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NURSE SCHEDULING MODEL IN THE INPATIENT DEPARTMENT WITH OVERTIME BALANCING: A CASE STUDY OF A HOSPITAL IN RATCHABURI, THAILAND

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Abstract. In this article, we formulate the nurse scheduling model in the inpatient department by using integer programming. The case study was conducted at a hospital in Ratchaburi, Thailand, with the objective of minimizing the number of overtime shifts while ensuring an equitable distribution of both regular and overtime work hours among nurses, in compliance with hospital policies. After obtaining the integer linear programming model, we solve the optimal solution using Python. The results show that the number of regular and overtime shifts of each nurse was equal, with reduced working hours compared to the manually arranged schedule by the nurse supervisor, and in accordance with the hospital's policies. In addition, we stimulate an increase in the number of nurses to reduce overtime. We also find the minimum number of nurses needed so that all nurses will not have to work overtime.

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1. Introduction

Scheduling problems are common in workplaces such as the operational flight and pilot scheduling problem [11,13,18], the production scheduling problem in many factories [8,10], and the duty scheduling problem for veterinarians, doctors, nurses, or pharmacists in hospitals or clinics [2–4,14]. Staff scheduling makes work systematic and efficient for both work quality and worker satisfaction in various departments, organizations, or companies.

Hospitals are important health care centers that provide essential medical care, treatment, and services. In a hospital, physicians, nurses, and pharmacists all play different but important roles in

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patient care. Nurses are qualified to help improve the quality of health care as caregivers by managing physical needs, preventing illness, and maintaining health conditions. Nurse scheduling is very crucial because if nurses work continuously for too long, they will not get enough rest, which can affect the efficiency of patient care. If some nurses work fewer hours, it can cause an inequality in work performance. In nurse scheduling, it is necessary to take into account hospital practices and policies, as well as the needs of nurses at different times in order to work efficiently. Generally, nurse scheduling in various departments is the responsibility of the head nurse or senior nurse in that department. The schedule will be arranged in accordance with the number of nurses in the department, as well as other related factors like the demand for nurses in each department at each time period or overtime work with the equal number of holidays for each nurse in that department. From studies, it was found that when staff scheduling by supervisors, there may sometimes be inequality in each nurse's working hours, or the hospital may not have enough nurses to take care of patients each day, causing nurses to work overtime.

In general, medical staff scheduling is a field that requires a lot of applied mathematics such as integer linear programming, goal programming, stochastic programming, and heuristic and metaheuristic optimization. The nurse scheduling problem is a problem that has been continuously studied and developed using mathematical methods. In 2011, Jenal et al. [9] proposed a cyclical nurse schedule using a 0-1 goal programming technique considering all constraints in the scheduling environment. In 2015, Agyei, Obeng-Denteh, and Andaam [2] also modeled a nurse scheduling using 0-1 goal programming with Lingo software for the Tafo government hospital, Kumasi-Ghana. In 2016, Widyastiti, Aman, and Bakhtiar [22] examined nurse qualification and modeled nursing scheduling using integer linear programming. In the same year, Thongsanit, Kantangkul, and Nithimethirot [19] improved nurses' shift balancing by modeling nurse scheduling for a local hospital in Ratchaburi, Thailand, using Solver in Microsoft Excel. In the same year, Cho et al. [6] proposed multilevel logistic regression models considering that nurses are clustered in hospitals in South Korea to analyze the impact of hospital nursing staff and overtime on patient safety, quality of care, and missed care. In 2017, Shahidin et al. [17] solved the nurse scheduling problem in an emergency ward of a private hospital using integer programming. In 2018, Zanda, Zuddas, and Seatzu [24] proposed a long-term nurse scheduling approach based on integer linear programming for the surgery department of the University hospital in Cagliari, Italy. In 2020, Emmanuel et al. [7] found relationships between long shifts, overtime, and staffing levels through generalized linear mixed models from a cross-sectional survey of 2990 registered nurses in 48 hospitals in England. In 2021, Thongsopa and Janjarassuk [20] applied a mixed-integer linear programming to examine the nurse scheduling problems with the case study problem. In 2022, Rizal and Aziz [16] presented a nurse scheduling model to reduce labor costs with Python. In 2023, Ouda, Sleptchenko, and Simsekler [15] formulated a mixed-integer programming workforce scheduling model as a nurse rostering problem with Python. In 2024, Badriyah [5] developed an optimized nurse scheduling model for the emergency department of a hospital in Malang, Indonesia. The nurse scheduling has received continued attention and has been extensively studied in recent years (see [1,12,21,23] for more details).

From the above previous studies, we are interested in studying linear programming to solve the nurse scheduling problem. Recently, we found that, in a hospital in Ratchaburi, nurses' work schedules have a lot of overtime hours due to insufficient nurses and unequal working hours. In this research, we formulate an integer linear programming model to solve the nurse scheduling problem to achieve equal staffing and minimize overtime hours using data from a hospital in Ratchaburi, Thailand.

