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AN INNOVATIVE APPROACH TO COMPLEX Q-FUZZY SET AND THEIR APPLICATIONS IN MEDICAL DIAGNOSIS

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Abstract. The Q-Fuzzy set (Q-FS) is a theoretical model created as an extension of the Fuzzy Set (FS), extending it to represent information with distributed uncertainty via the product between the universal set X and the set Q. This theoretical model has a domain of a citizen product between X and Q and has a codomain of real numbers.This model is widely used in processing one-dimensional data embedded in everyday problems. However, this concept is inadequate for handling two-dimensional data. Therefore, in this article, we propose a novel theoretical model called a complex Q-Fuzzy set (CQ-FS) model as a new extension of the Q-FS when the domain is a citizen product between X and Q, and the codomain is complex numbers. After that, we discovered the mean fundamental set theory of CQ-FSs. We formulated some basic theories and properties of these CQ-FSs. Ultimately, we will employ this tool to deal with a decision-making problem by using theoretical data.

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Key words and phrases. fuzzy set; Q-fuzzy set; complex fuzzy set; complex Q-fuzzy set; decision-making.

1. Introduction

This era is witnessing many developments across scientific and technological fields, marked by uncertainty and complexity. To overcome this obstacle, Zadeh [1] presented the concept of a fuzzy set (FS) as an extension of crisp sets. The FS is characterised by a function having a domain from the universal set X to a co-domain closed interval. A large number of developed contributions have been made regarding this set to address the uncertain problems that arise in models representing real-life phenomena. These include intuitionistic fuzzy sets, vague sets, rough sets, soft sets, Pythagorean fuzzy sets, and other advanced mathematical generalizations. Under these environments, several scholars

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and researchers have offered their theories and applications. For instance, Under these environments, several scholars and researchers have offered their theories and applications. For instance, Mendel [2] showed relationships between two FSs. Merge both fixed-point theorems with FSs, as presented by Hazaymeh and Bataihah [3,4]. The principle of probability was successfully applied with FSs by Ali et al. [7] under a soft environment. In addition to many effective innovations of high scientific value. One of these research contributions developed by researchers is called Q-FS [8], which expands the dynamics of the domain and codomain. The domain is the Cartesian product, and the codomain is the real numbers. This extension provided greater flexibility in interpreting the nature of the components of the universal set X. Q-fuzzy UP-ideals of UP-algebras and Q-fuzzy UP-subalgebras investigated by Tanamoon et al. [9].Başer and Uluçay [10] managed the virtual features of Q-FS with an expert method. Adam and Hassan [11] offer a soft parameterisation on Q-FS and apply these tools to problems that contain uncertainties. Al-Sharqi et al. [12] suggested the Q-neutrosophic matrix and showed its applications. On the other hand, it is worth noting that all the fuzzy mathematical models mentioned above deal with a single dimension. To address this deficiency, Ramot et al. [13] developed an innovative mathematical concept called the complex FS (CFS) to address two-dimensional uncertainty. In CFS there are two terms, namely, the amplitude term with the combination of a phase term, which gives a positive dynamic for dealing with an environment of uncertainty. Buckley [13] discussed the difference between FSs in real numbers and CFSs in complex numbers. Yang et al. [19] employed the bipolarity properties on CFS when they proposed that the CF subgroups were under bipolarity properties. Ma et al. [20] introduced some modern operations and laws of CFSs and show its applictions. Zeeshan et al. [21] established some particular examples on CFS and developed a new algorithm using a Cartesian product of CFSs. Hu et al. [22] established the orthogonality relation of CFSs. Al-Qudah et al. [23,24] initiated the definitions of the entropy, similarity distance measures between two CFS under a multi-set effect.Rani and Garg [25] developed some distance measures, such as Hamming and Hausdorff metrics on complex intuitionistic fuzzy sets (CIFSs) and showed their applications. The idea of a complex neutrosophic set was introduced by Ali and Smarandache [26] as an extension of the CFS and CIFS. Singh [27] defined algebraic properties on a complex plithogenic set and illustrated some numerical examples.

On the other hand, FSs provide a mathematical framework that can be employed in medical diagnosis, where patients and their associated symptoms are mathematically represented. The severity of the patient's symptoms is then assessed on a scale of 0 to 1; the closer the score is to 1, the more severe the illness. Numerous applied studies have been conducted in this field based on this principle. Gulzar et al. [28] employed the compositions of CF-relations in medical diagnosis. A new medical diagnosis form based on similarity measures of bipolar CFS, introduced by Rehman et al [29]. Nasir et al. [30] proposed the Cartesian product of two interval-valued CFSs and used it in medical diagnosis. And many research works that can be accessed through [31]– [35]. To illustrate the structural trajectory of our proposed concept, Figure 1 shows the point of convergence between previous concepts and how our proposed concept was generated. The results presented in this work are simply a development of the previous findings in the literature referenced in Figure 1.Therefore, the results presented in this work can be summarized as follows:

(1) The complex Q-fuzzy set (CQ-FS) presents as a new development of previous concepts mentioned in Figure 1, where the origin of the Cartesian product plays a role between X and Q plays a vital role in dealing with issues of uncertainty.
(2) The elemental algebraic properties and some related theorems of CQ-FS proven.
(3) A new algorithm has been designed to adapt this model to handle some of the decision-making problems.
(4) A systematic comparison of this model with other models has been presented to demonstrate the importance of the results presented in this work.

