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GENERALIZED BI-UNIVALENT FUNCTIONS DEFINED BY A PARAMETRIC SHEHU OPERATOR UNDER SUBORDINATION

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Abstract. This paper introduces a new subclass of bi-univalent functions defined via the Shehu operator combined with a generalized Ma–Minda type subordination. Initial coefficient bounds for $|a_2|$ and $|a_3|$are obtained. Furthermore, a sharp Fekete–Szegö inequality for the functional $\left|a_3 - \lambda {a_2}^2\right|$ is derived. Several special cases are discussed.

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1. Introduction

Let $\mathcal{A}$ denote the class of functions ℓ that are analytic and univalent in the open unit disk $\mathfrak{U} = \{\mathfrak{z} \in \mathbb{C} : |\mathfrak{z}| < 1\}$. Each function ℓ admits the Taylor series expansion

$$\ell\left(\mathfrak{z}\right) = \mathfrak{z} + \sum_{\Updownarrow=2}^{\infty} a_m \mathfrak{z}^{\Updownarrow} \tag{1}$$

The study of univalent functions has its roots in the classical coefficient problem initiated by Ludwig Bieberbach in 1916 [1], whose conjecture on coefficient bounds shaped much of the development of geometric function theory during the twentieth century. The final resolution of this conjecture by Louis de Branges in 1985 [2] marked a major milestone in the field. When we generalize these concepts to the bi-univalent setting, we encounter more significant difficulties, since geometric constraints must be satisfied simultaneously by a function and its inverse. Earlier, D. A. Brannan and J. G. Clunie [3] provided initial bounds for the coefficients a_2 and a_3 as key contributions to the coefficient problem for bi-univalent functions. Since then, numerous authors focused on subclasses of Σ which were specified as differential inequalities and subordination conditions to derive better estimates of coefficients while also looking for Fekete–Szegö type inequalities [4–6].

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This principle of subordination is at the heart of identifying such subclasses. If ℓ and $\mathfrak{B}$ are analytic in $\mathfrak{U}$, ℓ is said to be subordinate to $\mathfrak{B}$, and written $\ell(\mathfrak{z}) \prec \mathfrak{B}(\mathfrak{z})$, if there is an analytic Schwarz function $w : \mathfrak{U} \to \mathfrak{U}$, with $w(0) = 0$ and $|w(\mathfrak{z})| < 1$ such that [6–11]

$$\ell(\mathfrak{z}) = \mathfrak{B}(w(\mathfrak{z})), \mathfrak{z} \in \mathfrak{U} \tag{2}$$

When $\mathfrak{B}$ is univalent, this condition is equivalent to the geometric inclusion $\ell(\mathfrak{U}) \subset \mathfrak{B}(\mathfrak{U})$. The subordination framework was systematically developed and widely applied in geometric function theory, particularly in the formulation of Ma–Minda type classes, where a suitable dominant function governs the geometric behavior of the functions under consideration (see, for example, Ma and Minda, 1992) [12].

Parallel to these developments in geometric function theory, integral transforms have been introduced and studied extensively in applied analysis. Among them, the Shehu transform has recently attracted attention as a two-parameter integral transform defined by

$$\mathcal{S}\ell(t)(s,u) = \int_0^\infty \ell(t)e^{-\frac{s}{u}t}dt, s > 0, u > 0 \tag{3}$$

which serves as a natural bridge between continuous integral formulations and discrete coefficient representations. By expressing $\ell(t)$ in a power series form and applying term-by-term integration, the resulting integrals reduce to Gamma-type expressions by

$$\int_0^\infty t^{n-1}e^{-\alpha t}dt = \frac{(n-1)!}{\alpha^n}dt$$

By expressing the function $\ell(t)$ in its power series form and applying term-by-term integration, the integral transform reduces to some sequence Gamma-type integrals. Upon evaluation, this yields a coefficient multiplier representation involving factorial growth and exponential scaling. Consequently, the transform can be rewritten as a linear operator acting on analytic functions in the unit disk. Motivated by this perspective, we consider the integral transform defined by the following

