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EXTENSION OF THE REGIME METHOD TO GROUP DECISION-MAKING IN A Q-RUNG ORTHOPAIR FUZZY ENVIRONMENT

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Abstract. q-Rung orthopair fuzzy sets (qROFS) constitute a robust extension of Intuitionistic Fuzzy Sets (IFS) and Pythagorean Fuzzy Sets (PyFS). They provide an advanced framework for handling complex information in an uncertain environment. The REGIME method is an MCDM approach used to provide a definitive ranking of alternatives. In the literature, a few extensions have integrated REGIME with IFS and PyFS. However, not only do very few of these extensions address group decision-making problems, but they are also limited when dealing with q-ROFS type data. In this article, we propose an extension of the REGIME method to group decision-making in a qROFS environment. Our methodology proceeds in three phases: aggregation of fuzzy decision matrices, formulation of a non-linear programming model based on qROFS entropy for the objective weighting of criteria, and adaptation of REGIME to evaluate the options. This new methodology extends, for the first time, the REGIME method to a qROFS environment. The proposed method is applied to a numerical example on supplier selection to demonstrate its effectiveness.

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1. Introduction

Group decision-making (MCGDM) is often faced with decision problems in which the nature of the information is uncertain or ambiguous. Most often, the natural language used to express a judgment is subjective and vague. To address this vagueness, the fuzzy set theory was introduced by Zadeh [12–15], which is characterized by a membership degree. Since then, many studies have been devoted to improving the representation of uncertainty in the decision-making process. First, the Intuitionistic Fuzzy Sets (IFS) [16, 17] extend classical fuzzy sets. They are characterized by three parameters: the degree of membership, the degree of non-membership, and the degree of hesitation,

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such that the sum of the membership and non-membership degrees is less than or equal to 1. Next, IFS were extended to Pythagorean Fuzzy Sets (PyFS) [18–20]. In PyFS, the sum of the squares of the membership and non-membership degrees is less than or equal to 1. Finally, q-rung orthopair fuzzy sets (q-ROFS) [21] were proposed as a generalization of IFS and PyFS. For q-ROFS, the sum of the q-th power ($q \geqslant 1$) of the membership and non-membership degrees is less than or equal to 1, thus allowing flexibility in representing human judgments during a multicriteria decision-making (MCDM) process in a fuzzy environment.

Many MCDM methods such as TOPSIS [22], VIKOR [23], REGIME [7,8], ELECTRE [24], etc., have been extended to fuzzy frameworks. We focus particularly on the REGIME method, a compensatory MCDM methodology based on pairwise comparisons of alternatives. Each pair of alternatives receives $+1$ for the better, -1 for the worse, and 0 in case of equality for each criterion. These scores are combined with the weights of the criteria to determine the optimal alternative across all criteria. The main strength of REGIME is that it does not necessarily require normalization of decision matrices during the decision-making process. This characteristic, which avoids the tedious procedures present in other MCDM approaches, has motivated several fuzzy extensions of REGIME. Haktanir and Kahraman [26] proposed a CRITIC-REGIME methodology using Picture Fuzzy Sets (PFS) applied to the problem of selecting wearable medical technologies. Oztaysie et al. [25] suggested an extension of REGIME in a Pythagorean fuzzy environment for identifying the optimal waste disposal site.

In the MCDM process, the weighting of criteria has a significant impact on the final ranking of alternatives. In the literature, weighting methods are mainly divided into two major families: subjective methods such as AHP [27] and objective methods based on entropy measures such as Shannon's entropy [6]. Objective methods are rarely studied in the literature, although they are highly appreciated.

On one hand, the scope of REGIME extensions is limited to PyFS, and on the other hand, very few studies address the objective weighting of criteria in a q-ROFS environment. In this article, we propose an extension of the REGIME method (REGIME-q-ROFS) for group decision-making in a q-rung orthopair fuzzy environment. The advantage of the proposed method is that it can be used to solve problems in IFS, PyFS, and q-ROFS frameworks. In addition, a nonlinear programming model based on q-ROFS entropy is developed for the objective weighting of criteria.

The structure of this article is presented as follows: Section 2 briefly reviews the basic concepts of IFS, PyFS, q-ROFS, and q-ROFS entropy measures. Section 3 presents the proposed MCGDM methodology based on q-ROFS. In Section 4, a numerical example on supplier selection is used to illustrate the effectiveness of the proposed approach (REGIME-qROFS). Section 5 provides a comparative study followed by a discussion with some existing MCGDM methods to validate the obtained results. Finally, Section 6 concludes this work by presenting future research directions.

2. Some fundamental concepts

In this section $X = \{x_1, x_2, ..., x_i, ...\}$ refers to a set of objects also known as the discourse universe.

2.1. Classic fuzzy sets.

Definition 2.1. *An fuzzy set A in X [12, 30] is a set of ordered pairs of the form:*

$$A = \{(x_i, \mu_A(x_i)) : x_i \in X, \mu_A(x_i) \in [0,1]\} \tag{1}$$

where $\mu_A : X \longrightarrow [0,1]$ is the membership function of A. The value $\mu_A(x_i)$ represents the degree of membership of x_i in A, for all $x_i \in X$.
Using the definition 1, the basic concepts and operations defined on classical sets can be naturally extended to fuzzy sets [30]:
Consider A and B two fuzzy sets in X of functions with respective membership functions μ_A and μ_B.

(1) A fuzzy set is empty if and only if $\mu_\emptyset$ is identically zero on X;

(2) **Equality**: $A = B$ if and only if $\mu_A(x_i) = \mu_B(x_i), \quad \forall x_i \in X$;

(3) The **complement** of A in X note A^c is defined by $\mu_{A^c} = 1 - \mu_A$;

(4) **Inclusion**: $A \subset B$ if and only if $\mu_A \leqslant \mu_B$.

(5) **Union** :

$$A \cup B : \mu_{A \cup B} = max\,\{\mu_A, \mu_B\}$$

(6) **Intersection** :

$$A \cap B : \mu_{A \cap B} = min\,\{\mu_A, \mu_B\}$$

2.2. Intuitionistic fuzzy sets.

Definition 2.2. *An intuitionistic fuzzy set(IFS) A in X [16, 30] is a set defined by equation 2:*

$$A = \{(x_i, \mu_A(x_i), \nu_A(x_i)) : \forall x_i \in X, \mu_A(x_i), \nu_A(x_i) \in [0,1], 0 \leqslant \mu_A(x_i) + \nu_A(x_i) \leqslant 1\} \tag{2}$$

In the equation 2 $\mu_A : X \longrightarrow [0,1]$ is the membership function of A and $\nu_A : X \longrightarrow [0,1]$ the non-membership function of A and, $\mu_A(x_i), \nu_A(x_i)$ are respectively the degrees of membership and non-membership of x_i has A. For simplicity, we denote $A = (\mu_A, \nu_A)$. Moreover, $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i)$ is the hesitation degree of x_i to A. If $\pi_A(x_i) = 0$, then A is a fuzzy set. The properties and operations on fuzzy sets are extended to intuitionistic fuzzy sets. For more information, the reader may refer to [17].

2.3. **Pythagorean fuzzy sets.** In the definition of an intuitionistic fuzzy set, the condition $0 \leqslant \mu_A(x_i) + \nu_A(x_i) \leqslant 1$ is very restrictive, thereby limiting the flexibility regarding the membership and non-membership degrees. This motivated Yager [18] to propose the Pythagorean fuzzy sets.

