Asia Pac. J. Math. 2025 **12**:7

REAL HYPERSURFACES IN A NONFLAT COMPLEX SPACE FORM WHOSE SHAPE OPERATOR SATISFIES $\mathfrak{L}_\xi A + \nabla_\xi A = 0$

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Received Oct. 10, 2024

Abstract. Let M be a real hypersurface in a nonflat complex space form $M_n(c)$. In this paper, we prove that if $(\mathfrak{L}_\xi A) + (\nabla_\xi A) = 0$ holds on M, then M is a Hopf hypersurface, where $\mathfrak{L}_\xi$ is the Lie derivative in ξ direction and A is the shape operator of M in $M_n(c)$. We investigate the geometric structure of such Hopf hypersurfaces of $M_n(c)$.

2020 Mathematics Subject Classification. 53C40; 53C15.

Key words and phrases. real hypersurface; Lie derivative; covariant derivative; shape operator; Hopf hypersurface; model spaces.

1. Introduction

A complex n-dimensional Kaehlerian manifold of constant holomorphic sectional curvature c ($c \neq 0$) is called a *nonflat complex space form* and is denoted by $M_n(c)$. A complete and simply connected complex space form is complex analytically isometric to a complex projective space $P_n\mathbf{C}$, or a complex hyperbolic space $H_n\mathbf{C}$, depending on $c > 0$ or $c < 0$.

If a real hypersurface M is in a nonflat complex space form $M_n(c)$, then M has an almost contact metric structure (ϕ, g, ξ, η) induced from the Kaehler metric and a complex structure J on $M_n(c)$. For a shape operator A of M, $\alpha = \eta(A\xi)$ and $A\xi = \alpha\xi$, the Reeb vector field ξ is said to be *principal* and M is called a *Hopf hypersurface*. In this case, it is shown that α is locally constant [3].

Homogeneous Hopf hypersurfaces in $P_n\mathbf{C}$ are given as orbits under a subgroup of the projective unitary groups $PU(n+1)$. All these hypersurfaces are completely classified into six model spaces: A_1, A_2, B, C, D and E by Takagi [11]. Berndt [1] catagorized all homogeneous Hopf hypersurfaces in

DOI: 10.28924/APJM/12-7

$H_n\mathbf{C}$ as four model spaces which are said to be A_0, A_1, A_2 and B. If a real hypersurface M is of A_1 or A_2 in $P_n\mathbf{C}$ or of A_0, A_1 or A_2 in $H_n\mathbf{C}$, then M is said to be *type* A for simplicity.

The following theorem is a typical characterization of real hypersurfaces of type A.

Theorem 1.1. *Let M be a real hypersurface in a nonflat complex space form $M_n(c)$ for $n \geq 2$. It satisfies $A\phi - \phi A = 0$ on M if and only if M is locally congruent to one of the model spaces of type A.*

Theorem 1.1 is due to Okumura [10] for $c > 0$ and Montiel and Romero [6] for $c < 0$.

For the shape operator A on M, the Lie derivative $\mathfrak{L}_\xi A$ is defined by $(\mathfrak{L}_\xi A)X = [\xi, AX] - A[\xi, X]$ and $\nabla_\xi A$ is the covariant derivative with respect to the Reeb vector field ξ and a unit vector field X on M. Regarding the Lie derivative, real hypersurfaces in a nonflat complex space form have been studied by many geometricians and interesting results have been obtained ([2], [5], [7], [8] and [9] etc.). Among them, the following theorem is for the Lie derivative and the covariant derivative of the shape operator by Lim [4].

Theorem 1.2. *Let M be a real hypersurface in a nonflat complex space form $M_n(c)$. Then it satisfies $(\mathfrak{L}_\xi A) = (\nabla_\xi A)$ on M if and only if M is locally congruent to one of the model spaces of type A.*

In this paper, we study a real hypersurface M satisfying $\mathfrak{L}_\xi A + \nabla_\xi A = 0$ of a nonflat complex space form $M_n(c)$, where $\mathfrak{L}_\xi$ is the Lie derivative in ξ direction and A is the shape operator of M in $M_n(c)$. In section 4, we investigate the geometric structure of this hypersurface. In section 5, we prove that such hypersurface is a Hopf hypersurface and locally congruent to one of the model spaces of type A.

From now on, all manifolds are assumed to be connected and of class C^∞. And real hypersurfaces are supposed to be orientable.

