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A UNIFIED THEORY OF FAINT CONTINUITY FOR MULTIFUNCTIONS

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Abstract. This paper presents four classes of multifunctions, namely upper faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions, lower faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions, upper faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions and lower faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions. Furthermore, several characterizations and some properties concerning upper faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions, lower faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions, upper faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions and lower faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions are established.

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1. Introduction

In 1982, Long and Herrington [15] considered the class of faintly continuous functions. Nasef and Noiri [17] introduced and studied three strong forms of faint continuity under the names of strongly faint semi-continuity, strongly faint precontinuity and strongly faint β-continuity. Jafari and Noiri [12] introduced and investigated the notion of faintly α-continuous functions. Chanapun et al. [5] introduced a new class of continuous functions, namely faintly (m, μ)-continuous functions and established the relationships between faint (m, μ)-continuity and other related generalized forms of (m, μ)-continuity. In 2011, Jafari et al. [11] introduced and studied a new class of multifunctions called faintly δ-β-continuous multifunctions. Carpintero et al. [4] introduced and investigated the concept of faintly ω-continuous multifunctions as a generalization of ω-continuous multifunctions

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due to Zorlutuna [23]. On the other hand, the present authors introduced and investigated the notions of (τ_1, τ_2)-continuous multifunctions [20], almost (τ_1, τ_2)-continuous multifunctions [13], weakly (τ_1, τ_2)-continuous multifunctions [21], almost weakly (τ_1, τ_2)-continuous multifunctions [1], $m(\sigma_1, \sigma_2)$-continuous multifunctions [7], $(\tau_1, \tau_2)\mu$-continuous multifunctions [8], almost $(\tau_1, \tau_2)\mu$-continuous multifunctions [18], weakly $(\tau_1, \tau_2)\mu$-continuous multifunctions [6] and almost weakly $(\tau_1, \tau_2)\mu$-continuous multifunctions [14]. Pue-on et al. [19] introduced and studied the notions of upper faintly (τ_1, τ_2)-continuous multifunctions and lower faintly (τ_1, τ_2)-continuous multifunctions. In this paper, we introduce the notions of upper faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions, lower faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions, upper faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions and lower faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions. We also investigate some characterizations of upper faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions, lower faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions, upper faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions and lower faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions.

2. Preliminaries

Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called *$\tau_1\tau_2$-closed* [2] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called *$\tau_1\tau_2$-open*. The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the *$\tau_1\tau_2$-closure* [2] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the *$\tau_1\tau_2$-interior* [2] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 1. [2] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) $A \subseteq \tau_1\tau_2$-*Cl*(A) *and* $\tau_1\tau_2$-*Cl*$(\tau_1\tau_2$-*Cl*$(A)) = \tau_1\tau_2$-*Cl*(A).
(2) *If* $A \subseteq B$, *then* $\tau_1\tau_2$-*Cl*$(A) \subseteq \tau_1\tau_2$-*Cl*$(B)$.
(3) $\tau_1\tau_2$-*Cl*(A) *is* $\tau_1\tau_2$-*closed.*
(4) A *is* $\tau_1\tau_2$-*closed if and only if* $A = \tau_1\tau_2$-*Cl*(A).
(5) $\tau_1\tau_2$-*Cl*$(X - A) = X - \tau_1\tau_2$-*Int*$(A)$.

Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a *$(\tau_1, \tau_2)\theta$-cluster point* [22] of A if $\tau_1\tau_2$-Cl$(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U of X containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the *$(\tau_1, \tau_2)\theta$-closure* [22] of A and is denoted by $(\tau_1, \tau_2)\theta$-Cl(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be *$(\tau_1, \tau_2)\theta$-closed* [22] if $(\tau_1, \tau_2)\theta$-Cl$(A) = A$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be *$(\tau_1, \tau_2)\theta$-open*. The union of all $(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the *$(\tau_1, \tau_2)\theta$-interior* [22] of A and is denoted by $(\tau_1, \tau_2)\theta$-Int(A).