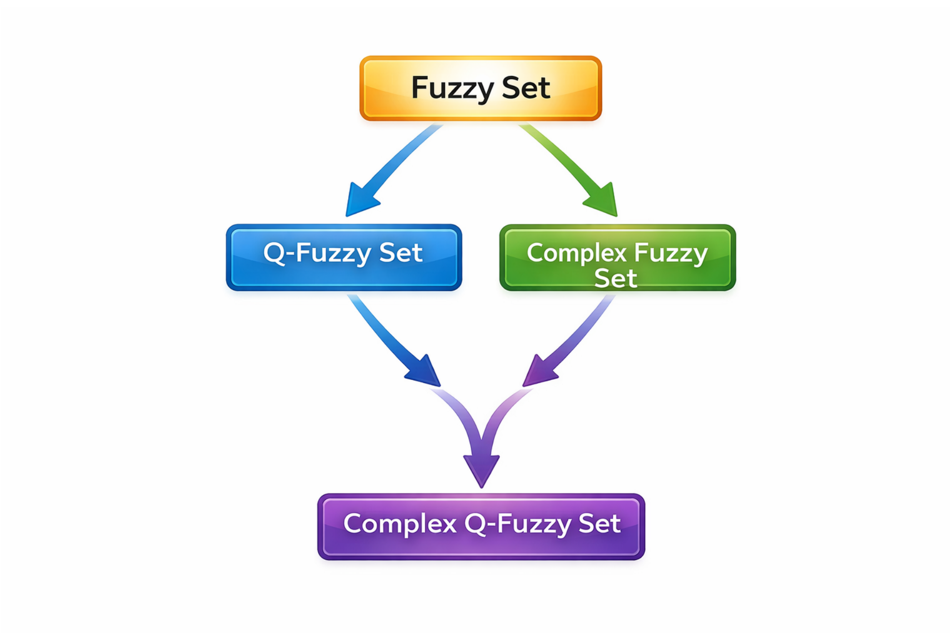

Figure 1. Structural interrelation illustrates our proposed concept

The paper is split into the following parts. To facilitate our discussion, in Section 2, we provide several background definitions regarding CQ-FSs. In Section 3, we characterize the concept of CQ-FSs as well as its theoretic operations. In Section 4, we present a practical application that illustrates how our proposed concept works with decision-making problems in the medical setting. Finally, we present a conclusion of this proposed work.

2. Preliminaries

Definition 2.1. [1] A FS Ψ on a reference set X is given in the following form:

$$\Psi = \{\langle x, \ddot{\mu}_{\Psi}(x)\rangle \mid x \in X\}$$

where the truth membership degree (TMD) $\ddot{\mu}_{\Psi}(x)$ of element x belongs to $[0,1]$.

Example 2.2. Let $X = \{x_1, x_2\}$ be a non-empty universe set. Then the FS Ψ is given by the following structure:

$$\Psi = \left\{\frac{\langle 0.1\rangle}{x_1}, \frac{\langle 0.4\rangle}{x_2}\right\}$$

Here, we denote that Ψ is a FS.

Definition 2.3. [1] Let $\Psi_1 = \{(x, \ddot{\mu}_{\Psi_1}(x)) \mid \forall x \in X\}$ and $\Psi_2 = \{(x, \ddot{\mu}_{\Psi_2}(x)) \mid \forall x \in X\}$ be two FSs on X. Then the following properties hold:

- **Complement:**

$$(\Psi_1)^C = \{(x, (\ddot{\mu}_{\Psi_1}(x))^C) \mid \forall x \in X\} = \{(x, 1 - \ddot{\mu}_{\Psi_1}(x)) \mid \forall x \in X\}$$

- **Union:**

$$\Psi_1 \cup \Psi_2 = \{\max[\ddot{\mu}_{\Psi_1}(x), \ddot{\mu}_{\Psi_2}(x)] \mid \forall x \in X\}$$

- **Intersection:**

$$\Psi_1 \cap \Psi_2 = \{\min[\ddot{\mu}_{\Psi_1}(x), \ddot{\mu}_{\Psi_2}(x)] \mid \forall x \in X\}$$

- **Subset:**

$$\Psi_1 \subseteq \Psi_2 \quad \text{if} \quad \ddot{\mu}_{\Psi_1}(x) \leq \ddot{\mu}_{\Psi_2}(x)$$

- **Equals:**

$$\Psi_1 = \Psi_2 \quad \text{if} \quad \ddot{\mu}_{\Psi_1}(x) = \ddot{\mu}_{\Psi_2}(x)$$

Definition 2.4. [10] A Q-FS Ψ_Q on the product of (X, Q) is given in the following form:

$$\Psi_Q = \left\{\langle x, \ddot{\mu}_{\Psi_Q}(x,q)\rangle \mid x \in X \text{ and } q \in Q\right\}$$

where the TMD $\ddot{\mu}_{\Psi_Q}(x,q)$ of element (x,q) belongs to $[0,1]$.

Example 2.5. Let $X = \{x_1, x_2\}$ be a non-empty universe set and $Q = \{q\}$. Then the Q-FS Ψ_Q is given by the following structure:

$$\Psi_Q = \left\{\frac{\langle 0.3\rangle}{(x_1, q)}, \frac{\langle 0.6\rangle}{(x_2, q)}\right\}$$

Here, we denote that Ψ_Q is a Q-FS.

