Asia Pac. J. Math. 2025 **12**:77

NEUTROSOPHIC INVERSE GAMMA DISTRIBUTION: PROPERTIES AND APPLICATION

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Received Jul. 5, 2025

Abstract. In the current study, we presented new continuous statistical distribution, a Neutrosophic Inverse Gamma Distribution. This distribution represents a combination of a classical inverse gamma distribution and the neutrosophic analysis, with the aim of studying the effect of the uncertainty part in the data in statistical analyses. The first part of the study included the most important theoretical properties of the new distribution, namely the first and second moments and variance, as well as the formula for the estimators of the Neutrosophic Inverse Gamma Distribution parameters using the maximum likelihood method. On the practical side, we used the R program version (2.3.4) to generate data distributed according to the inverse gamma distribution with size ($n = 100$) based on the simulation method. The parameters were estimated in two cases, the first is for the classic inverse gamma distribution (without adding the uncertainty part), The second case is estimating the parameters of the neutrosophic distribution (by adding the uncertainty part). It turns out that the inverse neutrosophic gamma distribution is more flexible and realistic than the classical model for data that are characterized by ambiguity or difficulty in measurement.
2020 Mathematics Subject Classification. 35Q35.
Key words and phrases. neutrosophic; inverse gamma distribution; a simulation method.

1. Introduction

Real-world data is fraught with ambiguity, unknown conditions, and challenges, and it is not possible to estimate a specific value for statistical features in this context, which lacks precision. In these cases, traditional probabilities fail to provide accurate results. Recent progress has been made in modeling these imprecise situations, using fuzzy logic and neutrosophic [1–6].

The inverse gamma distribution is a two-parameter family of continuous probability distributions defined on the positive real numbers [7].

In 1995, Florentine Smarandache defined the concept of neutrosophic logic using three symbols: T, I, and F. T represents a true value, I represents the indeterminate or uncertain value, and F represents

DOI: 10.28924/APJM/12-77

a false value.A field of philosophy that explores the origin, nature, and scope of neutrality. Huang emphasized that this concept alters the mechanistic understanding of human culture, implying that something cannot exist or not exist simultaneously due to certain uncertainties [8–10].

In 1998, the field of neutrosophic Statistics was established by by Florentine Smarandache, who further sophisticated and validated its theories in 2014. Neutrosophic statistics serves as a generalization of interval statistics, as it is based on the examination of intervals. Neutrosophic statistics pertains to a dataset in which some or all elements possess a degree of uncertainty. Ambiguous, together with the techniques employed to evaluate this data. Classical statistics is distinguished from neutrosophic statistics by its reliance on deterministic data, whereas neutrosophic statistics incorporates both deterministic and uncertain data. When the value of the uncertain data is zero, neutrosophic statistics corresponds precisely to classical statistics. Consequently, neutrosophic statistics is regarded as more adaptable than classical statistics. Given the prevalence of uncertain data over deterministic data in our environment, there is a greater necessity for neutrosophic statistical techniques compared to classical methods. The unspecified parameters of the classical inverse gamma distribution in neutrosophic logic allow for the management of all potential circumstances encountered during the analysis of real data [11–17].

The gamma distribution is a probability model used to represent data whose values are always positive, while the inverse gamma distribution remains under research and is limited in use in practical applications [18–21].

2. Inverse Gamma Distribution

The inverse_gamma distribution is a significant continuous statistical distribution in statistical models. The probability density function for this distribution is articulated as follows:

$$g(x,\gamma,\ \vartheta)=\frac{\vartheta^{\gamma}}{\Gamma(\gamma)}\,x^{-(\gamma+1)}exp(\,-\,{}^{\vartheta}\!/_{x}) \tag{1}$$

Such that:

γ: The shape parameter.