Lemma 2. [22] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) *If A is $\tau_1\tau_2$-open in X, then $\tau_1\tau_2$-$Cl(A) = (\tau_1, \tau_2)\theta$-$Cl(A)$.*

(2) *$(\tau_1, \tau_2)\theta$-$Cl(A)$ is $\tau_1\tau_2$-closed in X.*

Let $\mathcal{P}(X)$ denotes the power set of a nonempty set X. A class $\mu \subseteq \mathcal{P}(X)$ is called a *generalized topology* (briefly, GT) if $\emptyset \in \mu$ and an arbitrary union of elements of μ belongs to μ [10]. A set X with a generalized topology μ on X is called a *generalized topological space* (briefly, GTS) and is denoted by (X, μ). For a GTS (X, μ), the elements of μ are called *μ-open sets* and the complements of μ-open sets are called *μ-closed sets*. For $A \subseteq X$, the μ-closure of A [9], $c_\mu(A)$, is the intersection of all μ-closed sets containing A and the μ-interior of A [9], $i_\mu(A)$, is the union of all μ-open sets contained in A. Then, we have $i_\mu(i_\mu(A)) = i_\mu(A)$, $c_\mu(c_\mu(A)) = c_\mu(A)$ and $i_\mu(A) = X - c_\mu(X - A)$.

A subfamily m of the power set $\mathcal{P}(X)$ of a nonempty set X is called an *m-structure* [3] on X if m satisfies the following properties: (1) $\emptyset \in m$ and $X \in m$; (2) $\cup_{\alpha\in\nabla} A_\alpha \in m$ whenever $A_\alpha \in m$ for each $\alpha \in \nabla$. We call the pair (X, m) an *m-space*. Each member of m is said to be *m-open* and the complement of an m-open set is said to be *m-closed*. Let (X, m) be an m-space and A a subset of X. The *m-closure* mCl(A) and *m-interior* mInt(A) of A are defined in [16] as follows:

$$\text{mCl} = \cap\{F \mid A \subseteq F \text{ and } X - F \in m\}$$

and mInt $= \cup\{U \mid U \subseteq A$ and $U \in m\}$.

Lemma 3. [3] *Let (X, m) be an m-space. Then, for a subset A of X, the following properties hold:*

(1) *$A \in m$ if and only if $A = mInt(A)$;*

(2) *A is m-closed if and only if $A = mCl(A)$;*

(3) *$mCl(A)$ is m-closed and $mInt(A)$ is m-open.*

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x\in A} F(x)$.

3. Upper and lower faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions

In this section, we introduce new concepts of continuous multifunctions defined from a generalized topological space into a bitopological space, namely upper faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions and lower faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions. Furthermore, some characterizations of upper faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions and lower faintly $\mu(\sigma_1, \sigma_2)$-continuous multifunctions are discussed.

A net (x_γ) in a generalized topological space (X,μ) is said to be *μ-eventually* in the set $G \subseteq X$ if there exists $\gamma_0 \in \nabla$ such that $x_\gamma \in G$ for each $\gamma \geq \gamma_0$. A net (x_γ) is called *μ-converge* to a point x if for each μ-open set U of X containing x, there exists $\gamma_0 \in \nabla$ such that $x_\gamma \in U$ for each $\gamma \geq \gamma_0$.

