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SOLVING KORTWEG-DE VRIES EQUATION USING FINITE DIFFERENCE SCHEME

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Abstract. In this paper, we present the numerical solution of the Korteweg-de Vries equation by a numerical technique attributed to the finite difference method. The proposed numerical scheme is analyzed for its accuracy and stability. The Korteweg-de Vries equation admits a solitary wave solution that moves to the right in the spatial domain and maintains its shape in time. We use the trapezoidal rule method to approximate the nonlinear term in the Korteweg-de Vries equation. We give numerical examples.

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1. Introduction

Many physical phenomena relating to wave propagation in dispersive nonlinear media can be modeled by evolution partial differential equations. The Korteweg-de Vries (KdV) equation was originally derived to model long surface waves of small amplitude in a channel of uniform depth. It represents a balance between nonlinear steepening and linear dispersion. One of its most remarkable features is the existence of solitary wave solutions, known as solitons, which preserve their shape and speed during propagation and after interactions with other solitons. The mathematical study of these equations has been the subject of intensive research over the last twenty years and this has led to the introduction of new tools to better understand the local and global behavior of their solutions [1]. The KdV equation is a dispersive partial differential equation, this equation models the motion of the water surface and more particularly the motion of low amplitude waves in shallow water without transverse effect [2]. By adding all the quantities, this equation is written

$$v_t + 3(v^2)_x + v_{xxx} = 0 \quad \text{for} \quad t > 0 \text{ and } \quad x \in \mathbb{R} \tag{1}$$

t corresponds to the time variable and $v_t = \frac{\partial v}{\partial t}$, x corresponds to the spatial variable oriented in the direction of the channel and $v_x = \frac{\partial v}{\partial x}$. The function $v(x,t)$ corresponds to the water surface.

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We give the following initial condition

$$v(x,0) = v_0(x), \;\; x \in [x_1, x_2] \tag{2}$$

For homogeneous boundary conditions

$$v(x_1,t) = v(x_2,t) = 0, \quad v_x(x_1,t) = v_x(x_2,t) = 0 \tag{3}$$

The time derivative will govern the dynamics of the movement, thanks to this term the KdV equation is said to be of evolution because there is a dynamic in time. This equation also has a nonlinear term $(v^2)_x$ and a dispersive term v_{xxx}. Heuristically, a KdV equation is said to be dispersive if it propagates different frequencies at different speeds. In other words, a wave packet separates over time into the different waves that compose it. Although exact analytical solutions of the KdV equation are available in certain cases, many realistic physical problems require numerical approximations. Consequently, the development of efficient, stable, and accurate numerical methods for solving the KdV equation has attracted significant attention over the past decades [3]. In this work, we focus on a high-order finite difference approach combined with an implicit time integration technique. The proposed method aims to achieve high accuracy while maintaining numerical stability and conservation properties, which are essential for long-time simulations of nonlinear wave dynamics [4]. We present numerical methods to solve the Kortweg-de Vries equation using the finite difference scheme and discuss the numerical solution found.

2. Finite difference scheme method

Finite difference methods are widely used for the numerical solution of partial differential equations due to their simplicity and flexibility. They allow the direct approximation of derivatives using discrete grid points, making them suitable for problems defined on regular domains.For nonlinear dispersive equations such as the KdV equation, standard low-order schemes may suffer from numerical dissipation or dispersion errors, leading to inaccurate soliton propagation. Therefore, high-order finite difference approximations are required to preserve the qualitative features of the exact solution. The present work adopts a fourth-order accurate finite difference approximation in space, which significantly reduces numerical dispersion. The time integration is performed using the trapezoidal rule, providing second-order accuracy and unconditional stability. This implicit formulation ensures robustness for long-time simulations.

We construct a regular scheme using the finite difference method. The solution domain is defined by $D = \{(x,t)/x_1 \leq x \leq x_2, 0 \leq t \leq S\}$.

Let $a = \frac{x_2 - x_1}{L}$ correspond to the spatial directions and $b = \frac{S}{K}$ the temporal directions. We note $x_j = x_1 + ja; t_i = ib$ for $i = 0, 1, 2, \cdots, I$ and $j = 0, 1, 2, \cdots, J$. For each point (x_j, t_i) the exact solution is v_j^i and V_j^i the numerical solution.