In the next section, we discuss the method and process for data collection. In Section 3, we formulate the nurse scheduling model. The computational results are shown in Section 4. We also simulate an increase in the number of nurses in various cases in Section 5. Finally, we discuss and analyze our findings in Section 6.

2. Data Collection and Problem Description

To collect data, we contacted the director of a hospital in Ratchaburi, Thailand, to request information on nurse scheduling. After obtaining permission, we interviewed the nurse supervisor about all relevant information for scheduling nurses in the inpatient department (IPD). We have found that there were 11 nurses in the inpatient department, classified into two types based on their work experience as shown in Table 1.

Type	Work experience	Number of nurses
charge nurse	≥ 3 years	9
staff nurse	< 3 years	2

Table 1. The types of nurses in the inpatient department of a hospital in Ratchaburi, Thailand

According to Table 1, a charge nurse means a nurse who has worked for at least three years, has the duty to coordinate and determine the roles of nursing members. There exist nine charge nurses in the department including one nurse supervisor and two team leaders. A staff nurse means a nurse who has worked for less than three years, has the duty to receive orders from charge nurses, assess the patient's symptoms initially, and take care of the patient. There exist two staff nurses in the department.

In scheduling nursing work, the nurse supervisor has assigned work on a monthly basis. Work shifts are divided into three shifts: morning shift from 8:00 to 16:00, evening shift from 16:00 to 24:00, and night shift from 0:00 to 8:00. The number of each nurse's holidays in each month is equal to the number of Saturdays, Sundays, and public holidays in that month. However, the number of nurses is insufficient, so on some nurses' holidays, overtime shifts must be arranged for nurses. Hence, the number of days off for each nurse may be equal or less than the number of nurse's holidays. There exist guidelines for nurse scheduling in accordance with hospital policies as follows:

(1) On a working day, there must always be a nurse supervisor and two team leaders on duty in the morning.
(2) On a holiday, overtime work in the morning must be accompanied by at least one nurse supervisor or team leader.
(3) The number of days off for each nurse must be equal or less than the number of nurse's holidays.
(4) Each nurse gets at least one day off during a period of seven consecutive days.
(5) Each nurse must work at least one shift per day, otherwise it is a day off.
(6) Each nurse must work no more than one regular shift per day.
(7) Each nurse must work no more than one overtime shift per day.
(8) Nurses cannot work the night shift followed by the morning or evening shift of the following day.
(9) Nurses cannot work night shifts for more than three consecutive nights.
(10) In every shift, each staff nurse must always be accompanied by at least one charge nurse.
(11) The number of regular shifts for every nurse in each month is equal to the number of working days in that month.
(12) The demands for nurses to work the morning, evening and night shifts are five, four, and four nurses, respectively.
(13) During the same time, each nurse must work either on a regular or an overtime shift.

The work schedule of inpatient nurses in July 2024, organized by the nurse supervisor, is shown in Table 2. In addition, the number of working shifts of each nurse in the inpatient department arranged by the nurse supervisor for both regular and overtime shifts is shown in Table 3.

