

ROUGH-VIKOR FRAMEWORK FOR ROUGH MULTI-OBJECTIVE OPTIMIZATION

TAREK ABOU-EL-ENIEN¹, KAMILIA MOHAMMAD^{2,*}, ASHRAF ALMAHALAWY²¹Department of O.R. & D.S., Faculty of Computers and Artificial Intelligence, Cairo University, Egypt²Department of Physics and Engineering Mathematics, Faculty of Engineering, Tanta University, Egypt

*Corresponding author: Kamelia.Aboelfadl@f-eng.tanta.edu.eg

Received Jul. 2, 2026

ABSTRACT. Real-world multi-objective programming problems (MOPPs) often operate in imprecise decision regions, where rough approximations, rather than precise sets, define these regions. Although rough set theory (RST) offers a robust mathematical framework for addressing imprecision, its integration with a multi-objective optimization (MOO) approach to minimize individual regret remains theoretically underdeveloped. To address this disparity, we present an innovative extension of VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) for the conventional class of rough MOPPs (RMOPPs), where the decision sets are characterized by their lower and upper approximations. Our primary contribution is the reformulation of the metrics L_1 and L_∞ within a rough set-theoretic framework, guaranteeing that compromise solutions are obtained by examining the minimal and maximal probable decision sets. A numerical demonstration validates that the framework encapsulates the RST effect, providing a more robust solution than traditional crisp models.

2020 Mathematics Subject Classification. 90C29.

Key words and phrases. rough multi-objective; rough VIKOR; rough feasible set.

1. INTRODUCTION

The growing complexity of real-world systems has made decision-making among conflicting objectives and imprecise information a vital area of research. In certain instances, the decision set is imprecisely defined due to incomplete data, the vagueness of expert judgment, or the limitations of granular information systems; a phenomenon insufficiently represented by precise or probabilistic models. RST offers a rigorous set-theoretic framework for depicting and analyzing imprecision without requiring additional assumptions [12]. However, the integration of RST with an established MOO methodology for identifying a compromise solution that adaptively weighs the proximity to the ideal point against the individual regret, while systematically handling boundary uncertainties, remains largely unaddressed in the literature.

Although there is substantial literature on the study of imprecise parameters (intervals) in optimization problems [3–5, 8–10, 13–16], the roughness of the decision set itself has not been adequately researched. The investigation of optimization with rough decision sets originated with Youness [19], who defined rough single-objective programming. Subsequent work by Osman et al. [11] proposed classifications based on the roughness locus (decision set vs. objective). Atteya [2] extended these ideas to multi-objective settings by introducing a weighted-sum scalarization approach for RMOPPs with rough decision sets. However, this approach inherits well-known limitations, including sensitivity to objective scales, inability to capture non-convex Pareto fronts, and ad hoc weight selection that obscures the transparency of trade-offs. Recently, Abou-El-Enien et al. [1] developed a TOPSIS-based method in which the geometric distances to both ideal and anti-ideal reference points are evaluated within a rough decision set. This approach yields a compromise solution that is closest to the best and farthest from the worst.

The current methodologies fail to account for practical applications in which the cost of failing in a single objective is significantly higher than the value of achieving the best overall performance. Therefore, a method that preserves the structural properties of rough approximations is crucial in the search for a solution whose worst performance is acceptable. We choose VIKOR as the primary framework due to its emphasis on avoiding risk and minimizing negative deviations rather than maximizing positive performance. The VIKOR method, developed as a multi-criteria decision-making tool for discrete precise alternative spaces, aims to identify a compromise solution that is closest to the ideal point based on a specific aggregate measure [20], and has since been extended to continuous precise alternative spaces [21]. The fundamental philosophy of VIKOR, which reconciles maximizing collective utility and minimizing individual regret, is fully consistent with the principles of conflict resolution in some MOPPs.

2. MATHEMATICAL FOUNDATIONS AND PROBLEM FORMULATION

In this framework, we define an approximation space $\mathcal{A} = (\mathcal{U}, \mathcal{E})$, where $\mathcal{U}$ is a finite universe, and $\mathcal{E}$ is an equivalence relation [12]. $\mathcal{E}$ induces a partition $\mathcal{U}/\mathcal{E} = \{\mathcal{Y}_1, \mathcal{Y}_2, \dots, \mathcal{Y}_m\}$, representing the quotient (coarse) universe, while $\mathcal{U}$ remains the ground (fine) universe [18]. The function of $[x]_{\mathcal{E}}$ is dual: it acts as a subset in $\mathcal{U}$ and as a granule designated by a name in $\mathcal{U}/\mathcal{E}$ [6].

Any set $S \subseteq \mathcal{U}$ is characterized by its lower and upper approximations, S_L and S^v respectively [7].

$$S_L = \cup\{[x]_{\mathcal{E}} \mid \text{name}([x]_{\mathcal{E}}) \in \mathcal{U}/\mathcal{E}, [x]_{\mathcal{E}} \subseteq S\}$$

$$S^v = \cup\{[x]_{\mathcal{E}} \mid \text{name}([x]_{\mathcal{E}}) \in \mathcal{U}/\mathcal{E}, [x]_{\mathcal{E}} \cap S \neq \emptyset\}$$

The boundary region, $S_{BN} = S^v - S_L$, measures the epistemic vagueness of S . The set S is considered rough iff $S_{BN} \neq \emptyset$.

To transition between the fine and coarse universes, we employ two operators [17]. The operator $d : 2^{\mathcal{U}/\mathcal{E}} \rightarrow 2^{\mathcal{U}}$ maps the granular classes back to their constituent elements in the ground space. For any $\mathcal{H} \subseteq \mathcal{U}/\mathcal{E}$, $d(\mathcal{H}) = \bigcup_{\mathcal{Y} \in \mathcal{H}} d(\mathcal{Y})$, where $d(\mathcal{Y}) = [x]_{\mathcal{E}}$ for the unique $x \in \mathcal{U}$ such that $\mathcal{Y} = \text{name}([x]_{\mathcal{E}})$. The operator c coarsens elements into granules. For a rough set S , the abstraction is defined through its approximations as follows.

$$c(S_L) \subseteq c(S) \subseteq c(S^U)$$

where

$$c(S_L) = \{\mathcal{Y} \in \mathcal{U}/\mathcal{E} \mid d(\mathcal{Y}) \subseteq S_L\}$$

$$c(S^U) = \{\mathcal{Y} \in \mathcal{U}/\mathcal{E} \mid d(\mathcal{Y}) \subseteq S^U\}$$