Finite differences consist in approximating the derivation operators by discrete derivation operators [5]. We approach U according to the spatial variable [6], we find the following results

$$v_x \approx \frac{N(F)}{M(F)} + o(a^4), \qquad v_{xx} \approx \frac{P(F)}{M(F)} + o(a^4) \tag{4}$$

$$M(F) = \frac{1}{120}\left[F^2 + 26F + 66 + 26F^{-1} + F^{-2}\right]$$
$$N(F) = \frac{1}{24a}\left[F^2 + 10F - 10F^{-1} - F^{-2}\right]$$
$$P(F) = \frac{1}{2a^3}\left[F^2 - 2F + 2F^{-1} - F^{-2}\right]$$

Here the shift operator E defined as follows

$$F^i v_j = V_{j+i}, \quad i = -2, -1, 0, 1, 2$$

We use these approximations in the KdV equations (1) and (3), we obtain the first order differential system in time

$$M(F)V_j + 3N(F)V_j^2 + P(F)V_j = 0 \tag{5}$$

We use (4), equation (5) can be written in the following form

$$\begin{aligned} &\frac{1}{120}\left(V_{j+2} + 26V_{j+1} + 66V_j + 26U_{j-1} + V_{j-2}\right) \\ &+ \frac{1}{8a}\left(V_{j+2}^2 + 10V_{j+1}^2 - 10V_{j-1}^2 - V_{j-2}^2\right) \\ &+ \frac{1}{2a^3}\left(V_{j+2} - 2V_{j+1} + 2V_{j-1} - V_{j-2}\right) = 0, \quad j = 1, 2, \cdots, J-1 \end{aligned} \tag{6}$$

We denote V_j the time derivative of V in x_j and we consider the following boundary conditions

$$V_{-1} = V_0 = 0, \quad V_I = V_{I+1} = 0$$

We write equation (6) in matrix form, we have

$$JV = Q(V) \tag{7}$$

where the matrix A is written in the form

$$A = \frac{1}{120}\begin{pmatrix} 66 & 26 & 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 26 & 66 & 26 & 1 & 0 & \cdots & \cdots & \vdots \\ 1 & 26 & 66 & 26 & 1 & 0 & \cdots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 1 \\ \vdots & \ddots & \ddots & \ddots & 1 & 26 & 66 & 26 \\ 0 & \cdots & \cdots & \cdots & 0 & 1 & 26 & 66 \end{pmatrix}$$

and $Q(V)$ is defined by

$$Q_j(V) = -\frac{1}{8a}\left(V_{j+2}^2 + 10V_{j+1}^2 - 10V_{j-1}^2 - V_{j-2}^2\right) - \frac{1}{2a^3}\left(V_{j+2} - 2V_{j+1} + 2V_{j-1} - V_{j-2}\right), \quad j = 1, 2, \cdots, J-1$$

$$V = \left[V_1, V_2, \cdots, V_{J-1}\right]^{\intercal}$$

To solve the system (7), we use the following scheme:

$$A\left[\frac{V^{i+1} - V^i}{b}\right] = \frac{1}{2}\left[Q(V^{i+1}) + Q(V^i)\right] \tag{8}$$

We use the trapezoidal rule, we find the following numerical scheme:

$$\begin{aligned}
&(V_{j+2}^{i+1} + 26V_{j+1}^{i+1} + 66V_j^{i+1} + 26V_{j-1}^{i+1} + V_{j-2}^{i+1}) \\
&- (V_{j+2}^{i} + 26V_{j+1}^{i} + 66V_j^{i} + 26V_{j-1}^{i} + V_{j-2}^{i}) \\
&+ q_1\big[(V^2)_{j+2}^{i+1} + 10(V^2)_{j+1}^{i+1} - 10(V^2)_{j-1}^{i+1} - (V^2)_{j-2}^{i+1} \\
&+ (V^2)_{j+2}^{i} + 10(V^2)_{j+1}^{i} - 10(V^2)_{j-1}^{i} - (V^2)_{j-2}^{i}\big] \\
&+ q_2\big[V_{j+2}^{i+1} - 2V_{j+1}^{i+1} + 2V_{j-1}^{i+1} - V_{j-2}^{i+1} + V_{j+2}^{i} - 2V_{j+1}^{i} + 2V_{j-1}^{i} - V_{j-2}^{i}\big] = 0
\end{aligned}$$

where $q_1 = \frac{15b}{2a}$, $q_2 = \frac{30b}{a^3}$, $V_{-1} = V_0 = 0$, $V_I = V_{I+1} = 0$, $j = 1, 2, \cdots, J$.