Day	Nurse										
	1	2	3	4	5	6	7	8	9	10	11
1	M	M	M	$\widehat{N}$	N	$E\ \widehat{N}$	–	N	$E\ \widehat{M}$	$E\ \widehat{M}$	$E\ \widehat{M}$
2	M	M	M	$E\ N$	E	–	$N\ \widehat{M}$	$N\ \widehat{E}$	$M\ \widehat{N}$	E	–
3	M	M	M	–	$E\ \widehat{N}$	$M\ \widehat{E}$	$\widehat{N}$	$E\ \widehat{N}$	–	$E\ \widehat{M}$	$N\ \widehat{M}$
4	$M\ \widehat{E}$	M	M	$E\ \widehat{N}$	–	M	E	N	$N\ \widehat{M}$	$\widehat{M}$	E
5	$M\ \widehat{E}$	M	$M\ \widehat{N}$	N	–	$E\ \widehat{M}$	$\widehat{M}$	$N\ \widehat{E}$	$\widehat{E}$	M	$M\ \widehat{N}$
6	$\widehat{M}\ \widehat{E}$	–	–	$E\ \widehat{N}$	$N\ \widehat{M}$	$N\ \widehat{M}$	$E\ \widehat{M}$	E	–	–	–
7	$\widehat{M}$	–	$\widehat{M}\ \widehat{E}$	N	$E\ N$	$N\ \widehat{E}$	$E\ \widehat{M}$	M	–	$\widehat{N}$	M
8	M	M	M	–	E	$E\ \widehat{N}$	$M\ \widehat{N}$	$N\ \widehat{E}$	$\widehat{M}$	E	$N\ \widehat{M}$
9	M	M	M	$M\ \widehat{N}$	$E\ \widehat{N}$	E	–	$E\ \widehat{N}$	$N\ \widehat{M}$	$\widehat{M}$	E
10	M	M	M	$E\ \widehat{N}$	$E\ \widehat{N}$	$\widehat{M}$	E	–	$N\ \widehat{E}$	$N\ \widehat{M}$	$\widehat{M}$
11	$M\ \widehat{E}$	M	M	N	–	–	M	$N\ \widehat{M}$	$E\ \widehat{N}$	$N\ \widehat{E}$	$E\ \widehat{M}$
12	M	M	M	$\widehat{E}$	$\widehat{M}$	$M\ \widehat{N}$	$N\ \widehat{M}$	$\widehat{E}$	N	N	$E\ \widehat{M}$
13	$\widehat{M}\ \widehat{E}$	–	–	E	$\widehat{M}$	N	$\widehat{E}$	$N\ \widehat{M}$	$\widehat{N}$	$N\ \widehat{E}$	$\widehat{M}$
14	$\widehat{M}$	$\widehat{M}$	$\widehat{M}\ \widehat{E}$	–	–	$E\ \widehat{N}$	–	$E\ \widehat{N}$	–	$N\ \widehat{E}$	$N\ \widehat{M}$
15	M	M	M	$N\ \widehat{E}$	$N\ \widehat{M}$	E	–	N	$M\ E$	–	$E\ \widehat{N}$
16	$M\ \widehat{E}$	M	M	N	N	$\widehat{M}$	$E\ \widehat{M}$	$\widehat{E}$	$N\ \widehat{M}$	$M\ \widehat{N}$	E
17	M	M	M	$\widehat{E}$	E	$N\ \widehat{M}$	$M\ \widehat{N}$	$E\ \widehat{M}$	N	$E\ \widehat{N}$	–
18	M	M	M	E	M	N	$E\ \widehat{N}$	–	$N\ \widehat{E}$	N	$E\ \widehat{M}$
19	$M\ \widehat{E}$	M	$M\ \widehat{N}$	–	$N\ \widehat{M}$	$\widehat{E}$	N	–	E	$N\ \widehat{E}$	$\widehat{M}$
20	–	$\widehat{M}\ \widehat{E}$	–	$N\ \widehat{M}$	$\widehat{N}$	$E\ \widehat{M}$	$N\ \widehat{E}$	$\widehat{M}$	–	$E\ \widehat{N}$	–
21	$\widehat{M}\ \widehat{E}$	–	$\widehat{M}\ \widehat{E}$	$E\ \widehat{N}$	$\widehat{N}$	–	N	$E\ \widehat{M}$	–	–	$\widehat{M}$
22	–	–	$\widehat{M}$	$E\ \widehat{N}$	–	$N\ \widehat{E}$	$E\ \widehat{N}$	M	$N\ \widehat{M}$	–	$E\ \widehat{M}$
23	M	M	M	E	N	$E\ \widehat{N}$	–	$E\ \widehat{M}$	$N\ \widehat{E}$	M	$N\ \widehat{M}$
24	M	M	M	$N\ \widehat{E}$	N	–	$E\ \widehat{M}$	M	$E\ \widehat{N}$	$\widehat{M}$	$N\ \widehat{E}$
25	$M\ \widehat{E}$	M	M	–	$N\ \widehat{E}$	N	$M\ \widehat{N}$	–	E	$E\ \widehat{M}$	N
26	M	M	$M\ \widehat{E}$	–	E	$N\ \widehat{E}$	N	$\widehat{M}$	$E\ \widehat{N}$	$\widehat{M}$	N
27	$\widehat{M}\ \widehat{E}$	–	–	$M\ \widehat{N}$	–	$E\ \widehat{N}$	$N\ \widehat{E}$	$\widehat{M}$	–	$N\ \widehat{M}$	E
28	–	–	$\widehat{M}\ \widehat{E}$	N	–	–	$N\ \widehat{E}$	$N\ \widehat{M}$	$E\ \widehat{M}$	E	$N\ \widehat{M}$
29	–	–	$\widehat{M}$	$N\ \widehat{E}$	M	$M\ \widehat{E}$	$\widehat{N}$	$E\ \widehat{N}$	$M\ \widehat{N}$	$E\ \widehat{M}$	–
30	M	M	$M\ \widehat{E}$	$\widehat{E}$	M	$N\ \widehat{M}$	$E\ N$	E	$\widehat{N}$	$N\ \widehat{M}$	$\widehat{M}$
31	$M\ \widehat{E}$	M	M	–	$M\ \widehat{N}$	$\widehat{N}$	–	$\widehat{E}$	$E\ \widehat{N}$	$\widehat{E}$	$M\ \widehat{N}$

M : a morning shift E : an evening shift N : a night shift
$\widehat{M}$: an overtime morning shift $\widehat{E}$: an overtime evening shift $\widehat{N}$: an overtime night shift
– represents a day off

TABLE 2. Inpatient nurses' work schedule as arranged by the nurse supervisor.

Nurse	Regular Shift				Overtime				Day-off
	morning	afternoon	night	total	morning	afternoon	night	total	
1	21	0	0	21	6	11	0	17	4
2	21	0	0	21	2	1	0	3	8
3	21	0	0	21	6	6	2	14	4
4	2	9	9	20	1	6	8	15	7
5	4	8	8	20	3	1	6	10	7
6	3	8	9	20	8	7	7	22	5
7	3	9	9	21	8	4	7	19	6
8	1	9	9	19	11	6	4	21	4
9	3	8	9	20	7	3	8	18	7
10	3	9	9	21	11	3	4	18	4
11	3	10	8	21	15	1	3	19	5

TABLE 3. The numbers of nurses' shifts and days off as arranged by the nurse supervisor.

