ic}Cl^*_{Id}(\mu_{ic}Int^*_{Id}(\mathcal{K}))] \subset \mu_{ic}Int[\mu_{ic}Cl (\mu_{ic}Int(\mathcal{K}))]$,

□

Theorem 3.6. *Let* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *that is a* (MITS) *then the collections union of Micro* $\sqsupset_{Id}$*-open is a Micro* $\sqsupset_{Id}$*-open.*

Proof. Let $\chi_\alpha | \alpha \in \Delta$ be collection of Micro $\sqsupset_{Id}$-open set, $\chi_\alpha \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\chi_\alpha)) \cup (\mu_{ic}Int (\mu_{ic}Cl^*_{Id}(\chi_\alpha))$. Hence $\cup_{\alpha\in\Delta}\chi_\alpha \subset \cup_{\alpha\in\Delta} [\mu_{ic}Cl(\mu_{ic}Int^*_{Id}\ (\chi_\alpha)) \cup (\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\chi_\alpha))] \subset \cup_{\alpha\in\Delta}[\mu_{ic}Cl$

$(\mu_{ic}Int^*_{Id}(\chi_\alpha))] \cup \cup_{\alpha\in\Delta}[(\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\chi_\alpha))] \subset [\mu_{ic}Cl(\cup_{\alpha\in\Delta}(\mu_{ic}Int^*_{Id}(\chi_\alpha))] \cup (\mu_{ic}Int(\cup_{\alpha\in\Delta}(\mu_{ic}Cl^*_{Id}(\chi_\alpha))] \subset [\mu_{ic}Cl(\cup_{\alpha\in\Delta}(\mu_{ic}Int^*_{Id}(\chi_\alpha))] \cup [(\mu_{ic}Int(\cup_{\alpha\in\Delta}(\mu_{ic}Cl^*_{Id}(\chi_\alpha))] \subset [\mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\cup_{\alpha\in\Delta}\chi_\alpha))] \cup [(\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\cup_{\alpha\in\Delta}\chi_\alpha))]$. This $\cup_{\alpha\in\Delta}\chi_\alpha$ is Micro $\daleth_{Id}$-open. □

Theorem 3.7. *Let $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ that is a $(MITS)$ then the intersection of arbitrary collection of Micro $\daleth_{Id}$-closed sets is Micro $\daleth_{Id}$-closed.*

Proof. Let $\{\mathcal{K}_\alpha/\alpha \in \Delta\}$ be a collection of Micro $\daleth_{Id}$-closed. So $(\mathcal{K}_\alpha)^c/\alpha \in \Delta$ be a collection of Micro $\daleth_{Id}$-open set.then $(\cup_{\alpha\in\Delta})^c\chi_\alpha$ is Micro $\daleth_{Id}$-open. Hence $(\cap_{\alpha\in\Delta}\chi_\alpha)c = (\cup_{\alpha\in\Delta})^c\,\chi_\alpha$ is Micro $\daleth_{Id}$-open. This means $(\cap_{\alpha\in\Delta}\chi_\alpha)$ is Micro $\daleth_{Id}$-closed set. □

Theorem 3.8. *Let $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ that is a $(MITS)$ and let $\mathcal{K}, \chi \subseteq \aleph$. If $\mathcal{K}$ is Micro $\daleth_{Id}$-open set and $\chi \in \mu_{ic}\mathcal{Z}(\mathcal{X})$. Then $\mathcal{K} \cap \chi$ is a Micro $\daleth_{Id}$-open.*

Proof. Assuming $\mathcal{K} \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$ and $\chi \subseteq \mu_{ic}Int(\chi).\chi \in \mu_{ic}\mathcal{Z}(\mathcal{X}) \to \chi \cap \mathcal{K}^*_{\mu_{ic}} = \chi \cap (\chi \cap \mathcal{K})^*_{\mu_{ic}} \subseteq (\aleph \cap \mathcal{K})*$, $\mathcal{K} \cap \chi \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K}) \cap \mu_{ic}Int(\chi)) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}) \cap \mu_{ic}Int(\chi)) \subset (\mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cap \mu_{ic}Int(\chi)) \cup (\mu_{ic}Int(\mu_{ic}Cl(\mu_{ic}Cl^*_{Id}(\mathcal{K}))) \cap \mu_{ic}Cl(\mu_{ic}Cl(\mu_{ic}Int(\chi)))) \subset (\mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K} \cap \chi) \cup (\mu_{ic}Int(\mu_{ic}Cl(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \cap \mu_{ic}Cl(\mu_{ic}Int(\chi)))) \subset (\mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K} \cap \chi) \cup (\mu_{ic}Int(\mu_{ic}Cl(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \cap \mu_{ic}Int(\chi))) \subset (\mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K} \cap \aleph) \cup (\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K} \cap \aleph))), Thus \mathcal{K} \cap \chi$ is Micro $\daleth_{Id} - open$ □