ϑ: The scale parameter.

$$\Gamma(\gamma)=\int_{0}^{\infty}x^{\gamma-1}\exp\left(-x\right)\,dx=(\gamma\,\breve{}\,1)!,\gamma\in z^{+}.$$

The inverse_gamma distribution is an asymmetric distribution, which makes it suitable for modeling data with high variance or data containing outliers. It is widely used in Bayesian statistics as a prior distribution for the coefficients of regression models. The most important properties of the inverse_gamma distribution are the mathematical expectation, variance, and moment function, as we notice in Eqs. (2), (3), and (4).

$$E(X) = \frac{\vartheta}{\gamma - 1}, \gamma > 1. \tag{2}$$

$$Var(X) = \frac{\vartheta^2}{(\gamma - 2)^2 (\gamma - 1)}, \gamma > 2. \tag{3}$$

$$\begin{aligned}
E(X^n) &= \int_{-\infty}^{\infty} x^n \, g(x) \, dx \\
&= \int_0^{\infty} x^n \frac{\vartheta^\gamma}{\Gamma(\gamma)} \, x^{-(\gamma+1)} \exp(-\vartheta/x) dx \\
&= \frac{\vartheta^\gamma}{\Gamma(\gamma)} \int_0^{\infty} x^{n-\gamma-1} \exp(-\vartheta/x) dx \\
&= \frac{\vartheta^\gamma}{\Gamma(\gamma)} \int_0^{\infty} x^{-(\gamma-n+1)} \; \exp(-\vartheta/x) dx \\
&= \frac{\vartheta^\gamma}{\Gamma(\gamma)} \Gamma(\gamma - n) \\
&= \frac{\vartheta^\gamma}{(\gamma - 1)!} (\gamma - n + 1)!
\end{aligned} \tag{4}$$

3. Mathematical Foundations for Neutrosophic Modeling

This section presents the fundamental notion of the neutrosophic set, which pertains to statistical models that extend fuzzy set data models. Classical statistics typically involves distinct or precise values during data analysis. In neutrosophic theory, data can assume any form, as uncertainty might emerge in diverse forms contingent upon the nature of the problem being addressed. The elements of the Neutrosophic group are known by three independent compounds $x(T, I, F)$ where:

T: is the belonging (truth).

I: is the indeterminacy (vagueness).

F: is the non-belonging (falsehood).

The components (T, I, F) represent numbers or intervals. In the Neutrosophic model, these components are classified into three states as shown below:

$$S = \begin{cases} T + \ I + F = 1, & \textit{Steady situation} \\ T + \ I + F \ < 1, & \textit{Incomplete situation} \\ T + \ I + F \ > 1, & \textit{Paradoxical situation} \end{cases}$$

Because of the importance of neutrosophic analysis in determining the effect of the ambiguity part of any data in statistical analyses and making appropriate decisions, the neutrosophic formula was defined on many known statistical distributions as in the following examples:

3.1. A Neutrosophic Normal Distribution.

The density function of Neutrosophic (normal) distribution probability, is as follows [11]:

$$f_\eta\left(t\right) = \frac{1}{\sigma_\eta\sqrt{2\pi}} e^{-\frac{(t-\mu_\eta)^2}{2\sigma_\eta^2}} ; t,\ \mu_\eta, \sigma_\eta > 0. \tag{5}$$

3.2. A Neutrosophic Exponential Distribution.

The density function of Neutrosophic (Exponential) distribution probability, is as follows [22]:

$$f_\eta\left(t\right) = \lambda_\eta e^{-t\lambda_\eta}; t, \lambda_\eta > 0 \tag{6}$$

3.3. Neutrosophic Gamma Distribution.

The density function of Neutrosophic (Gamma) distribution probability, is as follows [23]:

$$f_\eta\left(t\right) = \frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta)} t^{u_\eta - 1} e^{-tv_\eta}; t, u_\eta, v_\eta > 0 \tag{7}$$

3.4. Neutrosophic Beta Distribution.

The density function of Neutrosophic (Beta) distribution probability, is as follows [24]:

$$f_\eta\left(t\right) = \frac{t^{u_\eta-1}(1-t)^{v_\eta-1}}{\beta(u_\eta, v_\eta)}; t > 0 \tag{8}$$

where u_η, v_η are the neutrosophic shape parameters.