2. Preliminaries

Let M be a real hypersurface immersed in a nonflat complex space form $M_n(c)$, and N be a unit normal vector field of M. By $\widetilde{\nabla}$, we denote the Levi-Civita connection with respect to the Fubini-Study metric tensor $\widetilde{g}$ of $M_n(c)$. Then the Gauss formula and the Weingarten formula are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)N, \qquad \widetilde{\nabla}_X N = -AX$$

respectively, where X and Y are any vector fields tangent to M, g denotes the Riemannian metric tensor of M induced from $\widetilde{g}$, and A is the shape operator of M in $M_n(c)$. For any vector field X on M, we put

$$JX = \phi X + \eta(X)N, \qquad JN = -\xi,$$

where J is the almost complex structure of $M_n(c)$. And we see that M induces an almost contact metric structure (ϕ, g, ξ, η), that is,

$$\phi^2 X = -X + \eta(X)\xi, \quad \phi\xi = 0, \quad \eta(\xi) = 1,$$
$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \eta(X) = g(X, \xi) \tag{1}$$

for any vector fields X and Y on M. Since the almost complex structure J is parallel, we can verify the followings from the Gauss and Weingarten formulas:

$$\nabla_X \xi = \phi AX, \tag{2}$$

and

$$(\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi.$$

As the ambient space has holomorphic sectional curvature c, the equations of Gauss and Codazzi are given, respectively, by:

$$\begin{aligned} R(X,Y)Z = \ & \tfrac{c}{4}\{g(Y,Z)X - g(X,Z)Y + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y \\ & -2g(\phi X, Y)\phi Z\} + g(AY, Z)AX - g(AX, Z)AY, \end{aligned} \tag{3}$$

and

$$(\nabla_X A)Y - (\nabla_Y A)X = \frac{c}{4}\{\eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi\}, \tag{4}$$

for any vector fields X, Y and Z on M, where R denotes the Riemannian curvature tensor of M.

Let $\alpha = \eta(A\xi)$ and Ω be an open subset of M defined by

$$\Omega = \{p \in M \mid A\xi - \alpha\xi \neq 0\}. \tag{5}$$

We put

$$A\xi = \alpha\xi + \mu W, \tag{6}$$

where W is a unit vector field orthogonal to ξ and μ does not vanish on Ω.

3. Real hypersurfaces satisfying $\mathfrak{L}_\xi A + \nabla_\xi A = 0$

Let M be a real hypersurface in a nonflat complex space form $M_n(c)$. In this section, we assume that M satisfies $\mathfrak{L}_\xi A + \nabla_\xi A = 0$ and the open subset Ω given in (5) is not empty. Then

$$2(\nabla_\xi A)X = \phi A^2 X - A\phi AX \tag{7}$$

is established from (2). By using the symmetric property of $\nabla_\xi A$, the equation of (7) becomes

$$(\phi A^2 - 2A\phi A + A^2\phi)X = 0 \tag{8}$$

for any vector field X on Ω.

Substituting $X = \xi$ into (7) and using (6), we have

$$2(\nabla_\xi A)\xi = \mu\phi AW - \mu A\phi W + \alpha\mu\phi W.$$

Since $(\nabla_\xi A)\xi = \nabla_\xi(\alpha\xi + \mu W) - A\nabla_\xi W$, we see that the covariant vector field in ξ direction of W is given by

$$2\mu\nabla_\xi W = -2(\xi\alpha)\xi - 2(\xi\mu)W - \alpha\mu\phi W + \mu\phi AW + \mu A\phi W.$$

If we take inner product of this equation with ξ and W respectively, then we obtain

$$\xi\alpha = \xi\mu = 0 \tag{9}$$

on Ω and hence the initial equation is reduced to

$$2\nabla_\xi W = -\alpha\phi W + \phi AW + A\phi W. \tag{10}$$

On the other hand, putting $X = \xi$ into (8) and using (6), we have

$$\phi AW - 2A\phi W + \alpha\phi W = 0. \tag{11}$$

Taking inner product of (11) with W and ϕW, we get

$$g(AW, \phi W) = 0 \quad \text{and} \quad \alpha + \gamma - 2g(A\phi W, \phi W) = 0. \tag{12}$$

Next, we will obtain some of the relationships that are important tools in this paper.