4. About different type of micro $\daleth_{Id}$ -open sets

Definition 4.1. *Let $\mathcal{K}$ be a sub set of $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ that is a $(MITS)$, then $\mathcal{K}$ is*

(1) *Micro $\daleth\daleth_{Id}$-open if $\mathcal{K} \subset \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$.*
(2) *Micro $\daleth\daleth\daleth_{Id}$- open if $\mathcal{K} \subseteq \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mu_{ic}Cl(\mathcal{K}))) \cup \mu_{ic}Cl(\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})))$.*

The family members of all Micro $\daleth\daleth\daleth_{Id}$ -open (Micro $\daleth\daleth_{Id}$-open) of the space $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ will be denoted by Micro $\daleth\daleth\daleth_{Id}O(\aleph)(Micro\daleth\daleth_{Id}O(\aleph))$.

A sub set $\mathcal{K}$ of $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ be a $(MITS)$ is said to be Micro $\daleth\daleth\daleth_{Id}$ -closed (Micro $\daleth\daleth\daleth_{Id}$-closed) if its complement is Micro $\daleth\daleth\daleth_{Id}$ -open (Micro $\daleth\daleth_{Id}$-open).

Proposition 4.2. *For a sub set of $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ that is a $(MITS)$, the following properties hold:*

(1) *$\mathcal{K}$ is Micro $\daleth\daleth_{Id}$-open $\to \mathcal{K}$ is Micro $\daleth$-open.*
(2) *$\mathcal{K}$ is Micro Id -open $\to \mathcal{K}$ is Micro $\daleth\daleth_{Id}$-open.*
(3) *$\mathcal{K}$ is Micro $\daleth\daleth_{Id}$-open $\to \mathcal{K}$ is Micro $\daleth_{Id}$-open.*
(4) *$\mathcal{K}$ is Micro $\daleth\daleth_{Id}$-open $\to \mathcal{K}$ is Micro $\daleth\daleth_{Id}$-open.*
(5) *$\mathcal{K}$ is Micro $\daleth\daleth_{Id}$-open $\to \mathcal{K}$ is Micro $\daleth_{Id}$-open.*

Proof.

(1) Take $\mathcal{K}$ be a Micro $\beth\beth_{Id}$-open set. Then $\mathcal{K} \subset \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \subset \mu_{ic}Int(\mu_{ic}Cl(\mathcal{K})) \cup \mu_{ic}Cl(\mu_{ic}Int(\mathcal{K}))$. This demonstrates that $\mathcal{K}$ is Micro $\beth$ -open.

(2) $\mathcal{K}$ is Micro Id -open $\rightarrow$ $\mathcal{K} \subset \mu_{ic}Int(\mathcal{K}^*_{\mu_{ic}}) \subseteq \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \subseteq \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$, which proves that $\mathcal{K}$ is Micro $\beth\beth_{Id}$-open.

(3) Take $\mathcal{K}$ be a Micro $\beth\beth_{Id}$-open set in $\aleph$. Then we have $\mathcal{K} \subset \mu_{ic}Cl^*(\mu_{ic}Int(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$. This leads that $\mathcal{K} \subset \mu_{ic}Cl(\mu_{ic}Int^*_{Id}(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$. This indicates that $\mathcal{K}$ is $\beth_{Id}$-open.

(4) Take $\mathcal{K}$ be a $\beth\beth_{Id}$-open set in $\aleph$. Then we have $\mathcal{K} \subset \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))$ this leads that $\mathcal{K} \subseteq \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mu_{ic}Cl(\mathcal{K}))) \cup \mu_{ic}Cl(\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})))$. This shows that $\mathcal{K}$ is $\beth\beth\beth_{Id}$- open.