We define an RMOPP where the objective functions are crisp, but the feasible region is a rough set:
RMOPP:

$$\begin{aligned} & \max \quad f_i(\mathbf{x}), \quad i \in I \\ & \min \quad f_j(\mathbf{x}), \quad j \in J \\ & \text{s.t.} \quad \mathbf{x} = (x_1, \dots, x_n) \in DS \\ & \quad \quad DS_L \subset DS \subset DS^U \\ & \quad \quad DS_L, DS^U \subseteq \mathcal{U} \end{aligned} \tag{1}$$

where the decision set is defined as a rough set $DS \subset \mathcal{U} \subseteq \mathbb{R}^n$, characterized by its lower and upper approximations, DS_L and DS^U respectively. $\mathbf{x}$ represents an n -dimensional decision vector that belongs to the ground universe $\mathcal{U}$. The objective functions $f_l : \mathcal{U} \rightarrow \mathbb{R}$ are $k = |I| + |J|$ precisely defined objective functions, with $f_i, i \in I$ to be maximized and $f_j, j \in J$ to be minimized.

3. A ROUGH-VIKOR FRAMEWORK

This section presents the mathematical framework to optimize precise objectives over an imprecise set of constraints. By combining RST with the geometric logic of VIKOR, we establish a method that preserves the structural roughness of the decision set throughout the optimization process.

3.1. Extending VIKOR Concepts to Rough Environments. To extend VIKOR to a rough environment, we must redefine some of its concepts.

Definition 1 (Rough Positive Ideal Solution). *For Problem (1), the rough positive ideal solution (PIS) is defined by the interval bounds $\underline{f}^*$ and $\overline{f}^*$ with components:*

$$\underline{f}_l^* = \begin{cases} \max \left\{ \max_{\mathbf{x} \in DS_L} f_i(\mathbf{x}), \max_{\mathcal{Y} \in c(DS_{BN})} \min_{\mathbf{x} \in d(\mathcal{Y})} f_i(\mathbf{x}) \right\} & , l = i \in I \\ \min_{\mathbf{x} \in DS^U} f_j(\mathbf{x}) & , l = j \in J \end{cases}$$

$$\overline{f}_l^* = \begin{cases} \max_{\mathbf{x} \in DS^U} f_i(\mathbf{x}) & , l = i \in I \\ \min \left\{ \min_{\mathbf{x} \in DS_L} f_j(\mathbf{x}), \min_{\mathcal{Y} \in c(DS_{BN})} \max_{\mathbf{x} \in d(\mathcal{Y})} f_j(\mathbf{x}) \right\} & , l = j \in J \end{cases}$$

Definition 2 (Rough Negative Ideal Solution). For Problem (1), the rough negative ideal solution (NIS) is defined by the interval bounds $\underline{f}^-$ and $\overline{f}^-$ with components:

$$\underline{f}_l^- = \begin{cases} \min_{\mathbf{x} \in DS^U} f_i(\mathbf{x}) & , l = i \in I \\ \max \left\{ \max_{\mathbf{x} \in DS_L} f_j(\mathbf{x}), \max_{\mathcal{Y} \in c(DS_{BN})} \min_{\mathbf{x} \in d(\mathcal{Y})} f_j(\mathbf{x}) \right\} & , l = j \in J \end{cases}$$

$$\overline{f}_l^- = \begin{cases} \min \left\{ \min_{\mathbf{x} \in DS_L} f_i(\mathbf{x}), \min_{\mathcal{Y} \in c(DS_{BN})} \max_{\mathbf{x} \in d(\mathcal{Y})} f_i(\mathbf{x}) \right\} & , l = i \in I \\ \max_{\mathbf{x} \in DS^U} f_j(\mathbf{x}) & , l = j \in J \end{cases}$$

Definition 3 (Rough Manhattan distance to the PIS). For Problem (1), the Manhattan distance $MD(\mathbf{x})$ from a point $\mathbf{x}$ to the ideal pole is specified by a dual-metric system comprising the pessimistic and optimistic $\mathcal{L}_1$ -norm distances to the PIS as follows:

$$MD_{\text{pess}}(\mathbf{x}) = \sum_{i \in I} w_i \cdot \frac{f_i^* - f_i(\mathbf{x})}{\underline{f}_i^* - \underline{f}_i^-} + \sum_{j \in J} w_j \cdot \frac{f_j(\mathbf{x}) - \overline{f}_j^*}{\underline{f}_j^- - \underline{f}_j^*}$$

$$MD_{\text{optim}}(\mathbf{x}) = \sum_{i \in I} w_i \cdot \frac{\overline{f}_i^* - f_i(\mathbf{x})}{\overline{f}_i^* - \underline{f}_i^-} + \sum_{j \in J} w_j \cdot \frac{f_j(\mathbf{x}) - \underline{f}_j^*}{\overline{f}_j^- - \underline{f}_j^*}$$

where $\sum_{l=1}^{|I|+|J|} w_l = 1$.

Definition 4 (Rough Chebyshev Distance to the PIS). For Problem (1), the Chebyshev distance $CD(\mathbf{x})$ from a point $\mathbf{x}$ to the ideal pole is specified by a dual-metric system comprising the pessimistic and optimistic $\mathcal{L}_\infty$ -norm distances to the PIS as follows:

$$CD_{\text{pess}}(\mathbf{x}) = \min_{\mathbf{x} \in DS^U} \max \left\{ w_i \cdot \frac{f_i^* - f_i(\mathbf{x})}{\underline{f}_i^* - \underline{f}_i^-}, w_j \cdot \frac{f_j(\mathbf{x}) - \overline{f}_j^*}{\underline{f}_j^- - \underline{f}_j^*} \right\}$$

$$CD_{\text{optim}}(\mathbf{x}) = \min_{\mathbf{x} \in DS^U} \max \left\{ w_i \cdot \frac{\overline{f}_i^* - f_i(\mathbf{x})}{\overline{f}_i^* - \underline{f}_i^-}, w_j \cdot \frac{f_j(\mathbf{x}) - \underline{f}_j^*}{\overline{f}_j^- - \underline{f}_j^*} \right\}$$

where $\sum_{l=1}^{|I|+|J|} w_l = 1$.