Definition 2.3. *An pythagorean fuzzy set (PyFS) A in X [18] is a set defined by equation 3:*

$$A = \left\{(x_i, \mu_A(x_i), \nu_A(x_i)) : \forall x_i \in X, \mu_A(x_i), \nu_A(x_i) \in [0,1], 0 \leqslant \mu_A^2(x_i) + \nu_A^2(x_i) \leqslant 1\right\} \tag{3}$$

The functions $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ are respectively the membership and non-membership functions of A. Also, $\pi_A(x_i) = \sqrt{1 - \mu_A^2(x_i) - \nu_A^2(x_i)}$ is called the Pythagorean index. For more details, see [18–20].

2.4. **The q-rung orthopair fuzzy sets (q-ROFS).** The q-rung orthopair fuzzy sets (q-ROFS) [21] generalize the intuitionistic fuzzy sets and the Pythagorean fuzzy sets. The q-ROFS significantly enhance the flexibility of the membership and non-membership functions, particularly in a multi-criteria decision-making process.

Definition 2.4. *An q-rung orthopair fuzzy sets (q-ROFS) A in X [21,30] is a set defined by equation 4:*

$$A = \left\{(x_i, \mu_A(x_i), \nu_A(x_i)) : \forall x_i \in X, \mu_A(x_i), \nu_A(x_i) \in [0,1], 0 \leqslant \mu_A^q(x_i) + \nu_A^q(x_i) \leqslant 1\right\} \tag{4}$$

Characterized by a membership function $\mu_A : X \longrightarrow [0,1]$ and a non-membership function $\nu_A : X \longrightarrow [0,1]$, with $q \in \mathbb{N}^*$. Also, $\pi_A(x_i) = \sqrt[q]{1 - \mu_A^q(x_i) - \nu_A^q(x_i)}$ is called the hesitation degree or the q-rung orthopair fuzzy index. For a given q-ROFS, the pair $(\mu_A(x_i), \nu_A(x_i))$ is called a q-rung orthopair fuzzy number (q-ROFN). By taking $q = 1$ and $q = 2$ in Definition 4, we obtain the IFS and PyFS, respectively.

Property 2.1. [28]. *Consider $\alpha_1 = (\mu_1, \nu_1)$ and $\alpha_2 = (\mu_2, \nu_2)$ two q-ROFN and $\sigma > 0$, then:*

(1) $\alpha_1^c = (\nu_1, \mu_1)$
(2) $\alpha_1 \vee \alpha_2 = (\max\{\mu_1, \nu_1\}, \min\{\mu_2, \nu_2\})$
(3) $\alpha_1 \wedge \alpha_2 = (\min\{\mu_1, \nu_1\}, \max\{\mu_2, \nu_2\})$
(4) $\alpha_1 \oplus \alpha_2 = \left((\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q)^{1/q}, \nu_1 \nu_2\right)$
(5) $\alpha_1 \otimes \alpha_2 = \left(\mu_1 \mu_2, (\nu_1^q + v_2^q - v_1^q v_2^q)^{1/q}\right)$
(6) $\sigma \alpha_1 = \left(\left(1 - (1 - \mu_1^q)^\sigma\right)^{1/q}, \nu_1^\sigma\right)$
(7) $\alpha_1^\sigma = \left(\mu_1^\sigma, \left(1 - (1 - v_1^q)^\sigma\right)^{1/q}\right)$

Property 2.2. *Consider two q-runq orthopair fuzzy sets A et B such as:*

$A = \left\{(x_i, \mu_A(x_i), \nu_A(x_i)) : \forall x_i \in X, \mu_A(x_i), \nu_A(x_i) \in [0,1], 0 \leqslant \mu_A^{q_2}(x_i) + \nu_A^{q_2}(x_i) \leqslant 1\right\}$
$B = \left\{(x_i, \mu_A(x_i), \nu_A(x_i)) : \forall x_i \in X, \mu_A(x_i), \nu_A(x_i) \in [0,1], 0 \leqslant \mu_A^{q_1}(x_i) + \nu_A^{q_1}(x_i) \leqslant 1\right\}$

If $q_2 > q_1$ alors A is greater than B.

Proof: See [30].

Definition 2.5. (***Score function for q-ROFNs***)
For all q-ROFN $\alpha = (\mu, \nu)$, Mi et al. [2] defined the score function of α (equation 5)

$$S_M(\alpha) = \frac{2 + \mu^q - \nu^q}{(2 - \mu^q + \nu^q)(1 + \pi^q)} \qquad with \qquad S_M(\alpha) \in \left[\frac{1}{3}, 3\right] \tag{5}$$

Definition 2.6. *For all $\alpha_1, \alpha_2 \in q - ROFS$, we have the preference and indifference relations of the q-ROFS [2]:*

$$\alpha_1 > \alpha_2 \Longleftrightarrow S_M(\alpha_1) > S_M(\alpha_2) \tag{6}$$

$$\alpha_1 = \alpha_2 \Longleftrightarrow S_M(\alpha_1) = S_M(\alpha_2) \tag{7}$$

Remark 2.1. *For more information on score functions, readers are referred to [28, 29]. We chose the score function proposed in [2] because, in addition to effectively distinguishing q-ROFSs, it reduces the comparison process.*

2.5. **Weighted Interactive Einstein Fuzzy Geometric Aggregation Operator.** The Weighted Interactive Einstein Fuzzy Geometric Aggregation Operator is based on the Einstein operational laws from Definition 2.7.

Definition 2.7. [1] ***Interactive Einstein Operations***
Consider $\alpha_1 = (\mu_1, \nu_1)$ and $\alpha_2 = (\mu_2, \nu_2)$ two q-ROFNs, and λ a positive real number.
The interactive Einstein operations on q-ROFNs are defined as follows:

$$\alpha_1 \oplus \alpha_2 = \left(\sqrt[q]{\frac{\prod_{s=1}^{2}(1+\mu_s^q) - \prod_{s=1}^{2}(1-\mu_s^q)}{\prod_{s=1}^{2}(1+\mu_s^q) + \prod_{s=1}^{2}(1-\mu_s^q)}}, \sqrt[q]{\frac{2\left(\prod_{s=1}^{2}(1-\mu_s^q) - \prod_{s=1}^{2}(1-\mu_s^q-\nu_s^q)\right)}{\prod_{s=1}^{2}(1+\mu_s^q) + \prod_{s=1}^{2}(1-\mu_s^q)}} \right) \tag{8}$$

$$\alpha_1 \otimes \alpha_2 = \left(\sqrt[q]{\frac{2\left(\prod_{s=1}^{2}(1-v_s^q) - \prod_{s=1}^{2}(1-v_s^q-\mu_s^q)\right)}{\prod_{s=1}^{2}(1+v_s^q) + \prod_{s=1}^{2}(1-v_s^q)}}, \sqrt[q]{\frac{\prod_{s=1}^{2}(1+v_s^q) - \prod_{s=1}^{2}(1-v_s^q)}{\prod_{s=1}^{2}(1+v_s^q) + \prod_{s=1}^{2}(1-v_s^q)}} \right) \tag{9}$$

$$\sigma\alpha = \left(\sqrt[q]{\frac{(1+\mu^q)^\sigma - (1-\mu^q)^\sigma}{(1+\mu^q)^\sigma + (1-\mu^q)^\sigma}}, \sqrt[q]{\frac{2\left((1-\mu^q)^\sigma - (1-\mu^q-\nu^q)^\sigma\right)}{(1+\mu^q)^\sigma + (1-\mu^q)^\sigma}} \right) \tag{10}$$

$$\alpha^\sigma = \left(\sqrt[q]{\frac{2\left((1-\nu^q)^\sigma - (1-\nu^q-\mu^q)^\sigma\right)}{(1+\nu^q)^\sigma + (1-\nu^q)^\sigma}}, \sqrt[q]{\frac{(1+\nu^q)^\sigma - (1-\nu^q)^\sigma}{(1+\nu^q)^\sigma + (1-\nu^q)^\sigma}} \right) \tag{11}$$

Based on the equations in Definition 2.7, Farid and Riaz [1] proposed the q-Rung Orthopair Fuzzy Einstein Interactive Weighted Geometric (q-ROFEIWG) Aggregation Operator as follows.