(5) Obvious.

□

Remark 4.3. *The converses of Proposition 4.2. are untrue, as demonstrated by the example that follows.*

Example 4.4. *Take $\aleph = \{P_1, P_2, P_3, P_4\}$ with $\aleph/\mathcal{Z} = \{\{P_1\}, \{P_3\}, \{P_2, P_4\}\}$ and $X = \{P_1, P_2\}$. Then $\tau\mathcal{Z}(\mathcal{X}) = \{\phi, \aleph, \{P_1\}, \{P_1, P_2, P_4\}\}$. If $\mu_{ic} = \{P_2\}$, then $\mu_{ic}\mathcal{Z}(\mathcal{X}), = \{\phi, \aleph, \{P_1\}, \{P_2\},$ $\{P_1, P_2, P_4\}\}$ and the Ideal be $Id = \{\phi, \{P_1\}\}$. $\mathcal{K} = \{P_1\}$ is Micro $\beth_{Id}$-open but not Micro Id –open and $\chi = \{P_1, P_3\}$ is Micro $\beth$ –open. However, it's not Micro $\beth\beth_{Id}$-open.*

Example 4.5. *Take $\aleph = \{P_1, P_2, P_3\}$, with $\aleph/\mathcal{Z} = \{\{P_1, P_2\}, \{P3\}\}$ and $\mathcal{X} = \{P1, P2\}, \tau\mathcal{Z}(\mathcal{X}) = \{\phi, \aleph, \{P_1, P_2\}\}$ and $Id = \{\phi, \{P_1\}\}$. Then $\mu_{ic} = P1$ then $\mu_{ic}\mathcal{Z}(\mathcal{X}) = \{\phi, \aleph, \{P_1\}, \{P_1, P_2\}\}$, $\mathcal{K} = \{P1, P3\}$ is Micro $\beth\beth\beth_{Id}$- open. However, it's not $\beth\beth_{Id}$-open*

Example 4.6. *Take $\aleph = \{P_1, P_2, P_3\}$ with $\aleph/\mathcal{Z} = \{\{P_1\}, \{P_3\}, \{P_2\}\}$ and $\mathcal{X} = \{P_1, P_2\}$. Then $\tau\mathcal{Z}(\mathcal{X}) = \{\phi, \aleph, \{P_1\}, \{P_2\}, \{P_1, P_2\}\}$ and $Id = \{\mathcal{O}, \{P_3\}\}, \mu_{ic} = \{P_2, P_3\}, \mu_{ic}\mathcal{Z}(\mathcal{X}) = \{\phi, \aleph, \{P_1\}, \{P_2\}, \{P_1, P_2\},$ $\{P_2, P_3\}\}.\mathcal{K} = \{P_1, P_3\}$ is Micro $\beth_{Id}$-open. However, it's not Micro $\phi\phi\phi_{Id}$-open.*

Theorem 4.7. *Take $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ that is a (MITS). If $\chi_\alpha \in$ Micro $\beth\beth_{Id}O(\chi) \forall \alpha \in \Delta$, then $\cup\{\chi_\alpha : \alpha \in \Delta\} \in$ Micro $\beth\beth_{Id}O(U)$.*