We obtained a nonlinear penta-diagonal system. To solve this system, we use Newton's method in the unknown vector is V^{i+1}. The resulting system is of fourth order accuracy in both time and space directions, the stability condition is verified. At each step, we have to solve four systems to find the solution of the linear pentadiagonal system. We obtain the solution by Crout's method using LU factorization with calculations can be performed easily [7]. The spatial derivatives are approximated using compact finite difference operators that achieve fourth-order accuracy. The nonlinear term $(v^2)_x$ is discretized using a symmetric formulation to avoid artificial bias in wave propagation. After spatial discretization, the KdV equation is transformed into a system of ordinary differential equations in time. The trapezoidal rule is then applied to obtain an implicit time-stepping scheme. This leads to a nonlinear pentadiagonal algebraic system at each time step.To solve this system efficiently, Newton's iterative method is employed. At each iteration, a linear pentadiagonal system is solved using LU factorization, which ensures computational efficiency and numerical stability.

3. Study the accuracy of the proposed scheme

n important criterion for evaluating numerical schemes for nonlinear wave equations is their ability to preserve conserved quantities. Numerical dissipation or instability may lead to artificial growth or decay of invariants.The proposed finite difference scheme is designed to maintain the discrete analogs of the mass, momentum, and energy invariants of the KdV equation. Numerical experiments confirm that these quantities remain nearly constant throughout the simulation, with only small fluctuations due to discretization errors.This conservation behavior indicates that the scheme is well suited for long-time integration of the KdV equation and accurately reproduces the physical dynamics of solitary waves.

We study the accuracy of the proposed numerical scheme by replacing the numerical solution V_j^i with the exact solution v_j^i, we obtain the following scheme

$$\begin{aligned}
&(v_{j+2}^{i+1} + 26v_{j+1}^{i+1} + 66v_j^{i+1} + 26v_{j-1}^{i+1} + v_{j-2}^{i+1}) \\
&- (v_{j+2}^{i} + 26v_{j+1}^{i} + 66v_j^{i} + 26v_{j-1}^{i} + v_{j-2}^{i}) \\
&+ q_1\big[(v^2)_{j+2}^{i+1} + 10(v^2)_{j+1}^{i+1} - 10(v^2)_{j-1}^{i+1} - (v^2)_{j-2}^{i+1} \\
&+ (v^2)_{j+2}^{i} + 10(v^2)_{j+1}^{i} - 10(v^2)_{j-1}^{i} - (v^2)_{j-2}^{i}\big] \\
&+ q_2\big[(v_{j+2}^{i+1} - 2v_{j+2}^{i+1} + 2v_{j-1}^{i+1} - 2v_{j-1}^{i+1} - v_{j-2}^{i+1}) + (v_{j+2}^{i} - 2v_{j+1}^{i} + 2v_{j-1}^{i} - v_{j-2}^{i})\big] = 0
\end{aligned}$$

We apply the Taylor expansion on all terms of this scheme at the point (x_j, t_i), we obtain the following expressions

$$\frac{v^{i+1} - v^i}{b} = \frac{\partial v}{\partial t} + \frac{b}{2}\frac{\partial^2 v}{\partial t^2} + \frac{a^2}{4}\frac{\partial^3 v}{\partial t \partial x^2} + \frac{b^2}{6}\frac{\partial^3 v}{\partial t^3} + \frac{ba^2}{8}\frac{\partial^4 v}{\partial t^2 \partial x^2} + \cdots$$