According to the hospital data, it was found that normally each nurse must work no more than 48 hours or no more than six shifts in a week. However, the number of nurses is not enough to meet the demand for nurses at each time. Therefore, it is necessary to hire additional nurses to work overtime, which requires different working hours for each nurse. Hence, we formulate an integer linear programming model to schedule nurses' work to have equal and minimum overtime hours.

3. Model Formulation

In this section, we start defining the notations for all indices and index sets used in this research. After that, we describe how to formulate the objective function and all required constraints of the nurse work schedule, as detailed below.

3.1. **Notations of indices, sets and decision variables.** The notations for all indices and index sets used in this research are shown as follows:

k represents the number of days in the month for which the work schedule is arranged.

n represents the total number of nurses.

D_M, D_E, D_N represent the demand for nurses for the morning, evening and night shifts.

A is the set of all days in the month, $A = \{1, 2, \ldots, k\}$

A_1 is the set of all working days, (every day from Monday to Friday, excluding public holidays).

A_2 is the set of all holidays (every day from Saturday to Sunday and public holidays). Note that A_1 and A_2 are disjoint sets and $A_1 \cup A_2 = A$.

B is the index set of all nurses such that $B = \{1, 2, 3, ..., n\}$.

B_1 is the index set of all charge nurses.

B_2 is the index set of all staff nurses.

Note that B_1 and B_2 are disjoint sets and $B_1 \cup B_2 = B$.

C is the index set of the nurse supervisor and team leaders such that $C \subset B_1$.

i represents a day index where $i \in A$.

j represents a nurse index where $j \in B$.

The cardinality of a set X is denoted by $|X|$.

For decision variables, they are binary variables that represent the work of each nurse on each day. The decision variables are defined as follows:

$$M_{i,j} = \begin{cases} 1 & \text{if a nurse number } j \text{ is assigned to work in the morning shift on day } i \\ 0 & \text{otherwise} \end{cases}$$

$$E_{i,j} = \begin{cases} 1 & \text{if a nurse number } j \text{ is assigned to work in the evening shift on day } i \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,j} = \begin{cases} 1 & \text{if a nurse number } j \text{ is assigned to work in the night shift on day } i \\ 0 & \text{otherwise} \end{cases}$$

$$H_{i,j} = \begin{cases} 1 & \text{if a nurse number } j \text{ takes day off on day } i \\ 0 & \text{otherwise} \end{cases}$$

$$\widehat{M}_{i,j} = \begin{cases} 1 & \text{if a nurse number } j \text{ is assigned to work overtime in the morning shift on day } i \\ 0 & \text{otherwise} \end{cases}$$

$$\widehat{E}_{i,j} = \begin{cases} 1 & \text{if a nurse number } j \text{ is assigned to work overtime in the evening shift on day } i \\ 0 & \text{otherwise} \end{cases}$$

$$\widehat{N}_{i,j} = \begin{cases} 1 & \text{if a nurse number } j \text{ is assigned to work overtime in the night shift on day } i \\ 0 & \text{otherwise} \end{cases}$$

3.2. **Objective Function.** The objective function is to minimize the number of overtime shifts in nurse scheduling, which is written as

$$\min Z = \sum_{i=1}^{k} \sum_{j=1}^{n} (\widehat{M}_{i,j} + \widehat{E}_{i,j} + \widehat{N}_{i,j})$$

3.3. **Constraints.** The constraints of the nurse scheduling model are as follows:

(1) On a working day, there must always be a nurse supervisor and two team leader on duty in the morning. The constraint can be written as

$$M_{i,j} = 1, \quad \text{for all } i \in A_1 \text{ and for all } j \in C.$$

(2) On a holiday, overtime work in the morning must be accompanied by at least one nurse supervisor or team leader. The constraint can be written as

$$\sum_{j \in C} \widehat{M}_{i,j} \geq 1, \quad \text{for all } i \in A_2.$$

(3) The number of days off for each nurse must be equal or less than the number of nurse's holidays, which can be written as

$$\sum_{i=1}^{k} H_{i,j} \leq |A_2|, \quad \text{for all } j \in B.$$

(4) Each nurse gets at least one day off during a period of seven consecutive days. The constraint can be written as

$$\sum_{l=0}^{6} H_{i+l,j} \geq 1,$$

for all $i \in \{1, 2, 3, ..., k-6\}$ and $j \in B$.

(5) Each nurse must work at least one shift per day, otherwise it is a day off. The constraint can be written as

$$M_{i,j} + E_{i,j} + N_{i,j} + \widehat{M}_{i,j} + \widehat{E}_{i,j} + \widehat{N}_{i,j} + H_{i,j} \geq 1,$$

for all $i \in A$ and $j \in B$.