If we put $X = W$ in (8), then we have

$$\phi A^2 W - 2A\phi AW + A^2\phi W = 0.$$

If we apply ϕ to the above equation and using the first equation of (1), then we gain

$$\phi A^2\phi W - 2\phi A\phi AW - A^2 W = -\eta(A^2 W)\xi. \tag{13}$$

Putting $X = \phi W$ into (8), we get

$$\phi A^2\phi W - 2A\phi A\phi W - A^2 W = 0. \tag{14}$$

Comparing (13) with (14), we gain

$$2(\phi A\phi AW - A\phi A\phi W) = \eta(A^2 W)\xi. \tag{15}$$

Differentiating the smooth function $\alpha = g(A\xi, \xi)$ along any vector field X on Ω and using (2), (4), and (6), we have

$$X\alpha = g((\nabla_\xi A)\xi - 2\mu A\phi W, X).$$

From this equation and $(\nabla_\xi A)\xi = \nabla_\xi(\alpha\xi + \mu W) - A\nabla_\xi W$, we see that the gradient vector field $\nabla\alpha$ of α is given by

$$\nabla\alpha = \mu\nabla_\xi W + (\xi\alpha)\xi + (\xi\mu)W + \alpha\mu\phi W - 3\mu A\phi W. \tag{16}$$

With a similar argument as above, we can verify that gradient vector fields of the smooth functions $\mu = g(A\xi, W)$ and $\gamma = g(AW, W)$ are given by

$$\nabla\mu = \mu\nabla_W W + (W\alpha)\xi + (W\mu)W + \{\frac{1}{2}(\alpha-\gamma)(2\gamma+\alpha) + \frac{c}{2}\}\phi W \tag{17}$$

and

$$\nabla\gamma = -(A-\gamma I)\nabla_W W + (W\mu)\xi + (W\gamma)W + \mu(2\gamma+\alpha)\phi W, \tag{18}$$

respectively. If we take inner product of (16) with W and (18) with ξ, then we obtain

$$W\alpha = \xi\mu \qquad \text{and} \qquad \xi\gamma = W\mu. \tag{19}$$

4. Some lemmas

We shall prove some lemmas, which will be used later.

Lemma 4.1. *Let M be a real hypersurface in a nonflat complex space form $M_n(c)$, satisfying $\mathfrak{L}_\xi A + \nabla_\xi A = 0$. If the open subset Ω is not empty, then the following properties hold;*

$$\begin{aligned} AW = \mu\xi + \gamma W, \qquad A\phi W &= \tfrac{\alpha+\gamma}{2}\phi W, \\ \nabla_\xi W &= \tfrac{1}{4}(3\gamma-\alpha)\phi W, \\ 4\mu^2 + (\alpha-\gamma)^2 &= 0. \end{aligned} \tag{20}$$

Proof. Since A is symmetric, we can choose a local orthogonal frame field $\{\xi, W, \phi W, X_4, \cdots, X_{2n-1}\}$ on Ω such that $AX_i = \lambda_i X_i$ for $4 \le i \le 2n-1$. The vector field $\nabla_\xi W$ can be expressed as

$$\nabla_\xi W = \frac{1}{4}(3\gamma-\alpha)\phi W + \sum_{i=4}^{2n-1} f_i X_i. \tag{21}$$

If we substitute (21) into (10), then we have

$$\phi AW + A\phi W = \frac{1}{2}(3\gamma+\alpha)\phi W + 2\sum_{i=4}^{2n-1} f_i X_i. \tag{22}$$

Comparing (22) with (11), we can show that

$$A\phi W = \frac{1}{2}(\gamma+\alpha)\phi W + \frac{2}{3}\sum_{i=4}^{2n-1} f_i X_i. \tag{23}$$

Substituting (23) into (11) and apply ϕ to this equation, we get

$$AW = \mu\xi + \gamma W - \frac{4}{3}\sum_{i=4}^{2n-1} f_i\phi X_i. \tag{24}$$

If we substitute (23) and (24) into (14) and make use of (1) and (6), then we obtain

$$\{-\frac{1}{4}(\alpha-\gamma)^2 - \mu^2\}W + \frac{1}{3}(\gamma - 3\alpha)\sum_{i=4}^{2n-1} f_i\phi X_i + \frac{2}{3}\sum_{i=4}^{2n-1} f_i\lambda_i\phi X_i = 0. \tag{25}$$

Taking inner product of (25) with W, we gain the fourth equation of (20) and hence (25) is reduced to

$$\frac{1}{3}(\gamma - 3\alpha)\sum_{i=4}^{2n-1} f_i\phi X_i + \frac{2}{3}\sum_{i=4}^{2n-1} f_i\lambda_i\phi X_i = 0. \tag{26}$$