3.5. Neutrosophic Weibull Distribution.

The density function of Neutrosophic (Weibull) distribution probability, is as follows [25]:

$$f_\eta\left(t\right) = \frac{v_\eta}{u_\eta} t^{v_\eta-1} e^{-\left(\frac{t}{u_\eta}\right)^{v_\eta}};\ t,\ u_\eta,\ v_\eta > 0 \tag{9}$$

In our current study, the Neutrosophic formula is defined as in the equation below:

$$t = u + \varepsilon \tag{10}$$

Where is the specific value and ε is the Unspecified value which is a real number.

$$\alpha_\eta \in [\alpha_L,\ \alpha_U].$$

4. Neutrosophic inverse_gamma Distribution(NIGD)

The neutrosophic inverse_gamma distribution represents an extension of the classical model by incorporating elements of uncertainty and ambiguity into the distribution parameters and data. In this framework, shape and scale parameters, as well as observed data, are expressed as neutrosophic numbers to facilitate modeling of incomplete or inconsistent data. This model is suitable for real-world applications such as survival analysis, risk modeling, and Bayesian inference, where data are often imprecise or partial.

The density function of random variable t for the neutrosophic inverse_gamma distribution with two parameters $u_\eta, v_\eta > 0$ as follows:

$$f_\eta(t) = \frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta) t^{1+u_\eta} e^{\frac{v_\eta}{t}}}, t > 0 \tag{11}$$

where $\Gamma(\cdot)$ is the neutrosophic gamma function. In the following theorems, we present the most important properties of the neutrosophic inverse_gamma distribution.

Theorem 4.1. *The first moment of NIGD is*

$$\frac{v_\eta}{u_\eta - 1}$$

Proof.

$$E_\eta(t) = \int_0^\infty t f_\eta(t)\, dt = \int_0^\infty t \frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta)\ t^{1+u_\eta}} e^{-\ \frac{v_\eta}{t}} dt \tag{12}$$

$$= \int_0^\infty \frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta)\ t^{u_\eta}} e^{-\ \frac{v_\eta}{t}} dt$$

$$= \int_0^\infty \frac{1}{\Gamma(u_\eta)} \left(\frac{v_\eta}{t}\right)^{u_\eta} e^{-\ \frac{v_\eta}{t}} dt \tag{13}$$

put $x = \frac{v_\eta}{t}$ in result (3), we have

$$\therefore E_\eta(t) = \frac{v_\eta}{u_\eta - 1}$$

□

Theorem 4.2. *The second moment of NIGD is*

$$\frac{v_\eta^2}{(u_\eta - 1)(u_\eta - 2)}.$$

Proof.

$$E_\eta\left(t^2\right) = \int_0^\infty t^2 f_\eta(t)\, dt = \int_0^\infty t^2 \frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta) t^{1+u_\eta}} e^{-\frac{v_\eta}{t}} dt \tag{14}$$

$$= \int_0^\infty \frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta) t^{-1+u_\eta}} e^{-\frac{v_\eta}{t}} dt$$

$$= v_\eta \int_0^\infty \frac{1}{\Gamma(u_\eta)} \left(\frac{v_\eta}{t}\right)^{u_\eta - 1} e^{-\frac{v_\eta}{t}} dt \tag{15}$$

put $x = \frac{v_\eta}{t}$ in result (5), we have

$$\therefore E_\eta\left(t^2\right) = \frac{v_\eta^2}{(u_\eta - 1)(u_\eta - 2)}.$$

□

Theorem 4.3. *The variance of NIGD is*

$$\frac{v_\eta^2}{(u_\eta - 1)^2 (u_\eta - 2)}$$

Proof.

$$\begin{aligned} Var_\eta(t) &= E_\eta\left(t^2\right) - E_\eta\left(t\right)^2 \\ &= \frac{v_\eta^2}{(u_\eta - 1)(u_\eta - 2)} - \frac{{v_\eta}^2}{(u_\eta - 1)^2} \\ &= \frac{{v_\eta}^2}{u_\eta - 1}\left[\frac{u_\eta - 1 - u_\eta + 2}{(u_\eta - 1)(u_\eta - 2)}\right] \end{aligned} \tag{16}$$

