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A NEW INVERSE STATISTICAL MODEL FOR FITTING DATA: DERIVATION AND STATISTICAL INFERENCE

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Abstract. This work examines the statistical and probabilistic characteristics of a novel probability distribution defined on the positive real line. The new distribution, called the inverse power modified mixture symmetric gamma distribution (IPMMSGD), was derived by modifying the mixture symmetric gamma distribution and applying the inverse power transformation to the modified distribution. This distribution can be used to describe heavy-tailed phenomena because of its rich structure, which combines inverse and polynomial terms with aspects of exponential decay. The probability density function (PDF), cumulative distribution function (CDF), hazard rate function (HRF), raw, incomplete, and conditional moments are among the important distributional features that we derived. Through graphical analysis, we observed that the PDF of the distribution is unimodal and positively skewed. Studying the model's limiting behaviour enabled us to discover that the distribution can be heavy-tailed. The maximum product of spacing estimation approach was established to be the best estimation procedure among twelve methods of point estimation for the parameters of the IPMMSGD compared through a simulation study. To demonstrate the adaptability and usefulness of the distribution, we fitted it to the lengths of 141 major North American rivers and compared its goodness-of-fit with those of the Weibull, Gompertz, Lindley, generalized Lindley, and Flexible Weibull distributions. Based on goodness-of-fit statistics, the IPMMSGD provides the best fit to the data among the distributions considered.

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1. Introduction

In a variety of fields, such as financial risk modelling, survival analysis, hydrology, and reliability engineering, probability distributions are essential for studying random phenomena. The pursuit of flexible probability models that can faithfully capture the stochastic nature of real-world phenomena continues to be an issue of major concern in statistical theory and applied probability. The creation of generalised distributions that can handle a variety of hazard rate behaviour, show skewness or heavy tails, and remain analytically tractable has drawn significant attention. The tendency of the probability density function (PDF) and hazard rate function (HRF) to have different shapes as a result of a change in parameter values, skewness, and tail behaviour, to a large extent, determines how flexible a distribution can be. There are methods for distribution construction, which do not lead to a change in the number of parameters. The inverse transformation belongs to this class of methods, whereas the inverse power transformation used in this paper adds one shape parameter. Both transformations are capable of producing distributions with upside-down bathtub-shaped hazard rate functions. For more details; see [23–27].

Distribution generalizing procedures, such as the exponentiation (Gupta et al. [16]), transmutation (Rahman et al. [17]), power transformation (Ghitany et al. [19]), Alpha Power Transformation (Mahdavi and Kundu [18]) and T-RY (Alzaatreh et al. [12]) are also widely used to introduce new and relatively more flexible distributions. It is noteworthy that the distributions constructed using these methods usually have more parameters than their associated baseline distributions. Chesneau et al. [11] suggested a new distribution that is a mixture of two symmetric distributions. Gemeay et al. [20] fitted industry data using the inverse-power generalized Maxwell distribution. Meraou et al. [21] used the T-X family of distributions to derive the exponential T-X Gompertz model for lifetime data.

The behavior of the tails of probability distributions plays a critical role in statistical modeling, particularly in applications involving extreme values, risk assessment, and reliability analysis. Classical distributions such as the normal, exponential, and Pareto offer canonical forms with well-understood tail properties: light-tailed in the case of the normal and exponential, and heavy-tailed in the case of the Pareto (Klebanov et al. [22]). However, real-world phenomena often exhibit asymmetric or "mixed" tail behaviors, where the left and right tails decay at different rates. Such cases are common in fields ranging from finance and insurance to environmental science and engineering. Despite the rich literature on heavy-tailed and light-tailed distributions, there is a relative paucity of flexible, analytically tractable continuous distributions that can accommodate both types of tail behavior simultaneously. Existing models often impose symmetric tail decay (e.g., Student's t-distribution) or restrict flexibility to within a single tail regime. This limitation motivates the construction of a new class of continuous

distributions defined explicitly in terms of their tail behavior, including the possibility of mixed tail behavior, for example, combining a heavy left tail with a light right tail, or vice versa.

In order to provide separate control over the left and right tails, this research presents a novel continuous distribution with parameterised tail decay functions. In order to maintain analytical tractability and flexibility in capturing a variety of empirical patterns, we define the distribution by altering the symmetric distribution of Chesneau et al. [11] using a combination of the inverse and power transformation procedures. Our method covers a broad range of real-world application scenarios by allowing modelling of data that can interpolate between exponential, polynomial, and even sub-exponential tail decay.

This is how the remainder of the paper is structured. The proposed model construction is presented in Section 2. Section 3 presents twelve methods for estimating the model parameters, and Section 4 compares their performance through a Monte Carlo simulation. The flexibility of the proposed model for fitting real data was presented in Section 5. In Section 6, the paper's conclusion is codified.

2. The Proposed Distribution and Its Properties

This section is devoted to the construction of the new model and the derivation of its notable properties.

2.1. **The Proposed Distribution Construction.**

Here, we derive the cumulative distribution function (CDF) and probability density function (PDF) of the distribution. Chesneau et al. [11] derived a new statistical model called the mixture symmetric gamma distribution (MSGD), its CDF and PDF are

$$G(x) = \begin{cases} \frac{1}{10}\left[4(1-\mu x)e^{\mu x} + e^{2\mu x}\right] & , x < 0, \\ 1 - \frac{1}{10}\left[4(1+\mu x)e^{-\mu x} + e^{-2\mu x}\right], & x \geq 0, \end{cases} \quad \mu > 0, \tag{1}$$

and

$$g(x) = \frac{\mu}{5}\left[2\mu|x|e^{-\mu|x|} + e^{-2\mu|x|}\right], \ x \in \mathbb{R}, \ \mu > 0.$$

First, we will derive a modified version of MSGD to have a new model with domain $x \geq 0$ called modified MSGD (MMSGD), its CDF is defined as follows

$$W(x) = 1 - \frac{2}{10}\left[4e^{-\mu x}(\mu x + 1) + e^{-2\mu x}\right]; x \geq 0, \ \mu > 0, \tag{2}$$

Now we will apply the inverse power transformation on the CDF (2), and then we will have a new model called inverse power MMSGD (IPMMSGD). Its CDF and PDF are

$$F(x) = \frac{1}{5}\left[e^{-\mu x^{-\lambda}}\left(4\mu x^{-\lambda} + 4\right) + e^{-2\mu x^{-\lambda}}\right]; x > 0, \ \mu > 0, \ \lambda > 0. \tag{3}$$

$$f(x) = \frac{2}{5}\lambda\mu x^{-2\lambda-1}e^{-2\mu x^{-\lambda}}\left(2\mu e^{\mu x^{-\lambda}} + x^{\lambda}\right); \ x > 0, \ \mu > 0, \ \lambda > 0. \tag{4}$$

With the combination of exponential and inverse power elements, this distribution emerges as a compound structure with rich mathematical features and adaptable modelling capabilities. The PDF guarantees that it defines a legitimate probability model since it is positive for all $x > 0$ and integrates to unity for all $\mu > 0$ and $\lambda > 0$, where $\mu^{1/\lambda}$ is a scale parameter and λ is a shape parameter. The graphical representation of the IPMMSGD's PDF for different parameter value combinations is shown in Figure 1. The distribution can display a variety of shapes, including unimodal forms that look decreasing (reversed-J shaped) when the mode is very close to zero, thanks to the combination of exponential and polynomial components, which makes it flexible enough to accommodate a variety of empirical data patterns.

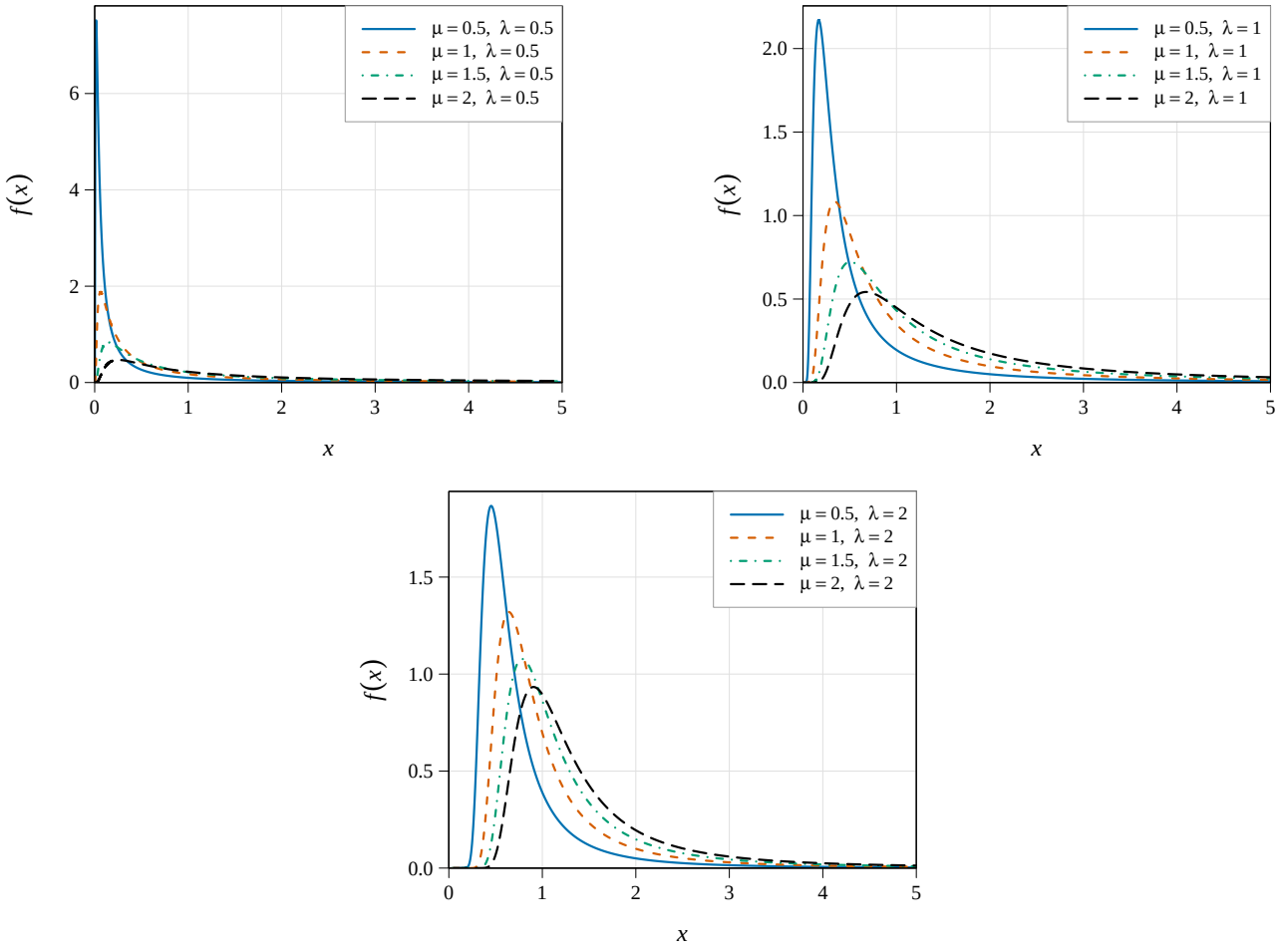


FIGURE 1. Plots of the PDF of the IPMMSGD for selected parameter values.

Next, we investigate the tail behaviour of the PDF as $x \to \infty$, pay attention to the role of λ. Now, if $x \to \infty$, then $x^{-\lambda} \to 0$, $e^{-2\mu x^{-\lambda}} \to 1$ and $x^{\lambda} \to \infty$. Hence, asymptotically, the PDF $f(x) \sim \frac{2}{5}\lambda\mu\left(2\mu x^{-2\lambda-1} + x^{-\lambda-1}\right)$. Here, the dominant term is $x^{-\lambda-1}$, implying a power-law decay for $\lambda > 0$. Again, as $x \to 0^{+}$, $x^{-\lambda} \to \infty$, $e^{-2\mu x^{-\lambda}} \to 0$ and $e^{\mu x^{-\lambda}} \to \infty$. In this case, $f(x) \approx \frac{4}{5}\lambda\mu^{2}x^{-2\lambda-1}e^{-\mu x^{-\lambda}}$,

which is super-exponentially decaying function dominated by $e^{-\mu x^{-\lambda}}$ term.

The likelihood that a given system or object will last past a given time (t) is indicated by the survival function (SF). The SF is useful for modelling and analysing time-to-event data, such as failure times and a host of other noteworthy occurrences that take place over time. The IPMMSGD's SF is given by

$$S(x) = 1 - \frac{1}{5}\left[e^{-\mu x^{-\lambda}}\left(4\mu x^{-\lambda} + 4\right) + e^{-2\mu x^{-\lambda}}\right]; x > 0,\ \mu > 0,\ \lambda > 0. \tag{5}$$

The related hazard rate function (HRF) is defined as

$$h(x) = \frac{2\lambda\mu x^{-2\lambda-1}e^{-2\mu x^{-\lambda}}\left(2\mu e^{\mu x^{-\lambda}} + x^{\lambda}\right)}{5\left(1 - \frac{1}{5}\left[e^{-\mu x^{-\lambda}}\left(4\mu x^{-\lambda} + 4\right) + e^{-2\mu x^{-\lambda}}\right]\right)}; x > 0,\ \mu > 0,\ \lambda > 0. \tag{6}$$

The HRF is graphed in Figure 2. Since $h(x) \to 0$ as $x \to 0^+$ and $h(x) \approx \lambda/x \to 0$ as $x \to \infty$, the HRF is always unimodal (upside-down bathtub-shaped); when its mode is very close to zero, as for $\mu = \lambda = 0.5$, it appears decreasing.

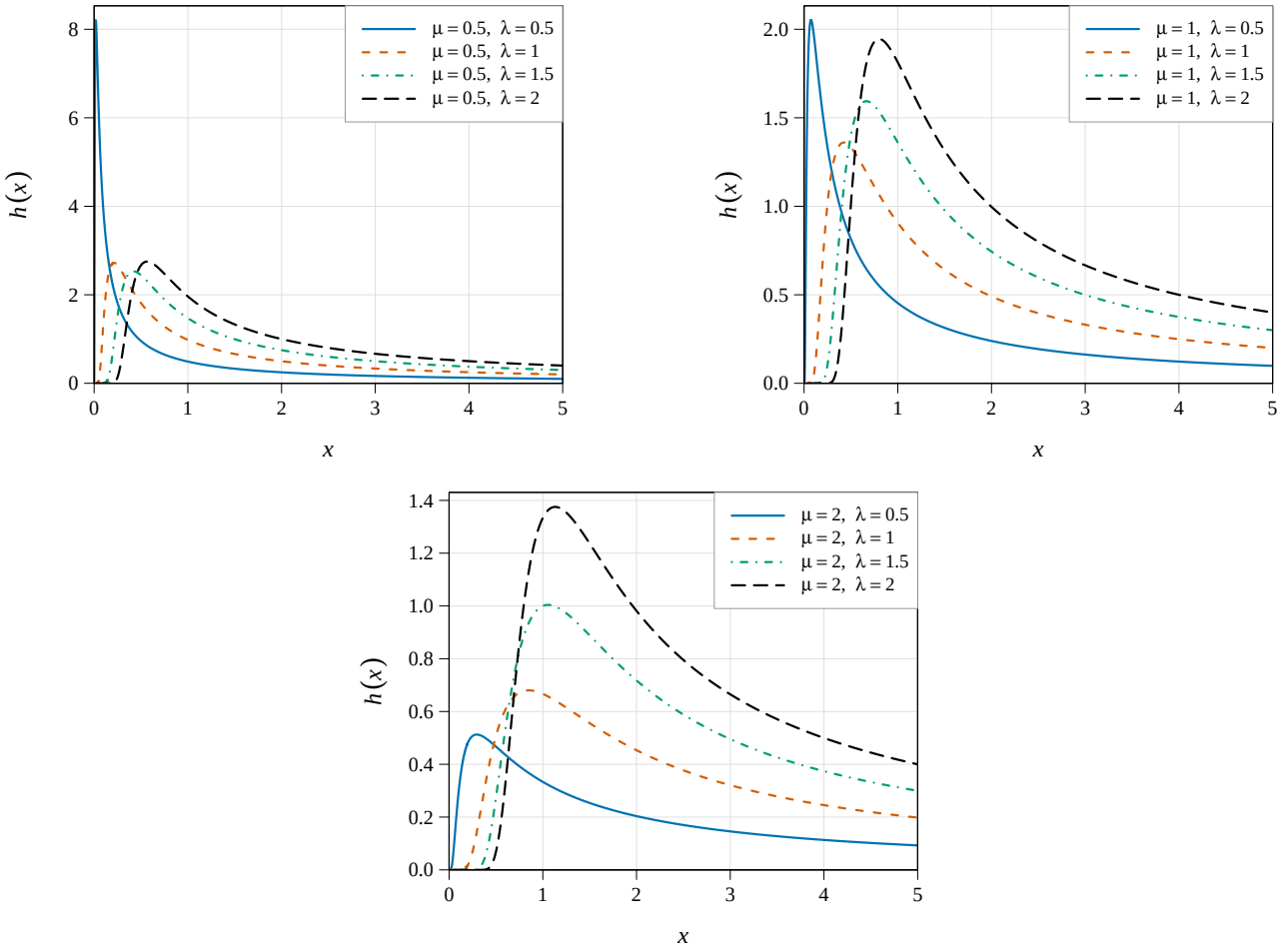


FIGURE 2. Plots of the HRF of the IPMMSGD for selected parameter values.

2.2. **Moments.** Raw moments of a random variable are an important tool for studying properties of the distribution of the random variable, including variance, skewness, and kurtosis.

$$\begin{aligned} M_r &= \int_0^\infty x^r f(x)dx = \frac{2}{5}\lambda\mu \int_0^\infty x^{r-2\lambda-1} e^{-2\mu x^{-\lambda}} \left(2\mu e^{\mu x^{-\lambda}} + x^\lambda\right) dx \\ &= \frac{2}{5}\lambda\mu \left[2\mu \int_0^\infty x^{r-2\lambda-1} e^{-\mu x^{-\lambda}} dx + \int_0^\infty x^{r-\lambda-1} e^{-2\mu x^{-\lambda}} dx\right] \\ &= \frac{2}{5}\lambda\mu \left[\frac{2\mu^{\frac{r}{\lambda}-1}}{\lambda}\left(1-\frac{r}{\lambda}\right)\Gamma\left(1-\frac{r}{\lambda}\right) + \frac{(2\mu)^{\frac{r}{\lambda}-1}}{\lambda}\Gamma\left(1-\frac{r}{\lambda}\right)\right] \end{aligned} \tag{7}$$

Simplifying (7), we obtain

$$M_r = \frac{\left[\lambda\left(2^{r/\lambda}+4\right)-4r\right]\mu^{r/\lambda}\Gamma\left(1-\frac{r}{\lambda}\right)}{5\lambda}, r < \lambda. \tag{8}$$

Hence, the mean, variance, coefficients of skewness and kurtosis of the distribution, denoted by M_1, $Var(X)$, S and K are defined as follows:

$$\begin{aligned} M_1 &= \frac{\left[\lambda\left(2^{1/\lambda}+4\right)-4\right]\mu^{1/\lambda}\Gamma\left(1-\frac{1}{\lambda}\right)}{5\lambda}; \\ Var(X) = M_2 - M_1^2 &= \frac{\left[\lambda\left(2^{2/\lambda}+4\right)-8\right]\mu^{2/\lambda}\Gamma\left(1-\frac{2}{\lambda}\right)}{5\lambda} - \left[\frac{\left[\lambda\left(2^{1/\lambda}+4\right)-4\right]\mu^{1/\lambda}\Gamma\left(1-\frac{1}{\lambda}\right)}{5\lambda}\right]^2; \\ S &= \frac{M_3 - 3M_1M_2 + 2M_1^3}{(Var(X))^{\frac{3}{2}}}; \\ K &= \frac{M_4 - 4M_1M_3 + 6M_1^2M_2 - 3M_1^4}{(Var(X))^2}. \end{aligned}$$

Since M_r is finite only for $r < \lambda$, the mean exists for $\lambda > 1$, the variance for $\lambda > 2$, the skewness for $\lambda > 3$, and the kurtosis for $\lambda > 4$. Here K is the (non-excess) kurtosis, so the excess kurtosis is $K - 3$. Moreover, $X = \mu^{1/\lambda}Z$, where Z follows the IPMMSGD with $\mu = 1$, so M_r is proportional to $\mu^{r/\lambda}$ and the coefficients S and K depend only on λ, not on μ. This is why the skewness and kurtosis surfaces in Figure 3 are flat in the μ direction. 3D plots of the mean, variance, skewness, and kurtosis of IPMMSGD are given in Figure 3.

