Asia Pac. J. Math. 2026 **13**:121

FUZZY IDEALS NEAR RING INDICATED WITH FUZZY LINE GRAPH

HUSSAIN ALI MUEEN*, AMEER A. J. AL-SWIDI

Department of Mathematics, College of Education for Pure Sciences, University of Babylon, Hillah, Iraq

*Corresponding author: hh6561290@gmail.com

Received Apr. 21, 2026

Abstract. Within the folds of this paper, we will introduce a new concept for line graph known as completely -equiprime fuzzy graph ($\mathcal{C}\amalg_{prime\ \mathcal{F}I}\mathfrak{S}$) of near ring $\mathbb{N}$ with level ideal, which we will denote by the symbol $L\left(\mathbb{N},\mathcal{F},\mathcal{P}_{\ c},\sigma\right)$ and also explore the properties of the chromatic number and maximum degree and the conditions that level ideal must adhere to in order to be considered a valid line graph, a complete, planar and a Hamiltonian graphs. We also showed the conditions under which $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ the line graph is bipartite or not and we studied the color number when the near ring is finished. Then we found the conditions that must be placed on $L\left(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma\right)$ cycle graph represents its period, and we proved the degree of each vertex in $L\left(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma\right)$ when it is $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$. We discussed the study placed certain conditions on the near loops so that the line graph is outer planar. We also showed how the line graph looks when we exclude elements level ideal.

2020 Mathematics Subject Classification. 05C72.

Key words and phrases. completely equiprime fuzzy ideal; fuzzy ideal; Hamiltonian graph and planar graph; line graph.

1. Introduction

Research that links algebraic structures to graphical representations is one of the most important contemporary trends in modern algebra, as this interaction between them allows for the identification of the deepest internal properties of algebraic structures using geometric and structural tools. This path began systematically with Pilz [1], who laid the theoretical foundations for near rings, which are considered as a natural generalization of rings, thus paving the way for exploring their algebraic properties without the need for the commutative property or the requirement of an identity element on both sides. In the context of working on developing the connection between the concepts of algebra and graphs, AL-Swidi and Omran [2,3] introduced the concept of c-equiprime graph of a near ring which is a graphical representation that reflects the substructure of the ideal within the near ring, and represents an important development in studies that study ideals using graph theory.

DOI: 10.28924/APJM/13-121

Jagadesha et al. [4] and Kuncham et al. [5] also contributed to blending fuzzy logic with modern algebra by introducing some new concepts. Examples of this are the fuzzy ideal L and the sets involved in the context of near rings, a trend that shows us the development of modern algebra towards integration with non-traditional theories to do the work of ambiguity in algebraic structures. In the realm of traditional rings. Alkass [6] presented a classic study on "commutative rings," which remains a fundamental reference in understanding the basic properties of units and zero elements. Based on this, Heydari and Nikmehr [7] explored the representation of unity in left-handed Artinian rings. Afkhami and Khosh-Ahang [8] further expanded these studies to include polynomial rings and power series, demonstrating the flexibility of these tools in diverse algebraic contexts. Regarding the structural analysis of the graphs themselves, Beineke [9] introduced a precise mathematical distinction for derived graphs, which is used today in the study of graph transformations associated with algebraic structures. This research paper introduces a new concept the line graph of completely -equip rime fuzzy graph ($\mathcal{C}\amalg_{prime\ FI}\mathfrak{S}$) of nearing $\mathbb{N}$ associated with level ideal $\mathcal{F}_\sigma$, where this graph is denoted by $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ we find the properties of this graph, including calculating its diameter, girth and determining and obtain the conditions necessary for $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ to be a complete graph, Hamiltonian and discussed outer planar graph, planar graph. Also investigates the relationship between the two graphs by $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ and $L(\mathcal{F}_\sigma.\mathcal{F}.\mathcal{P}_c.\sigma)$ and also may be generalized this paper in different topics [10–15].

2. Definitions and Preliminaries

Definition 2.1. [1] *Near ring called zero symmetric and symbolizes it $\mathbb{N}_0$ if $w_0 = 0$ for all $w \in \mathbb{N}$ and integral domain if $ab = 0$ then $a = 0$ or $b = 0$.*

Definition 2.2. [3] *Let I is an ideal of $\mathbb{N}$. Then $\mathbb{I}$ is called $\mathcal{C}\amalg_{prime\ ideal}$ of N if aw-ay $\in I$ then $w - y \in I$ for all $w, y \in \mathbb{N}$ and $a \in \mathbb{N}I$.*

Definition 2.3. [16] *Let I is an ideal of $\mathbb{N}$. Then I is called pseudo ideal of $\mathbb{N}$ if $0 = wy \in I$ then $w \in \mathbb{I}$ or $y \in I$ for $w, y \in \mathbb{N}$.*

Definition 2.4. [17] *A $F\mathfrak{S}H = (\mathcal{F}, P_c)$ of $(\mathbb{V}, \mathbb{E})$ is defined by a fuzzy sub set $\mathcal{F}$ of $\mathbb{V}$ and fuzzy sub set P_c of $\mathbb{E}$ such that $P_c(w, y) \leq \mathcal{F}(w)$ and $\mathcal{F}(y)$ $\forall\, w, y \in \mathbb{V}$.*

Definition 2.5. [18] *If $\mathbb{C} \neq \emptyset$ the mapping $F : \mathbb{C} \to [0, 1]$ is called a fuzzy of $\mathbb{C}$.*

Definition 2.6. [18] *Let $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$. Let $\mathcal{F}$ be $\mathcal{F}I$ with thresholds α, β if for all $w, y, i \in \mathbb{N}$:*

- $\alpha \vee \mathcal{F}(w - y) \geq \beta \wedge \mathcal{F}(x) \wedge \mathcal{F}(y)$.
- $\alpha \vee \mathcal{F}(y - w + y) \geq \beta \wedge \mathcal{F}(w)$.
- $\alpha \vee \mathcal{F}(wy) \geq \beta \wedge \mathcal{F}(w)$.
- $\alpha \vee \mathcal{F}(w(y + i) - wy) \geq \beta \wedge \mathcal{F}(i)$.