Definition 1.1. *Let $f \in \mathcal{A}$ be given by (1), we define the operator*

$$\mathcal{S}_{u.s} : \mathcal{A} \to \mathcal{A}$$

Such that

$$\mathcal{S}_{u.s}(\ell(\mathfrak{z})) = \mathfrak{z} + \sum_{2}^{\infty} a_n \frac{(n-1)!u^n}{s^n}\mathfrak{z}^n, s > 0, 0 < u < s$$

The study of operator theory in geometric function theory has proven to be an effective approach for generating new subclasses of analytic and bi-univalent functions and for obtaining these subclasses, it is natural to explore integral transforms and their corresponding series representations. Integral transforms provide a powerful bridge between continuous formulations and discrete coefficient structures, which are essential in geometric function theory.

transforming their Taylor coefficients in a manner reflecting the parametric structure of the Shehu transform [13]. This operator generates new analytic functions depending on parameters s and u, thereby introducing a controllable analytic deformation within the unit disk. Building upon this idea, a weighted operator of the form

$$G_{\alpha}^{s,u}(\ell)(\mathfrak{z}) = (1-\alpha)\frac{\mathfrak{z}\left(\mathcal{S}_{s,u}\ell(\mathfrak{z})\right)'}{\left(\mathcal{S}_{s,u}\ell(\mathfrak{z})\right)} + \alpha\left(1 + \frac{\mathfrak{z}\left(\mathcal{S}_{s,u}\ell(\mathfrak{z})\right)''}{\left(\mathcal{S}_{s,u}\ell(\mathfrak{z})\right)'}\right) \quad 0 \le \alpha \le 1, \tag{4}$$

With this in view, the new classes of bi-univalent functions can be achieved to use subordination conditions of Ma–Minda type. Specifically, for a suitable analytic function ϕ with positive real part in $\mathfrak{U}$ and $\phi(0) = 1$.

2. The class $M_{\propto}^{s,u}$ β, ϕ and the Coefficients Bounds

Definition 2.1. *A function $\ell \in \Sigma$ where $\ell(\mathfrak{z}) = \mathfrak{z} + \sum_{n=2}^{\infty} \mathrm{a}_n \mathfrak{z}^n$ is said to be in the class $M_{\propto}^{s,u}(\beta, \phi)$ if it satisfies the following subordination conditions*

$$(1-\beta) + \beta G_{\alpha}^{s,u}\ell(\mathfrak{z}) \prec \phi(\mathfrak{z}) \tag{5}$$

$$(1-\beta) + \beta G_{\alpha}^{s,u} g(w) \prec \phi(w) \tag{6}$$

For $0 \le \alpha \le 1$, For $0 < \beta \le 1, s > 0, u \epsilon \mathbb{C}$.

It uses: classical theory of bi-univalent functions, parametric structure inspired by the Shehu transform, and geometric control provided by subordination. Not only does the resulting framework yield several known subclasses serving as special cases, but it also allows new coefficient bounds to be derived, and Fekete–Szegö type inequalities to be included, in a unified framework.

Remark 2.2. *The class $M_{\alpha}^{s,u}(\beta, \phi)$ reduces to several well-known subclasses for particular choices of the parameters α, β, s and u, as follows:*

(1) *When $\alpha = 0$ the class $M_{\alpha}^{s,u}(\beta, \phi)$ reduces to the Bi–Shehu–starlike class $M^{s,u}(\beta, \phi)$.*
(2) *If $\alpha = 1$ the class $M_{\alpha}^{s,u}(\beta, \phi)$ reduces to the Bi–Shehu–convex class $M_1^{s,u}(\beta, \phi)$.*
(3) *If $\beta = 1$ The class $M_{\alpha}^{s,u}(\beta, \phi)$ becomes the Bi–Shehu–Ma–Minda class $M_{\alpha}^{s,u}(1, \phi)$.*
(4) *If $s = u = 1$: the class $M_{\alpha}^{s,u}(\beta, \phi)$ reduces to Classical bi–Ma–Minda class $M_{\alpha}^{1}(\beta, \phi)$*