$$\therefore Var_\eta(t) = \frac{v_\eta^2}{(u_\eta - 1)^2 (u_\eta - 2)}$$

R-th order statistics

$$f_{i,j}(t) = C_{i,j}\left[F(t)\right]^{i-1}\left[1 - F(t)\right]^{j-i} f_\eta(t)$$
$$f_{i,j}(t) = C_{i,j}\left[F(t;u,v)\right]^{i-1}\left[1 - F(t;u,v)\right]^{j-i} f_\eta(t;u,v)$$

$$\therefore f_{i,j}(t) = C_{i,j}\left[\frac{\Gamma(u_\eta, \frac{v_\eta}{t})}{\Gamma(u_\eta)}\right]^{i-1}\left[1 - \frac{\Gamma(u_\eta, \frac{v_\eta}{t})}{\Gamma(u_\eta)}\right]^{j-i}\frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta)\; t^{1+u_\eta}} e^{-\frac{v_\eta}{t}} \tag{17}$$

Smallest order statistics

$$f_{1,j}(t) = j_\eta\left[1 - \frac{\Gamma(u_\eta, \frac{v_\eta}{t})}{\Gamma(u_\eta)}\right]^{i-1}\frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta)\, t^{1+u_\eta}} e^{-\frac{v_\eta}{t}} \tag{18}$$

Largest order statistics

$$f_{j,j}(t) = j_\eta\left[\frac{\Gamma(u_\eta, \frac{v_\eta}{t})}{\Gamma(u_\eta)}\right]^{i-1}\frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta)\, t^{1+u_\eta}} e^{-\frac{v_\eta}{t}} \tag{19}$$

Reliability function

$$\Upsilon_\eta(t) = 1 - f_\eta(t) = 1 - \frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta)\; t^{1+u_\eta}} e^{-\frac{v_\eta}{t}}$$

$$\therefore \Upsilon_\eta(t) = \frac{\Gamma(u_\eta)\; t^{1+u_\eta} - {v_\eta}^{u_\eta} e^{-\frac{v_\eta}{t}}}{\Gamma(u_\eta)\; t^{1+u_\eta}} \tag{20}$$

Hazard function

$$\zeta_\eta(t) = \frac{f_\eta(t)}{\Upsilon_\eta(t)} = \frac{\frac{{v_\eta}^{u_\eta}}{\Gamma(u_\eta) t^{1+u_\eta}} e^{-\frac{v_\eta}{t}}}{\frac{\Gamma(u_\eta) t^{1+u_\eta} - {v_\eta}^{u_\eta} e^{-\frac{v_\eta}{t}}}{\Gamma(u_\eta)\; t^{1+u_\eta}}}$$

$$\therefore \zeta_\eta(t) = \frac{{v_\eta}^{u_\eta} t^{-1-u_\eta} e^{-\frac{v_\eta}{t}}}{\Gamma(u_\eta)\; - {v_\eta}^{u_\eta} t^{-1-u_\eta} e^{-\frac{v_\eta}{t}}} \tag{21}$$

□

5. Estimation Parameters of Neutrosophic inverse_gamma Distribution

The maximum likelihood method (ML) is considered one of the most important statistical estimation methods because it gives consistent and low-biased estimates. We adopted this method to find the estimates of the parameters for the Neutrosophic inverse_gamma Distribution.

Let $t_{\eta 1}, t_{\eta 2}, \ldots, t_{\eta m}$ Be a random sample from Neutrosophic inverse_gamma Distribution, then the logarithm of the likelihood function in result (1), becomes

$$L(u,v) = -mLn\Gamma(u_\eta) + mu_\eta Ln(v_\eta) - (u_\eta + 1) Ln\left(\prod_{i=1}^{m} t_i\right) - v_\eta \sum_{i=1}^{m} t_i^{-1} \tag{22}$$

$$\frac{\partial L}{\partial v_\eta} = \frac{mu_\eta}{v_\eta} - \sum_{i=1}^{m} t_i^{-1}, \frac{\partial L}{\partial v_\eta} = 0$$

$$\therefore v_\eta = \frac{mu_\eta}{\sum_{i=1}^{m} t_i^{-1}} \tag{23}$$

$$\frac{\partial L}{\partial u_\eta} = \frac{-m\Gamma'(u_\eta)}{\Gamma(u_\eta)} + mLn(v_\eta) - Ln\left(\prod_{i=1}^{m} t_i\right), \frac{\partial L}{\partial u_\eta} = 0$$

$$\frac{\Gamma'(u_\eta)}{\Gamma(u_\eta)} - Ln(v_\eta) + \frac{1}{n} Ln\left(\prod_{i=1}^{m} t_i\right) = 0$$