Definition 2.6. [10] Let $(\Psi_Q)_1 = \{(x, \ddot{\mu}_{(\Psi_Q)_1}(x,q)) \mid x \in X \text{ and } q \in Q\}$ and $(\Psi_Q)_2 = \{(x, \ddot{\mu}_{(\Psi_Q)_2}(x,q)) \mid x \in X \text{ and } q \in Q\}$ be two Q-FSs on X. Then the following properties hold:

- **Complement:**

$$((\Psi_Q)_1)^C = \{(x, (\ddot{\mu}_{(\Psi_Q)_1}(x,q))^C) \mid x \in X \text{ and } q \in Q\} = \{(x, 1 - \ddot{\mu}_{(\Psi_Q)_1}(x,q)) \mid x \in X \text{ and } q \in Q\}$$

- **Union:**

$$(\Psi_Q)_1 \cup (\Psi_Q)_2 = \{\max[\ddot{\mu}_{(\Psi_Q)_1}(x,q), \ddot{\mu}_{(\Psi_Q)_2}(x,q)] \mid x \in X \text{ and } q \in Q\}$$

- **Intersection:**

$$(\Psi_Q)_1 \cap (\Psi_Q)_2 = \{\min[\ddot{\mu}_{(\Psi_Q)_1}(x,q), \ddot{\mu}_{(\Psi_Q)_2}(x,q)] \mid x \in X \text{ and } q \in Q\}$$

- **Subset:**

$$(\Psi_Q)_1 \subseteq (\Psi_Q)_2 \quad \text{if} \quad \ddot{\mu}_{(\Psi_Q)_1}(x,q) \le \ddot{\mu}_{(\Psi_Q)_2}(x,q)$$

- **Equals:**

$$(\Psi_Q)_1 = (\Psi_Q)_2 \quad \text{if} \quad \ddot{\mu}_{(\Psi_Q)_1}(x,q) = \ddot{\mu}_{(\Psi_Q)_2}(x,q)$$

Definition 2.7. [13] A complex fuzzy set (CFS) $\mathcal{P}$ on a reference set X is given in the following form:

$$\mathcal{P} = \{\langle x, \ddot{\mu}_{\mathcal{P}}(x)\rangle \mid x \in X\}$$

where $\ddot{\mu}_{\mathcal{P}}(x) : X \longrightarrow \{\ddot{a} \mid \ddot{a} \in \mathbb{C}, |\ddot{a}| \le 1\}$ is a complex truth membership value represented as

$$\ddot{\mu}_{\mathcal{P}}(x) = \ddot{\rho}_{\mathcal{P}}(x) \cdot e^{i2\pi\left(\ddot{\varphi}_{\mathcal{P}}(x)\right)}$$

such that $\ddot{\rho}_{\mathcal{P}}(x) \in [0,1]$ and $\ddot{\varphi}_{\mathcal{P}}(x) \in [0, 2\pi]$.

3. Complex Q-Fuzzy Structure

Definition 3.1. A CQ–FS $\mathcal{R}_Q$ on the Cartesian product between a reference set and a Q-set (X, Q) is given in the following form:

$$\mathcal{P}_Q = \left\{\langle x,\ \ddot{\mu}_{\mathcal{P}_Q}(x,q)\rangle\ :\ x \in X,\ q \in Q\right\},$$

where

$$\ddot{\mu}_{\mathcal{P}_Q} : (X \times Q) \longrightarrow \left\{\,\ddot{a} : \ddot{a} \in \mathbb{C},\ |\ddot{a}| \le 1\,\right\}$$

is a complex TM value represented in polar form by

$$\ddot{\mu}_{\mathcal{P}_Q}(x,q) = \rho_{\mathcal{P}_Q}(x,q)\, e^{\,\mathrm{i}2\pi\varphi_{\mathcal{P}_Q}(x,q)},$$

such that

$$\rho_{\mathcal{P}_Q}(x,q) \in [0,1] \qquad \text{and} \qquad \varphi_{\mathcal{P}_Q}(x,q) \in [0,1],$$

so that the argument $2\pi\varphi_{\mathcal{P}_Q}(x,q)$ lies in $[0, 2\pi]$.

Example 3.2. From this perspective, we will present two analyses of the attractiveness of a number of Cars in this example, considering several characteristics represented by $\mathbb{Q}=\{q_1, q_2\}$. herefore, we assume there are two houses that are represent by reference set $\mathbb{X}=\{x_1, x_2\}$ and $\mathbb{Q}=\{q_1, q_2\}$ where $q_1 = white$ and *black* represent cities under consideration and decision parameters (attractiveness). Then, the complex Q-fuzzy set $\mathrm{P}_{\mathbb{Q}}$ is given by:

$$\mathrm{P}_{\mathbb{Q}}=\left\{\left(\frac{\left\langle 0,4.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_1,q_1)},\frac{\left\langle 0,4.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_1,q_2)}\ ,\ \frac{\left\langle 0,4.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_2,q_1)},\frac{\left\langle 0,4.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_2,q_2)}\right)\right.$$
$$\left.,\left(\frac{\left\langle 0,4.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_1,q_1)},\frac{\left\langle 0,4.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_1,q_2)}\ ,\ \frac{\left\langle 0,4.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_1,q_1)},\frac{\left\langle 0,4.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_1,q_2)}\right)\right\}$$

In this context, the $\mathrm{P}_{\mathbb{Q}}$ refers to the effect of indicates the percentage of users who prefer the color white over black. the closer the score is to 1, the greater the attractiveness. While the phrase 'term score' indicates a buyer's justifiable score.

Definition 3.3. Let $\mathrm{P}_{\mathbb{Q}}$be a $C\mathbb{Q}-FS$. Then $\mathrm{P}_{\mathbb{Q}}$ is called null $\mathrm{C}\mathbb{Q}-$FS and denotes as $\mathrm{P}_{\phi_{\mathbb{Q}}}$ and given as following

$$\mathrm{P}_{\phi_{\mathbb{Q}}}=\left\{\left(\frac{\left\langle 0.\mathfrak{e}^{2\pi(0)}\right\rangle}{(x,q)}\right)\right\}$$

Definition 3.4. Let $\mathrm{P}_{\mathbb{Q}}$be a $C\mathbb{Q}-FS$. Then $\mathrm{P}_{\mathbb{Q}}$ is called absolute $\mathrm{C}\mathbb{Q}-$FS and denotes as $\mathrm{P}_{\Xi_{\mathbb{Q}}}$ and given as following

$$\mathrm{P}_{\Xi_{\mathbb{Q}}}=\left\{\left(\frac{\left\langle 1.\mathfrak{e}^{2\pi(1)}\right\rangle}{(x,q)}\right)\right\}$$

Definition 3.5. Assume that $\mathrm{P}_{\mathbb{Q}_1}$ and $\mathrm{P}_{\mathbb{Q}_2}$ be $\mathrm{C}\mathbb{Q}-$FS.Then $\mathrm{P}_{\mathbb{Q}_1}$ is subset of $\mathrm{P}_{\mathbb{Q}_2}$ and this relation denotes as $\mathrm{P}_{\mathbb{Q}_1}\sqsubseteq\mathrm{P}_{\mathbb{Q}_2}$ if for every $(x,q)\in(\mathbb{X},\mathbb{Q})$.Then $\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q)\leq\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q)$.