We call α as the lower threshold of $\mathbb{N}$ and β as the upper threshold of $\mathbb{N}$. In this paper fuzzy ideal $\mathcal{F}$ means $\mathcal{F}I$ in association with lower threshold α and threshold β.

Theorem 2.7. [18] *Let $\mathcal{F}$ be a fuzzy sub set of $\mathbb{N}$ then $\mathcal{F}$ be $\mathcal{F}I$ of $\mathbb{N}$ if and only if for $\sigma \in (\alpha, \beta]$ the level sub set $\mathcal{F}_\sigma$ is ideal of $\mathbb{N}$, whenever $\mathcal{F}_\sigma = \{w|\mathcal{F}(w) \geq \ \sigma\}$.*

Theorem 2.8. [19] *Becomes graph $\mathfrak{G}$ is planner if and only if the subdivision of K_5 or $K_{3,3}$ not available in $\mathfrak{G}$.*

Theorem 2.9. [19] *Becomes graph $\mathfrak{G}$ is outer planner if and only if subdivision of K_4 or $K_{2,3}$, they don't exist in $\mathfrak{G}$.*

Also introduced some concepts and definitions of graph [20–23] the graph $\mathfrak{G} = (\mathbb{V}, \mathbb{E})$ contains two sets, a vertex set $(\mathbb{V})$ and an edge set $(\mathbb{E})$, such that every edge $\mathbb{e} \in \mathbb{E}$ can be identified with a pair of vertices $u, v \in V$. The vertices u and v are known as the terminal vertices of e and are referred as $\bar{uv}$. Adjacent vertices are vertices connected by the same edge called the number of edges that lie on any vertex u is said to be its degree, denoted $\deg(\mathbb{u})$ with maximum degree is $\triangle(\mathfrak{G})$. A path in graph G is a walk with no repetition of vertices and open, while a cyclic is a closed path. A clique is a set $\mathbb{S} \subseteq \mathbb{V}$ where every two distinct vertices in Sare adjacent, and the max size is the clique number $\omega(\mathfrak{G})$. The chromatic number as the assign adjacent vertices in different colors and the minimum number of assign colors needed for G is said the chromatic number and referred by $(\mathbb{C})$. A complete $\mathfrak{G}$ with n vertices is denoted by K_n. A null graph of degree m has no edges If we have two forms, $\mathfrak{G}_1 = (\mathbb{V}_1, \mathbb{E}_1)$ and $\mathfrak{G}_2 = (\mathbb{V}_2, \mathbb{E}_2)$ then the graph G is called a bipartite graph. If the set of vertices V can be divided into two separate sets V_1 and V_1, such that each edge in the graph connects a vertex in the first set to a vertex in the second. Subset B is induced subgraph of if all of the edges, from the $\mathfrak{G}$, connecting pairs of vertices in B and denoted by $< B >$. A dominating set B of its vertices, such that any vertex of $\mathfrak{G}B$ has a neighbor to B. Hamiltonian graph is a cycle that visits each vertex exactly once. A plane graph is a graph that can be represented (or drawn) on a plane without any overlapping edges. In other words, it can be drawn in a plane such that none of its edges intersect except at their common vertices. An outer plane graph is a plane graph that can be drawn such that all its vertices lie on the outer edges of the graph, and all its edges do not intersect. If the number nof vertices in the two groups is m and n, respectively, then this graph is denoted as $K_{m,n}$. A diagram of type $K_{1,n}$, is called a star graph S_n, where the single vertex in the first group $\mathbb{V}_1 = \{\mathbb{v}_1\}$ represents the center of the girth of the graph $\mathfrak{G}$, denoted as $gr(\mathfrak{G})$, is the length of the shortest cyclic within it, and is taken as ∞ if the graph has no cyclins.

3. Construct the Line Graph of Completely Equiprime Fuzzy Ideal

Definition 3.1. *Let $\mathcal{F}$ be $\mathcal{F}I$ of $\mathbb{N}$. Then is called $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ if for $\sigma \in (\alpha, \beta]$ with $\alpha \vee \mathcal{F}(a) \vee \mathcal{F}(w - y) \geq \beta \wedge \mathcal{F}(aw - ay)$ for all $w, y \in \mathbb{N}$ and $a \in \mathbb{N}\mathcal{F}_\sigma$.*

Definition 3.2. *Let $\mathcal{F}$ be a $\mathcal{F}I$ of $\mathbb{N}$. Then is called pseudo fuzzy ideal if for $\sigma \in (\alpha, \beta]$ with $\alpha \vee \mathcal{F}(w) \vee \mathcal{F}(y) \geq \beta \wedge \mathcal{F}(0 = wy)$ for all $w, y \in \mathbb{N}$.*

Remark 3.3.