Put $\psi(u_\eta) = \frac{\Gamma'(u_\eta)}{\Gamma(u_\eta)}$, we have

$$\psi(u_\eta) - Ln(v_\eta) + \frac{1}{n} Ln\left(\prod_{i=1}^{m} t_i\right) = 0$$

Let

$$Z(u_\eta) = \psi(u_\eta) - Ln(v_\eta) + \frac{1}{n} Ln\left(\prod_{i=1}^{m} t_i\right), \tag{24}$$

substitute result (18) in result (19), we have that

$$Z(u_\eta) = \psi(u_\eta) - Ln\left(\frac{mu_\eta}{\sum_{i=1}^{m} t_i^{-1}}\right) + \frac{1}{n} Ln\left(\prod_{i=1}^{m} t_i\right)$$

$$Z(u_\eta) = \psi(u_\eta) - Ln(m) - Ln(u_\eta) - Ln\left(\frac{1}{\sum_{i=1}^{m} t_i^{-1}}\right) + \frac{1}{n} Ln\left(\prod_{i=1}^{m} t_i\right)$$

By using Newton – Raphson method to find u_η, repeating until convergence

$$u_{\eta_{m+1}} = u_{\eta_m} - \frac{Z(u_\eta)}{Z\prime(u_\eta)}$$

Using method of moment estimate for u_η as initial guess, we have that

$$u_\eta = \psi^{-1}\left(logu_{\eta 0} - \sum_{i=1}^{m} t_i^{-1} - \overline{\log t}\right) \tag{25}$$

where ψ is the digamma function. Substituted the value of u_η in result (18) to get v_η.

6. The Application

In this aspect, we generated data distributed in the inverse_gamma distribution based on the simulation method. 100 values were generated when $\gamma = 3$ and $\vartheta = 10$, as shown in Table 1 in an appendix by using the R programming language, version (2.3.4).

Table 1. Classical data

u	u	u	u	u	u	u	u	u	u
2.951	3.9063	3.5249	10.6111	3.3301	4.1107	3.3629	18.371	3.1508	5.909
3.3346	2.1251	5.5969	7.0643	3.0685	3.3835	10.6088	10.0328	3.734	2.1115
3.7455	5.9002	3.4582	3.9855	3.8692	8.9565	4.5828	9.7629	5.888	18.4424
6.291	4.1164	6.2276	3.1783	2.3976	15.3642	4.4493	4.9516	2.0136	3.6919
2.3518	3.062	2.0158	5.1327	1.4723	9.138	2.0944	4.8657	2.5153	1.6815
2.4799	2.9391	5.4922	6.9804	5.6057	3.505	9.6195	4.3486	4.1447	3.047
3.1191	4.6529	1.8147	3.2764	7.8212	3.1885	3.1457	2.5203	5.53	11.1129
3.2611	5.9771	4.6702	2.2051	6.6472	5.6534	7.7228	2.6973	4.6076	19.4246
2.8313	3.0059	3.5729	3.0908	2.7972	1.4175	4.2196	2.8964	8.7649	2.079
1.5787	4.7676	1.6384	4.9721	4.0157	11.9714	5.3665	4.1613	4.2252	3.2245

To verify the data type, a Q-Q plot was used, as shown in Figure 1 below. It was found that the data approximated a straight line (which indicates the data conformity to the theoretical distribution). There was no deviation from the line, indicating that the data conformed to the distribution.