Definition 3.6. Assume that $\mathrm{P}_{\mathbb{Q}_1}$ and $\mathrm{P}_{\mathbb{Q}_2}$ be $\mathrm{C}\mathbb{Q}-$FS.Then $\mathrm{P}_{\mathbb{Q}_1}$ is equal of $\mathrm{P}_{\mathbb{Q}_2}$ and this relation denotes as $\mathrm{P}_{\mathbb{Q}_1}=\mathrm{P}_{\mathbb{Q}_2}$ if for every $(x,q)\in(\mathbb{X},\mathbb{Q})$.Then $\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q)=\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q)$.

Definition 3.7. Assume that $\mathrm{P}_{\mathbb{Q}}\in C\mathbb{Q}-FS(\mathbb{X})$. Then the complement of $\mathrm{P}_{\mathbb{Q}}$ denotes as $\mathrm{P}_{\mathbb{Q}}{}^{C}$and it given as following:

$$\mathrm{P}_{\mathbb{Q}}^{C}=\left\{\left\langle\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)=\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)\,,\mathfrak{e}^{i2\pi\left(\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)\right)}\right\rangle.(x,q)\in\mathbb{X}\times\mathbb{Q}\right\}$$

Then, for amplitude term

$$\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)=1-\dddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x,q)$$

and for phase term $\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)=1-\dddot{\mu}_{\Gamma_{\mathbb{Q}}(\mathfrak{a})}(x,q)$

Example 3.8. Take the part $\mathrm{P}_{\mathbb{Q}}(x_1,q_1)=\left(\frac{\left\langle 0,4.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_1,q_1)}\right)$ then the complement of this term given as following $\mathrm{P}_{\mathbb{Q}}^{C}(x_1,q_1)=\left(\frac{\left\langle 0,6.\mathfrak{e}^{2\pi(0.1)}\right\rangle}{(x_1,q_1)}\right)$.

Proposition 3.9. *Let* $\mathrm{P}_{\mathbb{Q}} \in CQ\!-\!FSS\,(\mathbb{X})$*. Then* $({\mathrm{P}_{\mathbb{Q}}}^{c})^{c}{=}\mathrm{P}_{\mathbb{Q}}$.

Proof. *This theory can be proven by relying on definitions 1 and 2 and some mathematical properties, as follows: Take*

$$\mathrm{P}_{\mathbb{Q}} = \left\{\left\langle x, \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x,q)\,, x{\in}\mathbb{X}\ and\ q{\in}\mathbb{Q}\right\rangle\right\}\ \mathrm{P}_{\mathbb{Q}}^{C}{=}\left\{\left\langle \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)\,{=}\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)\,, \mathfrak{e}^{i2\pi\left(\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)\right)}\right\rangle.\,(x,q)\in\mathbb{X}{\times}\mathbb{Q}\right\}$$

$$=\left\{\left\langle 1{-}\,\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x,q)\,, \mathfrak{e}^{i2\pi\left(1-\,\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x,q)\right)}\right\rangle\ .\,(x,q)\in\mathbb{X}{\times}\mathbb{Q}\right\}$$

Now take

$$({\mathrm{P}_{\mathbb{Q}}}^{c})^{c}{=}\left\{\left\langle 1{-}\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)\,, \mathfrak{e}^{i2\pi\left(1-\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}^{C}(x,q)\right)}\right\rangle\ (x,q)\in\mathbb{X}{\times}\mathbb{Q}\right\}$$

$$=\left\{\left\langle 1{-}\left(1{-}\,\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x,q)\right), \mathfrak{e}^{i2\pi\left(1-\left(1-\,\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x,q)\right)\right)}\right\rangle\ (x,q)\in\mathbb{X}{\times}\mathbb{Q}\right\}$$

$$=\left\{\left\langle \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x,q)\,, \mathfrak{e}^{i2\pi\left(\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}}}(x,q)\right)}\right\rangle.\,(x,q)\in\mathbb{X}{\times}\mathbb{Q}\right\}$$

$$=\mathrm{P}_{\mathbb{Q}}$$

Definition 3.10. Assume that $\mathrm{P}_{\mathbb{Q}_1}$ and $\mathrm{P}_{\mathbb{Q}_2}$ two $C\mathbb{Q}{-}FS$ define on $(\mathbb{X}{\times}\mathbb{Q})$. Then the union between $\mathrm{P}_{\mathbb{Q}_1}$ and $\mathrm{P}_{\mathbb{Q}_2}$ is given as following relation $\mathrm{P}_{\mathbb{Q}_1}\bigcup\mathrm{P}_{\mathbb{Q}_2}{=}\mathrm{P}_{\mathbb{Q}_3}$ and define as following formula:

$$\mathrm{P}_{\mathbb{Q}_3}\ = \left\{\left\langle x, \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_3}}\ (x,q)\,, x{\in}\mathbb{X}\ and\ q{\in}\mathbb{Q}\right\rangle\right\}$$

Where $\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_3}}(x,q) = max\left[\ \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q)\,,\ \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q)\right]$
and
$\dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_3}}(x,q) = max\left[\ \dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q)\,,\ \dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q)\right]$.