Definition 1. *A multifunction $F : (X,\mu) \to (Y,\sigma_1,\sigma_2)$ is said to be upper faintly $\mu(\sigma_1,\sigma_2)$-continuous at a point $x \in X$ if for each $(\sigma_1,\sigma_2)\theta$-open set V of Y containing $F(x)$, there exists a μ-open set U of X containing x such that $F(U) \subseteq V$. A multifunction $F : (X,\mu) \to (Y,\sigma_1,\sigma_2)$ is said to be upper faintly $\mu(\sigma_1,\sigma_2)$-continuous if F is upper faintly $\mu(\sigma_1,\sigma_2)$-continuous at each point x of X.*

Theorem 1. *For a multifunction $F : (X,\mu) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) *F is upper faintly $\mu(\sigma_1,\sigma_2)$-continuous;*
(2) *for each $x \in X$ and for every $(\sigma_1,\sigma_2)\theta$-open set V of Y such that $x \in F^+(V)$, there exists a μ-open set U of X containing x such that $U \subseteq F^+(V)$;*
(3) *for each $x \in X$ and for every $(\sigma_1,\sigma_2)\theta$-closed set K of Y such that $x \in F^+(Y-K)$, there exists a μ-closed set H of X containing $x \in X-H$ and $F^-(K) \subseteq H$;*
(4) *$F^+(V)$ is μ-open in X for every $(\sigma_1,\sigma_2)\theta$-open set V of Y;*
(5) *$F^-(K)$ is μ-closed in X for every $(\sigma_1,\sigma_2)\theta$-closed set K of Y;*
(6) *$F^-(Y-V)$ is μ-closed in X for every $(\sigma_1,\sigma_2)\theta$-open set V of Y;*
(7) *$F^+(Y-K)$ is μ-open in X for every $(\sigma_1,\sigma_2)\theta$-closed set K of Y;*
(8) *for each $x \in X$ and for each net (x_γ) which μ-converges to x in X and for each $(\sigma_1,\sigma_2)\theta$-open set V of Y such that $x \in F^+(V)$, the net (x_γ) is μ-eventually in $F^+(V)$.*

Proof. (1) $\Leftrightarrow$ (2): Obvious.

(2) $\Leftrightarrow$ (3): Let $x \in X$ and K be any $(\sigma_1,\sigma_2)\theta$-closed set of Y such that $x \in F^+(Y-K)$. By (2), there exists a μ-open set U of X containing x such that $U \subseteq F^+(Y-K)$. Then, $F^-(K) \subseteq X-U$. Put $H = X-U$. Then, we have H is μ-closed and $x \in X-H$. The converse is similar.

(1) $\Leftrightarrow$ (4): Let V be any $(\sigma_1,\sigma_2)\theta$-open set of Y and $x \in F^+(V)$. By (1), there exists a μ-open set U_x of X containing x such that $U_x \subseteq F^+(V)$. Thus, $F^+(V) = \cup_{x\in F^+(V)} U_x$ and so $F^+(V)$ is μ-open in X. The converse can be shown similarly.

(4) $\Rightarrow$ (5): Let K be any $(\sigma_1,\sigma_2)\theta$-closed set of Y. Then, $Y-K$ is $(\sigma_1,\sigma_2)\theta$-open in Y, by (4) we have $F^+(Y-K) = X - F^-(K)$ is μ-open in X. Thus, $F^-(K)$ is μ-closed in X.

(5) $\Rightarrow$ (4): It similar to that of (4) $\Rightarrow$ (5).

(4) $\Leftrightarrow$ (6) and (5) $\Leftrightarrow$ (7): It follows from the fact that $F^-(Y-B) = X - F^+(B)$ and $F^+(Y-B) = X - F^-(B)$ for every subset B of Y.

(1) $\Rightarrow$ (8): Let (x_γ) be a net which μ-converges to x in X and let V be any $(\sigma_1,\sigma_2)\theta$-open set of Y such that $x \in F^+(V)$. Since F is upper faintly $\mu(\sigma_1,\sigma_2)$-continuous, there exists a μ-open set U of X containing x such that $U \subseteq F^+(V)$. Since (x_γ) μ-converges to x, it follows that there exists $\gamma_0 \in \nabla$

such that $x_\gamma \in U$ for each $\gamma \geq \gamma_0$. Therefore, $x_\gamma \in U \subseteq F^+(V)$ for each $\gamma \geq \gamma_0$. Thus, the net (x_γ) is μ-eventually in $F^+(V)$.