Definition 2.8. [1] *Consider* $\{\alpha_i : i \in \overline{1,n}\}$ *a set of q-ROFN with* $\alpha_i = (\mu_i, \nu_i)$.
Let $w_i, i = \overline{1,n}$ *the weight of the fuzzy number* α_i *such as* $w_i \in [0,1]$ *and* $\sum_{i=1}^{n} w_i = 1$.
The opeartor $q - ROFEIWG : q - ROFN^n \longrightarrow q - ROFN$ *is defined by equation 12:*

$$\begin{aligned} q-\mathrm{ROFEIWG}\left(\alpha_1, \alpha_2, \ldots, \alpha_n\right) &= \otimes_{s=1}^{n} \alpha_s^{\omega_s} \\ &= \alpha_1^{\omega_1} \otimes \alpha_2^{\omega_2} \otimes \ldots \otimes \alpha_n^{\omega_n} \end{aligned} \tag{12}$$

Under the same assumptions as in Definition 12, the following theorem is stated.

Theorem 2.1. [1]

$$\begin{aligned} \text{q-ROFEIWG}\left(\alpha_1, \alpha_2, \ldots, \alpha_n\right) = & \left(\sqrt[q]{\frac{2\left(\prod_{s=1}^{n}\left(1-\nu_s^q\right)^{\omega_s} - \prod_{s=1}^{n}\left(1-\nu_s^q-\mu_s^q\right)^{\omega_s}\right)}{\prod_{s=1}^{n}\left(1+\nu_s^q\right)^{\omega_s} + \prod_{s=1}^{n}\left(1-\nu_s^q\right)^{\omega_s}}}, \right. \\ & \left. \sqrt[q]{\frac{\prod_{s=1}^{n}\left(1+\nu_s^q\right)^{\omega_s} - \prod_{s=1}^{n}\left(1-\nu_s^q\right)^{\omega_s}}{\prod_{s=1}^{n}\left(1+\nu_s^q\right)^{\omega_s} + \prod_{s=1}^{n}\left(1-\nu_s^q\right)^{\omega_s}}} \right) \end{aligned} \tag{13}$$

Proof: See [1]

Remark 2.2. *The q-ROFEIWG operator enhances the interaction between the membership degree and the non-membership degree during aggregation.*

2.6. **Knowledge Measure and Entropy Measure for q-ROFS.** The fuzzy knowledge measure determines the amount of knowledge and precision associated with a fuzzy set, while the entropy measure quantifies the amount of information conveyed in a fuzzy manner.

Definition 2.9. [3] (***Knowledge Measure***)
Consider $A = \{\alpha_i = (\mu(x_i), \nu(x_i)) : i = \overline{1,n}\}$ *a set q-ROFS in* X.
knowledge measure de A *is a function* $\mathcal{I}^q : q - ROFS \longrightarrow [0,1]$ *defined by:*

$$\mathcal{I}^q(A) = \left(\frac{1}{n} \sum_{i=1}^{n} \left\{ 1 - \frac{1}{2} \left(1 - \left(\mu^{2q}(x_i) + \nu^{2q}(x_i)\right) + \frac{4}{\pi} \tan^{-1}\left[\pi^{2q}(x_i)\right] \right) \right\} \right)^{\frac{1}{q}} \tag{14}$$

where $\pi(x_i) = (1 - \mu^q(x_i) - \nu^q(x_i))^{\frac{1}{q}}$

Property 2.3. [3] *The knowledge measures satisfy the following properties:*

$$\mathcal{I}^q(A) = 1 \quad if \quad A \quad est \quad un \quad ensemble \quad classique \tag{15}$$

$$\mathcal{I}^q(A) = 0 \quad \pi(x_i) = 1 \quad \forall x_i \in X \tag{16}$$

$$\mathcal{I}^q(A) \geqslant \mathcal{I}^q(B) \quad si \pi_A(x_i) \leqslant \pi_B(x_i) \quad \forall x_i \in X \tag{17}$$

$$\mathcal{I}^q(A) = \mathcal{I}^q(A^c) \tag{18}$$

where A^c *is the complement of* A.

Proof: See [3]

Definition 2.10. [3] (***Entropy measure***)
Consider $A = \left\{\alpha_i = (\mu(x_i), \nu(x_i)) : i = \overline{1,n}\right\}$ *a set q-ROFS in* X.
The entropy measure of A *is a function* $\mathcal{E}^q : q - ROFS \longrightarrow [0,1]$ *defined by:*

$$\mathcal{E}^q(A) = 1 - \left(\frac{1}{n}\sum_{i=1}^{n}\left\{1 - \frac{1}{2}\left(1 - \left(\mu^{2q}(x_i) + \nu^{2q}(x_i)\right) + \frac{4}{\pi}\tan^{-1}\left[\pi^{2q}(x_i)\right]\right)\right\}\right)^{\frac{1}{q}} \tag{19}$$

In other words:

$$\mathcal{E}^q(A) = 1 - \mathcal{I}^q(A)$$

Theorem 2.2. [3]
Consider A *a set q-ROFS in* X. *The mesure entropy measure satisfy the following properties:*

(1) $0 \leqslant \mathcal{E}^q(A) \leqslant 1$

(2) $\mathcal{E}^q(A) = 0$ *if* A *is a crisp set*

(3) $\mathcal{E}^q(A) = 1$ *if* $\pi(x_i) = 1, \forall x_i \in X$

(4) $\mathcal{E}^q(A) \geqslant \mathcal{E}^q(B)$ *if* $\pi_A(x_i) \leqslant \pi_B(x_i), \forall x_i \in X$

(5) $\mathcal{E}^q(A^c) = \mathcal{E}^q(A)$

3. Fuzzy REGIME Methodology in a q-ROFS Environment (REGIME-qROFS)

In this section, we have extended the classical REGIME method to q-rung orthopair fuzzy sets and proposed an entropic nonlinear programming model for the weighting of criteria. Our methodology unfolds in three phases. First, the aggregation of decision matrices using the Weighted Interactive Einstein Fuzzy Geometric Aggregation Operator [1]; second, our entropic nonlinear programming model in the second phase; and finally, the fuzzy REGIME method adapted to q-ROFS for the final ranking of alternatives in the third phase.

In a MCGDM (Multi-Criteria Group Decision Making) problem in a q-ROFS environment, we consider three fundamental sets. The set of n alternatives, denoted by $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_i, ..., \mathcal{A}_n\}$; the set of m conflicting criteria, denoted by $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_j, ..., \mathcal{C}_m\}$; and the set of p decision-makers, denoted by $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, ..., \mathcal{D}_k, ..., \mathcal{D}p\}$, who must interact and cooperate during the decision-making process. The REGIME-qROFS methodology is described below: Throughout the following, the criterion $\mathcal{C}j$ will be simply referred to as criterion j.