2.3. **Incomplete Moments.** Incomplete moments are statistical measures that have found widespread applications. In particular, the first incomplete moment is useful in deriving the mean deviations and income inequality measures like the Bonferroni and Lorenz curves. Symbolically, the rth incomplete moment is defined by

$$\begin{aligned} J_r(z) &= \int_0^z x^r f(x)dx = \frac{2}{5}\lambda\mu \int_0^z x^{r-2\lambda-1} e^{-2\mu x^{-\lambda}} \left(2\mu e^{\mu x^{-\lambda}} + x^\lambda\right) dx \\ &= \frac{2}{5}\lambda\mu \left[2\mu \int_0^z x^{r-2\lambda-1} e^{-\mu x^{-\lambda}} dx + \int_0^z x^{r-\lambda-1} e^{-2\mu x^{-\lambda}} dx\right] \end{aligned}$$

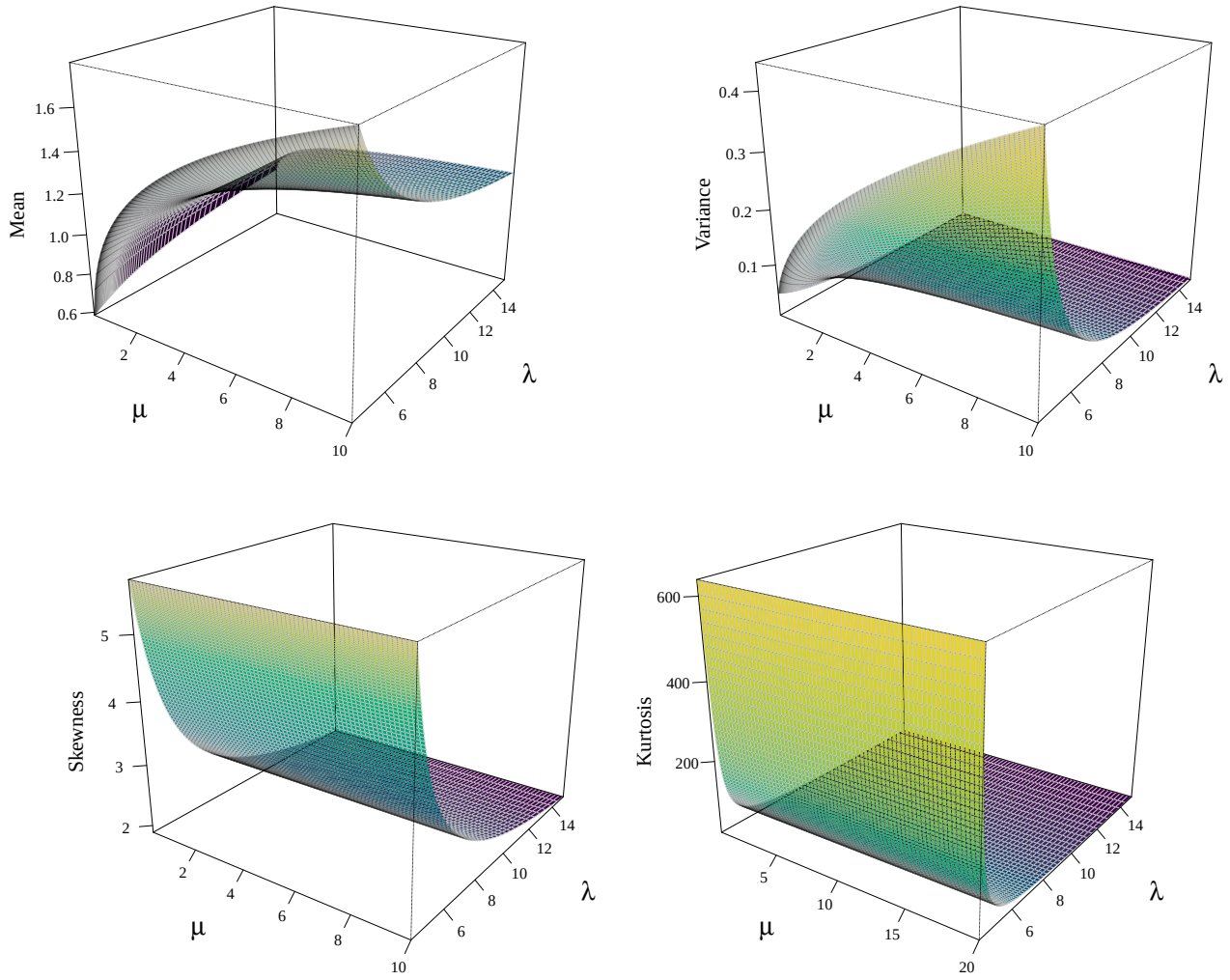


FIGURE 3. Plots of the descriptive statistics for the IPMMSGD for selected parameter values.

$$= \frac{\mu^{\frac{r}{\lambda}}}{5}\left[4\Gamma\left(2-\frac{r}{\lambda},\mu z^{-\lambda}\right)+2^{\frac{r}{\lambda}}\Gamma\left(1-\frac{r}{\lambda},2\mu z^{-\lambda}\right)\right], \tag{9}$$

where $\Gamma(a,z)=\int_z^\infty t^{a-1}e^{-t}\,dt$ is the upper incomplete gamma function.

2.4. **Conditional Moments.** In domains such as reliability analysis, finance, and extreme value theory, where interest frequently lies in the behaviour of variables under uncommon or extreme conditions, the conditional moment $\mathbb{E}\left(X^r|X>t\right)$ represents the expected value of the rth power of a random variable X, given that X exceeds a threshold t. The conditional moment is defined as follows for a continuous random variable X with probability density function $f(x)$:

$$\mathbb{E}\left(X^r|X>t\right)=\frac{\int_t^\infty x^r f(x)dx}{S(t)}.$$

Hence, the rth conditional moment of the IPMMSGD has the form

$$\begin{aligned}\mathbb{E}\left(X^r|X>t\right) &= \frac{2\lambda\mu\int_t^\infty x^{r-2\lambda-1}e^{-2\mu x^{-\lambda}}\left(2\mu e^{\mu x^{-\lambda}}+x^\lambda\right)dx}{5S(t)}\\ &= \frac{2\lambda\mu\left[2\mu\int_t^\infty x^{r-2\lambda-1}e^{-\mu x^{-\lambda}}dx+\int_t^\infty x^{r-\lambda-1}e^{-2\mu x^{-\lambda}}dx\right]}{5S(t)}\\ &= \frac{\mu^{\frac{r}{\lambda}}\left[4\gamma\left(2-\frac{r}{\lambda},\mu t^{-\lambda}\right)+2^{\frac{r}{\lambda}}\gamma\left(1-\frac{r}{\lambda},2\mu t^{-\lambda}\right)\right]}{5-\left[e^{-\mu t^{-\lambda}}\left(4\mu t^{-\lambda}+4\right)+e^{-2\mu t^{-\lambda}}\right]},\end{aligned}$$

where $\gamma(.)$ is the lower incomplete gamma function, $\gamma(a,z)=\int_0^z t^{a-1}e^{-t}\,dt$, and the result requires $r<\lambda$.

2.5. **Mean Residual Life and Mean Past Life function.** The useful extra life length for a unit that is alive at age x is known as the Mean Residual Life (MRL) or life expectancy at age x. It can be used in a wide range of fields, including insurance, economics, demography, product control, and life testing. The MRL for the IPMMSGD, which exists for $\lambda>1$, is defined thus:

$$m(x)=\mathbb{E}\left(X|X>x\right)-x=\frac{\mu^{\frac{1}{\lambda}}\left[4\gamma\left(2-\frac{1}{\lambda},\mu x^{-\lambda}\right)+2^{\frac{1}{\lambda}}\gamma\left(1-\frac{1}{\lambda},2\mu x^{-\lambda}\right)\right]}{5-\left[e^{-\mu x^{-\lambda}}\left(4\mu x^{-\lambda}+4\right)+e^{-2\mu x^{-\lambda}}\right]}-x.$$

The Mean Past Life function (MPLF), $\psi(x)=\mathbb{E}\left(x-X|X\le x\right)$, gives the expected time elapsed since the failure of a unit, given that it failed at or before time x. The definition of the MPLF of the IPMMSGD is as follows:

$$\psi(x)=x-\frac{J_1(x)}{F(x)}=x-\frac{\mu^{\frac{1}{\lambda}}\left[4\Gamma\left(2-\frac{1}{\lambda},\mu x^{-\lambda}\right)+2^{\frac{1}{\lambda}}\Gamma\left(1-\frac{1}{\lambda},2\mu x^{-\lambda}\right)\right]}{\left[e^{-\mu x^{-\lambda}}\left(4\mu x^{-\lambda}+4\right)+e^{-2\mu x^{-\lambda}}\right]}.$$

2.6. **Mean Deviations.** When robustness to outliers is desired in quality control, risk assessment, and economic modelling, mean deviation is a particularly helpful and understandable indicator of variability. Although it is less frequently employed in inferential statistics than variance-based measures, descriptive analysis benefits from its conceptual simplicity and robustness to extreme values. In what follows, we define two kinds of mean deviation based on the IPMMSGD. These are the mean deviation about the mean δ_1 and mean deviation about the median δ_2 defined below:

$$\delta_1=\int_0^\infty|x-M_1|f(x)dx=2M_1F(M_1)-2J_1(M_1). \tag{10}$$

Let ξ be the median of the IPMMSGD, obtained numerically from $F(\xi)=1/2$. The mean deviation about the median of the IPMMSGD is

$$\delta_2=\int_0^\infty|x-\xi|f(x)dx=M_1-2J_1(\xi). \tag{11}$$

2.7. **Bonferroni and Lorenz Curves.** Two essential instruments in inequality analysis are the Lorenz and Bonferroni curves. Their incorporation into empirical research improves knowledge of the dynamics of income distribution and helps create just social and economic policies. Let $q = F^{-1}(p)$ denote the pth quantile function of the IPMMSGD; since F has no closed-form inverse, q is obtained numerically by solving $F(q) = p$. The Bonferroni curve is given by

$$\begin{aligned} B(p) &= \frac{J_1(q)}{pM_1} \\ &= \frac{\lambda\mu^{\frac{1}{\lambda}}\left[4\Gamma\left(2-\frac{1}{\lambda},\mu q^{-\lambda}\right)+2^{\frac{1}{\lambda}}\Gamma\left(1-\frac{1}{\lambda},2\mu q^{-\lambda}\right)\right]}{p\left[\lambda\left(2^{1/\lambda}+4\right)-4\right]\mu^{1/\lambda}\Gamma\left(1-\frac{1}{\lambda}\right)}. \end{aligned}$$

The related Lorenz curve is of the form

$$\begin{aligned} L(p) &= \frac{J_1(q)}{M_1} \\ &= \frac{\lambda\mu^{\frac{1}{\lambda}}\left[4\Gamma\left(2-\frac{1}{\lambda},\mu q^{-\lambda}\right)+2^{\frac{1}{\lambda}}\Gamma\left(1-\frac{1}{\lambda},2\mu q^{-\lambda}\right)\right]}{\left[\lambda\left(2^{1/\lambda}+4\right)-4\right]\mu^{1/\lambda}\Gamma\left(1-\frac{1}{\lambda}\right)}. \end{aligned}$$

2.8. **Entropy and Extropy.** Two different measures of uncertainty serve as the foundation for this section. The terms entropy and extropy are used here. A crucial idea in statistics, thermodynamics, and information theory is entropy. Rényi, Arimoto, and Tsallis entropies, represented by R(γ), A(γ), and T(γ), respectively, are three different forms of entropy under consideration. All three depend on $I_\gamma = \int_0^\infty f^\gamma(x)\,dx$, $\gamma > 0$, $\gamma \neq 1$. When γ is a positive integer, expanding $\left(2\mu e^{\mu x^{-\lambda}} + x^\lambda\right)^\gamma$ by the binomial theorem gives a finite sum:

$$\begin{aligned} I_\gamma &= \left(\frac{2}{5}\lambda\mu\right)^\gamma \int_0^\infty x^{-(2\lambda+1)\gamma} e^{-2\gamma\mu x^{-\lambda}} \left(2\mu e^{\mu x^{-\lambda}} + x^\lambda\right)^\gamma dx \\ &= \left(\frac{2}{5}\lambda\mu\right)^\gamma \sum_{i=0}^{\gamma} \binom{\gamma}{i} (2\mu)^i \int_0^\infty x^{-\lambda i-\gamma(\lambda+1)} e^{-(2\gamma-i)\mu x^{-\lambda}} dx \\ &= \left(\frac{1}{5}\right)^\gamma \sum_{i=0}^{\gamma} 2^{i+\gamma} \binom{\gamma}{i} \lambda^{\gamma-1}\mu^{\frac{1-\gamma}{\lambda}} (2\gamma-i)^{-i-\gamma-\frac{\gamma-1}{\lambda}} \Gamma\left(i+\gamma+\frac{\gamma-1}{\lambda}\right). \end{aligned}$$

For non-integer γ, the corresponding infinite series diverges, because every term with $i > 2\gamma$ leads to a divergent integral; I_γ must then be computed numerically. As a consequence, we have

$$\begin{aligned} R(\gamma) &= \frac{1}{1-\gamma}\log I_\gamma, \\ A(\gamma) &= \frac{\gamma}{1-\gamma}\left[I_\gamma^{1/\gamma} - 1\right], \\ T(\gamma) &= \frac{1}{\gamma-1}\left[1 - I_\gamma\right], \end{aligned}$$

where $\gamma > 0$ and $\gamma \neq 1$.

The notion of extropy was created by Lad et al. [15] as a dual Shannon entropy metric. Furthermore, the concept of extropy of a continuous random variable has been discussed in quite a number of literary studies, such as Mohammed et al. [14] and Okereke et al. [13]. It is mathematically defined as

$$Ex(X) = -\frac{1}{2}\int_0^\infty f^2(x)dx. \tag{12}$$

Hence, in the case of IPMMSGD,

$$\begin{aligned} Ex(X) &= -\frac{2}{25}\lambda^2\mu^2\int_0^\infty x^{-4\lambda-2}e^{-4\mu x^{-\lambda}}\left(2\mu e^{\mu x^{-\lambda}}+x^{\lambda}\right)^2 dx \\ &= -\frac{2}{25}\lambda^2\mu^2\int_0^\infty \left(4\mu^2 x^{-4\lambda-2}e^{-2\mu x^{-\lambda}}+4\mu x^{-3\lambda-2}e^{-3\mu x^{-\lambda}}+x^{-2\lambda-2}e^{-4\mu x^{-\lambda}}\right)dx \\ &= -\frac{2}{25}\lambda\mu^{-\frac{1}{\lambda}}\Gamma\left(2+\frac{1}{\lambda}\right)\left[2^{-2-\frac{1}{\lambda}}\left(3+\frac{1}{\lambda}\right)\left(2+\frac{1}{\lambda}\right)+4\times 3^{-3-\frac{1}{\lambda}}\left(2+\frac{1}{\lambda}\right)+4^{-2-\frac{1}{\lambda}}\right]. \end{aligned}$$

3. Estimation Methods

In this section, we considered twelve different estimation methods to estimate the two unknown parameters of the IPMMSGD model. These methods are maximum likelihood estimation (MLE), Anderson–Darling estimation (ADE), Cramér–von Mises estimation (CVME), ordinary least squares estimation (LSE), weighted least squares estimation (WLSE), Kolmogorov estimation (KME), left-tail Anderson–Darling estimation (ADLTE), left-tail second-order Anderson–Darling estimation (ADLTSOE), right-tail Anderson–Darling estimation (ADRTE), maximum product of spacings estimation (MPSE), minimum spacing absolute distance estimation (MSADE), and minimum spacing absolute-log distance estimation (MSALDE). For more details on these estimators, see [7–10, 28–30]. In the next section, we conducted a Monte Carlo simulation to compare and evaluate the performance of these twelve estimation methods. Nine different scenarios are applied for this simulation study. λ and μ values for each scenario are presented in Table 1.

Table 1. Different parameter values with scenarios for simulation

Scenarios	λ	μ
Scenario 1	1.5	1.8
Scenario 2	1.6	1.9
Scenario 3	1.7	2.7
Scenario 4	1.8	2.5
Scenario 5	2.0	2.4
Scenario 6	2.2	2.7
Scenario 7	3.2	2.8
Scenario 8	3.3	1.8
Scenario 9	3.5	2.2

Assuming that $X_1, X_2, \ldots, X_n$ is a random sample from the IPMMSGD distribution and that $x_1, x_2, \ldots, x_n$ represent the observed values of the sample, the order statistics based on this sample are denoted as $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$. The likelihood (L) and log-likelihood (ℓ) functions are defined as follows:

$$L(\Psi) = \prod_{i=1}^{n} \frac{2}{5} \lambda \mu x_i^{-2\lambda-1} e^{-2\mu x_i^{-\lambda}} \left(2\mu e^{\mu x_i^{-\lambda}} + x_i^{\lambda}\right)$$

and

$$\begin{aligned} \ell(\Psi) &= n\left[\log(\mu) + \log(\lambda) + \log(2) - \log(5)\right] - (2\lambda+1)\sum_{i=1}^{n}\log(x_i) \\ &\quad -2\mu\sum_{i=1}^{n} x_i^{-\lambda} + \sum_{i=1}^{n}\log\left(2\mu e^{\mu x_i^{-\lambda}} + x_i^{\lambda}\right), \end{aligned} \tag{13}$$

where, $\boldsymbol{\Psi} = (\lambda, \mu)$. Now, to obtain the eleven other estimators, they are defined as follows:

$$\begin{aligned} ADE(\Psi) &= -n - \sum_{i=1}^{n}\left(\frac{2i-1}{n}\right)\left\{\log\left[\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right]\right]\right. \\ &\quad \left. +\log\left[1 - \frac{1}{5}\left[e^{-\mu x_{(n+1-i)}^{-\lambda}}\left(4\mu x_{(n+1-i)}^{-\lambda}+4\right) + e^{-2\mu x_{(n+1-i)}^{-\lambda}}\right]\right]\right\}, \end{aligned} \tag{14}$$

$$CVME(\Psi) = \frac{1}{12n} + \sum_{i=1}^{n}\left\{\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right] - \frac{2i-1}{2n}\right\}^2, \tag{15}$$

$$LSE(\Psi) = \sum_{i=1}^{n}\left\{\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right] - \frac{i}{n+1}\right\}^2, \tag{16}$$

$$WLSE(\Psi) = \sum_{i=1}^{n}\left[\frac{(n+1)^2(n+2)}{i(n-i+1)}\right]\left\{\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right] - \frac{i}{n+1}\right\}^2, \tag{17}$$

$$\begin{aligned} KME(\Psi) &= \max_{1\le i\le n}\left\{\frac{i}{n} - \frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right],\right. \\ &\quad \left.\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right] - \frac{i-1}{n}\right\}, \end{aligned} \tag{18}$$

$$\begin{aligned} ADLTE(\Psi) &= -\frac{3}{2}n + \sum_{i=1}^{n} 2\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right] \\ &\quad -\sum_{i=1}^{n}\left(\frac{2i-1}{n}\right)\log\left[\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right]\right], \end{aligned} \tag{19}$$

$$\begin{aligned} ADLTSOE(\Psi) &= 2\sum_{i=1}^{n}\log\left[\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right]\right] \\ &\quad +\sum_{i=1}^{n}\left(\frac{2i-1}{n}\right)\left\{\frac{5}{\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right) + e^{-2\mu x_{(i)}^{-\lambda}}\right]}\right\}, \end{aligned} \tag{20}$$

$$ADRTE(\Psi) = \frac{n}{2} - \sum_{i=1}^{n} 2\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right)+e^{-2\mu x_{(i)}^{-\lambda}}\right] - \sum_{i=1}^{n}\left(\frac{2i-1}{n}\right)\log\left[1-\frac{1}{5}\left[e^{-\mu x_{(n+1-i)}^{-\lambda}}\left(4\mu x_{(n+1-i)}^{-\lambda}+4\right)+e^{-2\mu x_{(n+1-i)}^{-\lambda}}\right]\right], \quad (21)$$

$$MPSE(\Psi) = \frac{1}{n+1}\sum_{i=1}^{n+1}\log\left\{\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right)+e^{-2\mu x_{(i)}^{-\lambda}}\right] - \frac{1}{5}\left[e^{-\mu x_{(i-1)}^{-\lambda}}\left(4\mu x_{(i-1)}^{-\lambda}+4\right)+e^{-2\mu x_{(i-1)}^{-\lambda}}\right]\right\}, \quad (22)$$

$$MSADE(\Psi) = \sum_{i=1}^{n+1}\left|\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right)+e^{-2\mu x_{(i)}^{-\lambda}}\right] - \frac{1}{5}\left[e^{-\mu x_{(i-1)}^{-\lambda}}\left(4\mu x_{(i-1)}^{-\lambda}+4\right)+e^{-2\mu x_{(i-1)}^{-\lambda}}\right] - \frac{1}{n+1}\right|, \quad (23)$$

$$MSALDE(\Psi) = \sum_{i=1}^{n+1}\left|\log\left[\frac{1}{5}\left[e^{-\mu x_{(i)}^{-\lambda}}\left(4\mu x_{(i)}^{-\lambda}+4\right)+e^{-2\mu x_{(i)}^{-\lambda}}\right] - \frac{1}{5}\left[e^{-\mu x_{(i-1)}^{-\lambda}}\left(4\mu x_{(i-1)}^{-\lambda}+4\right)+e^{-2\mu x_{(i-1)}^{-\lambda}}\right]\right] + \log(n+1)\right|, \quad (24)$$

where $x_{(0)}$ and $x_{(n+1)}$ are defined so that the corresponding CDF values equal 0 and 1, respectively.

Now, for the twelve estimators, the $\boldsymbol{\Psi}$ estimators are computed using optimization techniques such as Nelder-Mead and BFGS, which handle both maximization and minimization problems. Subsequently, the estimators are obtained using Equations (13)–(24).