(6) Each nurse must work no more than one regular shift per day. The constraint can be written as

$$H_{i,j} + M_{i,j} + E_{i,j} + N_{i,j} \leq 1,$$

for all $i \in A$ and $j \in B$.

(7) Each nurse must work no more than one overtime shift per day. The constraint can be written as

$$H_{i,j} + \widehat{M}_{i,j} + \widehat{E}_{i,j} + \widehat{N}_{i,j} \leq 1,$$

for all $i \in A$ and $j \in B$.

(8) Nurses cannot work the night shift followed by the morning or evening shift of the following day. The constraint can be written as

$$N_{i,j} + \widehat{N}_{i,j} + M_{i+1,j} + \widehat{M}_{i+1,j} \leq 1;$$

$$N_{i,j} + \widehat{N}_{i,j} + E_{i+1,j} + \widehat{E}_{i+1,j} \leq 1,$$

for all $i \in \{1, 2, 3, ..., k-1\}$ and $j \in B$.

(9) Nurses cannot work night shifts for more than three consecutive nights. The constraint can be written as

$$\sum_{l=0}^{3}\left(N_{i+l,j}+\widehat{N}_{i+l,j}\right)\leq 3$$

for all $i\in\{1,2,3,...,k-3\}$ and $j\in B$.

(10) In every shift, each staff nurse must always be accompanied by at least one charge nurse. The constraint can be written as

$$\sum_{j\in B_1}\left(M_{i,j}+\widehat{M}_{i,j}\right)\geq\sum_{j\in B_2}\left(M_{i,j}+\widehat{M}_{i,j}\right)\quad\text{for all } i\in A;$$
$$\sum_{j\in B_1}\left(E_{i,j}+\widehat{E}_{i,j}\right)\geq\sum_{j\in B_2}\left(E_{i,j}+\widehat{E}_{i,j}\right)\quad\text{for all } i\in A;$$
$$\sum_{j\in B_1}\left(N_{i,j}+\widehat{N}_{i,j}\right)\geq\sum_{j\in B_2}\left(N_{i,j}+\widehat{N}_{i,j}\right)\quad\text{for all } i\in A.$$

(11) The number of regular shifts for every nurse in each month is equal to the number of working days in that month. The constraint can be written as

$$\sum_{i=1}^{k}(M_{i,j}+E_{i,j}+N_{i,j})=|A_1|\quad\text{for all } j\in B.$$

(12) The demands for nurses to work the morning, evening and night shifts are $D_M=5$, $D_E=4$, and $D_N=4$, respectively. The constraint can be written as

$$\sum_{j=1}^{n}(M_{i,j}+\widehat{M}_{i,j})\geq D_M\quad\text{for all } i\in A;$$
$$\sum_{j=1}^{n}(E_{i,j}+\widehat{E}_{i,j})\geq D_E\quad\text{for all } i\in A;$$
$$\sum_{j=1}^{n}(N_{i,j}+\widehat{N}_{i,j})\geq D_N\quad\text{for all } i\in A.$$

(13) During the same time, each nurse must work either on a regular or an overtime shift. The constraint can be written as

$$M_{i,j}+\widehat{M}_{i,j}\leq 1;$$
$$E_{i,j}+\widehat{E}_{i,j}\leq 1;$$
$$N_{i,j}+\widehat{N}_{i,j}\leq 1,$$

for all $i\in A$ and $j\in B$.

(14) In overtime scheduling, each nurse must have the same number of shifts. The constraint can be written as

$$\sum_{i=1}^{n}\left(\widehat{M}_{i,l}+\widehat{E}_{i,l}+\widehat{N}_{i,l}\right)=\sum_{i=1}^{n}\left(\widehat{M}_{i,m}+\widehat{E}_{i,m}+\widehat{N}_{i,m}\right)$$

for all $l, m \in B$.

(15) The number of days off may be the same or slightly different. The constraint can be written as

$$\sum_{i=1}^{n} H_{i,l} - \sum_{i=1}^{n} H_{i,m} \leq 1;$$
$$\sum_{i=1}^{n} H_{i,l} - \sum_{i=1}^{n} H_{i,m} \geq -1,$$

for all $l, m \in B$.

(16) All decision variables are binary variables, that is

$$M_{i,j}, E_{i,j}, N_{i,j}, H_{i,j}, \widehat{M}_{i,j}, \widehat{E}_{i,j}, \widehat{N}_{i,j} \in \{0, 1\}$$

for all $i \in A$ and $j \in B$.

4. Computational Results

In the case study of the work schedule of inpatient nurses in July 2024 at a hospital in Ratchaburi, we applied the data to parameters as follows: In the inpatient department, there are 11 nurses $(n = 11)$ with the index set $B = \{1, 2, \ldots, 11\}$. All of these 11 nurses are divided into nine charge nurses with the index set $B_1 = \{1, 2, 3, \ldots, 9\}$ and two staff nurses with the index set $B_2 = \{10, 11\}$. The group of nine charge nurses contains a subgroup of one nurse supervisor and two team leaders with the index set $C = \{1, 2, 3\}$ such that $C \subset B_1$. The demands for nurses to work the morning, evening and night shifts are $D_M = 5$, $D_E = 4$, and $D_N = 4$, respectively. The number of days in the month, for which the work schedule is arranged, is equal to 31 $(k = 31)$. There exist 21 working days and ten holidays with the index sets $A_1 = \{1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 23, 24, 25, 26, 30, 31\}$ and $A_2 = \{6, 7, 13, 14, 20, 21, 22, 27, 28, 29\}$, respectively.