3.1. Aggregation of Decision Matrix.

Step 1: `Determination of the Decision-Makers' Weights`

The importance of the decision-makers is evaluated qualitatively using linguistic terms expressed by q-rung orthopair fuzzy numbers (q-ROFS). Let $\widetilde{E}_{\mathcal{D}_k} = (\tilde{\mu}^{(k)}, \tilde{\nu}^{(k)}, \tilde{\pi}^{(k)})$ be the q-ROFS evaluation of the k^{th} decision-maker. The weight of decision-maker $\mathcal{D}_k$ is calculated using the fuzzy knowledge measure from Definition 14.

$$\phi_k = \frac{I(\widetilde{E}_{\mathcal{D}_k})}{\sum_{k=1}^{p} I(\widetilde{E}_{\mathcal{D}_k})} \tag{20}$$

where:

$$I(\widetilde{E}_{\mathcal{D}_k}) = \left\{ \left[1 - \frac{1}{2}\left(1 - (\tilde{\mu}^{(k)})^{2q} - (\tilde{\nu}^{(k)})^{2q} + \frac{4}{\pi} \arctan\left[\left(1 - (\tilde{\mu}^{(k)})^{q} - (\tilde{\nu}^{(k)})^{q} \right)^2 \right] \right) \right] \right\}^{\frac{1}{q}} \tag{21}$$

is the knowledge measure of the fuzzy evaluation $\widetilde{E}_{\mathcal{D}_k}$.

We have:

$$\sum_{k=1}^{p} \phi_k = 1 \tag{22}$$

Step 2: `Aggregation of q-Rung Orthopair Fuzzy Decision Matrices`

Consider $\widetilde{\mathcal{R}}^{(k)} = \left(\tilde{\alpha}_{ij}^{(k)}\right)_{n\times m}$, where $\tilde{\alpha}_{ij}^{(k)} = \left(\tilde{\mu}_{ij}^{(k)}, \tilde{\nu}_{ij}^{(k)}\right)$ is the q-rung orthopair fuzzy decision matrix of decision-maker $\mathcal{D}_k$ (see Table 1) and ϕ_k is their weight.

The q-rung orthopair fuzzy decision matrices of all decision-makers are aggregated to obtain a global q-rung orthopair fuzzy matrix, denoted by $\widetilde{\mathcal{R}} = (\tilde{\alpha}_{ij})_{n\times m}$, where $\tilde{\alpha}_{ij} = (\tilde{\mu}_{ij}, \tilde{\nu}_{ij})$ (see Table 2).

The choice of the aggregation operator is based on that proposed by Farid and Riaz [1]. It is the q-ROFEIWG (q-Rung Orthopair Fuzzy Einstein Interactive Weighted Geometric) operator, which enhances the interaction between the membership and non-membership functions of the q-ROFS.

$$\begin{aligned} \tilde{\alpha}_{ij} = q\text{-}\,\mathrm{ROFEIWG}(\tilde{\alpha}_{ij}^{(1)}, \tilde{\alpha}_{ij}^{(2)}, \ldots, \tilde{\alpha}_{ij}^{(p)}) &= \bigotimes_{k=1}^{p} (\tilde{\alpha}_{ij}^{(k)})^{\phi_k} \\ &= (\tilde{\alpha}_{ij}^{(1)})^{\phi_k} \otimes (\tilde{\alpha}_{ij}^{(2)})^{\phi_k} \otimes \cdots \otimes (\tilde{\alpha}_{ij}^{(p)})^{\phi_p} \end{aligned} \tag{23}$$

Thus:

$$\begin{aligned} \tilde{\alpha}_{ij} = &\left(\sqrt[q]{\frac{2\left(\prod_{k=1}^{p}\left(1-\tilde{\nu}_{ij,k}^{q}\right)^{\phi_k} - \prod_{k=1}^{p}\left(1-\tilde{\nu}_{ij,k}^{q}-\tilde{\mu}_{ij,k}^{q}\right)^{\phi_k} \right)}{\prod_{k=1}^{p}\left(1+\tilde{\nu}_{ij,k}^{q}\right)^{\phi_k} + \prod_{k=1}^{p}\left(1-\tilde{\nu}_{ij,k}^{q}\right)^{\phi_k}}}, \right. \\ &\left. \sqrt[q]{\frac{\prod_{k=1}^{p}\left(1+\tilde{\nu}_{ij,k}^{q}\right)^{\phi_k} - \prod_{k=1}^{p}\left(1-\tilde{\nu}_{ij,k}^{q}\right)^{\phi_k}}{\prod_{k=1}^{p}\left(1+\tilde{\nu}_{ij,k}^{q}\right)^{\phi_k} + \prod_{k=1}^{p}\left(1-\tilde{\nu}_{ij,k}^{q}\right)^{\phi_k}}} \right) \end{aligned} \tag{24}$$

where $\tilde{\alpha}_{ij} = (\tilde{\mu}_{ij}, \tilde{\nu}_{ij})$ is the overall evaluation of alternative $\mathcal{A}_i$ according to the criterion $\mathcal{C}_j$.

Step 3: `Normalization of the Aggregated Fuzzy Matrix` $\widetilde{\mathcal{R}}$

It is important to normalize the aggregated fuzzy matrix $\widetilde{\mathcal{R}}$, especially for the weighting of criteria, since the criteria are often conflicting. On one hand, there are cost criteria, and on the other hand, benefit criteria. Using Equation 25, we obtain the normalized aggregated fuzzy matrix, denoted by $\widetilde{\mathcal{R}}_N$.

$$\tilde{\alpha}_{ij} = \begin{cases} (\tilde{\mu}_{ij}, \tilde{\nu}_{ij}) & for \quad the \quad benefit \quad criteria \\ (\tilde{\nu}_{ij}, \tilde{\mu}_{ij}) & for \quad the \quad cost \quad criteria \end{cases} \tag{25}$$

TABLE 1. Fuzzy matrix of $\mathcal{D}_k$

$$\widetilde{\mathcal{R}}^{(k)} = \begin{bmatrix} \left(\tilde{\mu}_{11}^{(k)}, \tilde{\nu}_{11}^{(k)}\right) & \left(\tilde{\mu}_{12}^{(k)}, \tilde{\nu}_{12}^{(k)}\right) & \cdots & \left(\tilde{\mu}_{1m}^{(k)}, \tilde{\nu}_{1m}^{(k)}\right) \\ \left(\tilde{\mu}_{21}^{(k)}, \tilde{\nu}_{21}^{(k)}\right) & \left(\tilde{\mu}_{22}^{(k)}, \tilde{\nu}_{22}^{(k)}\right) & \cdots & \left(\tilde{\mu}_{2m}^{(k)}, \tilde{\nu}_{2m}^{(k)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\tilde{\mu}_{n1}^{(k)}, \tilde{\nu}_{n1}^{(k)}\right) & \left(\tilde{\mu}_{n2}^{(k)}, \tilde{\nu}_{n2}^{(k)}\right) & \cdots & \left(\tilde{\mu}_{nm}^{(k)}, \tilde{\nu}_{nm}^{(k)}\right) \end{bmatrix}$$