With regard to the two subsequent problems, $CD_{\text{pess}}(\mathbf{x})$ and $CD_{\text{optim}}(\mathbf{x})$ can be defined as the terms to the left of the active constraint i^+ (or j^+) that achieve the minimum ideal distance while maximizing the anti-ideal distance

in the next two problems, respectively.

$$\begin{aligned}
 \min \quad & \lambda_{\text{pess}} \\
 \text{s.t.} \quad & \mathbf{x} \in DS^U \\
 & \overline{f_i^-} \leq f_i(\mathbf{x}) \leq \underline{f_i^*}, \forall i \\
 & \overline{f_j^*} \leq f_j(\mathbf{x}) \leq \underline{f_j^-}, \forall j \\
 & w_i \cdot \frac{\overline{f_i^*} - f_i(\mathbf{x})}{\overline{f_i^*} - \underline{f_i^-}} \leq \lambda_{\text{pess}}, \forall i \\
 & w_j \cdot \frac{f_j(\mathbf{x}) - \underline{f_j^*}}{\overline{f_j^-} - \underline{f_j^*}} \leq \lambda_{\text{pess}}, \forall j
 \end{aligned}$$

$$\begin{aligned}
 \min \quad & \lambda_{\text{optim}} \\
 \text{s.t.} \quad & \mathbf{x} \in DS^U \\
 & w_i \cdot \frac{\overline{f_i^*} - f_i(\mathbf{x})}{\overline{f_i^*} - \underline{f_i^-}} \leq \lambda_{\text{optim}}, \forall i \\
 & w_j \cdot \frac{f_j(\mathbf{x}) - \underline{f_j^*}}{\overline{f_j^-} - \underline{f_j^*}} \leq \lambda_{\text{optim}}, \forall j
 \end{aligned}$$

Definition 5 (Extreme Values of the Rough Manhattan Distance to the PIS). For Problem (1), the best-case and worst-case values of pessimistic and optimistic Manhattan distances to the PIS are defined as follows:

$$\begin{aligned}
 MD_{\text{pess}}^* &= \min MD_{\text{pess}}(\mathbf{x}) \\
 \text{s.t.} \quad & \mathbf{x} \in DS^U \\
 & \overline{f_i^-} \leq f_i(\mathbf{x}) \leq \underline{f_i^*}, \forall i \\
 & \overline{f_j^*} \leq f_j(\mathbf{x}) \leq \underline{f_j^-}, \forall j \\
 MD_{\text{optim}}^* &= \min MD_{\text{optim}}(\mathbf{x}) \\
 \text{s.t.} \quad & \mathbf{x} \in DS^U \\
 MD_{\text{pess}}^- &= \max MD_{\text{pess}}(\mathbf{x}) \\
 \text{s.t.} \quad & \mathbf{x} \in DS^U \\
 & \overline{f_i^-} \leq f_i(\mathbf{x}) \leq \underline{f_i^*}, \forall i \\
 & \overline{f_j^*} \leq f_j(\mathbf{x}) \leq \underline{f_j^-}, \forall j \\
 MD_{\text{optim}}^- &= \max MD_{\text{optim}}(\mathbf{x}) \\
 \text{s.t.} \quad & \mathbf{x} \in DS^U
 \end{aligned}$$

Definition 6 (Extreme Values of the Rough Chebyshev Distance to the PIS). For Problem (1), the best-case and worst-case values of both pessimistic and optimistic Chebyshev distances to the PIS are defined as follows:

$$\begin{aligned}
 CD_{pess}^* &= \min CD_{pess}(\mathbf{x}) \\
 \text{s.t. } \mathbf{x} &\in DS^U \\
 \overline{f_i^-} &\leq f_i(\mathbf{x}) \leq \underline{f_i^*}, \forall i \\
 \overline{f_j^*} &\leq f_j(\mathbf{x}) \leq \underline{f_j^-}, \forall j \\
 CD_{optim}^* &= \min CD_{optim}(\mathbf{x}) \\
 \text{s.t. } \mathbf{x} &\in DS^U \\
 CD_{pess}^- &= \max CD_{pess}(\mathbf{x}) \\
 \text{s.t. } \mathbf{x} &\in DS^U \\
 \overline{f_i^-} &\leq f_i(\mathbf{x}) \leq \underline{f_i^*}, \forall i \\
 \overline{f_j^*} &\leq f_j(\mathbf{x}) \leq \underline{f_j^-}, \forall j \\
 CD_{optim}^- &= \max CD_{optim}(\mathbf{x}) \\
 \text{s.t. } \mathbf{x} &\in DS^U
 \end{aligned}$$

Definition 7 (Rough VIKOR Function). For Problem (1), the least and greatest achievable VIKOR functions are defined as follows:

$$\begin{aligned}
 Q_{least}(\mathbf{x}) &= \rho \cdot \frac{MD_{pess}^* - MD_{pess}(\mathbf{x})}{MD_{pess}^* - MD_{pess}^-} + (1 - \rho) \cdot \frac{CD_{pess}^* - CD_{pess}(\mathbf{x})}{CD_{pess}^* - CD_{pess}^-} \\
 Q_{greatest}(\mathbf{x}) &= \rho \cdot \frac{MD_{optim}^* - MD_{optim}(\mathbf{x})}{MD_{optim}^* - MD_{optim}^-} + (1 - \rho) \cdot \frac{CD_{optim}^* - CD_{optim}(\mathbf{x})}{CD_{optim}^* - CD_{optim}^-} \\
 \text{where } \rho &\in]0, 1].
 \end{aligned}$$

Definition 8 (Extreme Values of the Rough VIKOR Function). For Problem (1), the best-case values of the least and greatest achievable VIKOR functions are defined as follows:

$$\begin{aligned}
 Q_{least}^* &= \min Q_{least}(\mathbf{x}) \\
 \text{s.t. } \mathbf{x} &\in DS^U \\
 \overline{f_i^-} &\leq f_i(\mathbf{x}) \leq \underline{f_i^*}, \forall i \\
 \overline{f_j^*} &\leq f_j(\mathbf{x}) \leq \underline{f_j^-}, \forall j \\
 Q_{greatest}^* &= \min Q_{greatest}(\mathbf{x}) \\
 \text{s.t. } \mathbf{x} &\in DS^U
 \end{aligned}$$

Definition 9 (Compromise Solution Classification). *For Problem (1), the intersection of VIKOR function optimality and feasibility certainty partitions the compromise solution space into quadripartite taxonomy as follows:*