We apply the formulated nurse scheduling model to find the optimal results in Python using Pulp library. The work schedule for each nurse in the inpatient department organized by using integer linear programming is shown in Table 4.

Table 4: Nursing shift work schedule in the inpatient department using integer linear programming

Day	Nurse										
	1	2	3	4	5	6	7	8	9	10	11
1	M	$M\ \widehat{E}$	M	E	N	$\widehat{N}$	$\widehat{M}$	E	$N\ \widehat{E}$	$N\ \widehat{E}$	M
2	M	$M\ \widehat{E}$	M	$M\ \widehat{E}$	$\widehat{N}$	$\widehat{N}$	$M\ \widehat{N}$	E	–	N	E

Continued on next page

Table 4 – continued from previous page

Day	Nurse										
	1	2	3	4	5	6	7	8	9	10	11
3	$M\ \widehat{E}$	$M\ \widehat{E}$	M	$M\ \widehat{N}$	–	N	N	E	E	$\widehat{N}$	$M\ \widehat{E}$
4	$M\ \widehat{E}$	$M\ \widehat{E}$	$M\ \widehat{E}$	–	M	–	N	$M\ \widehat{N}$	N	–	$E\ \widehat{N}$
5	$M\ \widehat{N}$	$M\ \widehat{N}$	M	E	E	$M\ \widehat{E}$	–	–	N	$E\ \widehat{M}$	N
6	$\widehat{N}$	–	$\widehat{M}$	$E\ \widehat{M}$	$E\ \widehat{N}$	M	$M\ \widehat{E}$	$E\ \widehat{N}$	$\widehat{N}$	M	–
7	–	$\widehat{M}$	–	$E\ \widehat{M}$	$\widehat{N}$	$E\ \widehat{N}$	$E\ \widehat{M}$	N	–	$M\ \widehat{E}$	$N\ \widehat{M}$
8	M	M	$M\ \widehat{E}$	$E\ \widehat{N}$	$\widehat{N}$	N	M	–	E	$M\ \widehat{E}$	N
9	$M\ \widehat{E}$	M	$M\ \widehat{E}$	$\widehat{N}$	–	N	M	$E\ \widehat{M}$	N	$E\ \widehat{N}$	–
10	M	M	$M\ \widehat{E}$	$\widehat{N}$	–	–	$E\ \widehat{M}$	$E\ \widehat{N}$	N	N	$M\ \widehat{E}$
11	M	$M\ \widehat{E}$	M	–	M	$E\ \widehat{N}$	$N\ \widehat{E}$	$\widehat{N}$	$\widehat{N}$	–	$M\ \widehat{E}$
12	$M\ \widehat{N}$	$M\ \widehat{N}$	$M\ \widehat{E}$	E	M	N	–	–	–	$E\ \widehat{N}$	$E\ \widehat{M}$
13	$\widehat{N}$	–	$\widehat{M}$	$N\ \widehat{M}$	$M\ \widehat{E}$	–	$M\ \widehat{N}$	$N\ \widehat{E}$	$M\ \widehat{E}$	–	E
14	–	$\widehat{M}$	–	N	$M\ \widehat{E}$	E	N	$\widehat{N}$	$M\ \widehat{E}$	$E\ \widehat{M}$	$M\ \widehat{N}$
15	M	$M\ \widehat{E}$	$M\ \widehat{E}$	$\widehat{N}$	$M\ \widehat{N}$	$E\ \widehat{N}$	–	–	$\widehat{M}$	$E\ \widehat{N}$	–
16	M	M	M	–	$\widehat{N}$	$\widehat{N}$	E	$E\ \widehat{M}$	$M\ \widehat{E}$	N	$E\ \widehat{N}$
17	M	M	M	$M\ \widehat{E}$	–	–	$E\ \widehat{N}$	$E\ \widehat{M}$	$E\ \widehat{N}$	$\widehat{N}$	$\widehat{N}$
18	$M\ \widehat{E}$	M	$M\ \widehat{E}$	$M\ \widehat{E}$	–	$M\ \widehat{N}$	N	E	$\widehat{N}$	–	$\widehat{N}$
19	$M\ \widehat{N}$	$M\ \widehat{N}$	$M\ \widehat{E}$	E	$M\ \widehat{E}$	$\widehat{N}$	–	$N\ \widehat{M}$	–	$\widehat{E}$	–
20	–	–	$\widehat{M}$	$M\ \widehat{N}$	$M\ \widehat{E}$	–	$E\ \widehat{N}$	N	$M\ \widehat{E}$	$M\ \widehat{N}$	E
21	$\widehat{N}$	$\widehat{M}$	–	–	$M\ \widehat{E}$	$E\ \widehat{M}$	N	N	$E\ \widehat{M}$	N	$M\ \widehat{E}$
22	–	$\widehat{M}$	–	$M\ \widehat{N}$	$M\ \widehat{E}$	$M\ \widehat{E}$	$\widehat{N}$	–	$M\ \widehat{E}$	$\widehat{N}$	$E\ \widehat{N}$
23	M	M	$M\ \widehat{E}$	$\widehat{N}$	$M\ \widehat{N}$	$M\ \widehat{E}$	–	$\widehat{E}$	$E\ \widehat{N}$	–	N
24	$M\ \widehat{E}$	M	M	–	N	M	$N\ \widehat{E}$	$M\ \widehat{E}$	N	$E\ \widehat{N}$	$\widehat{N}$
25	M	$M\ \widehat{E}$	$M\ \widehat{E}$	$E\ \widehat{M}$	–	$E\ \widehat{N}$	$\widehat{N}$	$M\ \widehat{N}$	N	–	–
26	$M\ \widehat{E}$	$M\ \widehat{N}$	$M\ \widehat{N}$	E	$E\ \widehat{M}$	–	–	N	–	M	$N\ \widehat{E}$