Theorem 2.3. *For $\ell \in M_{\alpha}^{s,u}(\beta, \phi)$ then:*

$$|a_2| \le \sqrt{\frac{|B_1|}{\beta\left[\frac{4(1+2\alpha)u^3}{s^3} - \frac{(1+3\alpha)u^4}{s^4}\right] - \frac{\beta^2(1+\alpha)^2 u^4}{s^4 B_1} B_2}}$$

$$|a_3| \le \frac{s^4}{\beta^2 (1+\alpha)^2 u^4} + \frac{s^3}{4\beta(1+2\alpha)u^3}|B_1|$$

Proof. Since $\ell \in M_{\alpha}^{s,u}(\beta, \phi)$, there exist regular functions $k, n : \mathfrak{U} \to \mathfrak{U}$, such that

$$k(\mathfrak{z}) = v_1 \mathfrak{z} + v_2 \mathfrak{z}^2 + \ldots \text{ such that } |k(\mathfrak{z})| \le 1 \tag{7}$$

$$n(\mathfrak{m}) = n_1 \mathfrak{m} + n_2 \mathfrak{m}^2 + \ldots \text{ such that } |n(\mathfrak{m})| \le 1 \tag{8}$$

and

$$\phi\left(\mathfrak{z}\right)=1+B_{1}\mathfrak{z}+B_{2}\mathfrak{z}^{2}+\ldots \tag{9}$$

$$[(1-\beta)+\beta G_{\alpha}^{s,u}\ell(\mathfrak{z})]=\phi(k(\mathfrak{z})),(\mathfrak{z}\in U) \tag{10}$$

$$[(1-\beta)+\beta G_{\alpha}^{s,u}g(w)]=\phi\left(n\left(\mathfrak{m}\right)\right),(\mathfrak{m}\in U) \tag{11}$$

$$[(1-\beta)+\beta G_{\alpha}^{s,u}\ell(\mathfrak{z})]=1+\beta\frac{(1+\alpha)\,u^{2}}{s^{2}}a_{2}\mathfrak{z}+\beta\left[\frac{(4+8\alpha)\,u^{3}}{s^{3}}a_{3}-\frac{(1+3\alpha)\,u^{4}}{s^{4}}a_{2}^{2}\right]\mathfrak{z}^{2}+\ldots \tag{12}$$

$$\begin{aligned}[(1-\beta)+\beta G_{\alpha}^{s,u}g(w)]=&1-\beta\frac{(1+\alpha)\,u^{2}}{s^{2}}a_{2}\mathfrak{m}\\&+\beta\left[(4+8\alpha)(2a_{2}^{2}-a_{3})(\frac{u^{3}}{s^{3}})-\frac{(1+3\alpha)\,u^{4}}{s^{4}}a_{2}^{2}\right]\mathfrak{m}^{2}+\ldots\end{aligned} \tag{13}$$

$$\phi(k(\mathfrak{z}))=1+B_{1}v_{1}\mathfrak{z}+[B_{1}v_{2}+B_{2}v_{1}^{2}]m2+\ldots \tag{14}$$

$$\phi(n\left(\mathfrak{m}\right))\;=1+B_{1}n_{1}\mathfrak{z}+[B_{1}n_{2}+B_{2}n_{1}^{2}]m2+\ldots \tag{15}$$

From (12), (13), (14), (15) we get

$$\beta\frac{(1+\alpha)\,u^{2}}{s^{2}}a_{2}=B_{1}v_{1} \tag{16}$$

$$\beta\left[\frac{(4+8\alpha)\,u^{3}}{s^{3}}a_{3}-\frac{(1+3\alpha)\,u^{4}}{s^{4}}a_{2}^{2}\right]=B_{1}v_{2}+B_{2}v_{1}^{2} \tag{17}$$

$$-\beta\frac{(1+\alpha)\,u^{2}}{s^{2}}a_{2}=B_{1}n_{1} \tag{18}$$

$$\beta\left[(4+8\alpha)(2a_{2}^{2}-a_{3})\left(\frac{u^{3}}{s^{3}}\right)-\frac{(1+3\alpha)\,u^{4}}{s^{4}}a_{2}^{2}\right]=B_{1}n_{2}+B_{2}n_{1}^{2} \tag{19}$$