4. Numerical Simulation

In this section of the study, a Monte Carlo simulation is conducted to compare the performance of the twelve previously introduced estimation techniques. To examine different cases, the simulation sample sizes are set to 25, 50, 75, 100, 125, 250, and 500, and the simulation is conducted over 10,000 replications. The numbers generated in the simulation study follow the algorithm outlined below:

Algorithm 1

Step 1: Generate data from IPMMSGD based on the acceptance-rejection algorithm.

Step 2: Compute the MLE, ADE, CVME, LSE, WLSE, ADLTE, ADLTSOE, ADRTE, KME, MPSE, MSADE, MSALDE.

Step 3: Steps 1 and 2 are each conducted 10,000 times.

Step 4: The MSEs, ABBs, and MREs for the λ parameter are calculated as follows:

$$MSEs_{\lambda}\left(\widehat{\lambda}_i\right) = \frac{1}{10000}\sum_{i=1}^{10000}\left(\widehat{\lambda}_i-\lambda\right)^2,$$

$$ABBs_{\lambda}\left(\widehat{\lambda}_i\right) = \frac{1}{10000}\sum_{i=1}^{10000}\left|\widehat{\lambda}_i - \lambda\right|,$$

$$MREs_{\lambda}\left(\widehat{\lambda}_i\right) = \frac{1}{10000}\sum_{i=1}^{10000}\frac{\left|\widehat{\lambda}_i - \lambda\right|}{\lambda}.$$

The same formulas are applied to compute the MSEs, ABBs, and MREs for the μ parameter.

The simulation results, including the rankings of twelve different estimators under scenarios generated for nine different λ and μ combinations, are presented in Tables 2–10. Each estimator is ranked from $^{(1)}$ (best) to $^{(12)}$ (worst). Table 11 reports the overall ranking results and total ranking scores across all scenarios. According to Table 11, MPSE achieved the highest overall ranking, demonstrating strong reliability under various conditions. WLSE followed closely, indicating its strong performance capability. ADE and LSE ranked third and fourth, respectively, highlighting their competence across different scenarios. On the other hand, CVME, MLE, ADLTSOE and ADRTE consistently exhibited the worst performance, ranking 9th, 10th, 11th, and 12th, respectively. ADLTE, MSALDE, MSADE and KME fell within the mid-range rankings, showing varying effectiveness depending on the scenario. The rankings also illustrate stability across different sample sizes, with the best-performing estimators maintaining their superiority. These results suggest that MPSE, WLSE, ADE and LSE are the most effective estimators, whereas CVME, MLE, ADLTSOE and ADRTE exhibit the weakest performance and are therefore not preferable for parameter estimation in this model.

5. Real Data Analysis

In this section, one real-world dataset is used to evaluate the performance of the IPMMSGD and to observe its applicability in real-world situations. The IPMMSGD is compared with five competitor distributions. The competitor distributions are as follows: Weibull (WB) [4], Gompertz (GP) [5], Lindley (LY) [3], Generalized Lindley (GL) [6], and flexible Weibull (FW) [2]. The pdfs of these competitor distributions are provided below.

WB distribution

$$f\left(x;p_1,p_2\right) = \frac{p_1\left(\frac{x}{p_2}\right)^{(p_1-1)}e^{-\left(\frac{x}{p_2}\right)^{p_1}}}{p_2}.$$

GP distribution

$$f\left(x;p_1,p_2\right) = p_1 e^{p_2 x}e^{\left(-\frac{p_1\left(e^{p_2 x}-1\right)}{p_2}\right)}$$

LY distribution

$$f\left(x;p_1\right) = \frac{p_1^2\left(1+x\right)e^{(-p_1 x)}}{p_1+1}$$

TABLE 2. Simulation results of MSE, ABB and MRE for Scenario 1

n		Parameter	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
25	MSE	λ	0.1156(8)	0.0922(4)	0.1457(11)	0.1077(6)	0.0956(5)	0.1151(7)	0.2119(12)	0.1402(10)	0.1354(9)	0.0588(1)	0.0858(3)	0.0734(2)
		μ	0.1262(8)	0.1185(2)	0.1523(11)	0.1241(6)	0.1200(4)	0.1219(5)	0.1387(9)	0.1904(12)	0.1423(10)	0.0945(1)	0.1250(7)	0.1186(3)
	ABB	λ	0.2566(8)	0.2283(4)	0.2781(11)	0.2457(6)	0.2299(5)	0.2511(7)	0.3174(12)	0.2741(10)	0.2714(9)	0.1884(1)	0.2222(3)	0.2075(2)
		μ	0.2744(8)	0.2653(4)	0.2902(10)	0.2652(3)	0.2619(2)	0.2714(7)	0.2919(11)	0.3100(12)	0.2876(9)	0.2388(1)	0.2669(5)	0.2681(6)
	MRE	λ	0.1711(8)	0.1522(4)	0.1854(11)	0.1638(6)	0.1532(5)	0.1674(7)	0.2116(12)	0.1828(10)	0.1809(9)	0.1256(1)	0.1482(3)	0.1383(2)
		μ	0.1524(8)	0.1474(3.5)	0.1612(10)	0.1474(3.5)	0.1455(2)	0.1508(7)	0.1622(11)	0.1722(12)	0.1598(9)	0.1327(1)	0.1483(5)	0.1490(6)
	$\sum$ Ranks		$48^{(8)}$	$21.5^{(3)}$	$64^{(10)}$	$30.5^{(6)}$	$23^{(4)}$	$40^{(7)}$	$67^{(12)}$	$66^{(11)}$	$55^{(9)}$	$6^{(1)}$	$26^{(5)}$	$21^{(2)}$
50	MSE	λ	0.0565(8)	0.0451(5)	0.0613(10)	0.0504(6)	0.0448(4)	0.0530(7)	0.0897(12)	0.0634(11)	0.0610(9)	0.0297(1)	0.0407(3)	0.0389(2)
		μ	0.0579(4)	0.0566(3)	0.0641(8)	0.0581(5)	0.0564(2)	0.0583(6)	0.0683(11)	0.0740(12)	0.0647(9)	0.0489(1)	0.0657(10)	0.0623(7)
	ABB	λ	0.1861(8)	0.1635(5)	0.1878(9)	0.1720(6)	0.1611(4)	0.1755(7)	0.2215(12)	0.1914(11)	0.1893(10)	0.1335(1)	0.1506(2)	0.1508(3)
		μ	0.1890(5)	0.1864(3)	0.1970(8)	0.1881(4)	0.1854(2)	0.1901(6)	0.2074(11)	0.2084(12)	0.1992(10)	0.1742(1)	0.1973(9)	0.1963(7)
	MRE	λ	0.1241(8)	0.1090(5)	0.1252(9)	0.1147(6)	0.1074(4)	0.1170(7)	0.1477(12)	0.1276(11)	0.1262(10)	0.0890(1)	0.1004(2)	0.1006(3)
		μ	0.1050(5)	0.1036(3)	0.1094(8)	0.1045(4)	0.1030(2)	0.1056(6)	0.1152(11)	0.1158(12)	0.1106(10)	0.0968(1)	0.1096(9)	0.1091(7)
	$\sum$ Ranks		$38^{(7)}$	$24^{(3)}$	$52^{(9)}$	$31^{(5)}$	$18^{(2)}$	$39^{(8)}$	$69^{(11.5)}$	$69^{(11.5)}$	$58^{(10)}$	$6^{(1)}$	$35^{(6)}$	$29^{(4)}$
75	MSE	λ	0.0406(9)	0.0314(5)	0.0401(8)	0.0343(6)	0.0306(4)	0.0355(7)	0.0626(12)	0.0430(11)	0.0413(10)	0.0223(1)	0.0282(2)	0.0300(3)
		μ	0.0366(4)	0.0362(3)	0.0396(7)	0.0372(5.5)	0.0360(2)	0.0372(5.5)	0.0449(10.5)	0.0449(10.5)	0.0412(8)	0.0325(1)	0.0452(12)	0.0416(9)
	ABB	λ	0.1602(11)	0.1366(5)	0.1534(8)	0.1427(6)	0.1339(4)	0.1442(7)	0.1914(12)	0.1594(10)	0.1563(9)	0.1149(1)	0.1250(2)	0.1325(3)
		μ	0.1510(4.5)	0.1495(3)	0.1554(7)	0.1510(4.5)	0.1490(2)	0.1520(6)	0.1681(12)	0.1639(11)	0.1586(8)	0.1426(1)	0.1635(10)	0.1608(9)
	MRE	λ	0.1068(11)	0.0911(5)	0.1023(8)	0.0952(6)	0.0893(4)	0.0962(7)	0.1276(12)	0.1063(10)	0.1042(9)	0.0766(1)	0.0833(2)	0.0883(3)
		μ	0.0839(4.5)	0.0831(3)	0.0864(7)	0.0839(4.5)	0.0828(2)	0.0845(6)	0.0934(12)	0.0911(11)	0.0881(8)	0.0792(1)	0.0909(10)	0.0893(9)
	$\sum$ Ranks		$44^{(8)}$	$24^{(3)}$	$45^{(9)}$	$32.5^{(4)}$	$18^{(2)}$	$38.5^{(7)}$	$70.5^{(12)}$	$63.5^{(11)}$	$52^{(10)}$	$6^{(1)}$	$38^{(6)}$	$36^{(5)}$
100	MSE	λ	0.0322(11)	0.0237(3)	0.0293(8)	0.0256(5)	0.0231(2)	0.0268(7)	0.0509(12)	0.0317(10)	0.0302(9)	0.0183(1)	0.0240(4)	0.0266(6)
		μ	0.0278(4)	0.0276(3)	0.0298(7)	0.0283(5)	0.0274(2)	0.0284(6)	0.0348(11)	0.0336(10)	0.0311(8)	0.0253(1)	0.0350(12)	0.0332(9)
	ABB	λ	0.1456(11)	0.1196(4)	0.1319(8)	0.1236(5)	0.1173(2)	0.1270(6)	0.1771(12)	0.1378(10)	0.1348(9)	0.1052(1)	0.1190(3)	0.1297(7)
		μ	0.1318(4)	0.1313(3)	0.1359(7)	0.1328(5)	0.1308(2)	0.1335(6)	0.1487(12)	0.1433(9)	0.1390(8)	0.1259(1)	0.1462(11)	0.1441(10)
	MRE	λ	0.0971(11)	0.0798(4)	0.0880(8)	0.0824(5)	0.0782(2)	0.0847(6)	0.1181(12)	0.0919(10)	0.0898(9)	0.0701(1)	0.0793(3)	0.0865(7)
		μ	0.0732(4)	0.0729(3)	0.0755(7)	0.0738(5)	0.0727(2)	0.0742(6)	0.0826(12)	0.0796(9)	0.0772(8)	0.0699(1)	0.0812(11)	0.0800(10)
	$\sum$ Ranks		$45^{(7.5)}$	$20^{(3)}$	$45^{(7.5)}$	$30^{(4)}$	$12^{(2)}$	$37^{(5)}$	$71^{(12)}$	$58^{(11)}$	$51^{(10)}$	$6^{(1)}$	$44^{(6)}$	$49^{(9)}$
125	MSE	λ	0.0283(11)	0.0198(3)	0.0234(6)	0.0207(4)	0.0194(2)	0.0222(5)	0.0465(12)	0.0263(10)	0.0243(8)	0.0168(1)	0.0241(7)	0.0257(9)
		μ	0.0218(4)	0.0217(3)	0.0233(7)	0.0224(5.5)	0.0216(2)	0.0224(5.5)	0.0280(11)	0.0264(9)	0.0244(8)	0.0201(1)	0.0296(12)	0.0275(10)
	ABB	λ	0.1378(11)	0.1102(3)	0.1188(6)	0.1120(4)	0.1083(2)	0.1158(5)	0.1722(12)	0.1261(9)	0.1212(7)	0.1021(1)	0.1242(8)	0.1310(10)
		μ	0.1174(4)	0.1170(3)	0.1208(7)	0.1185(5)	0.1167(2)	0.1190(6)	0.1335(11)	0.1275(9)	0.1236(8)	0.1131(1)	0.1362(12)	0.1326(10)
	MRE	λ	0.0919(11)	0.0735(3)	0.0792(6)	0.0747(4)	0.0722(2)	0.0772(5)	0.1148(12)	0.0841(9)	0.0808(7)	0.0681(1)	0.0828(8)	0.0873(10)
		μ	0.0652(4)	0.0650(3)	0.0671(7)	0.0658(5)	0.0648(2)	0.0661(6)	0.0742(11)	0.0708(9)	0.0686(8)	0.0628(1)	0.0757(12)	0.0737(10)
	$\sum$ Ranks		$45^{(7)}$	$18^{(3)}$	$39^{(6)}$	$27.5^{(4)}$	$12^{(2)}$	$32.5^{(5)}$	$69^{(12)}$	$55^{(9)}$	$46^{(8)}$	$6^{(1)}$	$59^{(10.5)}$	$59^{(10.5)}$
250	MSE	λ	0.0196(10)	0.0115(2)	0.0122(4)	0.0111(1)	0.0117(3)	0.0126(5)	0.0358(12)	0.0149(8)	0.0128(6)	0.0136(7)	0.0205(11)	0.0190(9)
		μ	0.0106(2)	0.0108(4)	0.0114(7)	0.0112(6)	0.0107(3)	0.0111(5)	0.0141(10)	0.0131(9)	0.0121(8)	0.0101(1)	0.0172(12)	0.0154(11)
	ABB	λ	0.1203(11)	0.0854(2)	0.0867(4)	0.0826(1)	0.0862(3)	0.0888(5)	0.1608(12)	0.0964(7)	0.0891(6)	0.0965(8)	0.1177(10)	0.1122(9)
		μ	0.0815(2)	0.0819(4)	0.0845(7)	0.0835(6)	0.0818(3)	0.0834(5)	0.0948(10)	0.0898(9)	0.0871(8)	0.0798(1)	0.1037(12)	0.0988(11)
	MRE	λ	0.0802(11)	0.0570(2)	0.0578(4)	0.0550(1)	0.0574(3)	0.0592(5)	0.1072(12)	0.0643(7.5)	0.0594(6)	0.0643(7.5)	0.0785(10)	0.0748(9)
		μ	0.0453(2)	0.0455(4)	0.0469(7)	0.0464(6)	0.0454(3)	0.0463(5)	0.0527(10)	0.0499(9)	0.0484(8)	0.0443(1)	0.0576(12)	0.0549(11)
	$\sum$ Ranks		$38^{(7)}$	$18^{(1.5)}$	$33^{(6)}$	$21^{(3)}$	$18^{(1.5)}$	$30^{(5)}$	$66^{(11)}$	$49.5^{(9)}$	$42^{(8)}$	$25.5^{(4)}$	$67^{(12)}$	$60^{(10)}$
500	MSE	λ	0.0160(11)	0.0079(4)	0.0072(2)	0.0068(1)	0.0087(6)	0.0082(5)	0.0312(12)	0.0099(7)	0.0077(3)	0.0128(9)	0.0148(10)	0.0102(8)
		μ	0.0054(2)	0.0056(3.5)	0.0059(7)	0.0058(6)	0.0056(3.5)	0.0057(5)	0.0079(11)	0.0067(9)	0.0062(8)	0.0053(1)	0.0089(12)	0.0078(10)
	ABB	λ	0.1148(11)	0.0730(4)	0.0679(2)	0.0654(1)	0.0779(6)	0.0741(5)	0.1595(12)	0.0810(7)	0.0699(3)	0.1008(9)	0.1030(10)	0.0820(8)
		μ	0.0586(2)	0.0591(3.5)	0.0605(7)	0.0601(6)	0.0591(3.5)	0.0600(5)	0.0711(11)	0.0646(9)	0.0621(8)	0.0579(1)	0.0748(12)	0.0705(10)
	MRE	λ	0.0765(11)	0.0487(4)	0.0453(2)	0.0436(1)	0.0520(6)	0.0494(5)	0.1063(12)	0.0540(7)	0.0466(3)	0.0672(9)	0.0687(10)	0.0547(8)
		μ	0.0325(2)	0.0328(3)	0.0336(7)	0.0334(6)	0.0329(4)	0.0333(5)	0.0395(11)	0.0359(9)	0.0345(8)	0.0322(1)	0.0415(12)	0.0392(10)
	$\sum$ Ranks		$39^{(8)}$	$22^{(2)}$	$27^{(3)}$	$21^{(1)}$	$29^{(4)}$	$30^{(5.5)}$	$69^{(12)}$	$48^{(9)}$	$33^{(7)}$	$30^{(5.5)}$	$66^{(11)}$	$54^{(10)}$