Definition 3.11. Assume that $\mathrm{P}_{\mathbb{Q}_1}$ and $\mathrm{P}_{\mathbb{Q}_2}$ two $C\mathbb{Q}{-}FS$ define on $(\mathbb{X}{\times}\mathbb{Q})$. Then the intersection between $\mathrm{P}_{\mathbb{Q}_1}$ and $\mathrm{P}_{\mathbb{Q}_2}$ is given as following relation $\mathrm{P}_{\mathbb{Q}_1}\bigcap\mathrm{P}_{\mathbb{Q}_2}{=}\mathrm{P}_{\mathbb{Q}_3}$ and define as following formula:

$$\mathrm{P}_{\mathbb{Q}_3}\ = \left\{\left\langle x, \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_3}}\ (x,q)\,, x{\in}\mathbb{X}\ and\ q{\in}\mathbb{Q}\right\rangle\right\}$$

Where $\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_3}}(x,q) = min\left[\ \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q)\,,\ \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q)\right]$ and $\dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_3}}(x,q) = min\left[\ \dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q)\,,\ \dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q)\right]$.

To illustrate how both union and intersection definitions work on CQ-FSs, we present the following numerical example.

Example 3.12. Let $\mathbb{X}{=}\{x_1,x_2\}$ be a reference set both $\mathbb{Q}{=}\{q_1,q_2\}$. Then, the CQ-FSSs$\mathrm{P}_{\mathbb{Q}_1}$ and $\mathrm{P}_{\mathbb{Q}_2}$ are given by:

$$\mathrm{P}_{\mathbb{Q}_1}{=}\left\{\left(\frac{\left\langle 0,1.\mathfrak{e}^{2\pi(0.9)}\right\rangle}{(x_1,q_1)},\frac{\left\langle 0,9.\mathfrak{e}^{2\pi(0.5)}\right\rangle}{(x_1,q_2)}\ ,\ \frac{\left\langle 0,6.\mathfrak{e}^{2\pi(0.4)}\right\rangle}{(x_2,q_1)},\frac{\left\langle 0,7.\mathfrak{e}^{2\pi(0.8)}\right\rangle}{(x_2,q_2)}\right)\right.$$
$$\left.,\left(\frac{\left\langle 0,2.\mathfrak{e}^{2\pi(0.3)}\right\rangle}{(x_1,q_1)},\frac{\left\langle 0,8.\mathfrak{e}^{2\pi(0.7)}\right\rangle}{(x_1,q_2)}\ ,\ \frac{\left\langle 0,6.\mathfrak{e}^{2\pi(0.8)}\right\rangle}{(x_1,q_1)},\frac{\left\langle 0,7.\mathfrak{e}^{2\pi(0.5)}\right\rangle}{(x_1,q_2)}\right)\right\}$$

$$\mathrm{P}_{\mathbb{Q}_2}=\left\{\left(\frac{\langle 0,3.\mathfrak{e}^{2\pi(0.5)}\rangle}{(x_1,q_1)},\frac{\langle 0,7.\mathfrak{e}^{2\pi(0.8)}\rangle}{(x_1,q_2)}\ ,\ \frac{\langle 0,8.\mathfrak{e}^{2\pi(0.1)}\rangle}{(x_2,q_1)},\frac{\langle 0,2.\mathfrak{e}^{2\pi(0.2)}\rangle}{(x_2,q_2)}\right)\right\}$$

Then,

$$\mathrm{P}_{\mathbb{Q}_3}=\left\{\left(\frac{\langle 0,3.\mathfrak{e}^{2\pi(0.9)}\rangle}{(x_1,q_1)},\frac{\langle 0,9.\mathfrak{e}^{2\pi(0.8)}\rangle}{(x_1,q_2)}\ ,\ \frac{\langle 0,8.\mathfrak{e}^{2\pi(0.4)}\rangle}{(x_2,q_1)},\frac{\langle 0,7.\mathfrak{e}^{2\pi(0.8)}\rangle}{(x_2,q_2)}\right)\right.$$
$$\left.\left(\frac{\langle 0,2.\mathfrak{e}^{2\pi(0.3)}\rangle}{(x_1,q_1)},\frac{\langle 0,8.\mathfrak{e}^{2\pi(0.7)}\rangle}{(x_1,q_2)}\ ,\ \frac{\langle 0,6.\mathfrak{e}^{2\pi(0.8)}\rangle}{(x_1,q_1)},\frac{\langle 0,7.\mathfrak{e}^{2\pi(0.5)}\rangle}{(x_1,q_2)}\right)\right\}$$

and

$$\mathrm{P}_{\mathbb{Q}_3}=\left\{\left(\frac{\langle 0,1.\mathfrak{e}^{2\pi(0.5)}\rangle}{(x_1,q_1)},\frac{\langle 0,7.\mathfrak{e}^{2\pi(0.5)}\rangle}{(x_1,q_2)}\ ,\ \frac{\langle 0,6.\mathfrak{e}^{2\pi(0.1)}\rangle}{(x_2,q_1)},\frac{\langle 0,2.\mathfrak{e}^{2\pi(0.2)}\rangle}{(x_2,q_2)}\right)\right.$$
$$\left.\left(\frac{\langle 0,2.\mathfrak{e}^{2\pi(0.3)}\rangle}{(x_1,q_1)},\frac{\langle 0,8.\mathfrak{e}^{2\pi(0.7)}\rangle}{(x_1,q_2)}\ ,\ \frac{\langle 0,6.\mathfrak{e}^{2\pi(0.8)}\rangle}{(x_1,q_1)},\frac{\langle 0,7.\mathfrak{e}^{2\pi(0.5)}\rangle}{(x_1,q_2)}\right)\right\}$$