- (1) *The set of surely-feasible, optimistic-compromise solutions:*

$$F_s C_o = \{ \mathbf{x} \in DS_L \mid Q_{\text{greatest}}(\mathbf{x}) = Q_{\text{greatest}}^* \}$$

- (2) *The set of probably-feasible, optimistic-compromise solutions:*

$$F_p C_o = \{ \mathbf{x} \in DS_{BN} \mid Q_{\text{greatest}}(\mathbf{x}) = Q_{\text{greatest}}^* \}$$

- (3) *The set of surely-feasible, pessimistic-compromise solutions:*

$$F_s C_p = \{ \mathbf{x} \in DS_L \mid Q_{\text{least}}(\mathbf{x}) = Q_{\text{least}}^* \}$$

- (4) *The set of probably-feasible, pessimistic-compromise solutions:*

$$F_p C_p = \{ \mathbf{x} \in DS_{BN} \mid Q_{\text{least}}(\mathbf{x}) = Q_{\text{least}}^* \}$$

3.2. The proposed Rough-VIKOR algorithm. We now present our proposed modified VIKOR method to obtain a compromise solution to Problem (1).

The algorithm:

- (1) Determine the positive and negative ideal solution bounds:
 - Calculate $\underline{f}^*$ and $\overline{f}^*$ using Definition (1).
 - Calculate $\underline{f}^-$ and $\overline{f}^-$ using Definition (2).
- (2) Determine the optimistic and pessimistic distances to PIS, $\forall \mathbf{x} \in DS^U$:
 - Calculate MD_{optim} and MD_{pess} using Definition (3).
 - Calculate CD_{optim} and CD_{pess} using Definition (4).
- (3) Determine the best- and worst-case distance values:
 - Calculate MD_{optim}^* , MD_{pess}^* , MD_{optim}^- , and MD_{pess}^- using Definition (5).
 - Calculate CD_{optim}^* , CD_{pess}^* , CD_{optim}^- , and CD_{pess}^- using Definition (6).
- (4) Construct the least and greatest achievable VIKOR functions, $Q_{\text{least}}(\mathbf{x})$ and $Q_{\text{greatest}}(\mathbf{x})$, using Definition (7).
- (5) Calculate the best-case VIKOR function values, Q_{least}^* and Q_{greatest}^* by solving the optimization problems in Definition (8).
- (6) Obtain the rough compromise solution sets, $F_s C_o$, $F_p C_o$, $F_s C_p$, and $F_p C_p$, using Definition (9).

4. NUMERICAL ILLUSTRATION

To validate the proposed Rough-VIKOR framework, we consider a bi-objective linear optimization problem characterized by a rough decision set.

Example:

Let $\mathcal{U} = \{\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2 \mid -5 \leq x_1 \leq 5, -5 \leq x_2 \leq 5\}$ represent the continuous fine universe. We define an equivalence relation $\mathcal{E}$ that partitions $\mathcal{U}$ into four disjoint granules, $\mathcal{U}/\mathcal{E} = \{\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3, \mathcal{Y}_4\}$. The ground universe representations of these granules are defined as follows:

$$d(\mathcal{Y}_1) = \{\mathbf{x} \in \mathcal{U} \mid 1.75 \leq x_1 \leq 2.5, 0.5 \leq x_2 \leq 1.5\}$$

$$d(\mathcal{Y}_2) = \{\mathbf{x} \in \mathcal{U} \mid x_1 - 0.5x_2 \geq 1, x_1 < 1.75, x_2 \geq 0.5\}$$

$$d(\mathcal{Y}_3) = \{\mathbf{x} \in \mathcal{U} \mid x_1 + x_2 \leq 4, x_1 + 0.25x_2 \leq 3, x_1 > 2.5, x_2 \geq 0.5\}$$

$$d(\mathcal{Y}_4) = \{\mathbf{x} \in \mathcal{U} \mid \mathbf{x} \notin d(\mathcal{Y}_1) \cup d(\mathcal{Y}_2) \cup d(\mathcal{Y}_3)\}$$

We seek to optimize a bi-objective vector $f(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x})]$ over a rough decision set DS , where:

$$\begin{aligned} \max \quad & f_1(\mathbf{x}) = -0.1x_1 + 1.1x_2 \\ \min \quad & f_2(\mathbf{x}) = 0.5x_1 + 0.3x_2 \\ \text{s.t.} \quad & \mathbf{x} \in DS \\ & DS_L \subset DS \subset DS^u \end{aligned} \tag{2}$$

In this scenario, the rough decision set is defined by the following approximations:

$$DS_L = d(\mathcal{Y}_1) \cup d(\mathcal{Y}_2)$$

$$DS^u = d(\mathcal{Y}_1) \cup d(\mathcal{Y}_2) \cup d(\mathcal{Y}_3)$$

Solution:

The boundary region of the decision set in the coarse universe is identified as:

$$DS_{BN} = DS^u - DS_L = d(\mathcal{Y}_3)$$

Based on Definition (1), the lower and upper bounds of the PIS are computed as:

$$\underline{f_1^*} = \max \left\{ \max_{\mathbf{x} \in DS_L} f_1(\mathbf{x}), \max_{\mathcal{Y} \in c(DS_{BN})} \min_{\mathbf{x} \in d(\mathcal{Y})} f_1(\mathbf{x}) \right\} = 1.475$$

$$\overline{f_1^*} = \max_{\mathbf{x} \in DS^u} f_1(\mathbf{x}) = 1.475$$

$$\underline{f_2^*} = \min_{\mathbf{x} \in DS^u} f_2(\mathbf{x}) = 0.775$$

$$\overline{f_2^*} = \min \left\{ \min_{\mathbf{x} \in DS_L} f_2(\mathbf{x}), \min_{\mathcal{Y} \in c(DS_{BN})} \max_{\mathbf{x} \in d(\mathcal{Y})} f_2(\mathbf{x}) \right\} = 1.025$$