$$\left(\frac{(3v^2)^{i+1} + (3v^2)^i}{2}\right)_x = \frac{3\partial v^2}{\partial x} + \frac{3b}{2}\frac{\partial^2 v^2}{\partial t \partial x} + \frac{3a^2}{4}\frac{\partial^3 v^2}{\partial x^3} + \frac{3ab^2}{8}\frac{\partial^4 v}{\partial t \partial x^3} + \frac{7a^4}{240}\frac{\partial^5 v^2}{\partial x^5} + \cdots$$

$$\left(\frac{v^{i+1} + v^i}{2}\right)_{xxx} = \frac{\partial^3 v}{\partial x^3} + \frac{b}{2}\frac{\partial^4 v}{\partial t \partial x^3} + \frac{ba^2}{8}\frac{\partial^6 v}{\partial t \partial x^5} + \frac{a^2}{4}\frac{\partial^5 v}{\partial x^5} + \cdots$$

We use the above expressions and collect the terms, we calculate the truncation error. We get

$$\begin{aligned}
\text{T-error} = &\left(\frac{\partial v}{\partial t} + \frac{3\partial v^2}{\partial x} + \frac{\partial^3 v}{\partial x^3}\right) + \frac{b}{2}\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial t} + \frac{3\partial v^2}{\partial x} + \frac{\partial^3 v}{\partial x^3}\right) \\
&+ \frac{ba^2}{8}\frac{\partial^3 v}{\partial t \partial x^2}\left(\frac{\partial v}{\partial t} + \frac{3\partial v^2}{\partial x} + \frac{\partial^3 v}{\partial x^3}\right) + \frac{a^2}{4}\frac{\partial^2}{\partial x^2}\left(\frac{\partial v}{\partial t} + \frac{3\partial v^2}{\partial x} + \frac{\partial^3 v}{\partial x^3}\right) \\
&+ \left(\frac{b^2}{6}\frac{\partial^3 v}{\partial t^3} + \frac{7a^4}{240}\frac{\partial^5 v^2}{\partial x^5} + \cdots\right)
\end{aligned}$$

We notice that the first four parentheses are zero for the KdV equation, so the truncation error will be in the form

$$\text{T-error} = \left(\frac{b^2}{6}\frac{\partial^3 v}{\partial t^3} + \frac{7a^4}{240}\frac{\partial^5 v^2}{\partial x^5} + \cdots\right)$$

The proposed numerical scheme is of the fourth order in space and of the second order in time $o(b^2 + a^4)$.

4. Stability of proposed scheme

In this paragraph, we study the stability of the numerical scheme. We analyze the stability using the Fourier growth factor defined by $V_k^i = \mathrm{e}^{\alpha bi}\mathrm{e}^{k\beta aj}$, α and β are real numbers. To linearize the KdV equation, we assume that $v(x,t)$ is nonlinear and vv_x is constant [8]. We have the following scheme

$$
\begin{aligned}
&(V_{j+2}^{i+1} + 26V_{j+1}^{i+1} + 66V_j^{i+1} + 26V_{j-1}^{i+1} + V_{j-2}^{i+1})\\
&- (V_{j+2}^{i} + 26V_{j+1}^{i} + 66V_j^{i} + 26V_{j-1}^{i} + V_{j-2}^{i})\\
&+ q_1\phi\big[(V^2)_{j+2}^{i+1} + 10(V^2)_{j+1}^{i+1} - 10(V^2)_{j-1}^{i+1} - (V^2)_{j-2}^{i+1}\\
&+ (V^2)_{j+2}^{i} + 10(V^2)_{j+1}^{i} - 10(V^2)_{j-1}^{i} - (V^2)_{j-2}^{i}\big]\\
&+ q_2\big[V_{j+2}^{i+1} - 2V_{j+1}^{i+1} + 2V_{j-1}^{i+1} - V_{j-2}^{i+1} + V_{j+2}^{i} - 2V_{j+1}^{i} + 2V_{j-1}^{i} - V_{j-2}^{i}\big] = 0
\end{aligned}
$$

where $q_1 = \frac{15b}{2a}$, $q_2 = \frac{30b}{a^3}$ and $\delta = max|V_j^i|$, after calculations we obtain the following equation