- *In general, $0 \in \mathcal{F}_\sigma, \sigma \in (\alpha, \beta]$ whenever is an ideal.*
- *$\mathcal{F}(0)$ is the maximum value in $\mathcal{F}(w)$ for all $w \in \mathbb{N}$.*

Definition 3.4. *The line graph of $C\amalg_{prime\ \mathcal{F}I}(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ as vertex set and any two distinct vertices, are adjacent in the graph if and only if their corresponding edges share common vertex in graph $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ we denote this graph by $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$.*

Example 3.5. *If we assume $\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be near ring. addition and multiplication defined as in Table 1.*

TABLE 1. $\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, 7\}$

+	0	1	2	3	4	5	6	7		*	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7		0	0	0	0	0	0	0	0	0
1	1	2	3	0	5	6	7	4		1	0	1	2	3	4	5	6	7
2	2	3	0	1	6	7	4	5		2	0	2	0	2	0	0	0	0
3	3	0	1	2	7	4	5	6		3	0	3	2	1	4	5	6	7
4	4	7	6	5	0	3	2	1		4	0	4	2	6	4	0	6	2
5	5	4	7	6	1	0	3	2		5	0	5	0	5	0	5	0	5
6	6	5	4	7	2	1	0	3		6	0	6	2	4	4	0	6	2
7	7	6	5	4	3	2	1	0		7	0	7	0	7	0	5	0	5

We define $\mathcal{F} : \mathbb{N} \to (0.1]$:

$$\mathcal{F}(x) = \begin{cases} 0.9 & if\ w = 0 \\ 0.1 & if\ w \in \{1, 3, 5, 7\} \\ 0.7 & if\ w = 4 \\ 0.3 & if\ w \in \{2, 6\} \end{cases}$$

If we take threshold $\alpha = 0, 1$ and $\beta = 0, 3$ then $\mathcal{F}$ is $\mathcal{F}I$ of $\mathbb{N}$. Let $\sigma = \beta then \mathcal{F}_\sigma = \{0, 2, 4, 6\}$. It can be verified that $\mathcal{F}$ is $C\amalg_{prime\ \mathcal{F}I}\mathbb{N}$. Then the values of $\mathcal{P}_c$ are as in Table 2.

TABLE 2. $\mathcal{P}_c(w, y)$ when $\mathcal{F}_\sigma = \{0, 2, 4, 6\}$

$\mathcal{P}_c(p, w)$	0	1	2	3	4	5	6	7
0	0	0.1	0.3	0.1	0.7	0.1	0.3	0.1
1	0.1	0	0.1	0	0.1	0	0.1	0
2	0.3	0.1	0	0.1	0.3	0.1	0.3	0.1
3	0.1	0	0.1	0	0.1	0	0.1	0
4	0.7	0.1	0.3	0.1	0	0.1	0.3	0.1
5	0.1	0	0.1	0	0.1	0	0.1	0
6	0.3	0.1	0.3	0.1	0.3	0.1	0	0.1
7	0.1	0	0.1	0	0.1	0	0.1	0

The graph and line graph $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$, is given in Figures 1 and 2.

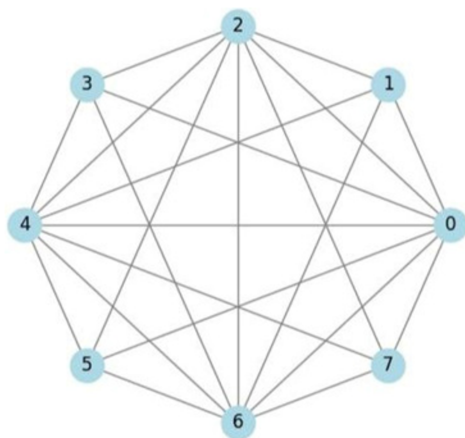

FIGURE 1. $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ when $\mathcal{F}_\sigma = \{0, 2, 4, 6\}$

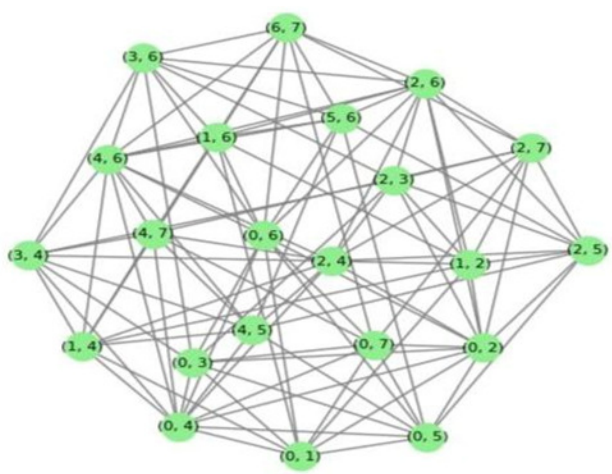

FIGURE 2. $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ when $\mathcal{F}_\sigma = \{0, 2, 4, 6\}$

Proposition 3.6. *Let $\mathcal{F}$ be a $\mathcal{F}I$ of $\mathbb{N}$. Then $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$ if and only if for every $\sigma\epsilon(\alpha, \beta]$ the level sub set $\mathcal{F}_\sigma$ is $\mathcal{C}\amalg_{prime\ ideal}$ of $\mathbb{N}$.*