Thus, the rough PIS is characterized by the interval bounds:

$$\underline{f}^* = (1.475, 0.775)$$

$$\overline{f}^* = (1.475, 1.025)$$

Based on Definition (2), the lower and upper bounds of the NIS are computed as:

$$\underline{f}_1^- = \min_{\mathbf{x} \in DS^U} f_1(\mathbf{x}) = 0.2625$$

$$\overline{f}_1^- = \min \left\{ \min_{\mathbf{x} \in DS_L} f_1(\mathbf{x}), \min_{\mathcal{Y} \in c(DS_{BN})} \max_{\mathbf{x} \in d(\mathcal{Y})} f_1(\mathbf{x}) \right\} = 0.3$$

$$\underline{f}_2^- = \max \left\{ \max_{\mathbf{x} \in DS_L} f_2(\mathbf{x}), \max_{\mathcal{Y} \in c(DS_{BN})} \min_{\mathbf{x} \in d(\mathcal{Y})} f_2(\mathbf{x}) \right\} = 1.7$$

$$\overline{f}_2^- = \max_{\mathbf{x} \in DS^U} f_2(\mathbf{x}) = 1.733$$

Thus, the rough NIS is characterized by the interval bounds:

$$\underline{f}^- = (0.2625, 1.7)$$

$$\overline{f}^- = (0.3, 1.733)$$

Based on Definition (3), the pessimistic and optimistic Manhattan distances to the PIS at $w_1 = w_2 = 0.5$ are specified as:

$$MD_{pess}(\mathbf{x}) = \frac{524}{1269}x_1 - \frac{104}{423}x_2 - \frac{167}{1269}$$

$$MD_{optim}(\mathbf{x}) = \frac{674}{2231}x_1 - \frac{3314}{11155}x_2 + \frac{4549}{22310}$$

Based on Definition (4), the pessimistic and optimistic Chebyshev distances to the PIS at $w_1 = w_2 = 0.5$ are specified as:

$$CD_{pess}(\mathbf{x}) = \frac{2}{47}x_1 - \frac{22}{47}x_2 + \frac{59}{94}$$

$$CD_{optim}(\mathbf{x}) = \frac{4}{97}x_1 - \frac{44}{97}x_2 + \frac{59}{97}$$

Based on Definition (5), the best-case and worst-case values of pessimistic and optimistic Manhattan distances to the PIS are specified as:

$$MD_{pess}^* = \frac{2}{9} \text{ at } (1.75, 1.5)$$

$$MD_{optim}^* = \frac{33}{115} \text{ at } (1.75, 1.5)$$

$$MD_{pess}^- = \frac{373}{405} \text{ at } \left(\frac{43}{15}, \frac{8}{15}\right)$$

$$MD_{optim}^- = \frac{85}{92} \text{ at } (2.875, 0.5)$$

Based on Definition (6), the best-case and worst-case values of pessimistic and optimistic Chebyshev distances to the PIS are specified as:

$$CD_{pess}^* = 0 \text{ at } (1.75, 1.5)$$

$$CD_{optim}^* = 0 \text{ at } (1.75, 1.5)$$

$$CD_{pess}^- = \frac{97}{188} \text{ at } (2.875, 0.5)$$

$$CD_{optim}^- = 0.5 \text{ at } (2.875, 0.5)$$

Based on Definition (7), the least and greatest achievable VIKOR functions at $\rho = 0.5$ are specified as:

$$\mathcal{Q}_{least}(\mathbf{x}) = 0.3367x_1 - 0.6295x_2 + 0.3551$$

$$\mathcal{Q}_{greatest}(\mathbf{x}) = 0.2784x_1 - 0.6868x_2 + 0.543$$

Based on Definition (8), the best-case values of the least and greatest achievable VIKOR functions are specified as:

$$\mathcal{Q}_{least}^* = 0 \text{ at } (1.75, 1.5)$$

$$\mathcal{Q}_{greatest}^* = 0 \text{ at } (1.75, 1.5)$$

Based on Definition (9), the compromise sets for Problem (2) are specified as:

$$F_s C_o = \{\mathbf{x} \in DS_L \mid \mathcal{Q}_{greatest}(\mathbf{x}) = 0\} = \{(1.75, 1.5)\}$$

$$F_p C_o = \{\mathbf{x} \in DS_{BN} \mid \mathcal{Q}_{greatest}(\mathbf{x}) = 0\} = \emptyset$$

$$F_s C_p = \{\mathbf{x} \in DS_L \mid \mathcal{Q}_{least}(\mathbf{x}) = 0\} = \{(1.75, 1.5)\}$$

$$F_p C_p = \{\mathbf{x} \in DS_{BN} \mid \mathcal{Q}_{least}(\mathbf{x}) = 0\} = \emptyset$$

Under conditions of equal objective importance and equal importance of decision-making strategy, the proposed algorithm identifies a compromise solution to Problem (2) located within the lower approximation of the decision set. This point optimizes the first objective while accepting suboptimal performance for the second, characterizing it as a surely-feasible optimistic compromise solution. The results obtained using our framework align with the weighted-sum approach proposed by [2] under similar weighting configurations, validating the numerical consistency of our model. In contrast, the TOPSIS-based approach described in [1] fails to identify any surely-feasible solution under equal weights. Instead, it yields a solution located within the boundary region of the decision set.

The subsequent tables (Table 1 and Table 2) detail the resulting optimistic-compromise and pessimistic-compromise solutions to Problem (2), respectively, obtained under various weight scenarios.

TABLE 1. optimistic-compromise solutions obtained under various weight scenarios

w_1	w_2	ρ	Q_{greatest}^*	x_1	x_2	f_1	f_2
0.1	0.9	0.1	0.03934	1.75	1.5	1.475	1.325
0.1	0.9	0.2	0.07868	1.75	1.5	1.475	1.325
0.1	0.9	0.3	0.11802	1.75	1.5	1.475	1.325
0.1	0.9	0.4	0.15736	1.75	1.5	1.475	1.325
0.1	0.9	0.5	0.19669	1.75	1.5	1.475	1.325
0.1	0.9	0.6	0.19497	1.4773	0.95455	0.90227	1.025
0.1	0.9	0.7	0.14623	1.4773	0.95455	0.90227	1.025
0.1	0.9	0.8	0.09749	1.4773	0.95455	0.90227	1.025
0.1	0.9	0.9	0.04874	1.4773	0.95455	0.90227	1.025
0.2	0.8	0.1	0.03284	1.75	1.5	1.475	1.325
0.2	0.8	0.2	0.06568	1.75	1.5	1.475	1.325
0.2	0.8	0.3	0.09853	1.75	1.5	1.475	1.325
0.2	0.8	0.4	0.13137	1.75	1.5	1.475	1.325
0.2	0.8	0.5	0.16421	1.75	1.5	1.475	1.325
0.2	0.8	0.6	0.19497	1.4773	0.95455	0.90227	1.025
0.2	0.8	0.7	0.14623	1.4773	0.95455	0.90227	1.025
0.2	0.8	0.8	0.09749	1.4773	0.95455	0.90227	1.025
0.2	0.8	0.9	0.04874	1.4773	0.95455	0.90227	1.025
0.3	0.7	0.1	0.02219	1.75	1.5	1.475	1.325
0.3	0.7	0.2	0.04437	1.75	1.5	1.475	1.325
0.3	0.7	0.3	0.06656	1.75	1.5	1.475	1.325
0.3	0.7	0.4	0.08875	1.75	1.5	1.475	1.325
0.3	0.7	0.5	0.11093	1.75	1.5	1.475	1.325
0.3	0.7	0.6	0.13312	1.75	1.5	1.475	1.325
0.3	0.7	0.7	0.14623	1.4773	0.95455	0.90227	1.025
0.3	0.7	0.8	0.09749	1.4773	0.95455	0.90227	1.025
0.3	0.7	0.9	0.04874	1.4773	0.95455	0.90227	1.025