TABLE 3. Simulation results of MSE, ABB and MRE for Scenario 2

n		Parameter	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
25	MSE	λ	0.1301(7)	0.1041(4)	0.1674(11)	0.1224(6)	0.1099(5)	0.1305(8)	0.2353(12)	0.1657(10)	0.1578(9)	0.0663(1)	0.0994(3)	0.0865(2)
		μ	0.1435(8)	0.1338(4)	0.1754(11)	0.1389(7)	0.1335(3)	0.1362(5)	0.1493(9)	0.2171(12)	0.1611(10)	0.1041(1)	0.1369(6)	0.1296(2)
	ABB	λ	0.2728(8)	0.2417(4)	0.2981(11)	0.2613(6)	0.2451(5)	0.2654(7)	0.3334(12)	0.2961(10)	0.2916(9)	0.1990(1)	0.2401(3)	0.2257(2)
		μ	0.2902(8)	0.2799(5)	0.3094(11)	0.2802(6)	0.2764(2)	0.2844(7)	0.3007(9)	0.3317(12)	0.3030(10)	0.2499(1)	0.2777(3)	0.2782(4)
	MRE	λ	0.1705(8)	0.1511(4)	0.1863(11)	0.1633(6)	0.1532(5)	0.1659(7)	0.2084(12)	0.1851(10)	0.1823(9)	0.1244(1)	0.1501(3)	0.1411(2)
		μ	0.1528(8)	0.1473(5)	0.1628(11)	0.1475(6)	0.1455(2)	0.1497(7)	0.1583(9)	0.1746(12)	0.1595(10)	0.1315(1)	0.1462(3)	0.1464(4)
	$\sum$ Ranks		$47^{(8)}$	$26^{(5)}$	$66^{(11.5)}$	$37^{(6)}$	$22^{(4)}$	$41^{(7)}$	$63^{(10)}$	$66^{(11.5)}$	$57^{(9)}$	$6^{(1)}$	$21^{(3)}$	$16^{(2)}$
50	MSE	λ	0.0652(8)	0.0526(5)	0.0720(9)	0.0593(6)	0.0524(4)	0.0619(7)	0.1046(12)	0.0738(11)	0.0727(10)	0.0346(1)	0.0468(3)	0.0438(2)
		μ	0.0639(5)	0.0621(3)	0.0715(9)	0.0643(6)	0.0620(2)	0.0634(4)	0.0726(11)	0.0846(12)	0.0720(10)	0.0531(1)	0.0712(8)	0.0672(7)
	ABB	λ	0.1996(8)	0.1761(5)	0.2033(9)	0.1867(6)	0.1740(4)	0.1897(7)	0.2392(12)	0.2063(11)	0.2056(10)	0.1436(1)	0.1605(3)	0.1592(2)
		μ	0.1979(6)	0.1947(3)	0.2066(9)	0.1969(4)	0.1940(2)	0.1976(5)	0.2134(11)	0.2207(12)	0.2089(10)	0.1812(1)	0.2051(8)	0.2031(7)
	MRE	λ	0.1248(8)	0.1101(5)	0.1271(9)	0.1167(6)	0.1088(4)	0.1186(7)	0.1495(12)	0.1289(11)	0.1285(10)	0.0898(1)	0.1003(3)	0.0995(2)
		μ	0.1042(6)	0.1025(3)	0.1087(9)	0.1036(4)	0.1021(2)	0.1040(5)	0.1123(11)	0.1162(12)	0.1100(10)	0.0954(1)	0.1079(8)	0.1069(7)
	$\sum$ Ranks		$41^{(8)}$	$24^{(3)}$	$54^{(9)}$	$32^{(5)}$	$18^{(2)}$	$35^{(7)}$	$69^{(11.5)}$	$69^{(11.5)}$	$60^{(10)}$	$6^{(1)}$	$33^{(6)}$	$27^{(4)}$
75	MSE	λ	0.0469(10)	0.0357(5)	0.0456(8)	0.0390(6)	0.0352(4)	0.0420(7)	0.0758(12)	0.0485(11)	0.0466(9)	0.0258(1)	0.0330(2)	0.0344(3)
		μ	0.0415(4)	0.0410(3)	0.0458(7)	0.0426(6)	0.0409(2)	0.0419(5)	0.0492(11)	0.0538(12)	0.0469(9)	0.0363(1)	0.0488(10)	0.0459(8)
	ABB	λ	0.1727(11)	0.1461(5)	0.1632(8)	0.1517(6)	0.1438(4)	0.1571(7)	0.2102(12)	0.1689(10)	0.1658(9)	0.1244(1)	0.1341(2)	0.1419(3)
		μ	0.1598(4)	0.1584(3)	0.1661(7)	0.1606(5.5)	0.1578(2)	0.1606(5.5)	0.1750(11)	0.1774(12)	0.1687(9)	0.1498(1)	0.1701(10)	0.1676(8)
	MRE	λ	0.1079(11)	0.0913(5)	0.1020(8)	0.0948(6)	0.0899(4)	0.0982(7)	0.1314(12)	0.1056(10)	0.1036(9)	0.0777(1)	0.0838(2)	0.0887(3)
		μ	0.0841(4)	0.0834(3)	0.0874(7)	0.0845(5.5)	0.0830(2)	0.0845(5.5)	0.0921(11)	0.0934(12)	0.0888(9)	0.0788(1)	0.0895(10)	0.0882(8)
	$\sum$ Ranks		$44^{(8)}$	$24^{(3)}$	$45^{(9)}$	$35^{(5)}$	$18^{(2)}$	$37^{(7)}$	$69^{(12)}$	$67^{(11)}$	$54^{(10)}$	$6^{(1)}$	$36^{(6)}$	$33^{(4)}$
100	MSE	λ	0.0363(11)	0.0261(3)	0.0320(8)	0.0278(4)	0.0257(2)	0.0302(7)	0.0593(12)	0.0355(10)	0.0335(9)	0.0206(1)	0.0281(5)	0.0297(6)
		μ	0.0305(4)	0.0302(3)	0.0331(7)	0.0313(6)	0.0301(2)	0.0308(5)	0.0368(10)	0.0386(12)	0.0344(8)	0.0273(1)	0.0383(11)	0.0361(9)
	ABB	λ	0.1544(11)	0.1260(3)	0.1385(8)	0.1297(5)	0.1240(2)	0.1344(6)	0.1897(12)	0.1457(10)	0.1416(9)	0.1119(1)	0.1292(4)	0.1372(7)
		μ	0.1379(4)	0.1371(3)	0.1427(7)	0.1390(6)	0.1366(2)	0.1387(5)	0.1523(10)	0.1529(11)	0.1456(8)	0.1308(1)	0.1531(12)	0.1500(9)
	MRE	λ	0.0965(11)	0.0787(3)	0.0865(8)	0.0811(5)	0.0775(2)	0.0840(6)	0.1186(12)	0.0910(10)	0.0885(9)	0.0699(1)	0.0807(4)	0.0857(7)
		μ	0.0726(4)	0.0722(3)	0.0751(7)	0.0731(6)	0.0719(2)	0.0730(5)	0.0802(10)	0.0805(11)	0.0766(8)	0.0689(1)	0.0806(12)	0.0789(9)
	$\sum$ Ranks		$45^{(6.5)}$	$18^{(3)}$	$45^{(6.5)}$	$32^{(4)}$	$12^{(2)}$	$34^{(5)}$	$66^{(12)}$	$64^{(11)}$	$51^{(10)}$	$6^{(1)}$	$48^{(9)}$	$47^{(8)}$
125	MSE	λ	0.0321(11)	0.0222(3)	0.0262(6)	0.0231(4)	0.0219(2)	0.0252(5)	0.0554(12)	0.0295(10)	0.0273(8)	0.0190(1)	0.0265(7)	0.0289(9)
		μ	0.0240(3)	0.0240(3)	0.0262(7)	0.0251(6)	0.0240(3)	0.0246(5)	0.0303(11)	0.0302(10)	0.0272(8)	0.0220(1)	0.0316(12)	0.0300(9)
	ABB	λ	0.1474(11)	0.1161(3)	0.1251(6)	0.1176(4)	0.1145(2)	0.1237(5)	0.1875(12)	0.1330(9)	0.1278(7)	0.1087(1)	0.1310(8)	0.1389(10)
		μ	0.1229(4)	0.1228(3)	0.1277(7)	0.1251(6)	0.1224(2)	0.1245(5)	0.1384(11)	0.1360(9)	0.1303(8)	0.1177(1)	0.1398(12)	0.1375(10)
	MRE	λ	0.0921(11)	0.0726(3)	0.0782(6)	0.0735(4)	0.0716(2)	0.0773(5)	0.1172(12)	0.0831(9)	0.0799(7)	0.0679(1)	0.0819(8)	0.0868(10)
		μ	0.0647(4)	0.0646(3)	0.0672(7)	0.0658(6)	0.0644(2)	0.0655(5)	0.0728(11)	0.0716(9)	0.0686(8)	0.0619(1)	0.0736(12)	0.0724(10)
	$\sum$ Ranks		$44^{(7)}$	$18^{(3)}$	$39^{(6)}$	$30^{(4.5)}$	$13^{(2)}$	$30^{(4.5)}$	$69^{(12)}$	$56^{(9)}$	$46^{(8)}$	$6^{(1)}$	$59^{(11)}$	$58^{(10)}$
250	MSE	λ	0.0223(10)	0.0131(2)	0.0138(4)	0.0125(1)	0.0133(3)	0.0143(5)	0.0414(12)	0.0167(8)	0.0144(6)	0.0155(7)	0.0236(11)	0.0219(9)
		μ	0.0119(2)	0.0121(3.5)	0.0129(7)	0.0125(6)	0.0121(3.5)	0.0122(5)	0.0150(9)	0.0151(10)	0.0136(8)	0.0113(1)	0.0186(12)	0.0170(11)
	ABB	λ	0.1289(11)	0.0914(2)	0.0923(3.5)	0.0878(1)	0.0923(3.5)	0.0952(6)	0.1725(12)	0.1027(7)	0.0945(5)	0.1036(8)	0.1268(10)	0.1210(9)
		μ	0.0861(2)	0.0865(4)	0.0890(7)	0.0879(6)	0.0863(3)	0.0873(5)	0.0978(10)	0.0959(9)	0.0915(8)	0.0839(1)	0.1081(12)	0.1031(11)
	MRE	λ	0.0806(11)	0.0571(2)	0.0577(3.5)	0.0549(1)	0.0577(3.5)	0.0595(6)	0.1078(12)	0.0642(7)	0.0591(5)	0.0648(8)	0.0792(10)	0.0756(9)
		μ	0.0453(2)	0.0455(4)	0.0469(7)	0.0463(6)	0.0454(3)	0.0460(5)	0.0515(10)	0.0505(9)	0.0482(8)	0.0442(1)	0.0569(12)	0.0543(11)
	$\sum$ Ranks		$38^{(7)}$	$17.5^{(1)}$	$32^{(5.5)}$	$21^{(3)}$	$19.5^{(2)}$	$32^{(5.5)}$	$65^{(11)}$	$50^{(9)}$	$40^{(8)}$	$26^{(4)}$	$67^{(12)}$	$60^{(10)}$
500	MSE	λ	0.0179(11)	0.0088(4)	0.0081(2)	0.0076(1)	0.0097(6)	0.0092(5)	0.0348(12)	0.0111(7)	0.0085(3)	0.0144(9)	0.0166(10)	0.0114(8)
		μ	0.0060(2)	0.0061(3)	0.0064(7)	0.0063(6)	0.0062(4.5)	0.0062(4.5)	0.0076(9.5)	0.0076(9.5)	0.0068(8)	0.0059(1)	0.0099(12)	0.0086(11)
	ABB	λ	0.1216(11)	0.0771(4)	0.0718(2)	0.0691(1)	0.0823(6)	0.0783(5)	0.1690(12)	0.0859(7)	0.0735(3)	0.1068(9)	0.1091(10)	0.0868(8)
		μ	0.0620(2)	0.0623(3)	0.0635(7)	0.0630(6)	0.0625(4)	0.0626(5)	0.0696(10)	0.0691(9)	0.0651(8)	0.0610(1)	0.0787(12)	0.0739(11)
	MRE	λ	0.0760(11)	0.0482(4)	0.0449(2)	0.0432(1)	0.0515(6)	0.0489(5)	0.1056(12)	0.0537(7)	0.0459(3)	0.0668(9)	0.0682(10)	0.0542(8)
		μ	0.0326(2)	0.0328(3)	0.0334(7)	0.0331(6)	0.0329(4)	0.0330(5)	0.0366(10)	0.0364(9)	0.0343(8)	0.0321(1)	0.0414(12)	0.0389(11)
	$\sum$ Ranks		$39^{(8)}$	$21^{(1.5)}$	$27^{(3)}$	$21^{(1.5)}$	$30.5^{(6)}$	$29.5^{(4)}$	$65.5^{(11)}$	$48.5^{(9)}$	$33^{(7)}$	$30^{(5)}$	$66^{(12)}$	$57^{(10)}$

TABLE 4. Simulation results of MSE, ABB and MRE for Scenario 3

n		Parameter	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
25	MSE	λ	0.1424(7)	0.1131(3)	0.1774(11)	0.1317(6)	0.1183(5)	0.1450(8)	0.2656(12)	0.1746(10)	0.1604(9)	0.0739(1)	0.1163(4)	0.0926(2)
		μ	0.3272(9)	0.2835(4)	0.4749(11)	0.3320(10)	0.3221(8)	0.2844(5)	0.2984(6)	0.6716(12)	0.3154(7)	0.1918(1)	0.2598(3)	0.2393(2)
	ABB	λ	0.2845(8)	0.2549(3)	0.3117(11)	0.2761(6)	0.2582(4)	0.2827(7)	0.3560(12)	0.3073(10)	0.2989(9)	0.2116(1)	0.2601(5)	0.2355(2)
		μ	0.4203(10)	0.3942(5)	0.4705(11)	0.4094(9)	0.3958(6)	0.3924(4)	0.3963(7)	0.5310(12)	0.4020(8)	0.3322(1)	0.3593(2)	0.3662(3)
	MRE	λ	0.1674(8)	0.1499(3)	0.1834(11)	0.1624(6)	0.1519(4)	0.1663(7)	0.2094(12)	0.1807(10)	0.1758(9)	0.1245(1)	0.1530(5)	0.1385(2)
		μ	0.1557(10)	0.1460(5)	0.1743(11)	0.1516(9)	0.1466(6)	0.1453(4)	0.1468(7)	0.1967(12)	0.1489(8)	0.1230(1)	0.1331(2)	0.1356(3)
	$\sum$ Ranks		$52^{(9)}$	$23^{(4)}$	$66^{(11.5)}$	$46^{(7)}$	$33^{(5)}$	$35^{(6)}$	$56^{(10)}$	$66^{(11.5)}$	$50^{(8)}$	$6^{(1)}$	$21^{(3)}$	$14^{(2)}$
50	MSE	λ	0.0715(8)	0.0570(4)	0.0795(10)	0.0655(6)	0.0580(5)	0.0687(7)	0.1178(12)	0.0821(11)	0.0741(9)	0.0376(1)	0.0522(3)	0.0488(2)
		μ	0.1414(8)	0.1321(6)	0.1792(11)	0.1487(10)	0.1368(7)	0.1275(5)	0.1256(4)	0.2370(12)	0.1437(9)	0.0967(1)	0.1230(3)	0.1215(2)
	ABB	λ	0.2086(9)	0.1834(5)	0.2130(10)	0.1955(6)	0.1822(4)	0.1984(7)	0.2519(12)	0.2169(11)	0.2075(8)	0.1496(1)	0.1712(3)	0.1681(2)
		μ	0.2860(9)	0.2762(7)	0.3111(11)	0.2881(10)	0.2761(6)	0.2717(4)	0.2725(5)	0.3515(12)	0.2817(8)	0.2406(1)	0.2596(2)	0.2657(3)
	MRE	λ	0.1227(9)	0.1079(5)	0.1253(10)	0.1150(6)	0.1072(4)	0.1167(7)	0.1482(12)	0.1276(11)	0.1221(8)	0.0880(1)	0.1007(3)	0.0989(2)
		μ	0.1059(9)	0.1023(6.5)	0.1152(11)	0.1067(10)	0.1023(6.5)	0.1006(4)	0.1009(5)	0.1302(12)	0.1043(8)	0.0891(1)	0.0962(2)	0.0984(3)
	$\sum$ Ranks		$52^{(10)}$	$33.5^{(5)}$	$63^{(11)}$	$48^{(7)}$	$32.5^{(4)}$	$34^{(6)}$	$50^{(8.5)}$	$69^{(12)}$	$50^{(8.5)}$	$6^{(1)}$	$16^{(3)}$	$14^{(2)}$
75	MSE	λ	0.0516(10)	0.0399(5)	0.0510(9)	0.0437(6)	0.0393(4)	0.0453(7)	0.0813(12)	0.0552(11)	0.0498(8)	0.0283(1)	0.0374(2)	0.0385(3)
		μ	0.0882(8)	0.0821(5)	0.1009(11)	0.0885(9)	0.0816(4)	0.0786(2)	0.0798(3)	0.1348(12)	0.0898(10)	0.0647(1)	0.0859(7)	0.0845(6)
	ABB	λ	0.1809(11)	0.1545(5)	0.1732(9)	0.1614(6)	0.1522(4)	0.1640(7)	0.2176(12)	0.1801(10)	0.1715(8)	0.1305(1)	0.1431(2)	0.1506(3)
		μ	0.2295(10)	0.2214(6)	0.2423(11)	0.2287(9)	0.2192(3)	0.2170(2)	0.2203(5)	0.2762(12)	0.2268(8)	0.1976(1)	0.2194(4)	0.2247(7)
	MRE	λ	0.1064(11)	0.0909(5)	0.1019(9)	0.0950(6)	0.0895(4)	0.0965(7)	0.1280(12)	0.1059(10)	0.1009(8)	0.0767(1)	0.0842(2)	0.0886(3)
		μ	0.0850(10)	0.0820(6)	0.0897(11)	0.0847(9)	0.0812(3)	0.0804(2)	0.0816(5)	0.1023(12)	0.0840(8)	0.0732(1)	0.0813(4)	0.0832(7)
	$\sum$ Ranks		$60^{(10.5)}$	$32^{(6)}$	$60^{(10.5)}$	$45^{(7)}$	$22^{(3)}$	$27^{(4)}$	$49^{(8)}$	$67^{(12)}$	$50^{(9)}$	$6^{(1)}$	$21^{(2)}$	$29^{(5)}$
100	MSE	λ	0.0420(11)	0.0306(3)	0.0373(9)	0.0325(5)	0.0299(2)	0.0348(7)	0.0681(12)	0.0414(10)	0.0368(8)	0.0239(1)	0.0320(4)	0.0346(6)
		μ	0.0703(8)	0.0652(5)	0.0780(11)	0.0699(7)	0.0650(4)	0.0623(2)	0.0636(3)	0.1039(12)	0.0709(9.5)	0.0530(1)	0.0709(9.5)	0.0693(6)
	ABB	λ	0.1662(11)	0.1367(3)	0.1498(9)	0.1403(5)	0.1344(2)	0.1451(6)	0.2048(12)	0.1573(10)	0.1487(8)	0.1211(1)	0.1381(4)	0.1482(7)
		μ	0.2054(10)	0.1970(5)	0.2128(11)	0.2029(8)	0.1954(3)	0.1930(2)	0.1968(4)	0.2429(12)	0.2013(6)	0.1788(1)	0.2014(7)	0.2041(9)
	MRE	λ	0.0978(11)	0.0804(3)	0.0881(9)	0.0825(5)	0.0791(2)	0.0854(6)	0.1205(12)	0.0925(10)	0.0875(8)	0.0712(1)	0.0812(4)	0.0872(7)
		μ	0.0761(10)	0.0730(5)	0.0788(11)	0.0751(8)	0.0724(3)	0.0715(2)	0.0729(4)	0.0899(12)	0.0745(6)	0.0662(1)	0.0746(7)	0.0756(9)
	$\sum$ Ranks		$61^{(11)}$	$24^{(3)}$	$60^{(10)}$	$38^{(6)}$	$16^{(2)}$	$25^{(4)}$	$47^{(9)}$	$66^{(12)}$	$45.5^{(8)}$	$6^{(1)}$	$35.5^{(5)}$	$44^{(7)}$
125	MSE	λ	0.0356(11)	0.0250(3)	0.0296(6.5)	0.0262(4)	0.0246(2)	0.0278(5)	0.0598(12)	0.0336(10)	0.0296(6.5)	0.0212(1)	0.0300(8)	0.0323(9)
		μ	0.0575(8)	0.0529(5)	0.0617(11)	0.0561(6)	0.0527(4)	0.0504(2)	0.0521(3)	0.0824(12)	0.0567(7)	0.0445(1)	0.0611(10)	0.0608(9)
	ABB	λ	0.1543(11)	0.1238(3)	0.1338(7)	0.1262(4)	0.1218(2)	0.1301(5)	0.1942(12)	0.1423(9)	0.1336(6)	0.1145(1)	0.1392(8)	0.1462(10)
		μ	0.1875(8)	0.1788(4)	0.1917(10)	0.1836(7)	0.1774(3)	0.1748(2)	0.1792(5)	0.2199(12)	0.1823(6)	0.1646(1)	0.1901(9)	0.1943(11)
	MRE	λ	0.0907(11)	0.0728(3)	0.0787(7)	0.0742(4)	0.0716(2)	0.0766(5)	0.1143(12)	0.0837(9)	0.0786(6)	0.0674(1)	0.0819(8)	0.0860(10)
		μ	0.0694(8)	0.0662(4)	0.0710(10)	0.0680(7)	0.0657(3)	0.0647(2)	0.0664(5)	0.0814(12)	0.0675(6)	0.0609(1)	0.0704(9)	0.0720(11)
	$\sum$ Ranks		$57^{(10)}$	$22^{(4)}$	$51.5^{(8)}$	$32^{(5)}$	$16^{(2)}$	$21^{(3)}$	$49^{(7)}$	$64^{(12)}$	$37.5^{(6)}$	$6^{(1)}$	$52^{(9)}$	$60^{(11)}$
250	MSE	λ	0.0254(10)	0.0151(2)	0.0159(4)	0.0145(1)	0.0152(3)	0.0160(5.5)	0.0458(12)	0.0197(8)	0.0160(5.5)	0.0177(7)	0.0270(11)	0.0247(9)
		μ	0.0312(9)	0.0267(3)	0.0292(8)	0.0273(6)	0.0269(4)	0.0253(1)	0.0271(5)	0.0403(11)	0.0279(7)	0.0256(2)	0.0404(12)	0.0387(10)
	ABB	λ	0.1365(11)	0.0978(2)	0.0996(5)	0.0950(1)	0.0983(3)	0.1002(6)	0.1817(12)	0.1114(8)	0.0992(4)	0.1097(7)	0.1348(10)	0.1277(9)
		μ	0.1400(9)	0.1280(3)	0.1332(8)	0.1289(6)	0.1281(4)	0.1247(1)	0.1299(7)	0.1558(10)	0.1287(5)	0.1258(2)	0.1588(12)	0.1564(11)
	MRE	λ	0.0803(11)	0.0575(2)	0.0586(5)	0.0559(1)	0.0578(3)	0.0589(6)	0.1069(12)	0.0655(8)	0.0583(4)	0.0645(7)	0.0793(10)	0.0751(9)
		μ	0.0519(9)	0.0474(3.5)	0.0493(8)	0.0477(5.5)	0.0474(3.5)	0.0462(1)	0.0481(7)	0.0577(10)	0.0477(5.5)	0.0466(2)	0.0588(12)	0.0579(11)
	$\sum$ Ranks		$59^{(10.5)}$	$15.5^{(1)}$	$38^{(7)}$	$20.5^{(3)}$	$20.5^{(3)}$	$20.5^{(3)}$	$55^{(8.5)}$	$55^{(8.5)}$	$31^{(6)}$	$27^{(5)}$	$67^{(12)}$	$59^{(10.5)}$
500	MSE	λ	0.0202(11)	0.0099(4)	0.0091(2)	0.0085(1)	0.0109(6)	0.0104(5)	0.0398(12)	0.0124(7)	0.0092(3)	0.0162(9)	0.0189(10)	0.0129(8)
		μ	0.0211(9)	0.0163(4)	0.0164(5)	0.0156(2)	0.0172(7)	0.0154(1)	0.0170(6)	0.0238(11)	0.0158(3)	0.0184(8)	0.0259(12)	0.0212(10)
	ABB	λ	0.1284(11)	0.0814(4)	0.0758(2)	0.0730(1)	0.0869(6)	0.0831(5)	0.1798(12)	0.0906(7)	0.0762(3)	0.1127(9)	0.1159(10)	0.0918(8)
		μ	0.1187(10)	0.1018(5)	0.1010(4)	0.0985(2)	0.1046(7)	0.0988(3)	0.1044(6)	0.1220(11)	0.0978(1)	0.1098(8)	0.1279(12)	0.1157(9)
	MRE	λ	0.0755(11)	0.0479(4)	0.0446(2)	0.0429(1)	0.0511(6)	0.0489(5)	0.1058(12)	0.0533(7)	0.0448(3)	0.0663(9)	0.0682(10)	0.0540(8)
		μ	0.0440(10)	0.0377(5)	0.0374(4)	0.0365(2)	0.0388(7)	0.0366(3)	0.0387(6)	0.0452(11)	0.0362(1)	0.0407(8)	0.0474(12)	0.0429(9)
	$\sum$ Ranks		$62^{(11)}$	$26^{(5)}$	$19^{(3)}$	$9^{(1)}$	$39^{(6)}$	$22^{(4)}$	$54^{(9.5)}$	$54^{(9.5)}$	$14^{(2)}$	$51^{(7)}$	$66^{(12)}$	$52^{(8)}$