Continued on next page

Table 4 – continued from previous page

Day	Nurse 1	2	3	4	5	6	7	8	9	10	11
27	$\widehat{M}$	–	$\widehat{N}$	$E\ \widehat{N}$	M	$M\ \widehat{E}$	M	$\widehat{N}$	E	$M\ \widehat{E}$	N
28	$\widehat{M}$	–	$\widehat{N}$	N	$M\ \widehat{E}$	M	$M\ \widehat{E}$	–	$M\ \widehat{E}$	$E\ \widehat{N}$	N
29	–	$\widehat{M}$	–	N	$M\ \widehat{N}$	$M\ \widehat{E}$	$M\ \widehat{E}$	$M\ \widehat{E}$	$E\ \widehat{N}$	N	–
30	$M\ \widehat{E}$	M	M	–	N	$E\ \widehat{M}$	$E\ \widehat{N}$	$E\ \widehat{M}$	–	N	$N\ \widehat{M}$
31	$M\ \widehat{E}$	M	M	E	N	M	$\widehat{N}$	E	$E\ \widehat{M}$	N	$\widehat{N}$

M: morning shift E: evening shift N: night shift

$\widehat{M}$: overtime morning shift $\widehat{E}$: overtime evening shift $\widehat{N}$: overtime night shift

– represents a day off

From the results in Table 4, we can summarize the number of working shifts of each nurse in the inpatient department using integer linear programming for both regular and overtime shifts, as shown in Table 5.

Nurse	Regular Shift				Overtime				Day-off
	morning	afternoon	night	total	morning	afternoon	night	total	
1	21	0	0	21	2	8	6	16	5
2	21	0	0	21	5	7	4	16	5
3	21	0	0	21	3	10	3	16	5
4	6	11	4	21	4	3	9	16	6
5	14	3	4	21	1	7	8	16	6
6	10	7	4	21	2	5	9	16	6
7	8	6	7	21	3	5	8	16	6
8	4	11	6	21	5	4	7	16	6
9	6	8	7	21	3	7	6	16	6
10	6	7	8	21	2	5	9	16	6
11	6	7	8	21	3	5	8	16	6

Table 5. The numbers of nurses' shifts and days off in the inpatient department using integer linear programming

According to Tables 4 and 5, all nurses were assigned to work 21 regular shifts, worked 16 overtime shifts, and had five or six days off in July 2024, which did not violate the hospital's practice. When the work schedule organized by the nurse supervisor in Table 3 is compared with Tables 5, it was found that each nurse was assigned to work different regular shifts and each nurse's overtime shifts were various, including the distinct number of days off for each nurse. This caused inequality in work performance and may affect the efficiency of patient care. It has been proven that the nursing shift work schedule using integer linear programming as shown in Table 4 can help the nurse supervisor organize nurses' work schedules efficiently and each nurse has the same number of regular shifts and overtime.

In addition, the nurse supervisor informed us that the department will soon be adding more staff, resulting in two more nurses, so we have simulated to improve the nurses' work schedules. From the simulation result, the number of regular shifts is 21 for each nurse as shown in Table 6, with the number of overtime shifts reduced to ten and the number of days off to five or six.

Nurse	Shift				Overtime				Day-off
	morning	afternoon	night	total	morning	afternoon	night	total	
1	21	0	0	21	1	3	6	10	5
2	21	0	0	21	5	1	4	10	5
3	21	0	0	21	4	5	1	10	5
4	9	9	3	21	1	1	8	10	6
5	6	12	3	21	3	0	7	10	6
6	4	13	4	21	1	2	7	10	6
7	4	9	8	21	2	2	6	10	6
8	3	11	7	21	2	0	8	10	6
9	8	11	5	21	1	2	7	10	6
10	5	12	4	21	3	0	7	10	6
11	9	12	0	21	2	3	5	10	6
12	7	9	5	21	2	1	7	10	6
13	5	8	8	21	5	1	4	10	5

TABLE 6. The numbers of nurses' shifts and days off in the inpatient department when adding two more nurses using integer linear programming

5. Simulation on the number of nurses to reduce overtime hours

From the nurse scheduling in the inpatient department in July 2024, it was found that the number of overtime hours worked by each nurse was in line with the hospital's practice. However, it may cause fatigue from work because of the high number of overtime hours worked by each nurse was quite high. The primary reason is the number of nurses in the department was not enough to work. We therefore simulated an increase in the number of nurses by adding one to seven staff nurses. In total, there were 12 to 18 inpatient nurses where the numbers of overtime shifts and days off are shown in Table 7.