From (16) and (18)

$$v_{1}=-n_{1} \tag{20}$$

and

$$2\beta^{2}\frac{(1+\alpha)^{2}\,u^{4}}{s^{4}}{a_{2}}^{2}=B_{1}^{2}(v_{1}^{2}+n_{1}^{2}) \tag{21}$$

add (17) and (19) and by (20)

$$\beta\left[\frac{8\,(1+2\alpha)\,u^{3}}{s^{3}}-\frac{2\,(1+3\alpha)\,u^{4}}{s^{4}}\right]a_{2}^{2}=B_{1}(v_{2}+n_{2})+B_{2}(v_{1}^{2}+n_{1}^{2})$$

From (21)

$$(v_{1}^{2}+n_{1}^{2})=\frac{2\beta^{2}\frac{(1+\alpha)^{2}u^{4}}{s^{4}}{a_{2}}^{2}}{B_{1}^{2}}$$

$$\beta\left[\frac{8\,(1+2\alpha)\,u^{3}}{s^{3}}-\frac{2\,(1+3\alpha)\,u^{4}}{s^{4}}\right]a_{2}^{2}=B_{1}(v_{2}+n_{2})+B_{2}\frac{2\beta^{2}\frac{(1+\alpha)^{2}u^{4}}{s^{4}}{a_{2}}^{2}}{B_{1}^{2}}$$

$$a_2^2\left[\beta\left(\frac{8\left(1+2\alpha\right)u^3}{s^3}-\frac{2\left(1+3\alpha\right)u^4}{s^4}\right)-\frac{2\beta^2\left(1+\alpha\right)^2u^4}{s^4B_1}B_2\right]=B_1(v_2+n_2)$$

$$a_2^2=\frac{B_1(v_2+n_2)}{\beta\left[\frac{8(1+2\alpha)u^3}{s^3}-\frac{2(1+3\alpha)u^4}{s^4}\right]-\frac{2\beta^2(1+\alpha)^2u^4}{s^4B_1}B_2}$$

$$|a_2|\leq\sqrt{\frac{|B_1|}{\beta\left[\frac{4(1+2\alpha)u^3}{s^3}-\frac{(1+3\alpha)u^4}{s^4}\right]-\frac{\beta^2(1+\alpha)^2u^4}{s^4B_1}B_2}}$$

From (17) and (19)

$$a_3=a_2^2+\frac{s^3}{8\beta\left(1+2\alpha\right)u^3}B_1(v_2-n_2)$$

$$a_3=\frac{s^4B_1^2(v_1^2+n_1^2)}{2\beta^2\left(1+\alpha\right)^2u^4}+\frac{s^3}{8\beta\left(1+2\alpha\right)u^3}B_1(v_2-n_2)$$

$$|a_3|\leq\frac{s^4}{\beta^2\left(1+\alpha\right)^2u^4}+\frac{s^3}{4\beta\left(1+2\alpha\right)u^3}\left|B_1\right|$$

These corollaries follow directly from Theorem 2.3 in conjunction with the parameter reductions stated in Remark 2.2. □

Corollary 2.4. *As a special case of Theorem 2.3, when $\alpha=1$, the class $M_\alpha^{s,u}(\beta,\phi)$ reduces to the bi-Shehu convex class. Consequently, the coefficient estimates become:*

$$|a_2|\leq\sqrt{\frac{|B_1|}{\beta\left[\frac{12u^3}{s^3}-\frac{4u^4}{s^4}\right]-\frac{4\beta^2u^4}{s^4B_1}B_2}}$$

$$|a_3|\leq\frac{s^4}{4\beta^2u^4}+\frac{s^3}{12\beta u^3}\left|B_1\right|$$

Corollary 2.5. *Similarly, when $\alpha=1$, the class $M_\alpha^{s,u}(\beta,\phi)$ reduces to the bi-Shehu starlike class. In this case, the bounds reduce accordingly to:*