TABLE 2. aggregated fuzzy matrix from the k decision-making

$$\widetilde{\mathcal{R}} = \begin{bmatrix} (\tilde{\mu}_{11}, \tilde{\nu}_{11}) & (\tilde{\mu}_{12}, \tilde{\nu}_{12}) & \cdots & (\tilde{\mu}_{1m}, \tilde{\nu}_{1m}) \\ (\tilde{\mu}_{21}, \tilde{\nu}_{21}) & (\tilde{\mu}_{22}, \tilde{\nu}_{22}) & \cdots & (\tilde{\mu}_{2m}, \tilde{\nu}_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ (\tilde{\mu}_{n1}, \tilde{\nu}_{n1}) & (\tilde{\mu}_{n2}, \tilde{\nu}_{n2}) & \cdots & (\tilde{\mu}_{nm}, \tilde{\nu}_{nm}) \end{bmatrix}$$

3.2. **Weighting of the Criteria.** The weights of the criteria significantly influence the final ranking of the alternatives. It is therefore important to properly evaluate the weights for an accepted decision. To this end, we have proposed a nonlinear programming model based on the entropy weighting method. This model builds upon those proposed in [4,5]. These models are inspired by Shannon entropy [6], which was used for the first time to weight criteria. The fuzzy entropy of a fuzzy set measures the amount of information conveyed in an uncertain manner by that set. The smaller the fuzzy entropy, the more precise the transmitted information. Our model belongs to the family of objective weightings, which significantly reduces subjectivity in the evaluation of weights. The proposed model is defined by Equation 26 below:

$$\begin{aligned} \min \quad & F = \sum_{j=1}^{m} \mathcal{W}_j \sum_{i=1}^{n} \mathcal{E}^q\left(\tilde{\alpha}_{ij}\right) \\ sc \quad & \begin{cases} \sum_{j=1}^{m} \mathcal{W}_j^2 = 1 \\ \mathcal{W}_j \geq 0 \quad , \quad j = 1, 2, ..., m \end{cases} \end{aligned} \tag{26}$$

The solution of madel is given by equation 27.

$$\mathcal{W}_j = \frac{\sum_{i=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij})}{\sqrt{\sum_{j=1}^{m} \left(\sum_{i=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij})\right)^2}} \quad , \quad j = 1, 2, ..., m \tag{27}$$

Proof:

Let $L(\mathcal{W}_j, \lambda)$ the Lagrangian as follows:

$$L(\mathcal{W}_j, \lambda) = \sum_{j=1}^{m} \mathcal{W}_j \sum_{1=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij}) + \frac{\lambda}{2}\left(\sum_{j=1}^{m} \mathcal{W}_j^2 - 1\right)$$

with $\mathcal{E}^q(\widetilde{\alpha}_{ij}) = 1 - \mathcal{I}^q(\widetilde{\alpha}_{ij})$ the entropy measure of the fuzzy number $\widetilde{\alpha}_{ij}$. We have:

$$\frac{\partial L(\mathcal{W}_j, \lambda)}{\partial \mathcal{W}_j} = \sum_{i=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij}) + \lambda \mathcal{W}_j \tag{28}$$

and

$$\frac{\partial L(\mathcal{W}_j, \lambda)}{\partial \lambda} = \frac{1}{2}\left(\sum_{j=1}^{m} \mathcal{W}_j^2 - 1\right) \tag{29}$$

From equation 28, we have $\mathcal{W}_j$:

$$\mathcal{W}_j = -\frac{\sum_{1=1}^{n} \mathcal{E}^q(\alpha_{ij})}{\lambda} \tag{30}$$

and from equation 29, we obtain:

$$\sum_{j=1}^{m} \mathcal{W}_j^2 = 1 \tag{31}$$

By replacing 30 in 31, we obtain:

$$\lambda^2 = \sum_{j=1}^{m} \left(\sum_{1=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij})\right)^2 \tag{32}$$

Given the equation 30 and $w_j \geqslant 0$, we take:

$$\lambda = -\sqrt{\sum_{j=1}^{m} \left(\sum_{1=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij})\right)^2} \tag{33}$$

Therefore,

$$\mathcal{W}_j = \frac{\sum_{1=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij})}{\sqrt{\sum_{j=1}^{m} \left(\sum_{1=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij})\right)^2}}, j = \overline{1, m} \tag{34}$$

The constraint of the proposed model (Equation 26) allows us to deduce the normalized weight, defined by Equation 35. :

$$\mathcal{W}_j^* = \mathcal{W}_j^2 = \frac{\left(\sum_{1=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij})\right)^2}{\sum_{j=1}^{m} \left(\sum_{1=1}^{n} \mathcal{E}^q(\widetilde{\alpha}_{ij})\right)^2}, j = \overline{1, m} \tag{35}$$

Hence the result.

3.3. **REGIME method modified.** The aggregated fuzzy matrix $\widetilde{\mathcal{R}}$ and the weight vector $\mathcal{W} = (\mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_j, ..., \mathcal{W}_m)$ obtained in Phases 1 and 2, respectively, are used in this final phase.

Step 1: `The superiority index based on a score function`

The score function proposed by Xiaomei Mi [2], denoted by R_{score}, is used to compare q-rung orthopair fuzzy sets. This function is defined by equation 36 as follows:

$$R_{score}(\tilde{\alpha}_{ij}) = \frac{2 + \tilde{\mu}_{ij}^q - \tilde{\nu}_{ij}^q}{\left(2 - \tilde{\mu}_{ij}^q + \tilde{\nu}_{ij}^q\right) \times \left(1 + \tilde{\pi}_{ij}^q\right)} = \frac{2 + \tilde{\mu}_{ij}^q - \tilde{\nu}_{ij}^q}{\left(2 - \tilde{\mu}_{ij}^q + \tilde{\nu}_{ij}^q\right) \times \left(2 - \tilde{\mu}_{ij}^q - \tilde{\nu}_{ij}^q\right)} \tag{36}$$

For two alternatives $\mathcal{A}_i$ and $\mathcal{A}_{i'}$, the superiority index $\mathcal{I}_{ii'}$ given by Equation 37 is the set of criteria for which alternative $\mathcal{A}_i$ is at least as good as alternative $\mathcal{A}_{i'}$.

$$\mathcal{I}_{ii'} = \left\{\mathcal{C}_j \in \mathcal{C} \mid R_{score}(\tilde{\alpha}_{ij}) \geq R_{score}(\tilde{\alpha}_{i'j})\right\} \tag{37}$$

Step 2: `The superiority identifier`

The superiority identifier is calculated using the following equation:

$$\widehat{\mathcal{I}_{ii'}} = \sum_{j \in \mathcal{I}_{ii'}} \mathcal{W}_j \tag{38}$$

where $\mathcal{W}_j$ is the weight assigned to criterion j.

Step 3: `Determination of the Impact Matrix`

The impact matrix, also called the ranking matrix, is derived from the information contained in the aggregated fuzzy matrix. In our case, since this information consists of q-rung orthopair fuzzy sets, we use the score function R_{score} defined by Equation 36 as the ranking function. Let $\succeq$ (at least as good as) be a total preorder defined on the q-ROFS.

Let $\tilde{\alpha}_{ij}$ and $\tilde{\alpha}_{i'j}$ be two q-rung orthopair fuzzy evaluations of the respective alternatives $\mathcal{A}_i$ and $\mathcal{A}_{i'}$ according to criterion j.

$$\tilde{\alpha}_{ij} \succeq \tilde{\alpha}_{i'j} \iff R_{score}(\tilde{\alpha}_{ij}) \geqslant R_{score}(\tilde{\alpha}_{i'j}) \tag{39}$$

Equation 39 allows the comparison of q-ROFS. The higher the score, the better the fuzzy set's position. We denote $\mathcal{R} = (\alpha_{ij})_{n\times m}$ as the impact matrix (see Table 3).