Theorem 3.13. *Let* $\mathrm{P}_{\mathbb{Q}_1}$, $\mathrm{P}_{\mathbb{Q}_2}$ *and* $\mathrm{P}_{\mathbb{Q}_3}$ *be three* $C\mathbb{Q}-FSs$ *define on* $(\mathbb{X}\times\mathbb{Q})$*.Then the following facts are fulfilled.*

(1) $\mathrm{P}_{\mathbb{Q}_1}\bigcup\mathrm{P}_{\phi_{\mathbb{Q}}}=\mathrm{P}_{\mathbb{Q}_1}$ *and* $\mathrm{P}_{\mathbb{Q}_1}\bigcup\mathrm{P}_{\Xi_{\mathbb{Q}}}=\mathrm{P}_{\Xi_{\mathbb{Q}}}$
(2) $\mathrm{P}_{\mathbb{Q}_1}\bigcap\mathrm{P}_{\phi_{\mathbb{Q}}}=\mathrm{P}_{\phi_{\mathbb{Q}}}$ *and* $\mathrm{P}_{\mathbb{Q}_1}\bigcap(\Xi_{\mathbb{Q}},\mathbb{A})=\mathrm{P}_{\mathbb{Q}_1}$
(3) $\mathrm{P}_{\mathbb{Q}_1}\bigcup\mathrm{P}_{\mathbb{Q}_2}=\mathrm{P}_{\mathbb{Q}_2}\bigcup\mathrm{P}_{\mathbb{Q}_1}$.
(4) $\mathrm{P}_{\mathbb{Q}_1}\bigcap\mathrm{P}_{\mathbb{Q}_2}=\mathrm{P}_{\mathbb{Q}_2}\bigcap\mathrm{P}_{\mathbb{Q}_1}$.
(5) $(\mathrm{P}_{\mathbb{Q}_1}\bigcup\mathrm{P}_{\mathbb{Q}_2})\bigcup\mathrm{P}_{\mathbb{Q}_3}=\mathrm{P}_{\mathbb{Q}_1}\bigcup(\mathrm{P}_{\mathbb{Q}_2}\bigcup\mathrm{P}_{\mathbb{Q}_3})$
(6) $(\mathrm{P}_{\mathbb{Q}_1}\bigcap\mathrm{P}_{\mathbb{Q}_2})\bigcap\mathrm{P}_{\mathbb{Q}_3}=\mathrm{P}_{\mathbb{Q}_1}\bigcap(\mathrm{P}_{\mathbb{Q}_2}\bigcap\mathrm{P}_{\mathbb{Q}_3})$

Proof (i)**.** *This limit of the theorem can be proven by relying on the definitions of union and intersection of* $C\mathbb{Q}-FSs$ *and some mathematical properties as following:*

$$\begin{aligned}\mathrm{P}_{\mathbb{Q}_1}\bigcup\mathrm{P}_{\phi_{\mathbb{Q}}}&=\left\{\left\langle(x,q),\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}\cup\mathrm{P}_{\phi_{\mathbb{Q}}}}(x,q),x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\rangle\right\}\\&=\left\{\left\langle(x,q),max\left[\dddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q),\ \dddot{\mu}_{\phi_{\mathbb{Q}}}(x,q)\right],\right.\right.\\&\qquad\left.\left.max\left[\dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q),\ \dddot{\vartheta}_{\phi_{\mathbb{Q}}}(x,q)\right]\right\rangle x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\}\\&=\left\{\left\langle(x,q),\dddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q).\ \dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q)\right\rangle x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\}\\&=\left\{\left\langle(x,q),\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q),x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\rangle\right\}=\mathrm{P}_{\mathbb{Q}_1}\end{aligned}$$

And

$$\begin{aligned}\mathrm{P}_{\mathbb{Q}_1}\bigcup\mathrm{P}_{\Xi_{\mathbb{Q}}}&=\left\{\left\langle(x,q),\ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}\cup\mathrm{P}_{\Xi_{\mathbb{Q}}}}(x,q),x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\rangle\right\}\\&=\left\{\left\langle(x,q),max\left[\dddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q),\ \dddot{\mu}_{\mathrm{P}_{\Xi_{\mathbb{Q}}}}(x,q)\right],\right.\right.\\&\qquad\left.\left.max\left[\dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q),\ \dddot{\vartheta}_{\mathrm{P}_{\Xi_{\mathbb{Q}}}}(x,q)\right]\right\rangle x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\}\\&=\left\{\left\langle(x,q),\dddot{\mu}_{\mathrm{P}_{\Xi_{\mathbb{Q}}}}(x,q).\ \dddot{\vartheta}_{\mathrm{P}_{\Xi_{\mathbb{Q}}}}(x,q)\right\rangle x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\}\\&=\left\{\left\langle(x,q),\ddot{\mu}_{\mathrm{P}_{\Xi_{\mathbb{Q}}}}(x,q),x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\rangle\right\}=\mathrm{P}_{\Xi_{\mathbb{Q}}}.\end{aligned}$$

***Proof* (ii).** *The proof is similar to i, but based on the definition of intersection.*

***Proof* (iii).** *This limit of the theorem can be proven by relying on the definitions of union and intersection of $C\mathbb{Q}-FSs$ and some mathematical properties as following:*

$$
\begin{aligned}
\mathrm{P}_{\mathbb{Q}_1}\bigcup\mathrm{P}_{\mathbb{Q}_2} &= \left\{\left\langle (x,q), \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}\bigcup\mathrm{P}_{\mathrm{P}_{\mathbb{Q}_2}}}\ (x,q), x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\rangle\right\} \\
&= \Big\{\Big\langle (x,q), max\left[\ \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q),\ \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q)\right], \\
& max\left[\ \dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q),\ \dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q)\right]\Big\rangle x\in\mathbb{X}\ and\ q\in\mathbb{Q}\Big\} \\
&= \Big\{\Big\langle (x,q), max\left[\ \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q),\ \ddot{\mu}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q)\right], \\
& max\left[\ \dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_2}}(x,q),\ \dddot{\vartheta}_{\mathrm{P}_{\mathbb{Q}_1}}(x,q)\right]\Big\rangle x\in\mathbb{X}\ and\ q\in\mathbb{Q}\Big\} \\
&= \mathrm{P}_{\mathbb{Q}_2}\bigcup\mathrm{P}_{\mathbb{Q}_1}
\end{aligned}
$$