Proposition 3.7. *Let $\mathcal{F}$ be $\mathcal{F}I$ of $\mathbb{N}$. Then $\mathcal{F}$ be a pseudo-x of $\mathbb{N}$ if and only if for every $\sigma\epsilon(\alpha, \beta]$ the level sub set $\mathcal{F}_\sigma$ is a pseudo deal of $\mathbb{N}$.*

Proposition 3.8. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$. Then $\mathcal{F}$ is pseudo fuzzy ideal of $\mathbb{N}$.*

Proposition 3.9. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N} = \mathcal{N}_0$. Then $w \in \mathcal{F}_\sigma$ if and only if $\deg(w) = \deg(0) = n - 1$ in $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ whenever $|\mathbb{N}| = n$.*

Theorem 3.10. [18] *Let $\mathcal{F}$ be $\mathcal{F}I$ of $\mathbb{N}$ if $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong S_n$ then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_n forn > 3, n \in Z^+$.*

Proof. Suppose $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong S_n$ for $\sigma \in (\alpha, \beta]$ then 0 is adjacent to all $w \in \mathbb{N}$ and $0w - 00 \in \mathcal{F}_\sigma$. Now:
Case 1: Consider w and $y \in \mathbb{N}$ then $\overline{0w} \in E(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ and $\overline{0y} \in E(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ that is $\mathcal{P}_c(0, w) > 0$ and $\mathcal{P}_c(0, y) > 0$ with $w \neq y$ and $\overline{wy} \notin E(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ in $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$. Now by definition of $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$, so $[0, w]$ and $[0, y]$ is joint with common vertex 0, so is complete of order 2 and $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_2$.
Case 2: Consider w, y and $z \in \mathbb{N}$ then $\overline{ow} \in E(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$, $\overline{oy} \in E(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ and $\overline{oz} \in L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ that is $\mathcal{P}_c(0, w) > 0$, $\mathcal{P}_c(0, y) > 0$ and $\mathcal{P}_c(0, z) > 0$ with $w \neq y \neq z$ and $\overline{wy} \notin E(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$, $\overline{wz} \notin E(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ and $\overline{yz} \notin E(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ in $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$. Now by definition of $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ then $[0, w], [0, y]$ and $[o, z]$ is joint with common vertex 0 so is complete of order 3 and $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_3$ continue same manner. We get $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_n$. □

Remark 3.11.

- *The convers of Theorem 3.10, is not work.*
- *If $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \ncong k_n$ then not necessary $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong k_n$.*

Proposition 3.12. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of nearing $\mathbb{N}$ and $\mathcal{F}_\sigma = 0$. Then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is completely graph.*

Proof. By Proposition 3.9 and $\mathcal{F}_\sigma = \{0\}$ is $\mathcal{C}\amalg_{prime\ ideal}$ of $\mathbb{N}$, so we get $\mathcal{P}_c(a, b) = 0 \forall 0 \neq a \neq b \in \mathbb{N}$ and $\mathcal{P}_c(0, a) > 0 \; \forall \; a \in \mathbb{N}$, thus implies $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is a star and the graph $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ has $n - 1$ edges, so $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is a complete graph with n-1 vertices that is isomorphic to k_{n-1}. □

Remark 3.13. *If $\mathcal{F}$ is not $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of near ring $\mathbb{N}$ then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is not complete graph.*

Example 3.14. *Let $\mathbb{N} = Z_6$ be the ring of integers modulo 6. We define $\mathcal{F} : \mathbb{N} \to (0, 1]$:*

$$\mathcal{F}(x) = \begin{cases} 0.9 & if\ w = 0 \\ 0.3 & if\ w \in \{1, 2, 3, 4, 5\} \end{cases}$$

If we take threshold $\alpha = 0.7$ and $\beta = 0.9$ then $\mathcal{F}$ is $\mathcal{FI}$ of $\mathbb{N}$. Let $\sigma = \beta then \mathcal{F}_\sigma = 0$ and the values of $\mathcal{P}_c$ are as in Table 3. The line graph $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is given in Figures 3 and 4.

TABLE 3. $\mathcal{P}_c(w, y)$ when $\mathcal{F}_\sigma = \{0\}$

$\mathcal{P}_c(p, w)$	0	1	2	3	4	5
0	0	0.3	0.3	0.3	0.3	0.3
1	0.3	0	0	0	0	0
2	0.3	0	0	0.3	0	0
3	0.3	0	0.3	0	0.3	0
4	0.3	0	0	0.3	0	0
5	0.3	0	0	0	0	0

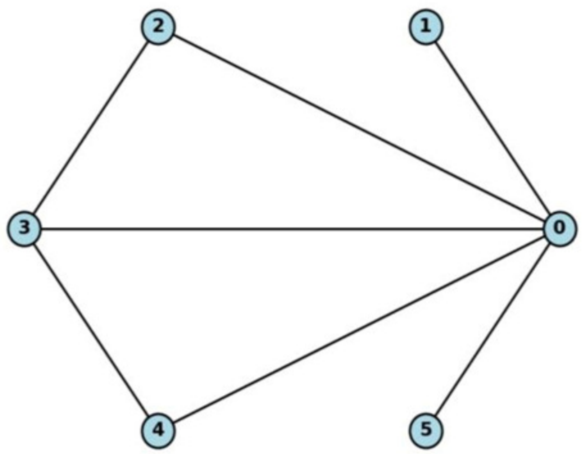

FIGURE 3. $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ when $\mathcal{F}_\sigma = \{0\}$