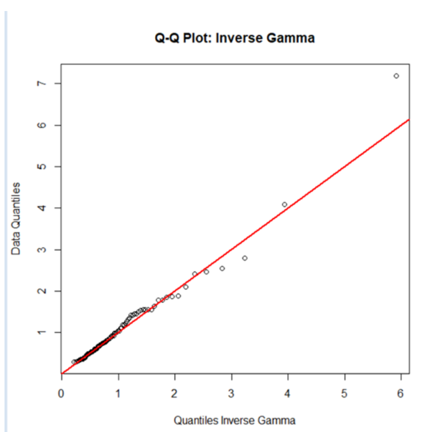

Figure 1. Q-Q plot of the data

The Kolmogorov-Smirnov test was also applied. The statistic value = 0.11972 showed that the p-value = 0.1138, which is greater than 0.05. It can be said that the data is distributed according to the inverse_gamma distribution.

After adding the uncertainty component to the generated data in Table 2 in an appendix, we re-estimated it, as the data became complete, as shown in Table 3:

Table 2. Neutrosophic data

$u+\varepsilon$	$u+\varepsilon$	$u+\varepsilon$	$u+\varepsilon$	$u+\varepsilon$	$u+\varepsilon$	$u+\varepsilon$	$u+\varepsilon$	$u+\varepsilon$	$u+\varepsilon$
3.0996	4.1031	3.7024	10.6111	3.4978	4.3178	3.3629	19.2964	3.3095	6.2066
3.5026	2.2321	5.8788	7.0643	3.2231	3.5539	10.6088	10.5382	3.9221	2.2179
3.9342	6.1974	3.6324	3.9855	4.0641	9.4076	4.5828	10.2547	6.1846	19.3713
6.2910	4.3237	6.5413	3.1783	2.5184	16.1381	4.4493	5.2010	2.1150	3.8779
2.3518	3.2162	2.1173	5.1327	1.5465	9.5983	2.0944	4.8657	2.6420	1.7662
2.4799	3.0871	5.7688	6.9804	5.8881	3.6815	9.6195	4.3486	4.3535	3.2005
3.1191	4.8873	1.9061	3.2764	8.2152	3.3491	3.1457	2.5203	5.8085	11.6727
3.4254	6.2782	4.9054	2.2051	6.6472	5.9382	8.1118	2.6973	4.8397	20.4030
2.9739	3.1573	3.7529	3.0908	2.7972	1.4889	4.4321	2.8964	9.2064	2.1837
1.6582	5.0077	1.7209	4.9721	4.0157	12.5744	5.6368	4.1613	4.4380	3.3869

Table 3. Parameter estimation by using MLE

Case	Distribution	Parameters		AIC
1	Inverse Gamma	$\hat{\gamma}$	462.5238	462.5238
		$\hat{\vartheta}$	13.80175	
2	Neutrosophic inverse_gamma	$\widehat{u_\eta}$	485.42	485.42
		$\widehat{v_\eta}$	15.47848	

In the first case, we estimated the parameters ($\hat{\gamma}$, $\hat{\vartheta}$) Without the uncertainty part (classical data). In the second case, we estimated the parameters ($\widehat{u_\eta}, \widehat{v_\eta}$) With the uncertainty part, i.e., the presence of the neutrosophic condition in the data. We found differences in the estimated values in both cases. The estimated parameters ($\widehat{u_\eta}, \widehat{v_\eta}$) for the neutrosophic data were larger than the parameters estimated in the data after deducting the uncertainty part. Hence, we conclude, according to neutrosophic logic, that neutrosophic statistics yield more accurate results than classical statistics, rather than ignoring the uncertainty. Therefore, it can be said that the inverse_gamma distribution for neutrosophic data is more accurate than the classical inverse_gamma distribution when the data contains uncertainty.

7. Conclusions

In reality, data encompasses ambiguity and indeterminacy. We postulated that certain data may follow an inverse gamma distribution, and we extended the traditional inverse gamma distribution to the neutrosophic inverse gamma distribution. We established multiple characteristics of the neutrosophic inverse_gamma distribution: expectation, variance, R-th moment, survival function, hazard rate function, and prevalent forms of classification statistics. We applied the neutrosophic inverse_gamma distribution to a dataset generated using the R programming language, version (2.3.4), and found that in real-world data characterized by uncertainty and inaccuracy, Parameter estimation using the neutrino inverse_gamma distribution is more accurate and realistic than parameter estimation using the classical inverse_gamma distribution for uncertain and indeterminate data.

Authors' Contributions. All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

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