TABLE 1. optimistic-compromise solutions obtained under various weight scenarios
(Continued)

w_1	w_2	ρ	Q_{greatest}^*	x_1	x_2	f_1	f_2
0.4	0.6	0.1	0.01010	1.75	1.5	1.475	1.325
0.4	0.6	0.2	0.02019	1.75	1.5	1.475	1.325
0.4	0.6	0.3	0.03029	1.75	1.5	1.475	1.325
0.4	0.6	0.4	0.04038	1.75	1.5	1.475	1.325
0.4	0.6	0.5	0.05048	1.75	1.5	1.475	1.325
0.4	0.6	0.6	0.06057	1.75	1.5	1.475	1.325
0.4	0.6	0.7	0.07067	1.75	1.5	1.475	1.325
0.4	0.6	0.8	0.08076	1.75	1.5	1.475	1.325
0.4	0.6	0.9	0.04874	1.75	1.5	1.475	1.325
0.5	0.5	0.1	0	1.75	1.5	1.475	1.325
0.5	0.5	0.2	0	1.75	1.5	1.475	1.325
0.5	0.5	0.3	0	1.75	1.5	1.475	1.325
0.5	0.5	0.4	0	1.75	1.5	1.475	1.325
0.5	0.5	0.5	0	1.75	1.5	1.475	1.325
0.5	0.5	0.6	0	1.75	1.5	1.475	1.325
0.5	0.5	0.7	0	1.75	1.5	1.475	1.325
0.5	0.5	0.8	0	1.75	1.5	1.475	1.325
0.5	0.5	0.9	0	1.75	1.5	1.475	1.325
0.6	0.4	0.1	0	1.75	1.5	1.475	1.325
0.6	0.4	0.2	0	1.75	1.5	1.475	1.325
0.6	0.4	0.3	0	1.75	1.5	1.475	1.325
0.6	0.4	0.4	0	1.75	1.5	1.475	1.325
0.6	0.4	0.5	0	1.75	1.5	1.475	1.325
0.6	0.4	0.6	0	1.75	1.5	1.475	1.325
0.6	0.4	0.7	0	1.75	1.5	1.475	1.325
0.6	0.4	0.8	0	1.75	1.5	1.475	1.325
0.6	0.4	0.9	0	1.75	1.5	1.475	1.325

TABLE 1. optimistic-compromise solutions obtained under various weight scenarios
(Continued)

w_1	w_2	ρ	$\mathcal{Q}_{greatest}^*$	x_1	x_2	f_1	f_2
0.7	0.3	0.1	0	1.75	1.5	1.475	1.325
0.7	0.3	0.2	0	1.75	1.5	1.475	1.325
0.7	0.3	0.3	0	1.75	1.5	1.475	1.325
0.7	0.3	0.4	0	1.75	1.5	1.475	1.325
0.7	0.3	0.5	0	1.75	1.5	1.475	1.325
0.7	0.3	0.6	0	1.75	1.5	1.475	1.325
0.7	0.3	0.7	0	1.75	1.5	1.475	1.325
0.7	0.3	0.8	0	1.75	1.5	1.475	1.325
0.7	0.3	0.9	0	1.75	1.5	1.475	1.325
0.8	0.2	0.1	0	1.75	1.5	1.475	1.325
0.8	0.2	0.2	0	1.75	1.5	1.475	1.325
0.8	0.2	0.3	0	1.75	1.5	1.475	1.325
0.8	0.2	0.4	0	1.75	1.5	1.475	1.325
0.8	0.2	0.5	0	1.75	1.5	1.475	1.325
0.8	0.2	0.6	0	1.75	1.5	1.475	1.325
0.8	0.2	0.7	0	1.75	1.5	1.475	1.325
0.8	0.2	0.8	0	1.75	1.5	1.475	1.325
0.8	0.2	0.9	0	1.75	1.5	1.475	1.325
0.9	0.1	0.1	0	1.75	1.5	1.475	1.325
0.9	0.1	0.2	0	1.75	1.5	1.475	1.325
0.9	0.1	0.3	0	1.75	1.5	1.475	1.325
0.9	0.1	0.4	0	1.75	1.5	1.475	1.325
0.9	0.1	0.5	0	1.75	1.5	1.475	1.325
0.9	0.1	0.6	0	1.75	1.5	1.475	1.325
0.9	0.1	0.7	0	1.75	1.5	1.475	1.325
0.9	0.1	0.8	0	1.75	1.5	1.475	1.325
0.9	0.1	0.9	0	1.75	1.5	1.475	1.325