(8) $\Rightarrow$ (1): Suppose that F is not upper faintly $\mu(\sigma_1, \sigma_2)$-continuous. There exists a point x of X and a $(\sigma_1, \sigma_2)\theta$-open set V of Y with $x \in F^+(V)$ such that $U \not\subseteq F^+(V)$ for each μ-open set U of X containing x. Let $x_U \in U$ and $x_U \notin F^+(V)$ for each μ-open set U of X containing x. Then, for each μ-neighbourhood net (x_U), (x_U) μ-converges to x, but (x_U) is not μ-eventually in $F^+(V)$. This is a contradiction. Thus, F is upper faintly $\mu(\sigma_1, \sigma_2)$-continuous. □

Definition 2. *A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be lower faintly $\mu(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a μ-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$. A multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$ is said to be lower faintly $\mu(\sigma_1, \sigma_2)$-continuous if F is lower faintly $\mu(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Theorem 2. *For a multifunction $F : (X, \mu) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is lower faintly $\mu(\sigma_1, \sigma_2)$-continuous;*
(2) *for each $x \in X$ and for each $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $x \in F^-(V)$, there exists a μ-open set U of X containing x such that $U \subseteq F^-(V)$;*
(3) *for each $x \in X$ and for each $(\sigma_1, \sigma_2)\theta$-closed set K of Y such that $x \in F^-(Y - K)$, there exists a μ-closed set H of X such that $x \in X - H$ and $F^+(K) \subseteq H$;*
(4) *$F^-(V)$ is μ-open in X for every $(\sigma_1, \sigma_2)\theta$-open set V of Y;*
(5) *$F^+(K)$ is μ-closed in X for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y;*
(6) *$F^+(Y - V)$ is μ-closed in X for every $(\sigma_1, \sigma_2)\theta$-open set V of Y;*
(7) *$F^-(Y - K)$ is μ-open in X for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y;*
(8) *for each $x \in X$ and for each net (x_γ) which μ-converges to x in X and for each $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $x \in F^-(V)$, the net (x_γ) is eventually in $F^-(V)$.*

Proof. The proof is similar to that of Theorem 1. □

4. Upper and lower faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions

In this section, we introduce new classes of continuous multifunctions defined from an m-space into a bitopological space, called upper faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions and lower faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions. Moreover, some characterizations of upper faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions and lower faintly $m(\sigma_1, \sigma_2)$-continuous multifunctions are investigated.

Definition 3. *A multifunction $F : (X, m) \to (Y, \sigma_1, \sigma_2)$ is said to be upper faintly $m(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)\theta$-open set V of Y containing $F(x)$, there exists an m-open set U of X containing x such that $F(U) \subseteq V$. A multifunction $F : (X, m) \to (Y, \sigma_1, \sigma_2)$ is said to be upper faintly $m(\sigma_1, \sigma_2)$-continuous if F is upper faintly $m(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Theorem 3. *For a multifunction $F : (X, m) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper faintly $m(\sigma_1, \sigma_2)$-continuous;*

(2) *$F^+(V)$ is m-open in X for every $(\sigma_1\sigma_2)\theta$-open set V of Y;*

(3) *$F^-(K)$ is m-closed in X for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y.*

Proof. (1) $\Rightarrow$ (2): Let V be any $(\sigma_1, \sigma_2)\theta$-open set of Y and $x \in F^+(V)$. Then, $F(x) \subseteq V$. Since F is upper faintly $m(\sigma_1, \sigma_2)$-continuous, there exists a an m-open set of X containing x such that $F(U) \subseteq V$. Thus, $x \in U \subseteq F^+(V)$ and so $x \in \mathrm{mInt}(F^+(V))$. Therefore, we have $F^+(V) \subseteq \mathrm{mInt}(F^+(V))$ and hence $F^+(V)$ is m-open in X.