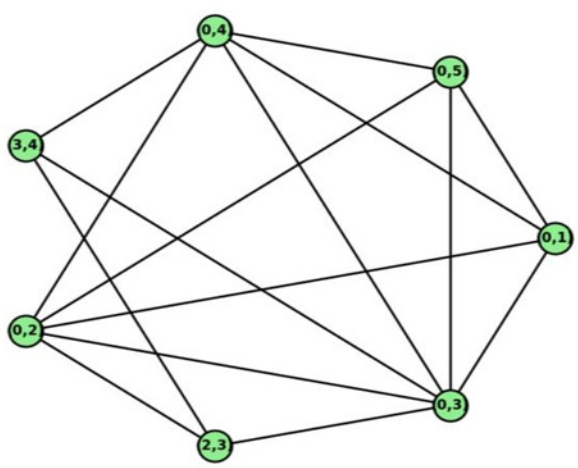

FIGURE 4. $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ when $\mathcal{F}_\sigma = \{0\}$

Not that not $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ *of* $\mathbb{N}\alpha \vee \mathcal{F}(3) \vee \mathcal{F}(2-4) = 0.7 \vee 0.3 \vee 0.3 = 0.7 \ngeq 0.9 = 0.9 \wedge 0.9 = \beta \wedge \mathcal{F}(aw - ay)$.

Corollary 3.15. *Let* $\mathcal{F}$ *be* $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ *of* $\mathbb{N}$ *and* $\mathcal{F}_\sigma = \{0\}$. *Then* $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ *is Hamiltonian.*

Proof. By Proposition 3.12, $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is complete graph because every edge in $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ has vertex $[0, a]$ is adjacent to every vertex in $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ for $a \in \mathbb{N}$. Hence $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ contains Hamiltonian cycle therefore $L\ (\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is Hamiltonian. □

Proposition 3.16. *Let* $\mathcal{F}$ *be* $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ *of* $\mathbb{N}$. *Then* $\triangle(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)) = \deg([0, a]) = m - 1$ *for all* $a \in \mathcal{F}_\sigma$ *where* $|L(\mathbb{N}, \mathcal{F}, \mathcal{P}, \sigma)| = m$.

Proof. As 0 is adjacent to all elements of $w \in \mathbb{N}$ then $\overline{0w} \in E(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ for $\sigma \in (\alpha, \beta]$. Then $\mathcal{P}_c(0, w) > 0$ and $[0, w] \in L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ and for all $a \in \mathcal{F}_\sigma$ and $w \in \mathbb{N}$, so by Proposition 3.9. Then $\mathcal{P}_c(a, y) > 0$ for all $y \in \mathbb{N}$ that is a is adjacent to all $y \in \mathbb{N}$ then $[0, a]$ is adjacent to all $[a, y]$ in $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$. So $\triangle(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) = \deg([0, a]) = m - 1$ for all $a \in \mathcal{F}_\sigma$ as simple graph. □

Proposition 3.17. *Let* $\mathcal{F}$ *be* $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ *of* $\mathbb{N}$ *and* $\chi(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)) = 2$. *Then* $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ *is not bipartite.*

Proof. Let $\chi\ L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) = 2$ then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma\ )$ is star graph as 0 is adjacent to all $w \in \mathbb{N}$ and $\mathcal{P}_c(0, w) > 0$ for $\sigma \in (\alpha.\beta]$ and $\omega(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_2$ therefore by Theorem 3.10, we get $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_n$, then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is not bipartite graph. □

Proposition 3.18. *Let* $\mathcal{F}$ *be* $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ *of* $\mathbb{N}$. *Then:*

$$\chi(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)) = \begin{cases} 1 & for\ |\mathbb{N}| = 2 \\ 3 & for\ |\mathbb{N}| \geq 3 \end{cases}$$

Proof. For $|\mathbb{N}| = 2, let \mathbb{N} = \{0, a\}$ with $a \neq 0$, since 0 is adjacent to all $a \in \mathbb{N}$ then $p_c(0, a) > 0$ for $\sigma \in (\alpha, \beta]$ and $\overline{0a} \in E(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) with\ [0., a] \in L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$. That is $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is singulation and $\chi(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)) = 1$.

For $|\mathbb{N}| \geq 3, let \mathbb{N} = \{0, a, b\}$ with $a \neq b \neq 0$, since 0 is adjacent to a and $b \in \mathbb{N}$ then $p_c(0, a) > 0 and p_c(0, b) > 0$ for $\sigma \in (\alpha, \beta]$ so $\overline{([0, a], [0, b])} \in E(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ if a is not adjacent to b then $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is star graph, then by Theorem 3.10, we get $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_3$ and $\chi(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)) = 3$.

Similarly, if a and b are adjacent in $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ then $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_3$, so that by definition of $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ we get $\overline{([a, b], [0, a])} \in E(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ and $\overline{([a, b], [0, b])} \in E(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ therefore, $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_3$ and $\chi(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) = 3$ from this we get in all cases for $|\mathbb{N}| \geq 3$ we get $\chi(L(\ \mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)) \geq 3$. □

Remark 3.19. *The* $\chi(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)) \neq 2$ *in all cases of* $|\mathbb{N}| = n,\ n \in Z^+$.