TABLE 2. pessimistic-compromise solutions obtained under various weight scenarios

w_1	w_2	ρ	Q_{least}^*	x_1	x_2	f_1	f_2
0.1	0.9	0.1	0	1.25	0.5	0.425	0.775
0.1	0.9	0.2	0	1.25	0.5	0.425	0.775
0.1	0.9	0.3	0	1.25	0.5	0.425	0.775
0.1	0.9	0.4	0	1.25	0.5	0.425	0.775
0.1	0.9	0.5	0	1.25	0.5	0.425	0.775
0.1	0.9	0.6	0	1.25	0.5	0.425	0.775
0.1	0.9	0.7	0	1.25	0.5	0.425	0.775
0.1	0.9	0.8	0	1.25	0.5	0.425	0.775
0.1	0.9	0.9	0	1.25	0.5	0.425	0.775
0.2	0.8	0.1	0	1.25	0.5	0.425	0.775
0.2	0.8	0.2	0	1.25	0.5	0.425	0.775
0.2	0.8	0.3	0	1.25	0.5	0.425	0.775
0.2	0.8	0.4	0	1.25	0.5	0.425	0.775
0.2	0.8	0.5	0	1.25	0.5	0.425	0.775
0.2	0.8	0.6	0	1.25	0.5	0.425	0.775
0.2	0.8	0.7	0	1.25	0.5	0.425	0.775
0.2	0.8	0.8	0	1.25	0.5	0.425	0.775
0.2	0.8	0.9	0	1.25	0.5	0.425	0.775
0.3	0.7	0.1	0	1.25	0.5	0.425	0.775
0.3	0.7	0.2	0	1.25	0.5	0.425	0.775
0.3	0.7	0.3	0	1.25	0.5	0.425	0.775
0.3	0.7	0.4	0	1.25	0.5	0.425	0.775
0.3	0.7	0.5	0	1.25	0.5	0.425	0.775
0.3	0.7	0.6	0	1.25	0.5	0.425	0.775
0.3	0.7	0.7	0	1.25	0.5	0.425	0.775
0.3	0.7	0.8	0	1.25	0.5	0.425	0.775
0.3	0.7	0.9	0	1.25	0.5	0.425	0.775

TABLE 2. pessimistic-compromise solutions obtained under various weight scenarios
(Continued)

w_1	w_2	ρ	$\mathcal{Q}_{least}^*$	x_1	x_2	f_1	f_2
0.4	0.6	0.1	0.00036	1.25	0.5	0.425	0.775
0.4	0.6	0.2	0.00072	1.25	0.5	0.425	0.775
0.4	0.6	0.3	0.00109	1.25	0.5	0.425	0.775
0.4	0.6	0.4	0.00145	1.25	0.5	0.425	0.775
0.4	0.6	0.5	0.00181	1.25	0.5	0.425	0.775
0.4	0.6	0.6	0.00217	1.25	0.5	0.425	0.775
0.4	0.6	0.7	0.00254	1.25	0.5	0.425	0.775
0.4	0.6	0.8	0.00290	1.25	0.5	0.425	0.775
0.4	0.6	0.9	0.00330	1.25	0.5	0.425	0.775
0.5	0.5	0.1	0	1.75	1.5	1.475	1.325
0.5	0.5	0.2	0	1.75	1.5	1.475	1.325
0.5	0.5	0.3	0	1.75	1.5	1.475	1.325
0.5	0.5	0.4	0	1.75	1.5	1.475	1.325
0.5	0.5	0.5	0	1.75	1.5	1.475	1.325
0.5	0.5	0.6	0	1.75	1.5	1.475	1.325
0.5	0.5	0.7	0	1.75	1.5	1.475	1.325
0.5	0.5	0.8	0	1.75	1.5	1.475	1.325
0.5	0.5	0.9	0	1.75	1.5	1.475	1.325
0.6	0.4	0.1	0	1.75	1.5	1.475	1.325
0.6	0.4	0.2	0	1.75	1.5	1.475	1.325
0.6	0.4	0.3	0	1.75	1.5	1.475	1.325
0.6	0.4	0.4	0	1.75	1.5	1.475	1.325
0.6	0.4	0.5	0	1.75	1.5	1.475	1.325
0.6	0.4	0.6	0	1.75	1.5	1.475	1.325
0.6	0.4	0.7	0	1.75	1.5	1.475	1.325
0.6	0.4	0.8	0	1.75	1.5	1.475	1.325
0.6	0.4	0.9	0	1.75	1.5	1.475	1.325

TABLE 2. pessimistic-compromise solutions obtained under various weight scenarios
(Continued)

w_1	w_2	ρ	$\mathcal{Q}_{least}^*$	x_1	x_2	f_1	f_2
0.7	0.3	0.1	0	1.75	1.5	1.475	1.325
0.7	0.3	0.2	0	1.75	1.5	1.475	1.325
0.7	0.3	0.3	0	1.75	1.5	1.475	1.325
0.7	0.3	0.4	0	1.75	1.5	1.475	1.325
0.7	0.3	0.5	0	1.75	1.5	1.475	1.325
0.7	0.3	0.6	0	1.75	1.5	1.475	1.325
0.7	0.3	0.7	0	1.75	1.5	1.475	1.325
0.7	0.3	0.8	0	1.75	1.5	1.475	1.325
0.7	0.3	0.9	0	1.75	1.5	1.475	1.325
0.8	0.2	0.1	0	1.75	1.5	1.475	1.325
0.8	0.2	0.2	0	1.75	1.5	1.475	1.325
0.8	0.2	0.3	0	1.75	1.5	1.475	1.325
0.8	0.2	0.4	0	1.75	1.5	1.475	1.325
0.8	0.2	0.5	0	1.75	1.5	1.475	1.325
0.8	0.2	0.6	0	1.75	1.5	1.475	1.325
0.8	0.2	0.7	0	1.75	1.5	1.475	1.325
0.8	0.2	0.8	0	1.75	1.5	1.475	1.325
0.8	0.2	0.9	0	1.75	1.5	1.475	1.325
0.9	0.1	0.1	0	1.75	1.5	1.475	1.325
0.9	0.1	0.2	0	1.75	1.5	1.475	1.325
0.9	0.1	0.3	0	1.75	1.5	1.475	1.325
0.9	0.1	0.4	0	1.75	1.5	1.475	1.325
0.9	0.1	0.5	0	1.75	1.5	1.475	1.325
0.9	0.1	0.6	0	1.75	1.5	1.475	1.325
0.9	0.1	0.7	0	1.75	1.5	1.475	1.325
0.9	0.1	0.8	0	1.75	1.5	1.475	1.325
0.9	0.1	0.9	0	1.75	1.5	1.475	1.325

5. CONCLUSION AND FUTURE WORK

This study establishes a mathematically rigorous extension of the VIKOR methodology specifically tailored for RMOPPs. By integrating the principles of individual regret and RST, the proposed framework successfully navigates the complexities of multi-objective optimization within a rough decision set. The core contribution of this study is the development of a systematic algorithmic pipeline that preserves the structural roughness of the decision space, from the definition of granular decoding operators to the construction of a four-part taxonomy of compromise solutions. Ultimately, this work demonstrates that "roughness" in a decision set is not a computational hurdle to be bypassed but a source of valuable information that, when modeled correctly, leads to more robust strategic choices.