$$|a_2|\leq\sqrt{\frac{|B_1|}{\beta\left[\frac{4u^3}{s^3}-\frac{u^4}{s^4}\right]-\frac{\beta^2u^4}{s^4B_1}B_2}}$$

$$|a_3|\leq\frac{s^4}{\beta^2u^4}+\frac{s^3}{4\beta u^3}\left|B_1\right|$$

Corollary 2.6. *If $\alpha=1$ and $\beta=1$ then the coefficient estimates reduce to:*

$$|a_2|\leq\sqrt{\frac{|B_1|}{\left[\frac{12u^3}{s^3}-\frac{4u^4}{s^4}\right]-\frac{4u^4}{s^4B_1}B_2}}$$

$$|a_3|\leq\frac{s^4}{4u^4}+\frac{s^3}{12u^3}\left|B_1\right|$$

Corollary 2.7. *If $\alpha = 0$ and $\beta = 1$, then the coefficient estimates reduce to:*

$$|a_2| \leq \sqrt{\frac{|B_1|}{\left[\frac{4u^3}{s^3} - \frac{u^4}{s^4}\right] - \frac{u^4}{s^4 B_1} B_2}}$$

$$|a_3| \leq \frac{s^4}{u^4} + \frac{s^3}{4u^3} |B_1|$$

Corollary 2.8. *If $u = s = 1$, then the coefficient estimates reduce to:*

$$|a_2| \leq \sqrt{\frac{|B_1|}{\beta(3 - 5\alpha) - \frac{\beta^2(1+\alpha)^2}{B_1} B_2}}$$

$$|a_3| \leq \frac{1}{\beta^2 (1+\alpha)^2} + \frac{1}{4\beta (1+2\alpha)} |B_1|$$

Corollary 2.9. *If $u = s = 1$ and $\alpha = 1$ and $\beta = 1$*

$$|a_2| \leq \sqrt{\frac{|B_1|}{-2 - \frac{4}{B_1} B_2}}$$

$$|a_3| \leq \frac{1}{4} + \frac{1}{12} |B_1|$$

3. Fekete-Szegö Inequality for the Class $M_\alpha^{s,u}(\beta, \phi)$

Theorem 3.1. *Let $\ell \in M_\alpha^{s,u}(\beta, \phi)$ then for any real number λ*

$$\left|a_3 - \lambda a_2^2\right| = \begin{cases} \frac{s^3}{4\beta(1+2\alpha)u^3} |B_1| & if\ |1-\lambda| \leq \zeta \\ |1-\lambda| \frac{|B_1|}{\left|\beta\left[\frac{8(1+2\alpha)u^3}{s^3} - \frac{2(1+3\alpha)u^4}{s^4}\right] - \frac{2\beta^2(1+\alpha)^2 u^4}{s^4 B_1} B_2\right|} & if\ |1-\lambda| > \zeta \end{cases}$$

were

$$\zeta = \frac{s^3}{8\beta(1+2\alpha)u^3}\left[\beta\left[\frac{8(1+2\alpha)u^3}{s^3} - \frac{2(1+3\alpha)u^4}{s^4}\right] - \frac{2\beta^2(1+\alpha)^2 u^4}{s^4 B_1} B_2\right]$$

Proof. From the subtraction of Eq. (17) and (19) we obtained

$$a_3 = a_2^2 + \frac{s^3}{8\beta(1+2\alpha)u^3} B_1(v_2 - n_2)$$

Thus

$$a_3 - \lambda a_2^2 = (1-\lambda)\frac{B_1(v_2+n_2)}{\beta\left[\frac{8(1+2\alpha)u^3}{s^3} - \frac{2(1+3\alpha)u^4}{s^4}\right] - \frac{2\beta^2(1+\alpha)^2 u^4}{s^4 B_1} B_2} + \frac{s^3}{8\beta(1+2\alpha)u^3} B_1(v_2 - n_2)$$