Proposition 3.20. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$. Then $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)\setminus L(\mathcal{F}_\sigma,\mathcal{F},\mathcal{P}_c,\sigma)$ is empty graph.*

Proof. Suppose $a,b\in\mathbb{N}\backslash\mathcal{F}_\sigma$ such that $\mathcal{P}_c(a,b)>0$ then $ab-a0\in\mathcal{F}_\sigma$ or $ba-b0\in\mathcal{F}_\sigma$ without loss of generality assume $ba-b0\in\mathcal{F}_\sigma$ by Proposition 3.6, $\mathcal{F}_\sigma$ is $\mathcal{C}\amalg_{prime\ ideal}$ of $\mathbb{N}$ and by Proposition3.2, $\mathcal{F}_\sigma$ pseudo ideal of $\mathbb{N}$ then $0=ab\in\mathcal{F}_\sigma$ this implies $a\in\mathcal{F}_\sigma$ or $b\in\mathcal{F}_\sigma$ a contradiction. So, we get $\mathcal{P}_c(a,b)=0$ hence $(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ does not containing edges, so by definition of $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ then $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)\setminus L(\mathcal{F}_\sigma.\mathcal{F}.\mathcal{P}_c.\sigma)$ is empty graph. □

Proposition 3.21. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}=\mathbb{N}_o$. Then $\omega(L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)=n-1,|\mathbb{N}|=n$.*

Proof. In graph $(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ has $\mathcal{P}_c(0,a)>0\ \forall\ a\in\mathbb{N}$ so inducing a spanning sub graph $K_{1,n-1}$. Thus, by Theorem 3.10, in $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ has $n-1$ vertices are adjacent to each other resulting a complete graph K_{n-1} thus $\omega(L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma))=n-1$. □

Theorem 3.22. [18] *The line graph $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ is cycle graph if and only if either $\mathbb{N}=\mathcal{F}_\sigma\cong Z_3$ or $\mathbb{N}\cong Z_4$ and $\mathcal{F}_\sigma=0$.*

Proof. Suppose $\mathbb{N}=\mathcal{F}_\sigma\cong Z_3$ from definition $L((\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma))$ is cycle of length 3 and hence the corresponding line graph is cycle graph is that C_3. Next suppose $\mathbb{N}\cong Z_4$ and $\mathcal{F}_\sigma=0$, in this case $(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ is complete bipartite graph $K_{1,3}$ by Theorem 3.10, implies that the corresponding line graph is a cycle of length 3.

Conversely, the line graph $L((\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma))$ be a cyclic graph if possible.

Case 1: If$\mathbb{N}=\mathcal{F}_\sigma\ncong Z_3$ from definition $(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ is complete graph if $\mathbb{N}=\mathcal{F}_\sigma$ that is $(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)=k_n$ hence the corresponding line graph contains more than one cyclic, a contradiction.

Case 2: If $\mathcal{F}_\sigma=0$ and $\mathbb{N}\not\approx Z_4$ with $\mathbb{N}\cong Z_3$ or $\mathbb{N}\cong Z_2$ then $N\ncong Z_n$, $n>5$ thus in $(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$, we get a spanning sub graph $K_{1,n-1}$ by Theorem 3.1, will induce complete graph K_{n-1} in $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ thus $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ contains more than one cyclic which is contradiction. □

Proposition 3.23. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of a simple integral near ring $\mathbb{N}$. Then $gr(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)=\infty$ if and only if $|\mathbb{N}|=2$ or $|\mathbb{N}|=3$.*

Proof. Suppose $gr(L((\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma))=\infty$ then $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ dos not contain a cycle. Suppose $|\mathbb{N}|>3$ then for $\mathcal{F}_\sigma=0$ we get $\mathcal{P}_c(0,w)>0\ \forall\ w\in\mathbb{N}$ then by Theorem 3.1 and Proposition 3.5, we get $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ contain a cyclic, a contradiction. Conversely, assume that $|\mathbb{N}|=2$ or $|\mathbb{N}|=3$ then $\mathbb{N}=0,a$ for some $a\neq 0$ then $\overline{0a}$ is an edge in $(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ implies $[0,a]$ is only one vertex in $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ implies $gr(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma))=\infty$, if $|\mathbb{N}|=3$ that is $\mathbb{N}=\{0,a,b\}$ then $[0,a]$ and $[0,b]$ are edges in $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ we get $\mathcal{P}_c(a,b)=0$ since $\mathcal{F}_\sigma=0$ hence $[0,a]$ and $[0,b]$ are only vertex in $L(\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma)$ this implies $gr((\mathbb{N},\mathcal{F},\mathcal{P}_c,\sigma))=\infty$. □

Corollary 3.24. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$ and $\mathcal{F}_\sigma = 0$. Then gr $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) = \infty$ if and only if $\mathbb{N} = Z_2$ or $\mathbb{N} = Z_3$.*

Proof. Same proof of Proposition 3.23. □

Theorem 3.25. [18] *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$. Then in $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is connected.*

Proof. Since $\mathcal{P}_c(0, a) > 0\,\forall\, a \in \mathbb{N}$ in graph $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$, then $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is connected, so $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is connected graph as $[0, a]$ is adjacent to all $a \in \mathbb{N}$. □

Theorem 3.26. [18] *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$ if $[a, b]$ is vertex in $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$. Then $\deg([a, b])$ is at least two.*