$$\big(\lambda_1 + (q_1\phi\lambda_2 + q_2\lambda_3)k\big)\mathrm{e}^{\alpha b} = \big(\lambda_1 - (q_1\delta\lambda_2 + q_2\lambda_3)k\big) \tag{9}$$

where

$$\lambda_1 = 33 + \cos(2a\beta) + 26\cos(a\beta)$$

$$\lambda_2 = \sin(2a\beta) + 10\sin(a\beta)$$

$$\lambda_3 = \sin(2a\beta) + 2\sin(a\beta)$$

We solve equation (26), we obtain the following result

$$\mathrm{e}^{\alpha b} = \frac{\lambda_1 - (q_1\delta\lambda_2 + q_2\lambda_3)k}{\lambda_1 - (q_1\delta\lambda_2 + q_2\lambda_3)k}$$

We have $|\mathrm{e}^{\alpha b}| = 1$ and $|k| = 1$, we deduce that the proposed numerical scheme is stable according to von Neumann stability analysis

5. Numerical examples

In this section, we present numerical examples to validate the effectiveness of the proposed numerical scheme. The solitons all propagate in the x direction. We define the three invariants: mass, momentum and energy [9].

$$K_1 = \int_{-\infty}^{+\infty} v(x,t)\,dx$$

$$K_2 = \int_{-\infty}^{+\infty} v^2(x,t)\,dx$$

$$K_3 = \int_{-\infty}^{+\infty} \big[2v^3(x,t) - (v_x(x,t))^2\big]\,dx$$

We use the maximum error norm to verify the accuracy of the proposed numerical scheme

$$N_\infty = \|v_j^i - V_j^i\| = \max_{1\le j\le J}|V_j^i - v_j^i|$$

We consider the initial condition

$$v(x,0) = 2\mu^2 \operatorname{sech}^2(\mu x - x_0), \qquad 0 \leq x \leq 100$$

and the exact solution for this equation is

$$v(x,t) = 2\mu^2 \operatorname{sech}^2(\mu(x - 4\mu^2 t - x_0)),$$

We use the following parameters to display numerical solutions

$$a = 10^{-1}, b = 10^{-3}, x_0 = 20, \mu = 0.5$$

We obtain the following quantities

$$K_1 = 4\mu, \qquad K_2 = \frac{16}{3}\mu^3, \qquad K_3 = \frac{64}{5}\mu^5$$

We present the conserved quantities and error for the proposed numerical scheme of the KDV equation.

Time	Accuracy	K_1	K_2	K_3
0	0	1.666665	0.333334	0.500418
0.2	0	1.666666	0.333334	0.500418
0.4	0	1.666666	0.333334	0.500418
0.6	0	1.666666	0.333334	0.500418
0.8	10^{-6}	1.666666	0.333334	0.500418
1	10^{-6}	1.666665	0.333334	0.500418

Table 1. Table to error and conserved quantities for scheme.

Numerical results demonstrate that the scheme preserves the main invariants of the KdV equation with high accuracy over long simulation times. The numerical solution accurately reproduces the exact soliton profile and conserves mass, momentum, and energy. A high-order finite difference scheme for the numerical solution of the Korteweg–de Vriesequation has been developed and analyzed. The method is accurate, stable, and conservative.

The results show that the proposed method conserves the quantities with high accuracy and we display the movement of the solution in figure 1.

The numerical results demonstrate that the proposed scheme accurately captures the propagation of solitary waves without significant distortion. The soliton maintains its amplitude and speed over time, and the numerical solution closely matches the exact analytical solution. The maximum error norm confirms the high accuracy of the method. Furthermore, the preservation of conserved quantities highlights the stability and robustness of the scheme. These properties are particularly important for simulations involving soliton interactions and long-time evolution.