TABLE 5. Simulation results of MSE, ABB and MRE for Scenario 4

n		Parameter	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
25	MSE	λ	0.1633(7)	0.1285(3)	0.2118(11)	0.1564(6)	0.1386(5)	0.1667(8)	0.3152(12)	0.2003(10)	0.1879(9)	0.0835(1)	0.1328(4)	0.1053(2)
		μ	0.2712(8)	0.2404(5)	0.3782(11)	0.2732(9)	0.2539(7)	0.2402(4)	0.2519(6)	0.5080(12)	0.2773(10)	0.1681(1)	0.2312(3)	0.2073(2)
	ABB	λ	0.3043(8)	0.2717(3)	0.3355(11)	0.2975(6)	0.2778(5)	0.3008(7)	0.3863(12)	0.3289(10)	0.3199(9)	0.2248(1)	0.2735(4)	0.2499(2)
		μ	0.3858(10)	0.3656(4)	0.4305(11)	0.3781(8)	0.3659(6)	0.3657(5)	0.3731(7)	0.4770(12)	0.3845(9)	0.3132(1)	0.3433(2)	0.3442(3)
	MRE	λ	0.1690(8)	0.1510(3)	0.1864(11)	0.1653(6)	0.1544(5)	0.1671(7)	0.2146(12)	0.1827(10)	0.1777(9)	0.1249(1)	0.1520(4)	0.1388(2)
		μ	0.1543(10)	0.1462(4)	0.1722(11)	0.1512(8)	0.1464(6)	0.1463(5)	0.1492(7)	0.1908(12)	0.1538(9)	0.1253(1)	0.1373(2)	0.1377(3)
	$\sum$ Ranks		$51^{(8)}$	$22^{(4)}$	$66^{(11.5)}$	$43^{(7)}$	$34^{(5)}$	$36^{(6)}$	$56^{(10)}$	$66^{(11.5)}$	$55^{(9)}$	$6^{(1)}$	$19^{(3)}$	$14^{(2)}$
50	MSE	λ	0.0810(8)	0.0652(5)	0.0893(10)	0.0735(6)	0.0650(4)	0.0770(7)	0.1316(12)	0.0927(11)	0.0857(9)	0.0428(1)	0.0595(3)	0.0552(2)
		μ	0.1158(8)	0.1095(5)	0.1412(11)	0.1192(9)	0.1112(6)	0.1062(2)	0.1071(4)	0.1893(12)	0.1249(10)	0.0832(1)	0.1135(7)	0.1063(3)
	ABB	λ	0.2222(8.5)	0.1961(5)	0.2264(10)	0.2080(6)	0.1941(4)	0.2115(7)	0.2696(12)	0.2300(11)	0.2222(8.5)	0.1593(1)	0.1829(3)	0.1790(2)
		μ	0.2609(8)	0.2534(6)	0.2810(11)	0.2614(9)	0.2524(5)	0.2504(3)	0.2543(7)	0.3154(12)	0.2662(10)	0.2235(1)	0.2496(2)	0.2507(4)
	MRE	λ	0.1234(8.5)	0.1090(5)	0.1258(10)	0.1156(6)	0.1078(4)	0.1175(7)	0.1498(12)	0.1278(11)	0.1234(8.5)	0.0885(1)	0.1016(3)	0.0995(2)
		μ	0.1044(8)	0.1014(6)	0.1124(11)	0.1046(9)	0.1010(5)	0.1002(3)	0.1017(7)	0.1262(12)	0.1065(10)	0.0894(1)	0.0998(2)	0.1003(4)
	$\sum$ Ranks		$49^{(8)}$	$32^{(6)}$	$63^{(11)}$	$45^{(7)}$	$28^{(4)}$	$29^{(5)}$	$54^{(9)}$	$69^{(12)}$	$56^{(10)}$	$6^{(1)}$	$20^{(3)}$	$17^{(2)}$
75	MSE	λ	0.0564(10)	0.0432(5)	0.0553(8)	0.0475(6)	0.0425(4)	0.0500(7)	0.0907(12)	0.0595(11)	0.0555(9)	0.0308(1)	0.0408(2)	0.0420(3)
		μ	0.0753(7)	0.0716(4)	0.0866(11)	0.0768(8)	0.0717(5)	0.0693(2)	0.0712(3)	0.1140(12)	0.0818(10)	0.0577(1)	0.0783(9)	0.0747(6)
	ABB	λ	0.1890(11)	0.1599(5)	0.1793(8)	0.1675(6)	0.1572(4)	0.1711(7)	0.2307(12)	0.1868(10)	0.1800(9)	0.1355(1)	0.1493(2)	0.1562(3)
		μ	0.2120(8)	0.2066(4)	0.2236(11)	0.2124(9)	0.2051(3)	0.2037(2)	0.2087(5)	0.2531(12)	0.2170(10)	0.1870(1)	0.2113(6)	0.2115(7)
	MRE	λ	0.1050(11)	0.0888(5)	0.0996(8)	0.0930(6)	0.0873(4)	0.0950(7)	0.1282(12)	0.1038(10)	0.1000(9)	0.0753(1)	0.0830(2)	0.0868(3)
		μ	0.0848(8)	0.0826(4)	0.0894(11)	0.0850(9)	0.0820(3)	0.0815(2)	0.0835(5)	0.1012(12)	0.0868(10)	0.0748(1)	0.0845(6)	0.0846(7)
	$\sum$ Ranks		$55^{(9)}$	$27^{(4)}$	$57^{(10.5)}$	$44^{(7)}$	$23^{(2)}$	$27^{(4)}$	$49^{(8)}$	$67^{(12)}$	$57^{(10.5)}$	$6^{(1)}$	$27^{(4)}$	$29^{(6)}$
100	MSE	λ	0.0467(11)	0.0338(3)	0.0412(9)	0.0359(5)	0.0331(2)	0.0387(7)	0.0755(12)	0.0455(10)	0.0411(8)	0.0265(1)	0.0354(4)	0.0385(6)
		μ	0.0548(7)	0.0518(4)	0.0601(10)	0.0546(6)	0.0516(3)	0.0501(2)	0.0519(5)	0.0785(12)	0.0580(8)	0.0434(1)	0.0609(11)	0.0588(9)
	ABB	λ	0.1756(11)	0.1441(3)	0.1574(8)	0.1474(5)	0.1414(2)	0.1529(6)	0.2147(12)	0.1664(10)	0.1575(9)	0.1274(1)	0.1449(4)	0.1555(7)
		μ	0.1816(7)	0.1761(4)	0.1881(10)	0.1805(6)	0.1750(3)	0.1735(2)	0.1786(5)	0.2125(12)	0.1847(8)	0.1622(1)	0.1880(9)	0.1891(11)
	MRE	λ	0.0976(11)	0.0801(3)	0.0874(8)	0.0819(5)	0.0786(2)	0.0849(6)	0.1193(12)	0.0925(10)	0.0875(9)	0.0708(1)	0.0805(4)	0.0864(7)
		μ	0.0726(7)	0.0704(4)	0.0753(10)	0.0722(6)	0.0700(3)	0.0694(2)	0.0714(5)	0.0850(12)	0.0739(8)	0.0649(1)	0.0752(9)	0.0756(11)
	$\sum$ Ranks		$54^{(10)}$	$21^{(3)}$	$55^{(11)}$	$33^{(5)}$	$15^{(2)}$	$25^{(4)}$	$51^{(8.5)}$	$66^{(12)}$	$50^{(7)}$	$6^{(1)}$	$41^{(6)}$	$51^{(8.5)}$
125	MSE	λ	0.0401(11)	0.0277(3)	0.0328(6)	0.0289(4)	0.0273(2)	0.0316(5)	0.0676(12)	0.0367(10)	0.0329(7)	0.0238(1)	0.0343(8)	0.0362(9)
		μ	0.0457(7)	0.0430(3.5)	0.0495(9)	0.0455(6)	0.0430(3.5)	0.0414(2)	0.0432(5)	0.0647(12)	0.0481(8)	0.0369(1)	0.0534(11)	0.0501(10)
	ABB	λ	0.1645(11)	0.1302(3)	0.1400(6)	0.1319(4)	0.1284(2)	0.1385(5)	0.2077(12)	0.1487(9)	0.1406(7)	0.1218(1)	0.1485(8)	0.1556(10)
		μ	0.1670(7)	0.1616(4)	0.1722(9)	0.1658(6)	0.1608(3)	0.1589(2)	0.1637(5)	0.1949(12)	0.1697(8)	0.1503(1)	0.1785(11)	0.1755(10)
	MRE	λ	0.0914(11)	0.0723(3)	0.0778(6)	0.0733(4)	0.0713(2)	0.0770(5)	0.1154(12)	0.0826(9)	0.0781(7)	0.0677(1)	0.0825(8)	0.0864(10)
		μ	0.0668(7)	0.0646(4)	0.0689(9)	0.0663(6)	0.0643(3)	0.0636(2)	0.0655(5)	0.0779(12)	0.0679(8)	0.0601(1)	0.0714(11)	0.0702(10)
	$\sum$ Ranks		$54^{(9)}$	$20.5^{(3)}$	$45^{(6.5)}$	$30^{(5)}$	$15.5^{(2)}$	$21^{(4)}$	$51^{(8)}$	$64^{(12)}$	$45^{(6.5)}$	$6^{(1)}$	$57^{(10)}$	$59^{(11)}$
250	MSE	λ	0.0287(10)	0.0171(2)	0.0181(4)	0.0165(1)	0.0173(3)	0.0184(6)	0.0517(12)	0.0222(8)	0.0182(5)	0.0199(7)	0.0302(11)	0.0282(9)
		μ	0.0251(9)	0.0225(4)	0.0245(8)	0.0231(6)	0.0227(5)	0.0215(2)	0.0224(3)	0.0329(11)	0.0240(7)	0.0213(1)	0.0338(12)	0.0315(10)
	ABB	λ	0.1456(11)	0.1043(2)	0.1058(4.5)	0.1007(1)	0.1051(3)	0.1081(6)	0.1933(12)	0.1179(8)	0.1058(4.5)	0.1173(7)	0.1428(10)	0.1369(9)
		μ	0.1249(9)	0.1177(3)	0.1221(8)	0.1188(6)	0.1179(4)	0.1154(2)	0.1186(5)	0.1405(10)	0.1206(7)	0.1149(1)	0.1443(12)	0.1409(11)
	MRE	λ	0.0809(11)	0.0579(2)	0.0588(4.5)	0.0560(1)	0.0584(3)	0.0600(6)	0.1074(12)	0.0655(8)	0.0588(4.5)	0.0652(7)	0.0794(10)	0.0760(9)
		μ	0.0500(9)	0.0471(3.5)	0.0489(8)	0.0475(6)	0.0471(3.5)	0.0461(2)	0.0474(5)	0.0562(10)	0.0482(7)	0.0460(1)	0.0577(12)	0.0564(11)
	$\sum$ Ranks		$59^{(10.5)}$	$16.5^{(1)}$	$37^{(7)}$	$21^{(2)}$	$21.5^{(3)}$	$24^{(4.5)}$	$49^{(8)}$	$55^{(9)}$	$35^{(6)}$	$24^{(4.5)}$	$67^{(12)}$	$59^{(10.5)}$
500	MSE	λ	0.0228(11)	0.0112(4)	0.0103(2)	0.0096(1)	0.0123(6)	0.0118(5)	0.0450(12)	0.0140(7)	0.0105(3)	0.0183(9)	0.0216(10)	0.0147(8)
		μ	0.0153(9)	0.0127(4)	0.0129(5.5)	0.0124(3)	0.0132(7)	0.0121(1)	0.0123(2)	0.0180(11)	0.0129(5.5)	0.0136(8)	0.0200(12)	0.0168(10)
	ABB	λ	0.1370(11)	0.0869(4)	0.0808(2)	0.0778(1)	0.0927(6)	0.0884(5)	0.1916(12)	0.0967(7)	0.0813(3)	0.1203(9)	0.1243(10)	0.0986(8)
		μ	0.0994(9)	0.0889(5)	0.0893(6)	0.0875(2)	0.0908(7)	0.0870(1)	0.0883(3)	0.1051(11)	0.0886(4)	0.0932(8)	0.1116(12)	0.1026(10)
	MRE	λ	0.0761(11)	0.0483(4)	0.0449(2)	0.0432(1)	0.0515(6)	0.0491(5)	0.1065(12)	0.0537(7)	0.0452(3)	0.0669(9)	0.0690(10)	0.0548(8)
		μ	0.0397(9)	0.0356(5)	0.0357(6)	0.0350(2)	0.0363(7)	0.0348(1)	0.0353(3)	0.0420(11)	0.0355(4)	0.0373(8)	0.0446(12)	0.0411(10)
	$\sum$ Ranks		$60^{(11)}$	$26^{(5)}$	$23.5^{(4)}$	$10^{(1)}$	$39^{(6)}$	$18^{(2)}$	$44^{(7)}$	$54^{(9.5)}$	$22.5^{(3)}$	$51^{(8)}$	$66^{(12)}$	$54^{(9.5)}$

TABLE 6. Simulation results of MSE, ABB and MRE for Scenario 5

n		Parameter	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
25	MSE	λ	0.2061(7)	0.1606(4)	0.2545(11)	0.1854(6)	0.1676(5)	0.2093(8)	0.3835(12)	0.2470(10)	0.2271(9)	0.1033(1)	0.1592(3)	0.1334(2)
		μ	0.2311(8)	0.2081(5)	0.3196(11)	0.2326(9)	0.2158(7)	0.2069(4)	0.2147(6)	0.4419(12)	0.2560(10)	0.1476(1)	0.2015(3)	0.1868(2)
	ABB	λ	0.3426(8)	0.3027(4)	0.3708(11)	0.3248(6)	0.3048(5)	0.3378(7)	0.4300(12)	0.3638(10)	0.3522(9)	0.2473(1)	0.3004(3)	0.2788(2)
		μ	0.3597(9)	0.3422(5)	0.3988(11)	0.3519(8)	0.3405(4)	0.3424(6)	0.3498(7)	0.4443(12)	0.3715(10)	0.2950(1)	0.3283(2)	0.3297(3)
	MRE	λ	0.1713(8)	0.1513(4)	0.1854(11)	0.1624(6)	0.1524(5)	0.1689(7)	0.2150(12)	0.1819(10)	0.1761(9)	0.1237(1)	0.1502(3)	0.1394(2)
		μ	0.1499(9)	0.1426(5)	0.1662(11)	0.1466(8)	0.1419(4)	0.1427(6)	0.1458(7)	0.1851(12)	0.1548(10)	0.1229(1)	0.1368(2)	0.1374(3)
	$\sum$ Ranks		$49^{(8)}$	$27^{(4)}$	$66^{(11.5)}$	$43^{(7)}$	$30^{(5)}$	$38^{(6)}$	$56^{(9)}$	$66^{(11.5)}$	$57^{(10)}$	$6^{(1)}$	$16^{(3)}$	$14^{(2)}$
50	MSE	λ	0.0992(8)	0.0790(5)	0.1084(10)	0.0892(6)	0.0781(4)	0.0937(7)	0.1578(12)	0.1105(11)	0.1065(9)	0.0525(1)	0.0728(3)	0.0684(2)
		μ	0.1061(9)	0.0993(4)	0.1228(11)	0.1048(8)	0.0991(3)	0.0977(2)	0.1005(6)	0.1550(12)	0.1142(10)	0.0785(1)	0.1040(7)	0.1003(5)
	ABB	λ	0.2461(8)	0.2164(5)	0.2498(10)	0.2297(6)	0.2134(4)	0.2340(7)	0.2923(12)	0.2522(11)	0.2477(9)	0.1768(1)	0.2013(3)	0.1990(2)
		μ	0.2510(9)	0.2427(5)	0.2639(11)	0.2469(7)	0.2405(2)	0.2413(3)	0.2471(8)	0.2918(12)	0.2568(10)	0.2186(1)	0.2426(4)	0.2443(6)
	MRE	λ	0.1230(8)	0.1082(5)	0.1249(10)	0.1149(6)	0.1067(4)	0.1170(7)	0.1461(12)	0.1261(11)	0.1239(9)	0.0884(1)	0.1007(3)	0.0995(2)
		μ	0.1046(9)	0.1011(4.5)	0.1099(11)	0.1029(7)	0.1002(2)	0.1005(3)	0.1030(8)	0.1216(12)	0.1070(10)	0.0911(1)	0.1011(4.5)	0.1018(6)
	$\sum$ Ranks		$51^{(8)}$	$28.5^{(5)}$	$63^{(11)}$	$40^{(7)}$	$19^{(2)}$	$29^{(6)}$	$58^{(10)}$	$69^{(12)}$	$57^{(9)}$	$6^{(1)}$	$24.5^{(4)}$	$23^{(3)}$
75	MSE	λ	0.0696(10)	0.0536(5)	0.0692(9)	0.0593(6)	0.0525(4)	0.0616(7)	0.1108(12)	0.0735(11)	0.0691(8)	0.0381(1)	0.0503(2)	0.0517(3)
		μ	0.0688(6)	0.0655(4)	0.0778(11)	0.0692(7)	0.0652(3)	0.0640(2)	0.0666(5)	0.0993(12)	0.0748(10)	0.0536(1)	0.0725(9)	0.0693(8)
	ABB	λ	0.2106(11)	0.1803(5)	0.2029(9)	0.1889(6)	0.1767(4)	0.1927(7)	0.2537(12)	0.2086(10)	0.2026(8)	0.1513(1)	0.1663(2)	0.1743(3)
		μ	0.2031(6)	0.1982(4)	0.2141(11)	0.2034(7)	0.1970(3)	0.1963(2)	0.2025(5)	0.2380(12)	0.2104(10)	0.1804(1)	0.2049(8)	0.2050(9)
	MRE	λ	0.1053(11)	0.0901(5)	0.1015(9)	0.0945(6)	0.0883(4)	0.0963(7)	0.1269(12)	0.1043(10)	0.1013(8)	0.0757(1)	0.0831(2)	0.0872(3)
		μ	0.0846(6)	0.0826(4)	0.0892(11)	0.0847(7)	0.0821(3)	0.0818(2)	0.0844(5)	0.0992(12)	0.0877(10)	0.0751(1)	0.0854(8.5)	0.0854(8.5)
	$\sum$ Ranks		$50^{(8)}$	$27^{(3.5)}$	$60^{(11)}$	$39^{(7)}$	$21^{(2)}$	$27^{(3.5)}$	$51^{(9)}$	$67^{(12)}$	$54^{(10)}$	$6^{(1)}$	$31.5^{(5)}$	$34.5^{(6)}$
100	MSE	λ	0.0568(11)	0.0415(3)	0.0509(8.5)	0.0444(5)	0.0405(2)	0.0474(7)	0.0931(12)	0.0555(10)	0.0509(8.5)	0.0322(1)	0.0434(4)	0.0464(6)
		μ	0.0513(6.5)	0.0488(4)	0.0561(10)	0.0513(6.5)	0.0487(3)	0.0475(2)	0.0500(5)	0.0716(12)	0.0554(9)	0.0416(1)	0.0586(11)	0.0553(8)
	ABB	λ	0.1928(11)	0.1582(3)	0.1741(9)	0.1631(5)	0.1552(2)	0.1683(6)	0.2395(12)	0.1819(10)	0.1734(8)	0.1395(1)	0.1613(4)	0.1715(7)
		μ	0.1764(7)	0.1720(4)	0.1827(9)	0.1756(5)	0.1710(3)	0.1700(2)	0.1760(6)	0.2052(12)	0.1825(8)	0.1595(1)	0.1860(11)	0.1842(10)
	MRE	λ	0.0964(11)	0.0791(3)	0.0870(9)	0.0816(5)	0.0776(2)	0.0842(6)	0.1197(12)	0.0909(10)	0.0867(8)	0.0698(1)	0.0807(4)	0.0858(7)
		μ	0.0735(7)	0.0717(4)	0.0761(9)	0.0732(5)	0.0712(3)	0.0708(2)	0.0733(6)	0.0855(12)	0.0760(8)	0.0665(1)	0.0775(11)	0.0767(10)
	$\sum$ Ranks		$53.5^{(10)}$	$21^{(3)}$	$54.5^{(11)}$	$31.5^{(5)}$	$15^{(2)}$	$25^{(4)}$	$53^{(9)}$	$66^{(12)}$	$49.5^{(8)}$	$6^{(1)}$	$45^{(6)}$	$48^{(7)}$
125	MSE	λ	0.0494(11)	0.0351(3)	0.0415(6)	0.0369(4)	0.0343(2)	0.0394(5)	0.0829(12)	0.0463(10)	0.0418(8)	0.0296(1)	0.0417(7)	0.0447(9)
		μ	0.0401(7)	0.0382(4)	0.0432(8)	0.0400(6)	0.0379(3)	0.0371(2)	0.0395(5)	0.0553(12)	0.0433(9)	0.0333(1)	0.0478(11)	0.0453(10)
	ABB	λ	0.1809(11)	0.1455(3)	0.1575(6)	0.1490(4)	0.1429(2)	0.1542(5)	0.2292(12)	0.1663(9)	0.1578(7)	0.1345(1)	0.1633(8)	0.1726(10)
		μ	0.1570(6)	0.1529(4)	0.1617(9)	0.1565(5)	0.1519(3)	0.1511(2)	0.1572(7)	0.1814(12)	0.1616(8)	0.1435(1)	0.1704(11)	0.1676(10)
	MRE	λ	0.0905(11)	0.0728(3)	0.0787(6)	0.0745(4)	0.0715(2)	0.0771(5)	0.1146(12)	0.0832(9)	0.0789(7)	0.0673(1)	0.0817(8)	0.0863(10)
		μ	0.0654(6)	0.0637(4)	0.0674(9)	0.0652(5)	0.0633(3)	0.0629(2)	0.0655(7)	0.0756(12)	0.0673(8)	0.0598(1)	0.0710(11)	0.0698(10)
	$\sum$ Ranks		$52^{(8)}$	$21^{(3.5)}$	$44^{(6)}$	$28^{(5)}$	$15^{(2)}$	$21^{(3.5)}$	$55^{(9)}$	$64^{(12)}$	$47^{(7)}$	$6^{(1)}$	$56^{(10)}$	$59^{(11)}$
250	MSE	λ	0.0350(10)	0.0205(2)	0.0217(4)	0.0197(1)	0.0209(3)	0.0225(6)	0.0649(12)	0.0264(8)	0.0222(5)	0.0243(7)	0.0369(11)	0.0341(9)
		μ	0.0222(7)	0.0205(3)	0.0224(8)	0.0212(6)	0.0206(4.5)	0.0197(2)	0.0206(4.5)	0.0293(11)	0.0227(9)	0.0193(1)	0.0308(12)	0.0289(10)
	ABB	λ	0.1613(11)	0.1142(2)	0.1155(4)	0.1100(1)	0.1154(3)	0.1190(6)	0.2163(12)	0.1288(7)	0.1166(5)	0.1296(8)	0.1577(10)	0.1505(9)
		μ	0.1169(8)	0.1117(3)	0.1163(7)	0.1135(6)	0.1120(4)	0.1099(2)	0.1131(5)	0.1327(10)	0.1173(9)	0.1089(1)	0.1388(12)	0.1346(11)
	MRE	λ	0.0807(11)	0.0571(2)	0.0578(4)	0.0550(1)	0.0577(3)	0.0595(6)	0.1082(12)	0.0644(7)	0.0583(5)	0.0648(8)	0.0789(10)	0.0753(9)
		μ	0.0487(8)	0.0466(3)	0.0485(7)	0.0473(6)	0.0467(4)	0.0458(2)	0.0471(5)	0.0553(10)	0.0489(9)	0.0454(1)	0.0579(12)	0.0561(11)
	$\sum$ Ranks		$55^{(10)}$	$15^{(1)}$	$34^{(6)}$	$21^{(2)}$	$21.5^{(3)}$	$24^{(4)}$	$50.5^{(8)}$	$53^{(9)}$	$42^{(7)}$	$26^{(5)}$	$67^{(12)}$	$59^{(11)}$
500	MSE	λ	0.0280(11)	0.0137(4)	0.0125(2)	0.0117(1)	0.0150(6)	0.0143(5)	0.0550(12)	0.0171(7)	0.0129(3)	0.0224(9)	0.0262(10)	0.0179(8)
		μ	0.0129(9)	0.0111(4)	0.0114(5)	0.0110(3)	0.0115(6)	0.0107(1.5)	0.0107(1.5)	0.0156(11)	0.0118(8)	0.0117(7)	0.0179(12)	0.0147(10)
	ABB	λ	0.1515(11)	0.0960(4)	0.0895(2)	0.0862(1)	0.1024(6)	0.0975(5)	0.2117(12)	0.1071(7)	0.0905(3)	0.1331(9)	0.1367(10)	0.1083(8)
		μ	0.0911(9)	0.0836(4)	0.0842(5)	0.0825(3)	0.0850(6)	0.0819(1)	0.0821(2)	0.0985(11)	0.0854(7)	0.0862(8)	0.1058(12)	0.0965(10)
	MRE	λ	0.0758(11)	0.0480(4)	0.0448(2)	0.0431(1)	0.0512(6)	0.0488(5)	0.1058(12)	0.0536(7)	0.0452(3)	0.0666(9)	0.0683(10)	0.0542(8)
		μ	0.0380(9)	0.0348(4)	0.0351(5)	0.0344(3)	0.0354(6)	0.0341(1)	0.0342(2)	0.0410(11)	0.0356(7)	0.0359(8)	0.0441(12)	0.0402(10)
	$\sum$ Ranks		$60^{(11)}$	$24^{(4)}$	$21^{(3)}$	$12^{(1)}$	$36^{(6)}$	$18.5^{(2)}$	$41.5^{(7)}$	$54^{(9.5)}$	$31^{(5)}$	$50^{(8)}$	$66^{(12)}$	$54^{(9.5)}$