Proof. Let $[a, b]$ be vertex in $L((\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma))$, since $\mathcal{P}_c(0, a) > 0\,\forall\, a \in \mathbb{N}$ thus there exist at least two edge $\overline{0a}$ and $\overline{0b}$ in $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ and as result in the corresponding line graph, we get two vertices $[0, a]$ and $[0, b]$ these two vertices are adjacent to $[a, b]$, then $\deg[a, b]$ is at least two. □

Proposition 3.27. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$ and $a, b \in \mathbb{N}$ such that $[a, b]$ be vertex of $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$. Then $a \in \mathcal{F}_\sigma$ or $b \in \mathcal{F}_\sigma$.*

Proof. If a=0 or b=0 the result is true since 0 is in every ideal $\mathcal{F}_\sigma$ of $\mathbb{N}$, let $a \neq 0$ and $b \neq 0$ and $[a, b]$ be vertex in $L((\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma))$, then $\mathcal{P}_c(a, b) > 0$ and $\bar{ab} \in E((\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$, so we get $ab - a0 \in \mathcal{F}_\sigma$ or $ba - b0 \in \mathcal{F}_\sigma$ as $\mathcal{F}$ c-equiprime fuzzy ideal, then $\mathcal{F}_\sigma$ $\mathcal{C}\amalg_{prime\ ideal}$, so we get $b - 0 \in \mathcal{F}_\sigma$ or $a - 0 \in \mathcal{F}_\sigma$. That is $a \in \mathcal{F}_\sigma$ or $b \in \mathcal{F}_\sigma$. □

Proposition 3.28. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of of $\mathbb{N}$. Then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is outer planar if and only if $\mathbb{N} \cong Z_2, Z_3$ or Z_4.*

Proof. Now we have the cases:
Case 1: If $|\mathbb{N}| \cong Z_4$ therefore $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_{1,3}$ by Theorem 3.1, we get $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_3$. Hence $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is outer planar.
Case 2: If $|\mathbb{N}| \cong Z_3$ then same case1 and Theorem 3.10, we get $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_3$ and hence by Theorem 2.9, so $L((\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma))$ is outer planar.
Case 3: If $|\mathbb{N}| \cong Z_2$ then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is a null graph that is singulation with one vertex. Hence $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is outer planar.
Conversely, directly from definition of outer planar and Theorem 2.9 then $\mathbb{N} \cong Z_2, Z_3$ or Z_4. □

Proposition 3.29. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$ and $|\mathbb{N}| = n, n \geq 5$. Then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is not outer planar.*

Proof. If $|\mathbb{N}| = 5$ then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_{1,4}$ and by Theorem 3.10, then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_4$, so $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ contains subdivision of K_4 thus by Theorem 2.9, we get $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is not outer planar and similarly for $n \geq 6$ that is $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is not is outer planar. □

Corollary 3.30. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$ with $|\mathbb{N}| > 5$ and $\mathcal{F}_\sigma = 0$. Then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is not planar.*

Proof. Let $|\mathbb{N}| > 5$, then $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is isomorphic to $k_{1.n-1}$, so by Theorem 3.10, then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_{1,n-1}$ and hence $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ contains a sub division of K_5 therefore by Theorem 2.9, thus $L(L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is not planar. □

Proposition 3.31. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$ and $\mathcal{F}_\sigma = 0$. Then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is planar if and only if $\mathbb{N} \cong Z_2, Z_3, Z_4$ or Z_5.*

Proof. Suppose the line graph $(\mathbb{N}, \mathcal{F}, \mathcal{P}\ _c, \sigma)$ is planar, let $V = \{a_1, a_2, a_3, a_4, a_5\}$ be the vertices, as every $a_i \neq 0$ then $\mathcal{P}(a_i, 0) > 0$ for index $i, 1 \leq i \leq 5$.

So, vertex 0 becomes a common vertex between every pair of edge in $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$. If $|\mathbb{N}| \geq 6$, let $\bar{V} = [0, a_1], [0, a_2], [0, a_3], [0, a_4], [0, a_5]$ be vertices subset of $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ therefore the induce subgraph V by Theorem 3.10, then $\bar{v}$ is complete graph isomorphic to k_5.

Then by Theorem 2.8, then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is not planar, a contradiction to supposition. Conversely, we have the cases:

Case 1: If $|\mathbb{N}| = 4$ or 3 then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_3$ thus $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is planar.

Case 2: If $|\mathbb{N}| = 2$ then $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ has singulation vertex, thus $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is planar.

Case 3: If $|\mathbb{N}| = 5$ then $(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong k_{1,4}$ ten by Theorem 3.10, we get $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma) \cong K_4$ which does not contain a sub division of K_5 or $K_{3,3}$ thus $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$ is planar. □

Proposition 3.32. *Let $\mathcal{F}$ be $\mathcal{C}\amalg_{prime\ \mathcal{F}I}$ of $\mathbb{N}$. Then $S = [a, n] | a \in \mathcal{F}_\sigma,\ n \in N$ is dominating set in $L(\mathbb{N}, \mathcal{F}, \mathcal{P}_c, \sigma)$.*

Proof. By Proposition 3.6, as $\triangle(L(\mathbb{N}, \mathcal{F}, \mathcal{P}, \sigma)) = \deg([0, a]) = m - 1$ for all $a \in \mathcal{F}_\sigma$ when $|L(\mathbb{N}, \mathcal{F}, \mathcal{P}, \sigma)| = m$ then $\ell L(\mathbb{N}, \mathcal{F}, \mathcal{P}, \sigma) = 1$. □

4. Conclusions

We have concluded that the maximum degree in line graph whenever the elements is containing in level set $\mathcal{F}_\sigma$ and also the chromatic number is not equal to two in all cases for finite or infinite elements of near ring also the line graph is planar and outer planar in some cases and in almost is not planar or outer planar graphs.