(2) $\Rightarrow$ (3): Let K be any $(\sigma_1, \sigma_2)\theta$-closed set of Y. Then, $Y - K$ is $(\sigma_1, \sigma_2)\theta$-open in Y, by (2) we have $F^+(Y - K) = X - F^-(K)$ is m-open in X. Thus, $F^-(K)$ is m-closed in X.

(3) $\Rightarrow$ (2): It similar to that of (2) $\Rightarrow$ (3).

(2) $\Rightarrow$ (1): Let $x \in X$ and V be any $(\sigma_1, \sigma_2)\theta$-open set of Y containing $F(x)$. By (2), $x \in F^+(V) = \mathrm{mInt}(F^+(V))$. There exists an m-open set U of X containing x such that $U \subseteq F^+(V)$. Thus, $F(U) \subseteq V$ and so F is upper faintly $m(\sigma_1, \sigma_2)$-continuous at x. This shows that F is upper faintly $m(\sigma_1, \sigma_2)$-continuous. $\square$

Definition 4. *A multifunction $F : (X, m) \to (Y, \sigma_1, \sigma_2)$ is said to be lower faintly $m(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists an m-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$. A multifunction $F : (X, m) \to (Y, \sigma_1, \sigma_2)$ is said to be lower faintly $m(\sigma_1, \sigma_2)$-continuous if F is lower faintly $m(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Theorem 4. *For a multifunction $F : (X, m) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is lower faintly $m(\sigma_1, \sigma_2)$-continuous;*

(2) *$F^-(V)$ is m-open in X for every $(\sigma_1, \sigma_2)\theta$-open set V of Y;*

(3) *$F^+(K)$ is m-closed in X for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y.*

Proof. (1) $\Rightarrow$ (2): Let V be any $(\sigma_1, \sigma_2)\theta$-open set of Y and $x \in F^-(V)$. Then, $F(x) \cap V \neq \emptyset$ and by (1), there exists an m-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$. Therefore, we have $U \subseteq F^-(V)$ and hence $U \subseteq \mathrm{mInt}(F^-(V))$. Thus, $F^-(V) \subseteq \mathrm{mInt}(F^-(V))$ and so $F^-(V)$ is m-open in X.

(2) $\Rightarrow$ (3): Let K be any $(\sigma_1, \sigma_2)\theta$-closed set of Y. Then, $Y - K$ is $(\sigma_1, \sigma_2)\theta$-open in Y. By (2), we have $F^+(Y - K) = X - F^-(K)$ is m-open in X. Therefore, $F^-(K)$ is m-closed in X.

(3) $\Rightarrow$ (1): Let $x \in X$ and V be any $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $F(x) \cap V \neq \emptyset$. Then, $x \in F^-(V)$ and $x \notin X - F^-(V) = F^+(Y - V)$, by (3) we have $x \notin \mathrm{mCl}(F^+(Y - V))$. There exists an m-open set U of X containing x such that $U \cap F^+(Y - V) = \emptyset$; hence $U \subseteq F^-(V)$. Thus, $F(z) \cap V \neq \emptyset$

for each $z \in U$. Thus, F is lower faintly $m(\sigma_1, \sigma_2)$-continuous at x. This shows that F is lower faintly $m(\sigma_1, \sigma_2)$-continuous. □

Definition 5. *A function $f : (X, m) \to (Y, \sigma_1, \sigma_2)$ is called faintly $m(\sigma_1, \sigma_2)$-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)\theta$-open set V of Y containing $f(x)$, there exists an m-open set U of X containing x such that $f(U) \subseteq V$. A function $f : (X, m) \to (Y, \sigma_1, \sigma_2)$ is called faintly $m(\sigma_1, \sigma_2)$-continuous if f is faintly $m(\sigma_1, \sigma_2)$-continuous at each point x of X.*

Corollary 1. *For a function $f : (X, m) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is faintly $m(\sigma_1, \sigma_2)$-continuous;*
(2) *$f^{-1}(V)$ is m-open in X for every $(\sigma_1, \sigma_2)\theta$-open set V of Y;*
(3) *$f^{-1}(K)$ is m-closed in X for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y.*

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