TABLE 3. Impact Matrix

$$\mathcal{R} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1m} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{nm} \end{bmatrix}$$

where α_{ij} is the ranking of alternative $\mathcal{A}_i$ according to criterion j.

Step 4: `Construction of the REGIME Matrix (MREG)`

From the impact matrix, the REGIME matrix is constructed using the REGIME vectors $VREG_{ii'}$ defined by Equation 40.

$$VREG_{ii'} = \left(VREG_{ii',1}, VREG_{ii',2}, ..., VREG_{ii',j}, ..., VREG_{ii',m}\right) \tag{40}$$

Each vector $VREG_{ii'}$, composed of the identifiers $VREG_{ii',j}$ (Equation 41), is the result of the pairwise comparison of alternatives $\mathcal{A}_i$ and $\mathcal{A}_{i'}$ across all criteria $\mathcal{C}_j$ [7–9].

$$VREG_{ii',j} = \begin{cases} +1 \quad si \quad \alpha_{ij} > \alpha_{i'j} \\ 0 \quad si \quad \alpha_{ij} = \alpha_{i'j} \\ -1 \quad si \quad \alpha_{ij} < \alpha_{i'j} \end{cases} \tag{41}$$

The set of REGIME vectors $VREG_{ii'}$ forms the REGIME matrix, given by Equation 42.

$$\begin{aligned} MREG = &\big[VREG_{12}, VREG_{13}, \ldots, VREG_{1n}, VREG_{21}, \\ &VREG_{23}, \ldots, VREG_{2n}, \ldots, VREG_{n1}, VREG_{n2}, \ldots, VREG_{n,n-1}\big] \end{aligned} \tag{42}$$

Step 5: `Calculation of guidage indices`

First technique: The guidage indice (Equation 43) allows obtaining the final ranking of the alternatives.

$$\overline{\text{VREG}}_{ii'} = \sum_{j=1}^{m} VREG_{ii',j} \times \mathcal{W}_j \tag{43}$$

If $\overline{\text{VREG}}_{ii'} > 0$ then option $\mathcal{A}_i$ is prefered $\mathcal{A}_{i'}$.

Second technique: The superiority identifier $\hat{\mathcal{I}} * ii'$ estimates the value of the better alternative. In fact, the REGIME approach is based on the subtraction $\hat{\mathcal{I}}_{ii'} - \hat{\mathcal{I}}_{i'i}$. A positive result of this subtraction indicates that alternative $\mathcal{A}_i$ is superior to alternative $\mathcal{A}_{i'}$, whereas a negative result shows the superiority of alternative $\mathcal{A}_{i'}$ over alternative $\mathcal{A}_i$.

Etape 6:: `Final Ranking of All Alternatives`

A comparison matrix $V = (v_{ii'})_{n\times n}$ of the alternatives is constructed using the following equation 44.

$$v_{ii'} = \begin{cases} +1 \quad si \quad \overline{\text{VREG}}_{ii'} > 0 \\ -1 \quad si \quad \overline{\text{VREG}}_{ii'} < 0 \end{cases} \tag{44}$$

Thus:

$$V = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \vdots \\ \mathcal{A}_n \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{A}_1 & \mathcal{A}_2 & \ldots & \mathcal{A}_n \end{array} \\ \begin{pmatrix} \cdot & v_{12} & \ldots & v_{1n} \\ v_{21} & \cdot & \ldots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1} & v_{n2} & \ldots & \cdot \end{pmatrix} \end{array}$$

4. Numerical Example

We consider a decision problem from [10].

An automotive company wishes to select the most suitable supplier for one of the key elements of its manufacturing process. After a preliminary assessment, five (05) suppliers$\{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5\}$ remain in contention for further evaluation. In order to evaluate the alternative suppliers, a committee composed of three (03) decision-makers $\{\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3\}$ was formed. Four (04) criteria are taken into consideration:

$\mathcal{C}_1$: Product quality;

$\mathcal{C}_2$: Relationship closeness;

$\mathcal{C}_3$: Delivery performance;

$\mathcal{C}_4$: Price.

Among then only $\mathcal{C}_4$ is the cost type attribute.

Take $q = 2$.

Linguistic terms and the corresponding q-ROFN used for the ratings of the decision makers are given in 4. The importance of the decision makers are given in 5.

Table 4. Linguistic terms for evaluating decision-makers

Linguistic terms	qROFN
Very important	(0.90,0.10)
Important	(0.75,0.20)
Medium	(0.50,0.45)
Unimportant	(0.35,0.60)
Very unimportant	(0.10,0.90)

Table 5. Importance of the three decision-makers

	DM_1	DM_2	DM_3
Linguistic terms	Very important	Medium	Important

Phase 1

Step 1: Calculating the weights of decision-makers

$\widetilde{E}_{\mathcal{D}_1} = (0.90, 0.10)$, $\widetilde{E}_{\mathcal{D}_2} = (0.50, 0.45)$ et $\widetilde{E}_{\mathcal{D}_3} = (0.75, 0.20)$ are the respective q-ROFN assessments of decision-makers $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$. Their respective weights are ϕ_1 , ϕ_2 and ϕ_3.

$$\phi_1 = \frac{\left\{\left[1 - \frac{1}{2}\left(1 - (\mu^{(1)})^2 - (\nu^{(1)})^2 + \frac{4}{\pi}\arctan\left(\left(1 - (\mu^{(1)})^1 - (\nu^{(1)})^1\right)^2\right)\right)\right]\right\}^{\frac{1}{1}}}{0.91 + 0.73 + 0.80}$$

$$= \frac{\left\{\left[1-\frac{1}{2}\left(1-(0.9)^2-(0.1)^2+\frac{4}{\pi}\arctan\left((1-0.9-0.1)^2\right)\right)\right]\right\}^{\frac{1}{1}}}{0.91+0.73+0.80}$$

$$= \frac{\left\{\left[1-\frac{1}{2}\left(1-0.81-0.01\right)\right]\right\}^{\frac{1}{1}}}{3}$$

$$= \frac{0.91}{0.91+0.73+0.80}$$

$$= 0.374$$

Similary, we find:

$$\phi_2 = 0.298 \qquad \phi_3 = 0.329$$

Step 2 : Aggregation of fuzzy matrix

The fuzzy matrix of the experts are given below:

$$\widetilde{\mathcal{R}}^{(1)} = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \\ \mathcal{A}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}_1 & \mathcal{C}_2 & \mathcal{C}_3 & \mathcal{C}_4 \end{array} \\ \begin{pmatrix} (0.7,0.2) & (0.6,0.3) & (0.8,0.1) & (0.7,0.2) \\ (0.6,0.3) & (0.5,0.4) & (0.7,0.2) & (0.6,0.3) \\ (0.9,0.1) & (0.8,0.1) & (0.8,0.1) & (0.8,0.1) \\ (0.6,0.3) & (0.5,0.4) & (0.8,0.1) & (0.7,0.2) \\ (0.5,0.4) & (0.4,0.5) & (0.7,0.2) & (0.5,0.4) \end{pmatrix} \end{array}$$