With a similar argument as above, if we substitute (23) and (24) into (15), and use (1) and (6), then we obtain

$$\sum_{i=4}^{2n-1} f_i(\lambda_i - \alpha)\phi X_i = 0. \tag{27}$$

Comparing (26) with (27), we can verify that

$$(\gamma - \alpha)\sum_{i=4}^{2n-1} f_i\phi X_i = 0.$$

Now, suppose there is a point p such that $\alpha(p) = \gamma(p)$ on Ω. According to the fourth equation of (20), the scalar function μ is zero. so it is a contradiction. Thus, we can easily show that $\sum_{i=4}^{2n-1} f_i\phi X_i = 0$, and hence we have $f_i = 0$ for $4 \leq i \leq 2n-1$. Substituting this equation into (21), (23) and (24), we get the third, second and first equations of (20). $\square$

Lemma 4.2. *Under the assumptions of Lemma 4.1,* $\xi\gamma = 0$ *and* $W\alpha = W\mu = W\gamma = 0$ *on* Ω.

Proof. Differentiating the fourth equation in (20), taking the inner product of ξ and using (9), we obtain

$$(\alpha - \gamma)\xi\gamma = 0. \tag{28}$$

Suppose that there is a point p of Ω such that $(\xi\gamma)(p) \neq 0$, the equation (28) means

$$\alpha - \gamma = 0.$$

Note that $\alpha - \gamma$ is smooth. Differentiating the equation $\alpha - \gamma = 0$ and taking inner product of ξ, we get $\xi\alpha = \xi\gamma$. Since $\xi\alpha = 0$, $\xi\gamma$ becomes zero, which contradicts the assumption. Thus, we have $\xi\gamma = 0$.

Since $\xi\mu = \xi\gamma = 0$, it is easily seen from (19) that $W\alpha = W\mu = 0$.

By the same reasoning as above, if we differentiate the fourth equation of (20) and use the inner product of W, then we obtain

$$4\mu W\mu + (\alpha - \gamma)(W\alpha - W\gamma) = 0.$$

From the fact that $W\alpha = W\mu = 0$, the above equation is reduced to

$$(\alpha - \gamma)W\gamma = 0. \tag{29}$$

Again, assuming that there is a point p of Ω such that $(W\gamma)(p) \neq 0$, we gain $\alpha - \gamma = 0$ from (29), so $\alpha - \gamma$ is a scalar function. Differentiating $\alpha - \gamma = 0$ with respect to W and making use of $W\alpha = 0$, we get $W\gamma = 0$. This is a contradiction. Therefore, $W\gamma = 0$ and this completes the proof. □

Lemma 4.3. *Under the assumptions of Lemma 4.1, the following equations hold on Ω.*

$$\begin{gathered}
\mu\nabla_W W = \{\tfrac{1}{2}\mu^2 - \tfrac{3}{4}\gamma(\alpha-\gamma) - \tfrac{c}{4}\}\phi W, \qquad \nabla\alpha = -\tfrac{3}{4}\mu(\alpha+\gamma)\phi W, \\
\nabla\mu = \{\tfrac{1}{2}\mu^2 + \tfrac{1}{4}(\alpha-\gamma)(2\alpha+\gamma) + \tfrac{c}{4}\}\phi W, \\
\mu\nabla\gamma = \{\tfrac{3}{4}\mu^2(\alpha+3\gamma) + \tfrac{1}{8}(\alpha-\gamma)[3(\alpha-\gamma)+c]\}\phi W.
\end{gathered} \tag{30}$$

Proof. If we put $X = W$ into (7) and make use of the first and second equations of (20), Then we obtain

$$2(\nabla_\xi A)W = \{\mu^2 + \frac{1}{2}\gamma(\alpha-\gamma)\}\phi W. \tag{31}$$

If we differentiate the smooth function $\mu = g(AW, \xi)$ along any vector field X on Ω, and use (2),(4) and (20), we have

$$X\mu = g((\nabla_w A)\xi + \frac{1}{2}(\alpha^2 - \gamma^2) + \frac{c}{2}\phi W, X). \tag{32}$$

Since we have $(\nabla_\xi A)W = \nabla_\xi(\mu\xi + \gamma W) - A\nabla_\xi W$, the equation of (31) and (32) is rewritten as

$$(A - \gamma I)\nabla_\xi W = \frac{1}{2}\{\mu^2 + \frac{1}{2}\gamma(\alpha-\gamma)\}\phi W \tag{33}$$

and

$$\nabla\mu = -(A-\gamma I)\nabla_\xi W + (\xi\mu)\xi + (\xi\gamma)W + \{\mu^2 + \frac{1}{2}(\alpha^2-\gamma^2) + \frac{c}{4}\}\phi W. \tag{34}$$