To further enhance the capabilities of the Rough-VIKOR framework, several promising research avenues are proposed:

- Although the current model utilizes crisp objective functions, future work could extend the framework to include rough-fuzzy or neutrosophic objectives. This would allow the model to simultaneously manage the boundary uncertainty of the decision set and the subjective vagueness of human preferences.
- Integrating the proposed logic with meta-heuristic solvers could maintain computational efficiency in large-scale industrial applications.
- A dynamic Rough-VIKOR model would be highly beneficial for real-time decision support systems where the equivalence relations are not static but evolve over time.

Authors' Contributions. All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

REFERENCES

- [1] T. Abou-El-Enien, Y. Abo-Elnaga, K. Mohammad, TOPSIS for Multiple Objective Programming with Rough Decision Set, *Yugosl. J. Oper. Res.* 35 (2025), 113–134. <https://doi.org/10.2298/yjor230614003a>.
- [2] T.E.M. Atteya, Rough Multiple Objective Programming, *Eur. J. Oper. Res.* 248 (2016), 204–210. <https://doi.org/10.1016/j.ejor.2015.06.079>.
- [3] E. Ammar, A. Al-Asfar, A Study of Uncertainty Multi-Objective Nonlinear Programming Problems for Rough Intervals, *J. Intell. Fuzzy Syst.* 42 (2022), 4821–4835. <https://doi.org/10.3233/jifs-202586>.
- [4] A. Hamzehee, M.A. Yaghoobi, M. Mashinchi, Linear Programming with Rough Interval Coefficients, *J. Intell. Fuzzy Syst.* 26 (2014), 1179–1189. <https://doi.org/10.3233/ifs-130804>.
- [5] D.K. Jana, K. Maity, T.K. Roy, Multi-Item Production Inventory Model with Fuzzy Rough Coefficients via Geometric Programming Approach, *OPSEARCH* 50 (2013), 475–490. <https://doi.org/10.1007/s12597-013-0122-9>.

- [6] T.Y. Lin, Topological and Fuzzy Rough Sets, in: R. Słowiński, (ed) Intelligent Decision Support, Springer, Netherlands, Dordrecht, pp. 287–304, (1992). https://doi.org/10.1007/978-94-015-7975-9_18.
- [7] T.Y. Lin, Q. Liu, Rough Approximate Operators: Axiomatic Rough Set Theory, in: W.P. Ziarko, (ed) Rough Sets, Fuzzy Sets and Knowledge Discovery, Workshops in Computing, Springer, London, pp. 256–260, (1994). https://doi.org/10.1007/978-1-4471-3238-7_31.
- [8] H. Lu, G. Huang, L. He, An Inexact Rough-Interval Fuzzy Linear Programming Method for Generating Conjunctive Water-Allocation Strategies to Agricultural Irrigation Systems, Appl. Math. Model. 35 (2011), 4330–4340. <https://doi.org/10.1016/j.apm.2011.03.008>.
- [9] D. Mardanya, G. Maity, S.K. Roy, V.F. Yu, Solving the Multi-modal Transportation Problem via the Rough Interval Approach, RAIRO Oper. Res. 56 (2022), 3155–3185. <https://doi.org/10.1051/ro/2022131>.
- [10] M. Omran, O. Emam, A. Mahmoud, On Solving Three-Level Fractional Programming Problem with Rough Coefficient in Constraints, Br. J. Math. Comput. Sci. 12 (2016), 1–13. <https://doi.org/10.9734/bjmcs/2016/21932>.
- [11] M.S. Osman, E.F. Lashein, E.A. Youness, T.E.M. Atteya, Mathematical Programming in Rough Environment, Optimization 60 (2011), 603–611. <https://doi.org/10.1080/02331930903536393>.
- [12] Z. Pawlak, Rough Sets, Int. J. Comput. Inf. Sci. 11 (1982), 341–356. <https://doi.org/10.1007/bf01001956>.
- [13] O. Saad, O. Emam, M. Sleem, On the Solution of a Rough Interval Three-level Quadratic Programming Problem, Br. J. Math. Comput. Sci. 5 (2015), 349–366. <https://doi.org/10.9734/bjmcs/2015/13430>.
- [14] Shivani, D. Rani, A. Ebrahimnejad, On Solving Fully Rough Multi-Objective Fractional Transportation Problem: Development and Prospects, Comput. Appl. Math. 42 (2023), 266. <https://doi.org/10.1007/s40314-023-02400-z>.
- [15] Shivani, D. Rani, R.M. Rizk-Allah, A Study of Uncertain 4D Transportation Problems with Rough Interval Parameters and Additional Real-Life Factors, Appl. Soft Comput. 163 (2024), 111920. <https://doi.org/10.1016/j.asoc.2024.111920>.
- [16] Shivani, D. Rani, A. Ebrahimnejad, G. Gupta, Multi-Objective Non-Linear Programming Problem with Rough Interval Parameters: an Application in Municipal Solid Waste Management, Complex Intell. Syst. 10 (2024), 2983–3002. <https://doi.org/10.1007/s40747-023-01305-y>.
- [17] Y. Yao, C.-J. Liao, N. Zhong, Granular Computing Based on Rough Sets, Quotient Space Theory, and Belief Functions, in: Foundations of Intelligent Systems, Lecture Notes in Computer Science, Springer, Berlin, Heidelberg, pp. 152–159, (2003). https://doi.org/10.1007/978-3-540-39592-8_21.
- [18] Y. Yao, J. Yang, Granular Rough Sets and Granular Shadowed Sets: Three-Way Approximations in Pawlak Approximation Spaces, Int. J. Approx. Reason. 142 (2022), 231–247. <https://doi.org/10.1016/j.ijar.2021.11.012>.
- [19] E.A. Youness, Characterizing Solutions of Rough Programming Problems, Eur. J. Oper. Res. 168 (2006), 1019–1029. <https://doi.org/10.1016/j.ejor.2004.05.019>.
- [20] M. Yazdani, F.R. Graeml, VIKOR and Its Applications, Int. J. Strateg. Decis. Sci. 5 (2014), 56–83. <https://doi.org/10.4018/ijds.2014040105>.
- [21] M. Heydari, M.K. Sayadi, K. Shahanaghi, Extended VIKOR as a New Method for Solving Multiple Objective Large-Scale Nonlinear Programming Problems, RAIRO Oper. Res. 44 (2010), 139–152. <https://doi.org/10.1051/ro/2010011>.