$$\widetilde{\mathcal{R}}^{(2)} = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \\ \mathcal{A}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}_1 & \mathcal{C}_2 & \mathcal{C}_3 & \mathcal{C}_4 \end{array} \\ \begin{pmatrix} (0.8,0.1) & (0.7,0.2) & (0.7,0.2) & (0.7,0.2) \\ (0.7,0.2) & (0.6,0.3) & (0.6,0.3) & (0.5,0.4) \\ (0.8,0.1) & (0.7,0.2) & (0.8,0.1) & (0.8,0.1) \\ (0.7,0.2) & (0.5,0.4) & (0.7,0.2) & (0.6,0.3) \\ (0.6,0.3) & (0.5,0.4) & (0.7,0.2) & (0.6,0.3) \end{pmatrix} \end{array}$$

$$\widetilde{\mathcal{R}}^{(3)} = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \\ \mathcal{A}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}_1 & \mathcal{C}_2 & \mathcal{C}_3 & \mathcal{C}_4 \end{array} \\ \begin{pmatrix} (0.7,0.2) & (0.6,0.3) & (0.8,0.1) & (0.7,0.2) \\ (0.5,0.4) & (0.7,0.2) & (0.6,0.3) & (0.6,0.3) \\ (0.8,0.1) & (0.8,0.1) & (0.7,0.2) & (0.7,0.2) \\ (0.7,0.2) & (0.6,0.3) & (0.7,0.2) & (0.6,0.3) \\ (0.6,0.3) & (0.5,0.4) & (0.6,0.3) & (0.5,0.4) \end{pmatrix} \end{array}$$

The three fuzzy matrix are merged using the equation 24:

For example:

$$\tilde{\alpha}_{23} = \left(\sqrt[q]{\frac{2\left(\prod_{k=1}^{3}\left(1-\tilde{\nu}_{23,k}^{q}\right)^{\phi_k} - \prod_{k=1}^{3}\left(1-\tilde{\nu}_{23,k}^{q}-\tilde{\mu}_{23,k}^{q}\right)^{\phi_k}\right)}{\prod_{k=1}^{3}\left(1+\tilde{\nu}_{23,k}^{q}\right)^{\phi_k} + \prod_{k=1}^{3}\left(1-\tilde{\nu}_{23,k}^{q}\right)^{\phi_k}}}, \right.$$
$$\left. \sqrt[q]{\frac{\prod_{k=1}^{3}\left(1+\tilde{\nu}_{23,k}^{q}\right)^{\phi_k} - \prod_{k=1}^{3}\left(1-\tilde{\nu}_{23,k}^{q}\right)^{\phi_k}}{\prod_{k=1}^{3}\left(1+\tilde{\nu}_{23,k}^{q}\right)^{\phi_k} + \prod_{k=1}^{3}\left(1-\tilde{\nu}_{23,k}^{q}\right)^{\phi_k}}} \right) \tag{45}$$
$$= (0.63, 0.27)$$

The aggregated matrix is shown below:

$$\widetilde{\mathcal{R}} = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \\ \mathcal{A}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}_1 & \mathcal{C}_2 & \mathcal{C}_3 & \mathcal{C}_4 \end{array} \\ \begin{pmatrix} (0.73, 0.17) & (0.63, 0.27) & (0.77, 0.13) & (0.70, 0.20) \\ (0.59, 0.31) & (0.60, 0.29) & (0.63, 0.27) & (0.57, 0.33) \\ (0.90, 0.10) & (0.77, 0.13) & (0.76, 0.14) & (0.76, 0.14) \\ (0.67, 0.23) & (0.54, 0.36) & (0.73, 0.17) & (0.63, 0.27) \\ (0.57, 0.33) & (0.47, 0.43) & (0.66, 0.24) & (0.53, 0.37) \end{pmatrix} \end{array}$$

Step 3 : Matrix normalisation $\widetilde{\mathcal{R}}$.

We obtain the matrix $\widetilde{\mathcal{R}}_N$ by normalising the matrix $\widetilde{\mathcal{R}}$ using the equation 25:

$$\widetilde{\mathcal{R}}_N = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \\ \mathcal{A}_5 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{C}_1 & \mathcal{C}_2 & \mathcal{C}_3 & \mathcal{C}_4 \end{array} \\ \begin{pmatrix} (0.73, 0.17) & (0.63, 0.27) & (0.77, 0.13) & (0.20, 0.70) \\ (0.59, 0.31) & (0.60, 0.29) & (0.63, 0.27) & (0.33, 0.57) \\ (0.90, 0.10) & (0.77, 0.13) & (0.76, 0.14) & (0.14, 0.76) \\ (0.67, 0.23) & (0.54, 0.36) & (0.73, 0.17) & (0.27, 0.63) \\ (0.57, 0.33) & (0.47, 0.43) & (0.66, 0.24) & (0.37, 0.53) \end{pmatrix} \end{array}$$

Phase 2: Weighting of criteria

Equation 26 makes it possible to obtain the weight vector of the criteria as follows:

$$\mathcal{W} = \begin{array}{c} \begin{array}{cccc} \mathcal{W}_1 & \mathcal{W}_2 & \mathcal{W}_3 & \mathcal{W}_4 \end{array} \\ \begin{pmatrix} 0.210 & 0.296 & 0.218 & 0.275 \end{pmatrix} \end{array}$$

Phase 3: Fuzzy REGIME method

Step 1 : Superiority index.

$$\mathcal{I}_{12} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}, \mathcal{I}_{13} = \{\mathcal{C}_3, \mathcal{C}_4\}, \mathcal{I}_{14} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}, \mathcal{I}_{15} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$$

$$\mathcal{I}_{23} = \{\mathcal{C}_4\}, \mathcal{I}_{24} = \{\mathcal{C}_2, \mathcal{C}_4\}, \mathcal{I}_{25} = \{\mathcal{C}_1, \mathcal{C}_2\}$$

$$\mathcal{I}_{34} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}, \mathcal{I}_{35} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}, \mathcal{I}_{45} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3\}$$

Step 2 : Superiority identifier.

$$\widehat{\mathcal{I}_{12}} = \mathcal{W}_1 + \mathcal{W}_2 + \mathcal{W}_3 = 0.724, \widehat{\mathcal{I}_{13}} = \mathcal{W}_3 + \mathcal{W}_4 = 0.514, \widehat{\mathcal{I}_{14}} = 0.724, \widehat{\mathcal{I}_{15}} = 0.724$$

$$\widehat{\mathcal{I}_{23}} = 0.275, \widehat{\mathcal{I}_{24}} = 0.571, \widehat{\mathcal{I}_{25}} = 0.506$$

$$\widehat{\mathcal{I}_{34}} = 0.724, \widehat{\mathcal{I}_{35}} = 0/724, \widehat{\mathcal{I}_{45}} = 0.724$$

Step 3 : Impact matrix

$$\mathcal{R} = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \\ \mathcal{A}_5 \end{array} \begin{array}{c} \begin{matrix} \mathcal{C}_1 & \mathcal{C}_2 & \mathcal{C}_3 & \mathcal{C}_4 \end{matrix} \\ \begin{pmatrix} 4 & 4 & 5 & 2 \\ 2 & 3 & 1 & 4 \\ 5 & 5 & 4 & 1 \\ 3 & 2 & 3 & 3 \\ 1 & 1 & 2 & 5 \end{pmatrix} \end{array}$$