Number of staff nurses added	Total nurses	Regular shift	Overtime shift	Day-off
1	12	21	13	5 or 6
2	13	21	10	5 or 6
3	14	21	8	5 or 6
4	15	21	6	6 or 7
5	16	21	5	6 or 7
6	17	21	4	6 or 7
7	18	21	4	6 or 7

Table 7. The numbers of overtime shifts and days off after adding staff nurses

According to Table 7, adding six more staff nurses will result in four overtime shifts for each nurse. From the simulation results of increasing the number of staff nurses, it can be found that when increasing the number of staff nurses to six or seven, the number of shifts and days off will remain the same. When analyzing the results of each work schedule, it is likely due to the condition of organizing the nursing work system that requires the nurse supervisor and team leaders to work in the morning. Therefore, we simulated the increase in the number of nurses in the case of adding six nurses including one team leader and five staff nurses. In total, there would be 17 nurses in the inpatient department, divided into ten charge nurses and seven staff nurses.

After that, we formulated a scheduling model to rearrange the 17 nurses' work schedules. The results showed that each nurse would work 21 regular shifts, have seven to eight days off, and three overtime shifts, as shown in Table 8. It can be seen that nurses work less overtime. On average, each nurse works six shifts per week, which is in line with current nursing work regulations.

Nurse	Regular Shift				Overtime				Day-off
	morning	afternoon	night	total	morning	afternoon	night	total	
1	18	1	2	21	3	0	0	3	8
2	15	4	2	21	3	0	0	3	7
3	17	1	3	21	2	0	1	3	7
4	18	2	1	21	3	0	0	3	7
5	2	7	12	21	0	0	3	3	7
6	6	12	3	21	0	0	3	3	7
7	3	9	9	21	1	0	2	3	8
8	3	14	4	21	0	0	3	3	8
9	4	11	6	21	0	0	3	3	8
10	7	6	8	21	0	1	2	3	8
11	9	6	6	21	0	0	3	3	7
12	9	6	6	21	1	1	1	3	8
13	6	10	5	21	0	0	3	3	8
14	9	5	7	21	0	1	2	3	7
15	9	9	3	21	0	0	3	3	7
16	6	10	5	21	1	0	2	3	8
17	3	9	9	21	0	0	3	3	8

Table 8. The numbers of nurses' shifts and days off in the inpatient department when adding six more nurses using integer linear programming

Besides, we have also modeled the expansion of the nursing members by adding nine nurses containing two charge nurses, one of whom is a team leader, and seven staff nurses. In total, there were 20 inpatient nurses, consisting of 11 charge nurses and nine staff nurses. When the linear programming model for nurse scheduling is optimized, it is found that if at least nine nurses are added to the inpatient department, all nurses will not have to work overtime, work the same 21 regular shifts, and have the same ten days off.

6. Conclusion and Discussion

In this research, we have organized the work schedule of nurses in the inpatient department of a hospital in Ratchaburi, Thailand. We obtained information on the work schedule in July 2024 from the nurse supervisor to formulate an integer linear programming model and find the optimal result using Python. In order to allocate overtime equally and minimize overtime, the results from the formulated model showed that each nurse's work schedule consisted of 21 regular shifts, 16 overtime shifts, and five or six days off. This is consistent with the nurses' work rules and policies where each nurse will have the same number of shifts, which is more efficient than the work schedule organized by the nurse supervisor. It can be seen that the integer linear programming model can be used in conjunction with the nurses' work schedule.

In addition, we have simulated the increase in the number of nurses in the inpatient department to reduce the number of overtime shifts due to the number of inpatient nurses not being enough to meet the demand for nurses each day. After that, we have formulated a linear programming model and rearranged the nurses' work schedules. The results have shown the minimum number of nurses needed in the hospital so that all nurses will not have to work overtime. These simulations can be valuable tools for hospitals in planning and allocating nursing staff in the future.

From this study, it can be seen that we can apply integer linear programming to organize nurse schedule in regular and overtime shifts under specified working conditions. Furthermore, we believe that integer linear programming can be applied to nurse scheduling in larger hospitals or with different operating conditions. If there are enough nurses without overtime but want to have as many holidays as possible, then we eliminate the nurse overtime variable and change the objective function to have the most holidays instead, with all relevant conditions adjusted. If there are additional policies or practices that differ from these, new conditions must be added or conditions with different practices must be adjusted, depending on the information of each hospital.

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