TABLE 7. Simulation results of MSE, ABB and MRE for Scenario 6

n		Parameter	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
25	MSE	λ	0.2422(7)	0.1933(3)	0.3124(11)	0.2307(6)	0.2025(5)	0.2443(8)	0.4392(12)	0.2985(10)	0.2666(9)	0.1242(1)	0.1939(4)	0.1578(2)
		μ	0.3399(9)	0.2988(5)	0.5154(11)	0.3622(10)	0.3317(7)	0.2929(4)	0.3073(6)	0.7130(12)	0.3370(8)	0.2007(1)	0.2797(3)	0.2479(2)
	ABB	λ	0.3701(8)	0.3322(3)	0.4091(11)	0.3622(6)	0.3369(5)	0.3664(7)	0.4607(12)	0.4013(10)	0.3827(9)	0.2737(1)	0.3328(4)	0.3061(2)
		μ	0.4267(10)	0.4028(5)	0.4835(11)	0.4217(8)	0.4048(7)	0.4001(4)	0.4042(6)	0.5425(12)	0.4234(9)	0.3401(1)	0.3689(2)	0.3717(3)
	MRE	λ	0.1682(8)	0.1510(3)	0.1859(11)	0.1646(6)	0.1531(5)	0.1665(7)	0.2094(12)	0.1824(10)	0.1740(9)	0.1244(1)	0.1513(4)	0.1391(2)
		μ	0.1580(10)	0.1492(5)	0.1791(11)	0.1562(8)	0.1499(7)	0.1482(4)	0.1497(6)	0.2009(12)	0.1568(9)	0.1259(1)	0.1366(2)	0.1377(3)
	$\sum$ Ranks		$52^{(8)}$	$24^{(4)}$	$66^{(11.5)}$	$44^{(7)}$	$36^{(6)}$	$34^{(5)}$	$54^{(10)}$	$66^{(11.5)}$	$53^{(9)}$	$6^{(1)}$	$19^{(3)}$	$14^{(2)}$
50	MSE	λ	0.1213(8)	0.0962(5)	0.1289(10)	0.1060(6)	0.0950(4)	0.1134(7)	0.1929(12)	0.1356(11)	0.1240(9)	0.0642(1)	0.0885(3)	0.0845(2)
		μ	0.1406(8)	0.1315(5)	0.1707(11)	0.1425(9)	0.1322(6)	0.1280(4)	0.1258(3)	0.2216(12)	0.1524(10)	0.0971(1)	0.1323(7)	0.1214(2)
	ABB	λ	0.2719(9)	0.2382(5)	0.2735(10)	0.2511(6)	0.2350(4)	0.2571(7)	0.3258(12)	0.2785(11)	0.2670(8)	0.1950(1)	0.2234(3)	0.2215(2)
		μ	0.2845(8)	0.2749(7)	0.3069(11)	0.2846(9)	0.2732(6)	0.2713(4)	0.2717(5)	0.3436(12)	0.2914(10)	0.2398(1)	0.2669(3)	0.2644(2)
	MRE	λ	0.1236(9)	0.1083(5)	0.1243(10)	0.1141(6)	0.1068(4)	0.1169(7)	0.1481(12)	0.1266(11)	0.1214(8)	0.0886(1)	0.1016(3)	0.1007(2)
		μ	0.1054(8.5)	0.1018(7)	0.1137(11)	0.1054(8.5)	0.1012(6)	0.1005(4)	0.1006(5)	0.1273(12)	0.1079(10)	0.0888(1)	0.0989(3)	0.0979(2)
	$\sum$ Ranks		$50.5^{(9)}$	$34^{(6)}$	$63^{(11)}$	$44.5^{(7)}$	$30^{(4)}$	$33^{(5)}$	$49^{(8)}$	$69^{(12)}$	$55^{(10)}$	$6^{(1)}$	$22^{(3)}$	$12^{(2)}$
75	MSE	λ	0.0844(10)	0.0651(5)	0.0824(9)	0.0709(6)	0.0637(4)	0.0754(7)	0.1354(12)	0.0889(11)	0.0807(8)	0.0465(1)	0.0616(2)	0.0620(3)
		μ	0.0921(8)	0.0858(5)	0.1057(11)	0.0923(9)	0.0855(4)	0.0824(2)	0.0832(3)	0.1419(12)	0.0978(10)	0.0671(1)	0.0916(7)	0.0875(6)
	ABB	λ	0.2303(11)	0.1979(5)	0.2211(9)	0.2067(6)	0.1942(4)	0.2118(7)	0.2802(12)	0.2284(10)	0.2184(8)	0.1669(1)	0.1829(2)	0.1911(3)
		μ	0.2345(9)	0.2258(5)	0.2472(11)	0.2329(8)	0.2238(3)	0.2214(2)	0.2243(4)	0.2821(12)	0.2386(10)	0.2011(1)	0.2269(6)	0.2277(7)
	MRE	λ	0.1047(11)	0.0900(5)	0.1005(9)	0.0939(6)	0.0883(4)	0.0963(7)	0.1274(12)	0.1038(10)	0.0993(8)	0.0759(1)	0.0831(2)	0.0869(3)
		μ	0.0868(9)	0.0836(5)	0.0915(11)	0.0863(8)	0.0829(3)	0.0820(2)	0.0831(4)	0.1045(12)	0.0884(10)	0.0745(1)	0.0840(6)	0.0843(7)
	$\sum$ Ranks		$58^{(10)}$	$30^{(6)}$	$60^{(11)}$	$43^{(7)}$	$22^{(2)}$	$27^{(4)}$	$47^{(8)}$	$67^{(12)}$	$54^{(9)}$	$6^{(1)}$	$25^{(3)}$	$29^{(5)}$
100	MSE	λ	0.0674(11)	0.0491(3)	0.0597(9)	0.0523(5)	0.0479(2)	0.0556(6)	0.1097(12)	0.0670(10)	0.0588(8)	0.0384(1)	0.0516(4)	0.0562(7)
		μ	0.0691(7)	0.0642(5)	0.0762(11)	0.0683(6)	0.0639(4)	0.0611(2)	0.0623(3)	0.1030(12)	0.0727(10)	0.0521(1)	0.0726(9)	0.0700(8)
	ABB	λ	0.2098(11)	0.1726(3)	0.1893(9)	0.1782(5)	0.1692(2)	0.1832(6)	0.2593(12)	0.2000(10)	0.1869(7)	0.1530(1)	0.1756(4)	0.1883(8)
		μ	0.2040(7)	0.1951(4.5)	0.2103(11)	0.2002(6)	0.1935(3)	0.1911(2)	0.1951(4.5)	0.2411(12)	0.2058(9.5)	0.1772(1)	0.2057(8)	0.2058(9.5)
	MRE	λ	0.0954(11)	0.0785(3)	0.0861(9)	0.0810(5)	0.0769(2)	0.0833(6)	0.1179(12)	0.0909(10)	0.0850(7)	0.0695(1)	0.0798(4)	0.0856(8)
		μ	0.0755(7)	0.0723(4.5)	0.0779(11)	0.0742(6)	0.0717(3)	0.0708(2)	0.0723(4.5)	0.0893(12)	0.0762(9)	0.0656(1)	0.0762(9)	0.0762(9)
	$\sum$ Ranks		$54^{(10)}$	$23^{(3)}$	$60^{(11)}$	$33^{(5)}$	$16^{(2)}$	$24^{(4)}$	$48^{(7)}$	$66^{(12)}$	$50.5^{(9)}$	$6^{(1)}$	$38^{(6)}$	$49.5^{(8)}$
125	MSE	λ	0.0594(11)	0.0417(3)	0.0493(7)	0.0437(4)	0.0410(2)	0.0464(5)	0.0988(12)	0.0567(10)	0.0488(6)	0.0353(1)	0.0509(8)	0.0538(9)
		μ	0.0570(7)	0.0524(5)	0.0612(9.5)	0.0557(6)	0.0523(4)	0.0500(2)	0.0517(3)	0.0815(12)	0.0586(8)	0.0441(1)	0.0636(11)	0.0612(9.5)
	ABB	λ	0.1993(11)	0.1595(3)	0.1723(7)	0.1624(4)	0.1569(2)	0.1679(5)	0.2513(12)	0.1842(9)	0.1703(6)	0.1475(1)	0.1807(8)	0.1886(10)
		μ	0.1865(8)	0.1780(4)	0.1905(9)	0.1828(6)	0.1767(3)	0.1742(2)	0.1787(5)	0.2172(12)	0.1859(7)	0.1642(1)	0.1945(11)	0.1940(10)
	MRE	λ	0.0906(11)	0.0725(3)	0.0783(7)	0.0738(4)	0.0713(2)	0.0763(5)	0.1142(12)	0.0837(9)	0.0774(6)	0.0670(1)	0.0821(8)	0.0857(10)
		μ	0.0691(8)	0.0659(4)	0.0706(9)	0.0677(6)	0.0654(3)	0.0645(2)	0.0662(5)	0.0805(12)	0.0688(7)	0.0608(1)	0.0720(11)	0.0718(10)
	$\sum$ Ranks		$56^{(9)}$	$22^{(4)}$	$48.5^{(7)}$	$30^{(5)}$	$16^{(2)}$	$21^{(3)}$	$49^{(8)}$	$64^{(12)}$	$40^{(6)}$	$6^{(1)}$	$57^{(10)}$	$58.5^{(11)}$
250	MSE	λ	0.0425(10)	0.0250(2)	0.0263(4.5)	0.0240(1)	0.0254(3)	0.0271(6)	0.0774(12)	0.0323(8)	0.0263(4.5)	0.0295(7)	0.0449(11)	0.0412(9)
		μ	0.0322(9)	0.0277(3)	0.0304(8)	0.0285(6)	0.0280(4)	0.0262(1)	0.0281(5)	0.0417(12)	0.0298(7)	0.0265(2)	0.0407(11)	0.0388(10)
	ABB	λ	0.1771(11)	0.1258(2)	0.1269(4)	0.1211(1)	0.1271(5)	0.1300(6)	0.2374(12)	0.1422(7)	0.1264(3)	0.1427(8)	0.1741(10)	0.1651(9)
		μ	0.1417(9)	0.1298(3)	0.1346(8)	0.1306(5)	0.1299(4)	0.1268(1)	0.1322(6)	0.1570(10.5)	0.1327(7)	0.1279(2)	0.1600(12)	0.1570(10.5)
	MRE	λ	0.0805(11)	0.0572(2)	0.0577(4)	0.0550(1)	0.0578(5)	0.0591(6)	0.1079(12)	0.0646(7)	0.0575(3)	0.0649(8)	0.0791(10)	0.0750(9)
		μ	0.0525(9)	0.0481(3.5)	0.0499(8)	0.0484(5)	0.0481(3.5)	0.0470(1)	0.0490(6)	0.0582(11)	0.0491(7)	0.0474(2)	0.0592(12)	0.0581(10)
	$\sum$ Ranks		$59^{(11)}$	$15.5^{(1)}$	$36.5^{(7)}$	$19^{(2)}$	$24.5^{(4)}$	$21^{(3)}$	$53^{(8)}$	$55.5^{(9)}$	$31.5^{(6)}$	$29^{(5)}$	$66^{(12)}$	$57.5^{(10)}$
500	MSE	λ	0.0340(11)	0.0166(4)	0.0152(2)	0.0142(1)	0.0182(6)	0.0175(5)	0.0670(12)	0.0206(7)	0.0153(3)	0.0272(9)	0.0318(10)	0.0215(8)
		μ	0.0210(9)	0.0164(3)	0.0167(5)	0.0159(2)	0.0172(7)	0.0155(1)	0.0170(6)	0.0239(11)	0.0166(4)	0.0183(8)	0.0259(12)	0.0211(10)
	ABB	λ	0.1672(11)	0.1059(4)	0.0982(3)	0.0944(1)	0.1131(6)	0.1078(5)	0.2337(12)	0.1176(7)	0.0981(2)	0.1469(9)	0.1514(10)	0.1192(8)
		μ	0.1181(10)	0.1015(5)	0.1010(4)	0.0986(1)	0.1043(7)	0.0987(2)	0.1042(6)	0.1213(11)	0.0997(3)	0.1093(8)	0.1286(12)	0.1153(9)
	MRE	λ	0.0760(11)	0.0481(4)	0.0446(2.5)	0.0429(1)	0.0514(6)	0.0490(5)	0.1062(12)	0.0534(7)	0.0446(2.5)	0.0668(9)	0.0688(10)	0.0542(8)
		μ	0.0437(10)	0.0376(5)	0.0374(4)	0.0365(1)	0.0386(6.5)	0.0366(2)	0.0386(6.5)	0.0449(11)	0.0369(3)	0.0405(8)	0.0476(12)	0.0427(9)
	$\sum$ Ranks		$62^{(11)}$	$25^{(5)}$	$20.5^{(4)}$	$7^{(1)}$	$38.5^{(6)}$	$20^{(3)}$	$54.5^{(10)}$	$54^{(9)}$	$17.5^{(2)}$	$51^{(7)}$	$66^{(12)}$	$52^{(8)}$