Proof. Since $\chi_\alpha \in$ Micro $\beth\beth_{Id}O(\chi)$, then $\chi_\alpha \subset \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\chi_\alpha)) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\chi_\alpha)) \forall \alpha \in \Delta$. So $\cup_{\alpha\in\Delta}\chi_\alpha \subset \cup_{\alpha\in\Delta} [\mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\chi_\alpha)) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\chi_\alpha))] \subset_{\alpha\in\Delta} [[\mu_{ic}Int(\chi_\alpha) \cup \mu_{ic}Int$ $(\chi_\alpha)^*_{\mu_{ic}}] \cup [\mu_{ic}Int(\chi_\alpha) \cup (\chi_\alpha)^*_{\mu_{ic}})]] \subset [\mu_{ic}Int(\cup_{\alpha\in\Delta}\chi_\alpha) \cup (\cup_{\alpha\in\Delta}\mu_{ic}Int(\chi_\alpha)^*_{\mu_{ic}})] \cup [\mu_{ic}Int(\cup_{\alpha\in\Delta}\chi_\alpha) \cup (\cup_{\alpha\in\Delta}(\chi_\alpha)^*_{\mu_{ic}}))] \cup [\mu_{ic}Int(\cup_{\alpha\in\Delta}\chi_\alpha) \cup (\mu_{ic}Int(\cup\alpha \in \Delta\chi_\alpha))^*_{\mu_{ic}}] \cup [\mu_{ic}Int((\cup_{\alpha\in\Delta}\chi_\alpha) \cup (\cup_{\alpha\in\Delta}\chi_\alpha)^*_{\mu_{ic}})] \subset \mu_{ic}Cl^*_{Id}[\mu_{ic}Int(\cup_{\alpha\in\Delta}\chi_\alpha)] \cup [\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\cup_{\alpha\in\Delta}\chi_\alpha))]$. As a result $\cup_{\alpha\in\Delta}\ \chi_\alpha$ is Micro $\beth\beth_{Id}$- open. □

The instance that follows shows that the finite numbers intersection of Micro $\beth\beth_{Id}$-open sets does not necessarily have to be Micro $\beth\beth_{Id}$-open.

Example 4.8. *Take* $\aleph = \{P_1, P_2, P_3, P_4\}$ *with* $\aleph/\mathcal{Z} = \{\{P_1\}, \{P_3\}, \{P_2, P_4\}\}$ *and* $\chi = \{P_1, P_2\}$. *Then* $\tau\mathcal{Z}(\mathcal{X}) = \{\phi, \aleph, \{P_1\}, \{P_1, P_2, P_4\}\}$. *If* $\mu_{ic} = \{P_2, P_4\}$, *then* $\mu_{ic}\mathcal{Z}(\mathcal{X}) = \{\phi, \aleph, \{P_1\}, \{P_2, P_4\}, \{P_1, P_2, P_4\}\}$ *and* $Id = \{\phi, \{P_1\}\}$. $H = \{P_1, P_2, P3\}$ *and* $\mathcal{X} = \{P_2, P_3, P_4\}$ *are Micro* $\beth\beth_{Id}$*- open. But* $H \cap \mathcal{X} = \{P_2, P_3\}$ *is not Micro* $\beth\beth_{Id}$*- open.*

Theorem 4.9. *Let* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *that is a (MITS) and* $\mathcal{K}, \chi \subset \aleph$. *If* $\mathcal{K} \in$ *Micro* $\beth\beth_{Id}O(\aleph)$ *and* $\chi \in \mu_{ic}\mathcal{Z}(\mathcal{X})$, *then* $\mathcal{K} \cap \chi \in$ *Micro* $\beth\beth_{Id}O(\aleph)$.

Proof. Let $\mathcal{K}$ Micro $\beth\beth_{Id}O(\aleph), \chi \in \mu_{ic}\mathcal{Z}(\mathcal{X})$. Then $\mathcal{K} \subset \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \cup \mu_{ic}Cl^*_{Id}\ (\mu_{ic}Int(\mathcal{K}))$ and $\chi \cap \mathcal{K} \subset \chi \cap [\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \cup \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K}))] = [\chi \cap \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K}))] \cup [\chi \cap \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K}))] = [\mu_{ic}Int(\chi) \cap \mu_{ic}Int(\mu_{ic}Cl^*_{Id}\ (\mathcal{K})) \cup [\chi \cap \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K}))] \subset [\mu_{ic}Int(\chi \cap \mu_{ic}Cl^*_{Id}(\mathcal{K}))] \cup [\mu_{ic}Cl^*_{Id}\ (\chi \cap \mu_{ic}Int(\mathcal{K}))]$. Thus, $\chi \cap \mathcal{K} \subset [\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\chi \cap \mathcal{K}))] \cup [\mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\chi \cap \mathcal{K}))]$. This indicates that $\chi \cap \mathcal{K} \in$ Micro $\beth\beth_{Id}O(\aleph)$. □

Theorem 4.10. *If a sub set* $\mathcal{K}$ *of a space* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id)$ *is Micro* $\beth_{Id}$*-closed, then* $\mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\mathcal{K})) \cap \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K})) \subset \mathcal{K}$.