Step 4 : REGIME matrix

$$MREG = \begin{array}{c} \\ VREG_{12} \\ VREG_{13} \\ VREG_{14} \\ VREG_{15} \\ VREG_{23} \\ VREG_{24} \\ VREG_{25} \\ VREG_{34} \\ VREG_{35} \\ VREG_{45} \end{array} \begin{pmatrix} \mathcal{C}_1 & \mathcal{C}_2 & \mathcal{C}_3 & \mathcal{C}_4 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & -1 \end{pmatrix} \begin{array}{c} \\ VREG_{51} \\ VREG_{52} \\ VREG_{53} \\ VREG_{54} \\ VREG_{31} \\ VREG_{41} \\ VREG_{42} \\ VREG_{43} \\ VREG_{44} \\ VREG_{45} \end{array} \begin{pmatrix} \mathcal{C}_1 & \mathcal{C}_2 & \mathcal{C}_3 & \mathcal{C}_4 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 \end{pmatrix}$$

Step 5 : Calculation of guidage index

First technique:

$\overline{\text{VREG}}_{12} = VREG_{12,1} \times \mathcal{W}_1 + VREG_{12,2} \times \mathcal{W}_2 + VREG_{12,3} \times \mathcal{W}_3 + VREG_{12,4} \times \mathcal{W}_4 = 1 \times 0.210 + 1 \times 0.296 + 1 \times 0.218 - 1 \times 0.275 = 0.450$

Similarly, we obtain:

$\overline{\text{VREG}}_{13} = -0.013, \overline{\text{VREG}}_{14} = 0.450, \overline{\text{VREG}}_{15} = 0.450$

$\overline{\text{VREG}}_{23} = -0.450, \overline{\text{VREG}}_{24} = 0.14, \overline{\text{VREG}}_{25} = 0.013$

$\overline{\text{VREG}}_{34} = 0.450, \overline{\text{VREG}}_{35} = 0.450, \overline{\text{VREG}}_{45} = 0.450$

Second technique:

$\widehat{\mathcal{I}_{12}} - \widehat{\mathcal{I}_{21}} = 0.724 - 0.276 = 0.448, \widehat{\mathcal{I}_{13}} - \widehat{\mathcal{I}_{31}} = 0.514 - 0.486 = 0.028$

$\widehat{\mathcal{I}_{14}} - \widehat{\mathcal{I}_{14}} = 0.448, \widehat{\mathcal{I}_{15}} - \widehat{\mathcal{I}_{51}} = 0.448, \widehat{\mathcal{I}_{23}} - \widehat{\mathcal{I}_{23}} = -0.45, \widehat{\mathcal{I}_{24}} - \widehat{\mathcal{I}_{42}} = 0.142$, $\widehat{\mathcal{I}_{25}} - \widehat{\mathcal{I}_{52}} = 0.012$,
$\widehat{\mathcal{I}_{34}} - \widehat{\mathcal{I}_{43}} = 0.448, \widehat{\mathcal{I}_{35}} - \widehat{\mathcal{I}_{53}} = 0.448, \widehat{\mathcal{I}_{45}} - \widehat{\mathcal{I}_{54}} = 0.448$

Step 6 : Ranking of alternatives.

The comparison matrix V is:

$$V = \begin{array}{c} \\ \mathcal{A}_1 \\ \mathcal{A}_2 \\ \mathcal{A}_3 \\ \mathcal{A}_4 \\ \mathcal{A}_5 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathcal{A}_1 & \mathcal{A}_2 & \mathcal{A}_3 & \mathcal{A}_4 & \mathcal{A}_5 \end{array} \\ \begin{pmatrix} 0 & 1 & -1 & 1 & 1 \\ -1 & 0 & -1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ -1 & -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & -1 & 0 \end{pmatrix} \end{array}$$

The final ranking of suppliers from best to worst is given below:

$$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_2 \succ \mathcal{A}_4 \succ \mathcal{A}_5$$

Option $\mathcal{A}_3$ is the best supplier.

q	**Operators / Authors**	**Method**	**Environment**	**Ranking**
–	IFWA / Boran et al. [10]	TOPSIS	IFSs	$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_2 \succ \mathcal{A}_4 \succ \mathcal{A}_5$
2	q-ROFWFA / Saha et al. [11]	Not determined	q-ROFSs	$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_4 \succ \mathcal{A}_2 \succ \mathcal{A}_5$
2	**Proposed method**	REGIME	q-ROFSs	$\mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_2 \succ \mathcal{A}_4 \succ \mathcal{A}_5$

Table 6. Comparison of results with existing methods

5. Comparison and Discussion

To evaluate the validity of the proposed REGIME-qROFS methodology, we applied our approach to a numerical example previously solved by [10,11]. The obtained results (Table 6) were compared with those reported by [10,11]. All three approaches yield similar results ($\mathcal{A}_3$ is the best supplier and $\mathcal{A}_5$ is the worst supplier). These results confirm the validity of the method proposed in this paper. The ranking of alternatives is identical to that obtained using the TOPSIS method in an intuitionistic fuzzy (IFS) environment as proposed by [10]. Regarding the comparison with the method using the q-ROFWFA operator of [11], the order of alternatives $\mathcal{A}_2$ and $\mathcal{A}_4$ is reversed. This inversion is strongly related to the choice of aggregation operator used in the decision-making process. The following points provide further insights to better understand the results of our study:

(1) The q-ROFWFA aggregation operator used by Saha et al. [11] is fair with respect to membership and non-membership degrees, thus consolidating preferences. However, it may produce erroneous

results in certain cases. Specifically, when at least one of the membership degrees is zero, the aggregated membership degree via the q-ROFWFA operator becomes zero. This significantly affects the outcome of the decision-making process. The integration of the q-ROFEIWG operator into the REGIME method proposed in this paper addresses this limitation.

(2) Unlike the TOPSIS method in [10], which can only handle intuitionistic fuzzy (IFS) data, the proposed REGIME-qROFS method applies not only in an intuitionistic fuzzy context by taking $q = 1$, but also can handle Pythagorean fuzzy data (for $q = 2$) and other types of data ($q > 2$). Therefore, our method generalizes the approach proposed by [10].

(3) In [10], each expert (decision-maker) evaluates the weighting of the criteria, which are then aggregated to obtain a global weighting. This evaluation approach does not reduce the subjective nature of judgments related to human perception. Saha et al. [11] improved this limitation by determining the criterion weights from the decision matrices using a linear programming model where the objective function is a weighted sum of the scores of each alternative. In this paper, we further improve this technique by integrating a nonlinear programming model and entropy measures [1], allowing us to obtain a weighting vector that minimizes subjectivity during the evaluation of criterion weights. Our criterion weighting model belongs to the family of objective weighting methods, which are highly appreciated in the literature.

(4) Our method also differs from existing approaches by evaluating decision-maker weights based on knowledge measures, better reflecting their degree of expertise in the decision-making process.

6. Conclusion

In this paper, we proposed a new MCGDM method in a q-Rung Orthopair Fuzzy (q-ROFS) environment, integrating the REGIME method and q-Rung Orthopair Fuzzy Numbers. The REGIME-qROFS methodology is the first extension of the classical REGIME method to a q-Rung Orthopair Fuzzy context. We also developed a quadratic optimization model based on entropy measures for criterion weighting. Moreover, our method considers the diverse opinions of decision-makers by incorporating a weighting of their expertise based on knowledge measures. The results obtained on a supplier selection example demonstrate that the proposed method is valid and consistent with existing approaches, while being stable and thereby improving the decision-making outcome. In addition to confirming the validity of the REGIME-qROFS method, it expands the applicability of the classical REGIME method to various decision-making problems in uncertain environments.

A limitation of this study is that it cannot handle q-ROFS type fuzzy information where membership and non-membership degrees are represented by linguistic variables. This limitation will be addressed in our future work.

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