TABLE 8. Simulation results of MSE, ABB and MRE for Scenario 7

n		Parameter	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
25	MSE	λ	0.5240(8)	0.4216(4)	0.6924(11)	0.5082(6)	0.4479(5)	0.5204(7)	0.9373(12)	0.6620(10)	0.5475(9)	0.2653(1)	0.3864(3)	0.3322(2)
		μ	0.3934(8)	0.3438(5)	0.5912(11)	0.4124(9)	0.3775(7)	0.3420(4)	0.3596(6)	0.8193(12)	0.4479(10)	0.2259(1)	0.3270(3)	0.2838(2)
	ABB	λ	0.5473(9)	0.4906(4)	0.6057(11)	0.5340(7)	0.4996(5)	0.5406(8)	0.6770(12)	0.5914(10)	0.5314(6)	0.4004(1)	0.4601(3)	0.4438(2)
		μ	0.4508(9)	0.4259(6)	0.5177(11)	0.4497(8)	0.4308(7)	0.4227(4)	0.4241(5)	0.5759(12)	0.4758(10)	0.3562(1)	0.3997(3)	0.3947(2)
	MRE	λ	0.1710(9)	0.1533(4)	0.1893(11)	0.1669(7)	0.1561(5)	0.1689(8)	0.2116(12)	0.1848(10)	0.1661(6)	0.1251(1)	0.1438(3)	0.1387(2)
		μ	0.1610(9)	0.1521(6)	0.1849(11)	0.1606(8)	0.1539(7)	0.1510(4)	0.1515(5)	0.2057(12)	0.1699(10)	0.1272(1)	0.1428(3)	0.1410(2)
	$\sum$ Ranks		$52^{(9.5)}$	$29^{(4)}$	$66^{(11.5)}$	$45^{(7)}$	$36^{(6)}$	$35^{(5)}$	$52^{(9.5)}$	$66^{(11.5)}$	$51^{(8)}$	$6^{(1)}$	$18^{(3)}$	$12^{(2)}$
50	MSE	λ	0.2544(9)	0.2024(5)	0.2744(10)	0.2267(6)	0.2017(4)	0.2424(7)	0.4187(12)	0.2835(11)	0.2490(8)	0.1344(1)	0.1812(3)	0.1748(2)
		μ	0.1548(8)	0.1412(5)	0.1879(11)	0.1553(9)	0.1424(6)	0.1361(3)	0.1377(4)	0.2483(12)	0.1755(10)	0.1043(1)	0.1439(7)	0.1347(2)
	ABB	λ	0.3920(9)	0.3452(5)	0.3975(10)	0.3661(6)	0.3412(4)	0.3737(8)	0.4774(12)	0.4051(11)	0.3727(7)	0.2830(1)	0.3148(2)	0.3188(3)
		μ	0.3002(9)	0.2880(7)	0.3228(11)	0.2981(8)	0.2861(6)	0.2829(4)	0.2843(5)	0.3659(12)	0.3158(10)	0.2502(1)	0.2815(3)	0.2803(2)
	MRE	λ	0.1225(9)	0.1079(5)	0.1242(10)	0.1144(6)	0.1066(4)	0.1168(8)	0.1492(12)	0.1266(11)	0.1165(7)	0.0884(1)	0.0984(2)	0.0996(3)
		μ	0.1072(9)	0.1029(7)	0.1153(11)	0.1064(8)	0.1022(6)	0.1010(4)	0.1015(5)	0.1307(12)	0.1128(10)	0.0893(1)	0.1005(3)	0.1001(2)
	$\sum$ Ranks		$53^{(10)}$	$34^{(5.5)}$	$63^{(11)}$	$43^{(7)}$	$30^{(4)}$	$34^{(5.5)}$	$50^{(8)}$	$69^{(12)}$	$52^{(9)}$	$6^{(1)}$	$20^{(3)}$	$14^{(2)}$
75	MSE	λ	0.1781(10)	0.1377(5)	0.1777(9)	0.1526(6)	0.1356(4)	0.1596(7)	0.2871(12)	0.1892(11)	0.1642(8)	0.0979(1)	0.1227(2)	0.1305(3)
		μ	0.0992(8)	0.0922(5)	0.1161(11)	0.1011(9)	0.0927(6)	0.0879(2)	0.0882(3)	0.1569(12)	0.1122(10)	0.0708(1)	0.0971(7)	0.0916(4)
	ABB	λ	0.3362(11)	0.2881(5)	0.3236(9)	0.3020(6)	0.2833(4)	0.3073(7)	0.4087(12)	0.3344(10)	0.3085(8)	0.2430(1)	0.2591(2)	0.2772(3)
		μ	0.2447(9)	0.2348(7)	0.2589(11)	0.2437(8)	0.2331(4)	0.2299(2)	0.2323(3)	0.2947(12)	0.2574(10)	0.2079(1)	0.2332(5)	0.2345(6)
	MRE	λ	0.1051(11)	0.0900(5)	0.1011(9)	0.0944(6)	0.0885(4)	0.0960(7)	0.1277(12)	0.1045(10)	0.0964(8)	0.0759(1)	0.0810(2)	0.0866(3)
		μ	0.0874(9)	0.0839(7)	0.0925(11)	0.0871(8)	0.0833(4.5)	0.0821(2)	0.0830(3)	0.1052(12)	0.0919(10)	0.0743(1)	0.0833(4.5)	0.0837(6)
	$\sum$ Ranks		$58^{(10)}$	$34^{(6)}$	$60^{(11)}$	$43^{(7)}$	$26.5^{(4)}$	$27^{(5)}$	$45^{(8)}$	$67^{(12)}$	$54^{(9)}$	$6^{(1)}$	$22.5^{(2)}$	$25^{(3)}$
100	MSE	λ	0.1451(11)	0.1041(3)	0.1275(9)	0.1116(4)	0.1019(2)	0.1198(7)	0.2446(12)	0.1416(10)	0.1183(6)	0.0828(1)	0.1128(5)	0.1228(8)
		μ	0.0757(7)	0.0691(5)	0.0828(11)	0.0740(6)	0.0686(4)	0.0657(2)	0.0682(3)	0.1125(12)	0.0816(10)	0.0562(1)	0.0803(9)	0.0767(8)
	ABB	λ	0.3083(11)	0.2517(3)	0.2763(8)	0.2597(5)	0.2475(2)	0.2687(7)	0.3861(12)	0.2899(10)	0.2638(6)	0.2250(1)	0.2572(4)	0.2781(9)
		μ	0.2133(7)	0.2034(4.5)	0.2208(11)	0.2104(6)	0.2014(3)	0.1987(2)	0.2034(4.5)	0.2539(12)	0.2201(10)	0.1840(1)	0.2166(9)	0.2145(8)
	MRE	λ	0.0963(11)	0.0787(3)	0.0864(8)	0.0811(5)	0.0773(2)	0.0840(7)	0.1207(12)	0.0906(10)	0.0824(6)	0.0703(1)	0.0804(4)	0.0869(9)
		μ	0.0762(7)	0.0726(4)	0.0789(11)	0.0751(6)	0.0719(3)	0.0710(2)	0.0727(5)	0.0907(12)	0.0786(10)	0.0657(1)	0.0774(9)	0.0766(8)
	$\sum$ Ranks		$54^{(10)}$	$22.5^{(3)}$	$58^{(11)}$	$32^{(5)}$	$16^{(2)}$	$27^{(4)}$	$48.5^{(8)}$	$66^{(12)}$	$48^{(7)}$	$6^{(1)}$	$40^{(6)}$	$50^{(9)}$
125	MSE	λ	0.1273(11)	0.0889(3)	0.1057(7)	0.0934(4)	0.0873(2)	0.0991(5)	0.2127(12)	0.1206(10)	0.0998(6)	0.0757(1)	0.1073(8)	0.1160(9)
		μ	0.0618(7)	0.0561(5)	0.0668(11)	0.0605(6)	0.0559(4)	0.0532(2)	0.0549(3)	0.0897(12)	0.0654(9)	0.0465(1)	0.0663(10)	0.0640(8)
	ABB	λ	0.2925(11)	0.2340(3)	0.2541(7)	0.2392(4)	0.2307(2)	0.2459(6)	0.3679(12)	0.2703(9)	0.2436(5)	0.2171(1)	0.2609(8)	0.2782(10)
		μ	0.1932(7)	0.1834(5)	0.1984(9)	0.1898(6)	0.1820(3)	0.1787(2)	0.1828(4)	0.2277(12)	0.1980(8)	0.1676(1)	0.1994(11)	0.1989(10)
	MRE	λ	0.0914(11)	0.0731(3)	0.0794(7)	0.0748(4)	0.0721(2)	0.0768(6)	0.1150(12)	0.0845(9)	0.0761(5)	0.0678(1)	0.0815(8)	0.0869(10)
		μ	0.0690(7)	0.0655(5)	0.0709(9)	0.0678(6)	0.0650(3)	0.0638(2)	0.0653(4)	0.0813(12)	0.0707(8)	0.0598(1)	0.0712(11)	0.0710(10)
	$\sum$ Ranks		$54^{(9)}$	$24^{(4)}$	$50^{(8)}$	$30^{(5)}$	$16^{(2)}$	$23^{(3)}$	$47^{(7)}$	$64^{(12)}$	$41^{(6)}$	$6^{(1)}$	$56^{(10)}$	$57^{(11)}$
250	MSE	λ	0.0902(10)	0.0524(2)	0.0550(5)	0.0500(1)	0.0533(3)	0.0570(6)	0.1675(12)	0.0676(8)	0.0534(4)	0.0628(7)	0.0942(11)	0.0870(9)
		μ	0.0372(9)	0.0310(3)	0.0336(7)	0.0314(5)	0.0313(4)	0.0295(1)	0.0325(6)	0.0463(11)	0.0340(8)	0.0302(2)	0.0479(12)	0.0445(10)
	ABB	λ	0.2590(11)	0.1831(3)	0.1847(4)	0.1760(1)	0.1851(5)	0.1898(6)	0.3481(12)	0.2070(7)	0.1803(2)	0.2086(8)	0.2526(10)	0.2415(9)
		μ	0.1532(9)	0.1380(3)	0.1427(7)	0.1381(4)	0.1383(5)	0.1346(1)	0.1422(6)	0.1671(10)	0.1434(8)	0.1369(2)	0.1733(12)	0.1685(11)
	MRE	λ	0.0809(11)	0.0572(3)	0.0577(4)	0.0550(1)	0.0578(5)	0.0593(6)	0.1088(12)	0.0647(7)	0.0564(2)	0.0652(8)	0.0789(10)	0.0755(9)
		μ	0.0547(9)	0.0493(3.5)	0.0510(7)	0.0493(3.5)	0.0494(5)	0.0481(1)	0.0508(6)	0.0597(10)	0.0512(8)	0.0489(2)	0.0619(12)	0.0602(11)
	$\sum$ Ranks		$59^{(10.5)}$	$17.5^{(2)}$	$34^{(7)}$	$15.5^{(1)}$	$27^{(4)}$	$21^{(3)}$	$54^{(9)}$	$53^{(8)}$	$32^{(6)}$	$29^{(5)}$	$67^{(12)}$	$59^{(10.5)}$
500	MSE	λ	0.0713(11)	0.0347(4)	0.0315(3)	0.0293(1)	0.0381(6)	0.0360(5)	0.1402(12)	0.0440(7)	0.0313(2)	0.0572(9)	0.0660(10)	0.0456(8)
		μ	0.0245(10)	0.0186(3)	0.0188(4)	0.0179(2)	0.0196(6)	0.0175(1)	0.0201(7)	0.0275(11)	0.0194(5)	0.0211(8)	0.0299(12)	0.0242(9)
	ABB	λ	0.2425(11)	0.1532(4)	0.1419(3)	0.1364(1)	0.1636(6)	0.1554(5)	0.3392(12)	0.1710(7)	0.1405(2)	0.2133(9)	0.2172(10)	0.1731(8)
		μ	0.1284(10)	0.1088(4)	0.1082(3)	0.1055(2)	0.1118(6)	0.1054(1)	0.1140(7)	0.1308(11)	0.1094(5)	0.1182(8)	0.1377(12)	0.1230(9)
	MRE	λ	0.0758(11)	0.0479(4)	0.0443(3)	0.0426(1)	0.0511(6)	0.0486(5)	0.1060(12)	0.0534(7)	0.0439(2)	0.0667(9)	0.0679(10)	0.0541(8)
		μ	0.0459(10)	0.0389(4)	0.0387(3)	0.0377(2)	0.0399(6)	0.0376(1)	0.0407(7)	0.0467(11)	0.0391(5)	0.0422(8)	0.0492(12)	0.0439(9)
	$\sum$ Ranks		$63^{(11)}$	$23^{(5)}$	$19^{(3)}$	$9^{(1)}$	$36^{(6)}$	$18^{(2)}$	$57^{(10)}$	$54^{(9)}$	$21^{(4)}$	$51^{(7.5)}$	$66^{(12)}$	$51^{(7.5)}$

TABLE 9. Simulation results of MSE, ABB and MRE for Scenario 8

n		Parameter	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
25	MSE	λ	0.5643(9)	0.4463(4)	0.7021(11)	0.5183(6)	0.4630(5)	0.5470(8)	1.0109(12)	0.6889(10)	0.5381(7)	0.2849(1)	0.3788(3)	0.3572(2)
		μ	0.1307(7)	0.1224(4)	0.1542(11)	0.1256(5)	0.1220(3)	0.1262(6)	0.1439(9)	0.1855(12)	0.1440(10)	0.0981(1)	0.1344(8)	0.1212(2)
	ABB	λ	0.5634(9)	0.4990(4)	0.6060(11)	0.5360(7)	0.5024(5)	0.5447(8)	0.6957(12)	0.6059(10)	0.5044(6)	0.4106(1)	0.4502(2)	0.4575(3)
		μ	0.2789(8)	0.2691(4)	0.2923(10)	0.2678(3)	0.2653(2)	0.2754(6)	0.2977(11)	0.3100(12)	0.2855(9)	0.2434(1)	0.2783(7)	0.2700(5)
	MRE	λ	0.1707(9)	0.1512(4)	0.1836(10.5)	0.1624(7)	0.1522(5)	0.1651(8)	0.2108(12)	0.1836(10.5)	0.1528(6)	0.1244(1)	0.1364(2)	0.1386(3)
		μ	0.1549(8)	0.1495(4)	0.1624(10)	0.1488(3)	0.1474(2)	0.1530(6)	0.1654(11)	0.1722(12)	0.1586(9)	0.1352(1)	0.1546(7)	0.1500(5)
	$\sum$ Ranks		$50^{(9)}$	$24^{(4)}$	$63.5^{(10)}$	$31^{(6)}$	$22^{(3)}$	$42^{(7)}$	$67^{(12)}$	$66.5^{(11)}$	$47^{(8)}$	$6^{(1)}$	$29^{(5)}$	$20^{(2)}$
50	MSE	λ	0.2711(9)	0.2163(5)	0.2952(10)	0.2435(6)	0.2159(4)	0.2539(8)	0.4311(12)	0.3093(11)	0.2504(7)	0.1436(1)	0.1781(2)	0.1879(3)
		μ	0.0579(5)	0.0563(3)	0.0637(8)	0.0578(4)	0.0562(2)	0.0580(6)	0.0682(10)	0.0727(12)	0.0640(9)	0.0490(1)	0.0690(11)	0.0623(7)
	ABB	λ	0.4049(9)	0.3560(5)	0.4115(10)	0.3794(7)	0.3523(4)	0.3817(8)	0.4826(12)	0.4205(11)	0.3639(6)	0.2912(1)	0.3091(2)	0.3289(3)
		μ	0.1894(5)	0.1864(3)	0.1959(7)	0.1874(4)	0.1853(2)	0.1897(6)	0.2069(12)	0.2065(11)	0.1973(9)	0.1747(1)	0.2020(10)	0.1968(8)
	MRE	λ	0.1227(9)	0.1079(5)	0.1247(10)	0.1150(7)	0.1067(4)	0.1157(8)	0.1463(12)	0.1274(11)	0.1103(6)	0.0883(1)	0.0937(2)	0.0997(3)
		μ	0.1052(5)	0.1035(3)	0.1088(7)	0.1041(4)	0.1030(2)	0.1054(6)	0.1149(12)	0.1147(11)	0.1096(9)	0.0970(1)	0.1122(10)	0.1093(8)
	$\sum$ Ranks		$42^{(7.5)}$	$24^{(3)}$	$52^{(10)}$	$32^{(4.5)}$	$18^{(2)}$	$42^{(7.5)}$	$70^{(12)}$	$67^{(11)}$	$46^{(9)}$	$6^{(1)}$	$37^{(6)}$	$32^{(4.5)}$
75	MSE	λ	0.1880(10)	0.1439(5)	0.1854(9)	0.1589(6)	0.1416(4)	0.1655(8)	0.3035(12)	0.1985(11)	0.1601(7)	0.1025(1)	0.1242(2)	0.1404(3)
		μ	0.0376(4)	0.0371(3)	0.0408(7)	0.0382(5)	0.0370(2)	0.0383(6)	0.0469(12)	0.0463(11)	0.0413(8)	0.0333(1)	0.0454(10)	0.0422(9)
	ABB	λ	0.3434(11)	0.2916(5)	0.3275(9)	0.3056(7)	0.2864(4)	0.3106(8)	0.4195(12)	0.3404(10)	0.2930(6)	0.2467(1)	0.2567(2)	0.2855(3)
		μ	0.1532(4.5)	0.1518(3)	0.1579(7)	0.1532(4.5)	0.1513(2)	0.1545(6)	0.1719(12)	0.1665(11)	0.1591(8)	0.1445(1)	0.1650(10)	0.1616(9)
	MRE	λ	0.1041(11)	0.0884(5)	0.0992(9)	0.0926(7)	0.0868(4)	0.0941(8)	0.1271(12)	0.1032(10)	0.0888(6)	0.0748(1)	0.0778(2)	0.0865(3)
		μ	0.0851(4.5)	0.0843(3)	0.0877(7)	0.0851(4.5)	0.0840(2)	0.0859(6)	0.0955(12)	0.0925(11)	0.0884(8)	0.0803(1)	0.0917(10)	0.0898(9)
	$\sum$ Ranks		$45^{(9)}$	$24^{(3)}$	$48^{(10)}$	$34^{(4)}$	$18^{(2)}$	$42^{(7)}$	$72^{(12)}$	$64^{(11)}$	$43^{(8)}$	$6^{(1)}$	$36^{(5.5)}$	$36^{(5.5)}$
100	MSE	λ	0.1565(11)	0.1140(4)	0.1384(9)	0.1208(5)	0.1109(2)	0.1296(8)	0.2558(12)	0.1548(10)	0.1226(6)	0.0890(1)	0.1138(3)	0.1278(7)
		μ	0.0276(4)	0.0275(3)	0.0298(7)	0.0284(5.5)	0.0274(2)	0.0284(5.5)	0.0350(11)	0.0341(10)	0.0307(8)	0.0251(1)	0.0355(12)	0.0332(9)
	ABB	λ	0.3211(11)	0.2645(5)	0.2894(9)	0.2715(6)	0.2592(3)	0.2812(7)	0.3972(12)	0.3055(10)	0.2616(4)	0.2341(1)	0.2568(2)	0.2854(8)
		μ	0.1321(4)	0.1314(3)	0.1363(7)	0.1332(5)	0.1310(2)	0.1338(6)	0.1491(12)	0.1444(10)	0.1383(8)	0.1262(1)	0.1477(11)	0.1439(9)
	MRE	λ	0.0973(11)	0.0801(5)	0.0877(9)	0.0823(6)	0.0786(3)	0.0852(7)	0.1204(12)	0.0926(10)	0.0793(4)	0.0709(1)	0.0778(2)	0.0865(8)
		μ	0.0734(4)	0.0730(3)	0.0757(7)	0.0740(5)	0.0728(2)	0.0743(6)	0.0828(12)	0.0802(10)	0.0768(8)	0.0701(1)	0.0820(11)	0.0799(9)
	$\sum$ Ranks		$45^{(8)}$	$23^{(3)}$	$48^{(9)}$	$32.5^{(4)}$	$14^{(2)}$	$39.5^{(6)}$	$71^{(12)}$	$60^{(11)}$	$38^{(5)}$	$6^{(1)}$	$41^{(7)}$	$50^{(10)}$
125	MSE	λ	0.1386(11)	0.0964(3)	0.1135(8)	0.1000(4)	0.0949(2)	0.1061(6)	0.2268(12)	0.1311(10)	0.1021(5)	0.0825(1)	0.1101(7)	0.1231(9)
		μ	0.0225(2.5)	0.0226(4)	0.0243(7)	0.0233(6)	0.0225(2.5)	0.0232(5)	0.0288(11)	0.0275(9)	0.0249(8)	0.0208(1)	0.0298(12)	0.0279(10)
	ABB	λ	0.3056(11)	0.2418(4)	0.2606(7)	0.2454(5)	0.2385(2)	0.2532(6)	0.3816(12)	0.2806(9)	0.2387(3)	0.2255(1)	0.2625(8)	0.2871(10)
		μ	0.1194(4)	0.1193(3)	0.1232(7)	0.1208(5)	0.1189(2)	0.1212(6)	0.1358(11)	0.1302(9)	0.1249(8)	0.1149(1)	0.1361(12)	0.1328(10)
	MRE	λ	0.0926(11)	0.0733(4)	0.0790(7)	0.0744(5)	0.0723(2.5)	0.0767(6)	0.1156(12)	0.0850(9)	0.0723(2.5)	0.0683(1)	0.0795(8)	0.0870(10)
		μ	0.0663(3.5)	0.0663(3.5)	0.0684(7)	0.0671(5)	0.0661(2)	0.0673(6)	0.0754(11)	0.0723(9)	0.0694(8)	0.0639(1)	0.0756(12)	0.0738(10)
	$\sum$ Ranks		$43^{(7.5)}$	$21.5^{(3)}$	$43^{(7.5)}$	$30^{(4)}$	$13^{(2)}$	$35^{(6)}$	$69^{(12)}$	$55^{(9)}$	$34.5^{(5)}$	$6^{(1)}$	$59^{(10.5)}$	$59^{(10.5)}$
250	MSE	λ	0.0977(10)	0.0581(3)	0.0616(5)	0.0561(1)	0.0591(4)	0.0626(6)	0.1751(12)	0.0755(8)	0.0577(2)	0.0681(7)	0.1010(11)	0.0944(9)
		μ	0.0108(2)	0.0109(3.5)	0.0116(7)	0.0113(6)	0.0109(3.5)	0.0112(5)	0.0146(10)	0.0131(9)	0.0120(8)	0.0104(1)	0.0174(12)	0.0157(11)
	ABB	λ	0.2688(11)	0.1921(3)	0.1950(5)	0.1857(2)	0.1941(4)	0.1985(6)	0.3555(12)	0.2179(8)	0.1832(1)	0.2163(7)	0.2595(10)	0.2505(9)
		μ	0.0829(2)	0.0831(4)	0.0855(7)	0.0846(6)	0.0830(3)	0.0845(5)	0.0965(10)	0.0905(9)	0.0869(8)	0.0812(1)	0.1054(12)	0.0999(11)
	MRE	λ	0.0814(11)	0.0582(3)	0.0591(5)	0.0563(2)	0.0588(4)	0.0601(6)	0.1077(12)	0.0660(8)	0.0555(1)	0.0655(7)	0.0786(10)	0.0759(9)
		μ	0.0460(2)	0.0462(4)	0.0475(7)	0.0470(6)	0.0461(3)	0.0469(5)	0.0536(10)	0.0503(9)	0.0483(8)	0.0451(1)	0.0586(12)	0.0555(11)
	$\sum$ Ranks		$38^{(8)}$	$20.5^{(1)}$	$36^{(7)}$	$23^{(3)}$	$21.5^{(2)}$	$33^{(6)}$	$66^{(11)}$	$51^{(9)}$	$28^{(5)}$	$24^{(4)}$	$67^{(12)}$	$60^{(10)}$
500	MSE	λ	0.0780(11)	0.0382(4)	0.0350(3)	0.0326(1)	0.0420(6)	0.0402(5)	0.1532(12)	0.0475(7)	0.0339(2)	0.0627(9)	0.0718(10)	0.0499(8)
		μ	0.0054(2)	0.0055(3.5)	0.0058(7)	0.0057(5.5)	0.0055(3.5)	0.0057(5.5)	0.0080(11)	0.0066(9)	0.0060(8)	0.0053(1)	0.0089(12)	0.0079(10)
	ABB	λ	0.2535(11)	0.1608(4)	0.1494(3)	0.1436(1)	0.1718(6)	0.1636(5)	0.3539(12)	0.1779(7)	0.1440(2)	0.2232(9)	0.2261(10)	0.1816(8)
		μ	0.0585(2)	0.0589(3.5)	0.0603(7)	0.0599(6)	0.0589(3.5)	0.0598(5)	0.0716(11)	0.0642(9)	0.0615(8)	0.0579(1)	0.0749(12)	0.0703(10)
	MRE	λ	0.0768(11)	0.0487(4)	0.0453(3)	0.0435(1)	0.0521(6)	0.0496(5)	0.1072(12)	0.0539(7)	0.0437(2)	0.0676(9)	0.0685(10)	0.0550(8)
		μ	0.0325(2)	0.0327(3.5)	0.0335(7)	0.0333(6)	0.0327(3.5)	0.0332(5)	0.0398(11)	0.0357(9)	0.0342(8)	0.0322(1)	0.0416(12)	0.0391(10)
	$\sum$ Ranks		$39^{(8)}$	$22.5^{(2)}$	$30^{(5)}$	$20.5^{(1)}$	$28.5^{(3)}$	$30.5^{(7)}$	$69^{(12)}$	$48^{(9)}$	$30^{(5)}$	$30^{(5)}$	$66^{(11)}$	$54^{(10)}$