***Proof* (iv).** *The proof is similar to iii, but based on the definition of intersection.*

Proposition 3.14. *Let$\mathrm{P}_{\mathbb{Q}_1}$ and $\mathrm{P}_{\mathbb{Q}_2}$ be three $C\mathbb{Q}-FSs$ define on $(\mathbb{X}\times\mathbb{Q})$.Then the following De Morgan's laws are fulfilled.*

(1) $(\mathrm{P}_{\mathbb{Q}_1}\cup\mathrm{P}_{\mathbb{Q}_2})^c = (\mathrm{P}_{\mathbb{Q}_1})^c\cap(\mathrm{P}_{\mathbb{Q}_1})^c$.

(2) $(\mathrm{P}_{\mathbb{Q}_1}\cap\mathrm{P}_{\mathbb{Q}_2})^c = (\mathrm{P}_{\mathbb{Q}_1})^c\cup(\mathrm{P}_{\mathbb{Q}_1})^c$.

Proof: *The proof is clear based on the definitions of the basic operations on CQ-FSs.*

4. Application in Medical diagnosis under the Complex Q-Fuzzy Environment

The coronavirus that emerged in 2019 is a highly contagious respiratory virus that poses a significant risk due to its rapid spread and the severity of its symptoms. These symptoms include chest pain, abdominal pain, sore throat, fever, and mouth ulcers. The intensity and duration of these symptoms vary from patient to patient. This risk can be minimized or even eliminated by seeking medical attention as soon as possible, where a doctor can assess the severity of the infection and determine whether hospitalization is necessary. In this section we will test the proposed model for dealing with the medical diagnosis process for a number of patients based on a number of different symptoms.

Algorithm:

To implement this performance in accordance with the mathematical structure of our proposed concept, we will introduce the following algorithm.

Step 1. The reference set $\mathbb{X}$ represent the number of patients while the set $\mathbb{Q}$ represents the symptoms.

Step 2. Building a CQ-FS model that assesses the interaction between each patient and their associated set of symptoms.

Step 3. Converting units of CQ-FS $\mathrm{P}_{\mathbb{Q}}$ from complex to Q-FS $\Psi_{\mathbb{Q}}$ real area according to the following formula:

$$\Psi_{\mathbb{Q}} = \left\{\left\langle x, \ddot{\mu}_{\Psi_{\mathbb{Q}}}(x,q), x\in\mathbb{X}\ and\ q\in\mathbb{Q}\right\rangle\right\}$$

Where $\ddot{\mu}_{\Psi_{\mathbb{Q}}}(x,q) = \omega_1\, \ddot{\rho}_{\mathrm{P}_{\mathbb{Q}}}(x,q) + \omega_2 \left(\frac{1}{2\pi}\right) \ddot{\varphi}_{\mathrm{P}_{\mathbb{Q}}}(x,q)$

Such that $\omega_1 + \omega_2 = 1$.

Step 4. Find the value Z_i where $Z_i = \frac{\ddot{\mu}_{\Psi_{\mathbb{Q}_n}}(x,q)}{n}$, where $i, n = 1, 2, 3, \ldots m$

Step 5. Perform a comparison between the values of Z_i where if $Z_i \geq$0.5 then We conclude that the patient is in a critical stage and requires him to remain in the hospital.

At a government hospital in Kuala Lumpur, a respiratory specialist examined five patients $\mathbb{X} = \{x_1, x_2, x_3, x_4, x_5\}$with a range of the following symptoms chest pain, abdominal pain, sore throat, fever, oral cavity ulceration represented by $\mathbb{Q} = \{q_1, q_2, q_3, q_4, q_5\}$. Accordingly, the doctor's opinion on each case was translated into the data shown in Table 1, which describes the process by which each symptom set's effect on each patient is mediated by the interaction between the $\mathbb{X}$ and $\mathbb{Q}$

Table 1. Present $\mathrm{P}_{\mathbb{Q}}$ values

$\mathbb{X}/\mathbb{Q}$	q_1	q_2	q_3	q_4	q_5
x_1	$\left\langle 0,2.e^{2\pi\left(\frac{3}{7}\right)}\right\rangle$	$\left\langle 0,5.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$	$\left\langle 0,5.e^{2\pi\left(\frac{1}{7}\right)}\right\rangle$	$\left\langle 0,5.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$	$\left\langle 0,2.e^{2\pi\left(\frac{2}{7}\right)}\right\rangle$
x_2	$\left\langle 0,7.e^{2\pi\left(\frac{2}{7}\right)}\right\rangle$	$\left\langle 0,3.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$	$\left\langle 0,3.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$	$\left\langle 0,1.e^{2\pi\left(\frac{2}{7}\right)}\right\rangle$	$\left\langle 0,8.e^{2\pi\left(\frac{5}{7}\right)}\right\rangle$
x_3	$\left\langle 0,5.e^{2\pi\left(\frac{6}{7}\right)}\right\rangle$	$\left\langle 0,8.e^{2\pi\left(\frac{3}{7}\right)}\right\rangle$	$\left\langle 0,8.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$	$\left\langle 0,3.e^{2\pi\left(\frac{5}{7}\right)}\right\rangle$	$\left\langle 0,3.e^{2\pi\left(\frac{3}{7}\right)}\right\rangle$
x_4	$\left\langle 0,1.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$	$\left\langle 0,7.e^{2\pi\left(\frac{5}{7}\right)}\right\rangle$	$\left\langle 0,7.e^{2\pi\left(\frac{5}{7}\right)}\right\rangle$	$\left\langle 0,7.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$	$\left\langle 0,4.e^{2\pi\left(\frac{5}{7}\right)}\right\rangle$
x_5	$\left\langle 0,1.e^{2\pi\left(\frac{2}{7}\right)}\right\rangle$	$\left\langle 0,5.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$	$\left\langle 0,5.e^{2\pi\left(\frac{6}{7}\right)}\right\rangle$	$\left\langle 0,6.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$	$\left\langle 0,7.e^{2\pi\left(\frac{4}{7}\right)}\right\rangle$