Proof. Since $\mathcal{K}$ is Micro $\beth_{Id}$-closed, $\aleph - \mathcal{K} \in$ Micro $\beth_{Id}O(\aleph)$, we have $\aleph - \mathcal{K} \subset \mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\aleph - \mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^*_{Id}(\aleph - \mathcal{K})) \subset \mu_{ic}Cl(\mu_{ic}Int(\aleph - \mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl(\aleph - \mathcal{K})) = [\aleph^{\smile}(\mu_{ic}Int(\mu_{ic}Cl(\mathcal{K})))] \cup [\aleph^{\smile}(\mu_{ic}Cl(\mu_{ic}Int(\mathcal{K})))] \subset [\aleph^{\smile}(\mu_{ic}(\mu_{ic}Cl^*_{Id}(\mathcal{K})))] \cup [\aleph^{\smile}(\mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K})))] = \aleph^{\smile}[(\mu_{ic}Int(\mu_{ic} Cl^*_{Id}(\mathcal{K}))) \cap (\mu_{ic}Cl^*_{Id}(\mu_{ic}Int(\mathcal{K}))]$ □

Theorem 4.11. *If a sub set* $\mathcal{K}$ *of a space* $(\aleph, \tau\mathcal{Z}(\mathcal{X}), \mu_{ic}\mathcal{Z}(\mathcal{X}), Id) \forall \aleph - [\mu_{ic}Int(\mu_{ic}Cl^\star(\mathcal{K}))] = \mu_{ic}Cl^\star(\mu_{ic}Int(\aleph - \mathcal{K}))$ *and* $\aleph - [\mu_{ic}Cl^\star(\mu_{ic}Int(\mathcal{K}))] = \mu_{ic}Int(\mu_{ic}Cl^\star(\aleph - \mathcal{K}))$. *Then* $\mathcal{K}$ *is Micro* $\beth\beth_Id$*-closed* $\Leftrightarrow \mu_{ic}Int(\mu_{ic}Cl^\star(\mathcal{K})) \cap \mu_{ic}Cl^\star(\mu_{ic}Int(\mathcal{K})) \subset \mathcal{K}$.

Proof. This is the immediate consequence of the Theorem 4.10 Let $\mu_{ic}Int(\mu_{ic}Cl^\star(\mathcal{K})) \cap \mu_{ic}Cl^\star (\mu_{ic}Int(\mathcal{K})) \subset \mathcal{K}$. Then $\aleph - \mathcal{K} \subset \aleph - [(\mu_{ic}Int(\mu_{ic}Cl^\star(\mathcal{K}))) \cap (\mu_{ic}Cl^\star(\mu_{ic}Int(\mathcal{K}))] \subset [\aleph^{\smile}(\mu_{ic}Int(\mu_{ic}Cl^\star (\mathcal{K})))] \cup [\aleph^{\smile}(\mu_{ic}Cl^\star(\mu_{ic}Int(\mathcal{K})))] = \mu_{ic}Cl^\star(\mu_{ic}Int(\aleph - \mathcal{K})) \cup \mu_{ic}Int(\mu_{ic}Cl^\star(\aleph - \mathcal{K}))$, Thus $\aleph - \mathcal{K}$ is Micro $\beth\beth_Id$- open and so $\mathcal{K}$ is Micro $\beth\beth_Id$-closed.

□

5. Conclusion

We can see from the above that none of these meanings can be inverted, as demonstrated by the aforementioned examples.

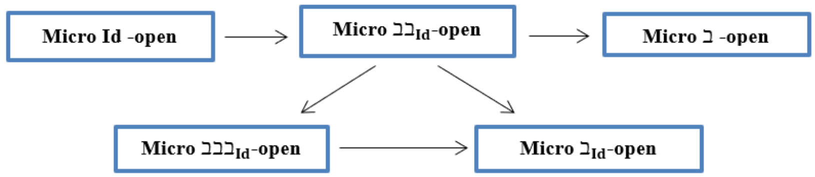

Figure 1. The relationships between the various types of Micro open sets introduced in this article, and the opposite direction of the implication in the above diagram may be incorrect

Authors' Contributions. All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

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