Authors' Contributions. All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] G. Pilz, Near-Rings: The Theory and Its Applications, Elsevier, 2011.

[2] A.A.J. Al-Swidi, A. Omran, Uniquely Colorable Completely Equiprime Graph of a Near Ring, AIP Conf. Proc. 2899 (2023), 050045. https://doi.org/10.1063/5.0157185.

[3] A.A.J. Al-Swidi, A.A. Omran, Completely Equiprime Graph of a Near Ring, AIP Conf. Proc. 2516 (2023), 080011. https://doi.org/10.1063/5.0102632.

[4] B. Jagadeesha, K.B. Srinivas, K.S. Prasad, Interval Valued L-Fuzzy Ideals Based on t-Norms and t-Conorms, J. Intell. Fuzzy Syst. 28 (2015), 2631–2641. https://doi.org/10.3233/IFS-151541.

[5] S.P. Kuncham, B. Jagadeesha, B.S. Kedukodi, Interval Valued L-Fuzzy Cosets of Nearrings and Isomorphism Theorems, Afr. Mat. 27 (2015), 393–408. https://doi.org/10.1007/s13370-015-0348-1.

[6] D. Alkass, Finite Posets as Prime Spectra of Commutative Noetherian Rings, Rose-Hulman Undergrad. Math. J. 25 (2024), 1–15.

[7] F. Heydari, M.J. Nikmehr, The Unit Graph of a Left Artinian Ring, Acta Math. Hung. 139 (2012), 134–146. https://doi.org/10.1007/s10474-012-0250-3.

[8] M. Afkhami, F. Khosh-Ahang, Unit Graphs of Rings of Polynomials and Power Series, Arab. J. Math. 2 (2013), 233–246. https://doi.org/10.1007/s40065-013-0067-0.

[9] L.W. Beineke, Characterizations of Derived Graphs, J. Comb. Theory 9 (1970), 129–135. https://doi.org/10.1016/S0021-9800(70)80019-9.

[10] A. Al-Alawani, A. Al-Swidi, Union of Subgraphs with Respect to Some Ideals on Near Rings, J. Discrete Math. Sci. Cryptogr. 28 (2025), 399–408. https://doi.org/10.47974/JDMSC-2048.

[11] A.A. Al-Alwani, A.A.J. AL-Swidi, New Types of Ideals on Near Rings, AIP Conf. Proc. 3264 (2025), 050047. https://doi.org/10.1063/5.0259003.

[12] A.A.J. Al-Swidi, E.H. Al-Saadi, L.H. Al-Saadi, Soft Public Key Cipher, Period. Eng. Nat. Sci. 7 (2019), 1433–1438. https://doi.org/10.21533/pen.v7.i3.1643.

[13] A.A. Al-Swidi, E.H. Al-Saadi, R.A.A. Shekan, Huffman Code via Fuzzy Generators, J. Eng. Appl. Sci. 13 (2018), 9735–9738.

[14] A.M. Almamoori, H.A. Kadhum, I.H. Ibrahim, Removal of Two Textile Dyes Using Aspergillus Niger, Bionatura 8 (2023), 1–8. https://doi.org/10.21931/RB/CSS/2023.08.04.15.

[15] M.M. Khadairi, A.M. Almamoori, A.I.H. Al-Janabi, M.J.Y. Al-amari, Genotoxic Effects of Lead and Cadmium on DNA of Some Fuel Stations Workers Blood in Hilla City, IOP Conf. Ser. Earth Environ. Sci. 910 (2021), 012032. https://doi.org/10.1088/1755-1315/910/1/012032.

[16] M.S. Shnain, A.A. AL-Swidi, Pseudo Weakly Completely Prime Ideal in Near Ring, J. Discrete Math. Sci. Cryptogr. 28 (2025), 1245–1255. https://doi.org/10.47974/JDMSC-2263.

[17] J.N. Mordeson, P.S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica-Verlag, 2012.

[18] B. Davvaz, $(\in, \in \vee q)$-Fuzzy Subnear-Rings and Ideals, Soft Comput. 10 (2006), 206–211. https://doi.org/10.1007/s00500-005-0472-1.

[19] M. Aijaz, S. Pirzada, Annihilating-Ideal Graphs of Commutative Rings, Asian-Eur. J. Math. 13 (2019), 2050121. https://doi.org/10.1142/S1793557120501211.

[20] I. Beck, Coloring of Commutative Rings, J. Algebra 116 (1988), 208–226. https://doi.org/10.1016/0021-8693(88)90202-5.

[21] N. Deo, Graph Theory with Applications to Engineering and Computer Science, Courier Dover Publications, 2016.

[22] C. Vasudev, Graph Theory with Applications, New Age International, 2006.

[23] T. Wu, On Directed Zero-Divisor Graphs of Finite Rings, Discret. Math. 296 (2005), 73–86. https://doi.org/10.1016/j.disc.2005.03.006.