By substituting (33) into (34), we obtain the third equation of (30). If we compare (17) with the third equation of (30) and use Lemma 4.2, then we get the first equation of (30). If we apply the third equation of (20) into (16) and use (9), then we have the second equation of (30). Comparing (18) with the first equation of (30) and using Lemma 4.2, we have the fourth equation of (30). □

5. Characterizations of Real Hypersurfaces

Theorem 5.1. *Let M be a real hypersurface satisfying $\mathfrak{L}_\xi A + \nabla_\xi A = 0$ in a nonflat complex space form $M_n(c)$. Then M is a Hopf hypersurface in $M_n(c)$.*

Proof. Assume that the open set $\Omega = \{p \in M \mid A\xi - \alpha\xi \neq 0\}$ is not empty. By Lemma 4.1, $4\mu^2 + (\alpha - \gamma)^2 = 0$ holds on Ω. After differentiating this equation and multiplying by μ, we have

$$4\mu^2 \nabla\mu + (\mu\nabla\alpha - \mu\nabla\gamma) = 0. \tag{35}$$

Substituting the second, third and fourth equation of (30) into (35), we can find

$$4\mu^2 + (\alpha - \gamma)^2 + \frac{3c}{4} = 0. \tag{36}$$

Comparing the fourth equation (20) with (36), we reach the conclusion that $c = 0$, which is a contradiction.

Thus the set Ω is empty, so M is a Hopf hypersurface. □

Theorem 5.2. *Let M be a real hypersurface in a nonflat complex space form $M_n(c)$. Then it is satisfies $\mathfrak{L}_\xi A + \nabla_\xi A = 0$ on M if and only if M is locally congruent to one of the model spaces of type A.*

Proof. By Theorem 5.1, M is a Hopf hypersurface in $M_n(c)$, that is, $A\xi = \alpha\xi$. Therefore the assumption $\mathfrak{L}_\xi A + \nabla_\xi A = 0$ is given by

$$(\phi A^2 - 2A\phi A + A^2\phi)X = 0. \tag{37}$$

On the other hand, if we differentiate $A\xi = \alpha\xi$ covariantly and make use of the equation (4) of Codazzi, then we have

$$A\phi A - \frac{\alpha}{2}(\phi A + A\phi) - \frac{c}{4}\phi = 0. \tag{38}$$

For any vector field X on M such that $AX = \lambda X$, it follows from (38) that

$$(\lambda - \frac{\alpha}{2})A\phi X = \frac{1}{2}(\alpha\lambda + \frac{c}{2})\phi X. \tag{39}$$

We can choose an orthonormal frame field $\{\xi, X_1, X_2, \cdots, X_{2n-1}\}$ on M such that $AX_i = \lambda_i X_i$ for $1 \leq i \leq 2(n-1)$.

If $\lambda_i \neq \frac{\alpha}{2}$ for $1 \leq i \leq p \leq 2(n-1)$, then we see from (39) that ϕX_i is also a principal direction, say $A\phi X_i = \mu_i \phi X_i$. From (37), we have $\mu_i = \lambda_i$, so $A\phi X_i = \phi A X_i$ for $1 \leq i \leq p$.

If $\lambda_i \neq \frac{\alpha}{2}$ and $\lambda_j = \frac{\alpha}{2}$ for $1 \leq i \leq p$ and $p+1 \leq j \leq 2(n-1)$ respectively, then it follows from (37) that

$$A^2\phi X_j - \alpha A\phi X_j + \frac{\alpha^2}{4}\phi X_j = 0.$$

For $p+1 \leq j \leq 2(n-1)$, taking inner product of (39) with X_i, we obtain $g(\phi X_j, X_i) = 0$ for $1 \leq i \leq p$. Thus the vector field ϕX_j is expressed by a linear combination of $X_j{}'s$ only, which implies $A\phi X_j = \frac{\alpha}{2}\phi X_j = \phi A X_j$.

If $\lambda_i = \frac{\alpha}{2}$ for $1 \leq i \leq 2(n-1)$, then it is easily seen that $\phi A X_i = A\phi X_i$ for all i.

Therefore we have $\phi A - A\phi = 0$ on M and the proof is completed from Theorem 1.1. □

Acknowledgements. This paper was supported by the Sehan University Research Fund in 2025.

Authors' Contributions. All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

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