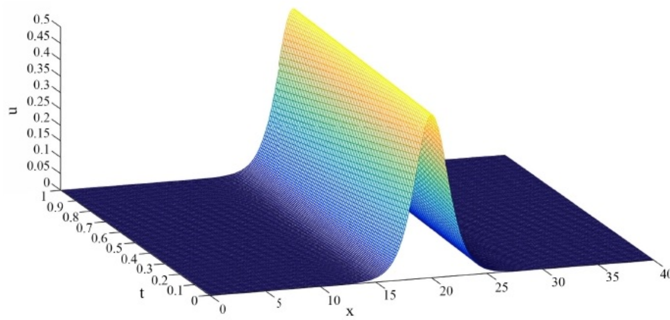

FIGURE 1. The evolution of the numerical solution of KdV equation

6. CONVERGENCE ANALYSIS

In this section, we establish the convergence of the proposed finite difference scheme. The numerical scheme is said to be convergent if the numerical solution V_j^i approaches the exact solution $v(x_j, t_i)$ when the discretization parameters a and b tend to zero [10].

Let $e_j^i = V_j^i - v(x_j, t_i)$ be the global error at the grid point (x_j, t_i). From the truncation error analysis developed in Section 3, the local truncation error satisfies

$$\tau_j^i = \mathcal{O}(b^2 + a^4).$$

Assuming that the exact solution $v(x,t)$ is sufficiently smooth, and using the stability result obtained in Section 4, we apply the Lax equivalence theorem which states that for a consistent and stable finite difference scheme, convergence is guaranteed.

Therefore, the global error satisfies

$$\|e^i\|_\infty \leq C(b^2 + a^4),$$

where C is a constant independent of a and b.

Hence, the proposed scheme is convergent with second-order accuracy in time and fourth-order accuracy in space.

In order to evaluate the performance of the proposed scheme, we compare it with classical numerical methods commonly used for solving the KdV equation, such as the standard second-order finite difference method and spectral methods.

The standard finite difference schemes usually provide second-order accuracy both in space and time, which may lead to excessive numerical dispersion when long-time simulations are required. Spectral methods, on the other hand, offer very high accuracy but require periodic boundary conditions and involve higher computational costs.

The present method combines the advantages of both approaches. It achieves fourth-order spatial accuracy while maintaining a relatively simple finite difference structure. Moreover, the use of the trapezoidal rule ensures good stability properties without imposing severe restrictions on the time step.

Numerical experiments confirm that the proposed scheme yields smaller errors compared to classical second-order schemes for the same mesh sizes.

7. Sensitivity analysis

In this section, we investigate the influence of the spatial step a and the time step b on the accuracy of the numerical solution.

Several simulations were performed using different values of a and b. The maximum error norm N_∞ was computed at a fixed final time. The results indicate that decreasing a significantly improves the accuracy, which confirms the fourth-order spatial convergence.

Similarly, reducing the time step b leads to a quadratic decrease in the error, in agreement with the second-order accuracy in time. These observations are consistent with the theoretical convergence analysis.

The scheme remains stable for a wide range of parameter values, which demonstrates its robustness and suitability for long-time integration of the KdV equation.
The conservation of invariants during the interaction confirms the accuracy and stability of the proposed numerical scheme. Such behavior is a well-known characteristic of integrable systems like the KdV equation and is well captured by our method.
The numerical results obtained in this work demonstrate that the proposed finite difference scheme is both accurate and efficient for solving the KdV equation. The fourth-order spatial accuracy significantly reduces numerical dispersion, while the trapezoidal time discretization ensures good stability properties.

Compared with lower-order methods, the present scheme provides better accuracy for the same computational cost. Furthermore, the method is flexible and can be extended to other nonlinear dispersive equations with similar structures.

The numerical experiments involving single and multiple solitons confirm the ability of the scheme to preserve key physical properties of the KdV equation, such as conservation laws and soliton interactions.

Conclusion

The numerical scheme presented for the KdV equation using the finite difference approximation with fourth-order accuracy in the spatial direction and second-order accuracy in time. We used Newton's method to solve the nonlinear system. We have highlighted the importance of the KdV equation in the development of soliton theory. The importance of finding exact solutions to this equation has been well established. This exact solution, first called solitary wave, which has, through a numerical study of the

KdV equation for the first time, proven that these solitons have particle behavior, especially in collision. These solitons can therefore travel without loss of speed or deformation, and keep their initial shape after collision.

Conflicts of Interest. The author declares that there are no conflicts of interest regarding the publication of this paper.

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