$$\begin{aligned} a_3 - \lambda a_2^2 =& v_2\left[\frac{s^3}{8\beta(1+2\alpha)u^3} B_1 + (1-\lambda)\frac{B_1}{\beta\left[\frac{8(1+2\alpha)u^3}{s^3} - \frac{2(1+3\alpha)u^4}{s^4}\right] - \frac{2\beta^2(1+\alpha)^2 u^4}{s^4 B_1} B_2}\right] \\ &+ \left[(1-\lambda)\frac{B_1}{\beta\left[\frac{8(1+2\alpha)u^3}{s^3} - \frac{2(1+3\alpha)u^4}{s^4}\right] - \frac{2\beta^2(1+\alpha)^2 u^4}{s^4 B_1} B_2} - \frac{s^3}{8\beta(1+2\alpha)u^3} B_1\right] \end{aligned}$$

$$\left|a_3-\lambda a_2^2\right|=\begin{cases}\frac{s^3}{4\beta(1+2\alpha)u^3}\left|B_1\right| & if\ \ |1-\lambda|\le\zeta\\ |1-\lambda|\frac{|B_1|}{\left|\beta\left[\frac{8(1+2\alpha)u^3}{s^3}-\frac{2(1+3\alpha)u^4}{s^4}\right]-\frac{2\beta^2(1+\alpha)^2u^4}{s^4B_1}B_2\right|} & if\ \ |1-\lambda|>\zeta\end{cases}$$

Were

$$\zeta=\frac{s^3}{8\beta\left(1+2\alpha\right)u^3}\left[\beta\left[\frac{8\left(1+2\alpha\right)u^3}{s^3}-\frac{2\left(1+3\alpha\right)u^4}{s^4}\right]-\frac{2\beta^2\left(1+\alpha\right)^2u^4}{s^4B_1}B_2\right]$$

□

Corollary 3.2. *if* $\alpha=1$*, then the Fekete-Szegö inequality reduce to:*

$$\left|a_3-\lambda a_2^2\right|=\begin{cases}\frac{s^3}{12\beta u^3}\left|B_1\right| & if\ \ |1-\lambda|\le\zeta\\ |1-\lambda|\frac{|B_1|}{\left|\beta\left[\frac{24u^3}{s^3}-\frac{8u^4}{s^4}\right]-\frac{8\beta^2u^4}{s^4B_1}B_2\right|} & if\ |1-\lambda|>\zeta\end{cases}$$

were

$$\zeta=\frac{s^3}{24\beta u^3}\left[\beta\left[\frac{24u^3}{s^3}-\frac{8u^4}{s^4}\right]-\frac{8\beta^2u^4}{s^4B_1}B_2\right]$$

Corollary 3.3. *if* $\alpha=0$*, then the Fekete-Szegö inequality reduce to:*

$$\left|a_3-\lambda a_2^2\right|=\begin{cases}\frac{s^3}{4\beta u^3}\left|B_1\right| & if\ \ |1-\lambda|\le\zeta\\ |1-\lambda|\frac{|B_1|}{\left|\beta\left[\frac{8u^3}{s^3}-\frac{2u^4}{s^4}\right]-\frac{2\beta^2u^4}{s^4B_1}B_2\right|} & if\ |1-\lambda|>\zeta\end{cases}$$

were

$$\zeta=\frac{s^3}{8\beta u^3}\left[\beta\left[\frac{8u^3}{s^3}-\frac{2u^4}{s^4}\right]-\frac{2\beta^2u^4}{s^4B_1}B_2\right]$$

Corollary 3.4. *If* $\alpha=1$ *and* $\beta=1$*, then the Fekete-Szegö inequality reduce to:*

$$\left|a_3-\lambda a_2^2\right|=\begin{cases}\frac{s^3}{12u^3}\left|B_1\right| & if\ \ |1-\lambda|\le\zeta\\ |1-\lambda|\frac{|B_1|}{\left|\frac{24u^3}{s^3}-\frac{8u^4}{s^4}-\frac{8u^4}{s^4B_1}B_2\right|} & if\ |1-\lambda|>\zeta\end{cases}$$

were

$$\zeta=\frac{s^3}{24u^3}\left[\frac{24u^3}{s^3}-\frac{8u^4}{s^4}-\frac{8u^4}{s^4B_1}B_2\right]$$

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