The CQ-FS data presented in Table 1 indicate the interaction between each patient and their symptom cluster. The term "amplitude" refers to the intensity of the patient's symptoms, while the term "phase" refers to the duration of the symptoms. A 7-day period was estimated as the duration of the symptoms. For example, a numerical value of $\left\langle 0,2.e^{2\pi\left(\frac{3}{7}\right)}\right\rangle$ indicates that patient $x_1$experienced a fever of $0,2$ for a period of 3 days. Next by applying the second step of the algorithm mentioned above, we obtain the Q-FS values shown in Table 2.

Table 2. PresentΨ_Q values

$\mathbb{X}/\mathbb{Q}$	q_1	q_2	q_3	q_4	q_5
x_1	$\langle 0.32\rangle$	$\langle 0.54\rangle$	$\langle 0.28\rangle$	$\langle 0.54\rangle$	$\langle 0.24\rangle$
x_2	$\langle 0.44\rangle$	$\langle 0.46\rangle$	$\langle 0.46\rangle$	$\langle 0.20\rangle$	$\langle 0.74\rangle$
x_3	$\langle 0.71\rangle$	$\langle 0.57\rangle$	$\langle 0.66\rangle$	$\langle 0.71\rangle$	$\langle 0.37\rangle$
x_4	$\langle 0.38\rangle$	$\langle 0.70\rangle$	$\langle 0.70\rangle$	$\langle 0.62\rangle$	$\langle 0.58\rangle$
x_5	$\langle 0.21\rangle$	$\langle 0.54\rangle$	$\langle 0.71\rangle$	$\langle 0.58\rangle$	$\langle 0.62\rangle$

Based on the values shown in Table 2, we obtain the values of Z_i respectively and as following: $Z_1 = 0.38$, $Z_2 = 0.46$, $Z_3 = 0.61$, $Z_4 = 0.69$, $Z_5 = 0.53$. Therefore, those who are infected are $[x_3, x_4$ and $x_5]$, while those who are not infected are $[x_1$ and $x_2]$.

In addition, we can illustrate the variation in the severity of the injury among patients through the following statistical chart.

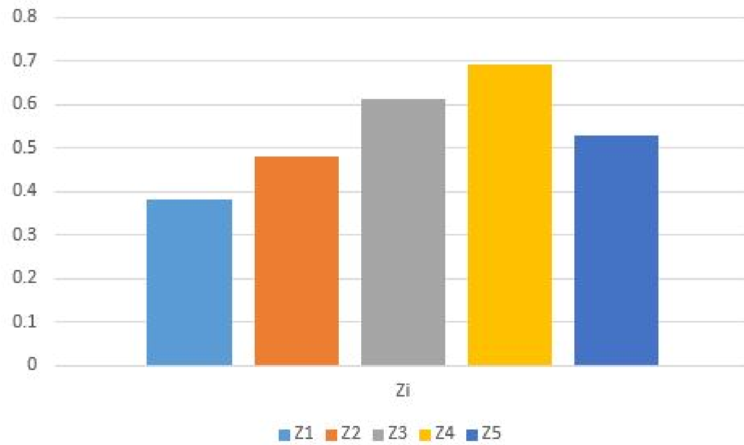

Fig. 2 The contrast between injury severity illustrates

4.1. **Structural Comparison.** In this subsection, we will work to show the difference between the proposed concept and some previous concepts to illustrate the novelty of this concept. Our proposed concept, CQ-FS, is a combination of FS and the Q-set, expressed in terms of complex numbers. The organic membership function has a domain consisting of the product of the universal set and the Q-set, with a stable element called the complex level. This performance differs from both FSs and CFSs. This concept provides the appropriate key to handling uncertainty data effectively, where each element in the global set is related to a corresponding element in the L set within a Cartesian product framework. Each resulting component is given a complex fuzzy score that measures the degree of correlation between the two elements. To make this comparison clearer, we present the following comparison table where this comparison focuses on certain structural criteria.

TABLE 3. Comparative between our model and existing models

Concept	Domain	Co-domain	Q-set effect	Periodic
FS [1]	X	$[0, 1]$	$\times$	$\times$
Q-FS [11]	X×Q	$[0, 1]$	$\surd$	$\times$
CFS [13]	X	Complex unit interval	$\times$	$\surd$
The proposed method CQFS [14]	X×Q	Complex unit interval	$\surd$	$\surd$

5. Conclusion

This study established a concept of a CQ-FS by extending the theories of CFS under the impact of Q set. The basic operations on CQ-FS, namely complement, subset, union, and intersection operations, were defined. Subsequently, the basic properties of these operations, such as De Morgan's laws and other relevant laws about the concept of CQ-FS were proved. Finally, a practical application that illustrates how our proposed concept works with decision-making problems in the medical environment. This new extension will provide a significant addition to existing theories for handling uncertainty data under time effects, where time plays a vital role in the decision process and spurs more developments of further research and pertinent applications. For further research, we intend to take into account unknown weight information to develop some real applications of CQ-FS in other environments, where the phase term may represent other variables.

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