TABLE 10. Simulation results of MSE, ABB and MRE for Scenario 9

n		Parameter	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
25	MSE	λ	0.6315(8)	0.5076(4)	0.8102(11)	0.6019(6)	0.5395(5)	0.6445(9)	1.1511(12)	0.7897(10)	0.6104(7)	0.3280(1)	0.4597(3)	0.4095(2)
		μ	0.2022(8)	0.1854(4)	0.2722(11)	0.2050(9)	0.1949(7)	0.1851(3)	0.1929(6)	0.3976(12)	0.2377(10)	0.1353(1)	0.1863(5)	0.1681(2)
	ABB	λ	0.5967(9)	0.5364(4)	0.6574(11)	0.5830(7)	0.5461(5)	0.5911(8)	0.7435(12)	0.6466(10)	0.5473(6)	0.4452(1)	0.4957(3)	0.4905(2)
		μ	0.3379(9)	0.3238(5)	0.3698(11)	0.3299(7)	0.3226(4)	0.3253(6)	0.3361(8)	0.4098(12)	0.3586(10)	0.2817(1)	0.3191(3)	0.3113(2)
	MRE	λ	0.1705(9)	0.1533(4)	0.1878(11)	0.1666(7)	0.1560(5)	0.1689(8)	0.2124(12)	0.1848(10)	0.1564(6)	0.1272(1)	0.1416(3)	0.1402(2)
		μ	0.1536(9)	0.1472(5)	0.1681(11)	0.1500(7)	0.1467(4)	0.1479(6)	0.1528(8)	0.1863(12)	0.1630(10)	0.1280(1)	0.1450(3)	0.1415(2)
	$\sum$	Ranks	$52^{(9)}$	$26^{(4)}$	$66^{(11.5)}$	$43^{(7)}$	$30^{(5)}$	$40^{(6)}$	$58^{(10)}$	$66^{(11.5)}$	$49^{(8)}$	$6^{(1)}$	$20^{(3)}$	$12^{(2)}$
50	MSE	λ	0.3031(9)	0.2416(5)	0.3295(10)	0.2705(6)	0.2409(4)	0.2890(8)	0.4975(12)	0.3434(11)	0.2744(7)	0.1588(1)	0.2000(2)	0.2061(3)
		μ	0.0855(6)	0.0822(3)	0.1009(11)	0.0882(8)	0.0831(4)	0.0818(2)	0.0880(7)	0.1270(12)	0.0995(10)	0.0670(1)	0.0919(9)	0.0848(5)
	ABB	λ	0.4309(9)	0.3787(5)	0.4374(10)	0.4017(7)	0.3749(4)	0.4108(8)	0.5227(12)	0.4435(11)	0.3857(6)	0.3091(1)	0.3305(2)	0.3465(3)
		μ	0.2267(6)	0.2224(3)	0.2410(10)	0.2275(7)	0.2216(2)	0.2228(4)	0.2339(9)	0.2632(12)	0.2414(11)	0.2025(1)	0.2314(8)	0.2266(5)
	MRE	λ	0.1231(9)	0.1082(5)	0.1250(10)	0.1148(7)	0.1071(4)	0.1174(8)	0.1493(12)	0.1267(11)	0.1102(6)	0.0883(1)	0.0944(2)	0.0990(3)
		μ	0.1030(5.5)	0.1011(3)	0.1095(10)	0.1034(7)	0.1007(2)	0.1013(4)	0.1063(9)	0.1197(12)	0.1097(11)	0.0921(1)	0.1052(8)	0.1030(5.5)
	$\sum$	Ranks	$44.5^{(8)}$	$24^{(3)}$	$61^{(10.5)}$	$42^{(7)}$	$20^{(2)}$	$34^{(6)}$	$61^{(10.5)}$	$69^{(12)}$	$51^{(9)}$	$6^{(1)}$	$31^{(5)}$	$24.5^{(4)}$
75	MSE	λ	0.2166(10)	0.1669(5)	0.2144(9)	0.1834(6)	0.1644(4)	0.1909(8)	0.3404(12)	0.2324(11)	0.1836(7)	0.1182(1)	0.1451(2)	0.1601(3)
		μ	0.0556(5)	0.0540(3.5)	0.0627(9)	0.0569(6)	0.0540(3.5)	0.0534(2)	0.0581(7)	0.0777(12)	0.0639(10)	0.0458(1)	0.0642(11)	0.0598(8)
	ABB	λ	0.3696(11)	0.3175(5)	0.3563(9)	0.3319(7)	0.3118(4)	0.3358(8)	0.4478(12)	0.3690(10)	0.3182(6)	0.2668(1)	0.2789(2)	0.3070(3)
		μ	0.1843(5)	0.1813(4)	0.1929(9)	0.1850(6)	0.1805(2)	0.1810(3)	0.1906(7)	0.2118(12)	0.1944(11)	0.1684(1)	0.1942(10)	0.1907(8)
	MRE	λ	0.1056(11)	0.0907(5)	0.1018(9)	0.0948(7)	0.0891(4)	0.0960(8)	0.1279(12)	0.1054(10)	0.0909(6)	0.0762(1)	0.0797(2)	0.0877(3)
		μ	0.0838(5)	0.0824(4)	0.0877(9)	0.0841(6)	0.0820(2)	0.0823(3)	0.0866(7)	0.0963(12)	0.0884(11)	0.0765(1)	0.0883(10)	0.0867(8)
	$\sum$	Ranks	$47^{(8)}$	$26.5^{(3)}$	$54^{(10)}$	$38^{(7)}$	$19.5^{(2)}$	$32^{(4)}$	$57^{(11)}$	$67^{(12)}$	$51^{(9)}$	$6^{(1)}$	$37^{(6)}$	$33^{(5)}$
100	MSE	λ	0.1730(10)	0.1273(4)	0.1584(9)	0.1389(6)	0.1255(2)	0.1428(7)	0.2841(12)	0.1762(11)	0.1385(5)	0.0983(1)	0.1272(3)	0.1431(8)
		μ	0.0404(5)	0.0395(4)	0.0448(8)	0.0417(6)	0.0394(3)	0.0392(2)	0.0434(7)	0.0552(12)	0.0463(10)	0.0345(1)	0.0481(11)	0.0453(9)
	ABB	λ	0.3359(11)	0.2776(4)	0.3067(9)	0.2888(6)	0.2733(3)	0.2924(7)	0.4176(12)	0.3224(10)	0.2777(5)	0.2449(1)	0.2722(2)	0.3017(8)
		μ	0.1579(5)	0.1560(4)	0.1651(7)	0.1600(6)	0.1553(2)	0.1557(3)	0.1655(8)	0.1812(12)	0.1678(10)	0.1464(1)	0.1706(11)	0.1673(9)
	MRE	λ	0.0960(11)	0.0793(4.5)	0.0876(9)	0.0825(6)	0.0781(3)	0.0835(7)	0.1193(12)	0.0921(10)	0.0793(4.5)	0.0700(1)	0.0778(2)	0.0862(8)
		μ	0.0718(5)	0.0709(4)	0.0751(7)	0.0727(6)	0.0706(2)	0.0708(3)	0.0752(8)	0.0824(12)	0.0763(10)	0.0666(1)	0.0775(11)	0.0760(9)
	$\sum$	Ranks	$47^{(8)}$	$24.5^{(3)}$	$49^{(9)}$	$36^{(5)}$	$15^{(2)}$	$29^{(4)}$	$59^{(11)}$	$67^{(12)}$	$44.5^{(7)}$	$6^{(1)}$	$40^{(6)}$	$51^{(10)}$
125	MSE	λ	0.1509(11)	0.1055(3)	0.1251(8)	0.1109(5)	0.1035(2)	0.1187(6)	0.2516(12)	0.1413(10)	0.1108(4)	0.0897(1)	0.1227(7)	0.1371(9)
		μ	0.0324(5)	0.0314(4)	0.0350(8)	0.0328(6)	0.0313(3)	0.0311(2)	0.0344(7)	0.0437(12)	0.0362(9)	0.0280(1)	0.0407(11)	0.0375(10)
	ABB	λ	0.3180(11)	0.2548(4)	0.2754(7)	0.2596(5)	0.2506(2)	0.2680(6)	0.3993(12)	0.2921(9)	0.2535(3)	0.2360(1)	0.2783(8)	0.3022(10)
		μ	0.1419(5)	0.1397(4)	0.1467(7)	0.1425(6)	0.1391(2)	0.1392(3)	0.1476(8)	0.1622(12)	0.1492(9)	0.1323(1)	0.1587(11)	0.1534(10)
	MRE	λ	0.0908(11)	0.0728(4)	0.0787(7)	0.0742(5)	0.0716(2)	0.0766(6)	0.1141(12)	0.0835(9)	0.0724(3)	0.0674(1)	0.0795(8)	0.0864(10)
		μ	0.0645(5)	0.0635(4)	0.0667(7)	0.0648(6)	0.0632(2)	0.0633(3)	0.0671(8)	0.0737(12)	0.0678(9)	0.0602(1)	0.0722(11)	0.0697(10)
	$\sum$	Ranks	$48^{(8)}$	$23^{(3)}$	$44^{(7)}$	$33^{(5)}$	$13^{(2)}$	$26^{(4)}$	$59^{(10.5)}$	$64^{(12)}$	$37^{(6)}$	$6^{(1)}$	$56^{(9)}$	$59^{(10.5)}$
250	MSE	λ	0.1089(10)	0.0640(3)	0.0678(5)	0.0617(1)	0.0654(4)	0.0697(6)	0.2006(12)	0.0832(8)	0.0621(2)	0.0757(7)	0.1132(11)	0.1038(9)
		μ	0.0170(5)	0.0164(3)	0.0178(8)	0.0171(6)	0.0165(4)	0.0162(2)	0.0177(7)	0.0225(10)	0.0185(9)	0.0154(1)	0.0257(12)	0.0230(11)
	ABB	λ	0.2849(11)	0.2018(3)	0.2046(5)	0.1946(2)	0.2044(4)	0.2097(6)	0.3807(12)	0.2287(7)	0.1913(1)	0.2293(8)	0.2764(10)	0.2624(9)
		μ	0.1029(5)	0.1010(4)	0.1050(7)	0.1030(6)	0.1009(3)	0.1004(2)	0.1057(8)	0.1170(10)	0.1071(9)	0.0979(1)	0.1269(12)	0.1208(11)
	MRE	λ	0.0814(11)	0.0576(3)	0.0584(4.5)	0.0556(2)	0.0584(4.5)	0.0599(6)	0.1088(12)	0.0653(7)	0.0546(1)	0.0655(8)	0.0790(10)	0.0750(9)
		μ	0.0468(5.5)	0.0459(3.5)	0.0477(7)	0.0468(5.5)	0.0459(3.5)	0.0456(2)	0.0481(8)	0.0532(10)	0.0487(9)	0.0445(1)	0.0577(12)	0.0549(11)
	$\sum$	Ranks	$47.5^{(8)}$	$19.5^{(1)}$	$36.5^{(7)}$	$22.5^{(2)}$	$23^{(3)}$	$24^{(4)}$	$59^{(10)}$	$52^{(9)}$	$31^{(6)}$	$26^{(5)}$	$67^{(12)}$	$60^{(11)}$
500	MSE	λ	0.0860(11)	0.0421(4)	0.0387(3)	0.0361(1)	0.0463(6)	0.0444(5)	0.1700(12)	0.0524(7)	0.0371(2)	0.0691(9)	0.0799(10)	0.0546(8)
		μ	0.0093(8)	0.0087(2.5)	0.0091(7)	0.0088(4)	0.0089(5.5)	0.0085(1)	0.0089(5.5)	0.0116(10.5)	0.0095(9)	0.0087(2.5)	0.0140(12)	0.0116(10.5)
	ABB	λ	0.2657(11)	0.1681(4)	0.1568(3)	0.1509(2)	0.1794(6)	0.1715(5)	0.3728(12)	0.1859(7)	0.1505(1)	0.2336(9)	0.2378(10)	0.1893(8)
		μ	0.0772(9)	0.0742(2)	0.0753(7)	0.0743(3)	0.0750(5)	0.0735(1)	0.0751(6)	0.0851(10)	0.0768(8)	0.0744(4)	0.0939(12)	0.0857(11)
	MRE	λ	0.0759(11)	0.0480(4)	0.0448(3)	0.0431(2)	0.0513(6)	0.0490(5)	0.1065(12)	0.0531(7)	0.0430(1)	0.0667(9)	0.0679(10)	0.0541(8)
		μ	0.0351(9)	0.0337(2)	0.0342(6.5)	0.0338(3.5)	0.0341(5)	0.0334(1)	0.0342(6.5)	0.0387(10)	0.0349(8)	0.0338(3.5)	0.0427(12)	0.0389(11)
	$\sum$	Ranks	$59^{(11)}$	$18.5^{(3)}$	$29.5^{(5)}$	$15.5^{(1)}$	$33.5^{(6)}$	$18^{(2)}$	$54^{(9)}$	$51.5^{(8)}$	$29^{(4)}$	$37^{(7)}$	$66^{(12)}$	$56.5^{(10)}$

TABLE 11. Ranking scores and overall rank scores for all scenarios

Scenarios	n	MLE	ADE	CVME	LSE	WLSE	ADLTE	ADLTSOE	ADRTE	KME	MPSE	MSADE	MSALDE
Scenario 1	25	8	3	10	6	4	7	12	11	9	1	5	2
	50	7	3	9	5	2	8	11.5	11.5	10	1	6	4
	75	8	3	9	4	2	7	12	11	10	1	6	5
	100	7.5	3	7.5	4	2	5	12	11	10	1	6	9
	125	7	3	6	4	2	5	12	9	8	1	10.5	10.5
	250	7	1.5	6	3	1.5	5	11	9	8	4	12	10
	500	8	2	3	1	4	5.5	12	9	7	5.5	11	10
Scenario 2	25	8	5	11.5	6	4	7	10	11.5	9	1	3	2
	50	8	3	9	5	2	7	11.5	11.5	10	1	6	4
	75	8	3	9	5	2	7	12	11	10	1	6	4
	100	6.5	3	6.5	4	2	5	12	11	10	1	9	8
	125	7	3	6	4.5	2	4.5	12	9	8	1	11	10
	250	7	1	5.5	3	2	5.5	11	9	8	4	12	10
	500	8	1.5	3	1.5	6	4	11	9	7	5	12	10
Scenario 3	25	9	4	11.5	7	5	6	10	11.5	8	1	3	2
	50	10	5	11	7	4	6	8.5	12	8.5	1	3	2
	75	10.5	6	10.5	7	3	4	8	12	9	1	2	5
	100	11	3	10	6	2	4	9	12	8	1	5	7
	125	10	4	8	5	2	3	7	12	6	1	9	11
	250	10.5	1	7	3	3	3	8.5	8.5	6	5	12	10.5
	500	11	5	3	1	6	4	9.5	9.5	2	7	12	8
Scenario 4	25	8	4	11.5	7	5	6	10	11.5	9	1	3	2
	50	8	6	11	7	4	5	9	12	10	1	3	2
	75	9	4	10.5	7	2	4	8	12	10.5	1	4	6
	100	10	3	11	5	2	4	8.5	12	7	1	6	8.5
	125	9	3	6.5	5	2	4	8	12	6.5	1	10	11
	250	10.5	1	7	2	3	4.5	8	9	6	4.5	12	10.5
	500	11	5	4	1	6	2	7	9.5	3	8	12	9.5
Scenario 5	25	8	4	11.5	7	5	6	9	11.5	10	1	3	2
	50	8	5	11	7	2	6	10	12	9	1	4	3
	75	8	3.5	11	7	2	3.5	9	12	10	1	5	6
	100	10	3	11	5	2	4	9	12	8	1	6	7
	125	8	3.5	6	5	2	3.5	9	12	7	1	10	11
	250	10	1	6	2	3	4	8	9	7	5	12	11
	500	11	4	3	1	6	2	7	9.5	5	8	12	9.5
Scenario 6	25	8	4	11.5	7	6	5	10	11.5	9	1	3	2
	50	9	6	11	7	4	5	8	12	10	1	3	2
	75	10	6	11	7	2	4	8	12	9	1	3	5
	100	10	3	11	5	2	4	7	12	9	1	6	8
	125	9	4	7	5	2	3	8	12	6	1	10	11
	250	11	1	7	2	4	3	8	9	6	5	12	10
	500	11	5	4	1	6	3	10	9	2	7	12	8
Scenario 7	25	9.5	4	11.5	7	6	5	9.5	11.5	8	1	3	2
	50	10	5.5	11	7	4	5.5	8	12	9	1	3	2
	75	10	6	11	7	4	5	8	12	9	1	2	3
	100	10	3	11	5	2	4	8	12	7	1	6	9
	125	9	4	8	5	2	3	7	12	6	1	10	11
	250	10.5	2	7	1	4	3	9	8	6	5	12	10.5
	500	11	5	3	1	6	2	10	9	4	7.5	12	7.5
Scenario 8	25	9	4	10	6	3	7	12	11	8	1	5	2
	50	7.5	3	10	4.5	2	7.5	12	11	9	1	6	4.5
	75	9	3	10	4	2	7	12	11	8	1	5.5	5.5
	100	8	3	9	4	2	6	12	11	5	1	7	10
	125	7.5	3	7.5	4	2	6	12	9	5	1	10.5	10.5
	250	8	1	7	3	2	6	11	9	5	4	12	10
	500	8	2	5	1	3	7	12	9	5	5	11	10
Scenario 9	25	9	4	11.5	7	5	6	10	11.5	8	1	3	2
	50	8	3	10.5	7	2	6	10.5	12	9	1	5	4
	75	8	3	10	7	2	4	11	12	9	1	6	5
	100	8	3	9	5	2	4	11	12	7	1	6	10
	125	8	3	7	5	2	4	10.5	12	6	1	9	10.5
	250	8	1	7	2	3	4	10	9	6	5	12	11
	500	11	3	5	1	6	2	9	8	4	7	12	10
∑ Ranks		560.5	213.5	526.5	287.5	200.5	302	615.5	678.5	473.5	146.5	470.5	439
Overall Rank		10	3	9	4	2	5	11	12	8	1	7	6

GL distribution

$$f\left(x;p_1,p_2\right)=\frac{p_1p_2^2\left(1+x\right)\left[1-\frac{(1+p_2+p_2x)e^{(-p_2x)}}{1+p_2}\right]^{(p_1-1)}e^{(-p_2x)}}{1+p_2}$$

FW distribution

$$f\left(x;p_1,p_2\right)=\left(p_1+\frac{p_2}{x^2}\right)e^{\left(p_1x-\frac{p_2}{x}\right)}e^{\left[-e^{\left(p_1x-\frac{p_2}{x}\right)}\right]}$$

The MLE method is applied to estimate the λ and μ parameters of the distributions introduced above. To evaluate whether these distributions provide a good fit to the dataset, various metrics are considered and presented, including standard error (se), ℓ, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Consistent Akaike Information Criterion (CAIC), Hannan-Quinn Information Criterion (HQIC), Kolmogorov-Smirnov (KS) test, Anderson-Darling (AD) test, Cramer-von-Mises (CvM) test, and the p-values obtained for these statistics (KS_{p-val}, AD_{p-val}, and CvM_{p-val}).

The dataset consists of the lengths (in miles) of 141 major rivers in North America, as compiled by the U.S. Geological Survey and reported in *The World Almanac and Book of Facts 1975*. It was introduced by McNeil [1] and is available in R as the built-in dataset `rivers`. For the analysis, the lengths are expressed in thousands of miles. The dataset is as follows: 735, 320, 325, 392, 524, 450, 1459, 135, 465, 600, 330, 336, 280, 315, 870, 906, 202, 329, 290, 1000, 600, 505, 1450, 840, 1243, 890, 350, 407, 286, 280, 525, 720, 390, 250, 327, 230, 265, 850, 210, 630, 260, 230, 360, 730, 600, 306, 390, 420, 291, 710, 340, 217, 281, 352, 259, 250, 470, 680, 570, 350, 300, 560, 900, 625, 332, 2348, 1171, 3710, 2315, 2533, 780, 280, 410, 460, 260, 255, 431, 350, 760, 618, 338, 981, 1306, 500, 696, 605, 250, 411, 1054, 735, 233, 435, 490, 310, 460, 383, 375, 1270, 545, 445, 1885, 380, 300, 380, 377, 425, 276, 210, 800, 420, 350, 360, 538, 1100, 1205, 314, 237, 610, 360, 540, 1038, 424, 310, 300, 444, 301, 268, 620, 215, 652, 900, 525, 246, 360, 529, 500, 720, 270, 430, 671, 1770.

Some descriptive statistics for the dataset are presented in Table 12. Additionally, parameter estimations, goodness-of-fit test results, and other goodness-of-fit measures for this dataset are provided in Table 13. According to Table 13, the IPMMSGD has the largest log-likelihood and the smallest AIC, BIC, CAIC, and HQIC values among the six distributions; its AIC is 28.2476, compared with 73.8150 for the second-best model (GL). It also gives the smallest KS, AD, and CvM statistics and the largest corresponding p-values (0.9521, 0.9924, and 0.9916, respectively), whereas all five competitor distributions are rejected at the 5% significance level by all three tests. These results indicate that the IPMMSGD provides the best fit to this dataset among the distributions considered. Furthermore, the same fit is visually confirmed. The graphical representation of the fit between the IPMMSGD and the dataset is presented in Figure 4.

The plots are structured as follows: the top-left plot represents the fitted PDF, the top-right plot represents the CDF, the bottom-left plot represents the survival function $S(x)$, and the bottom-right

plot represents the observed versus expected probability-probability (P-P) plot. It is observed that the empirical data exhibit an excellent fit with the theoretical IPMMSGD.

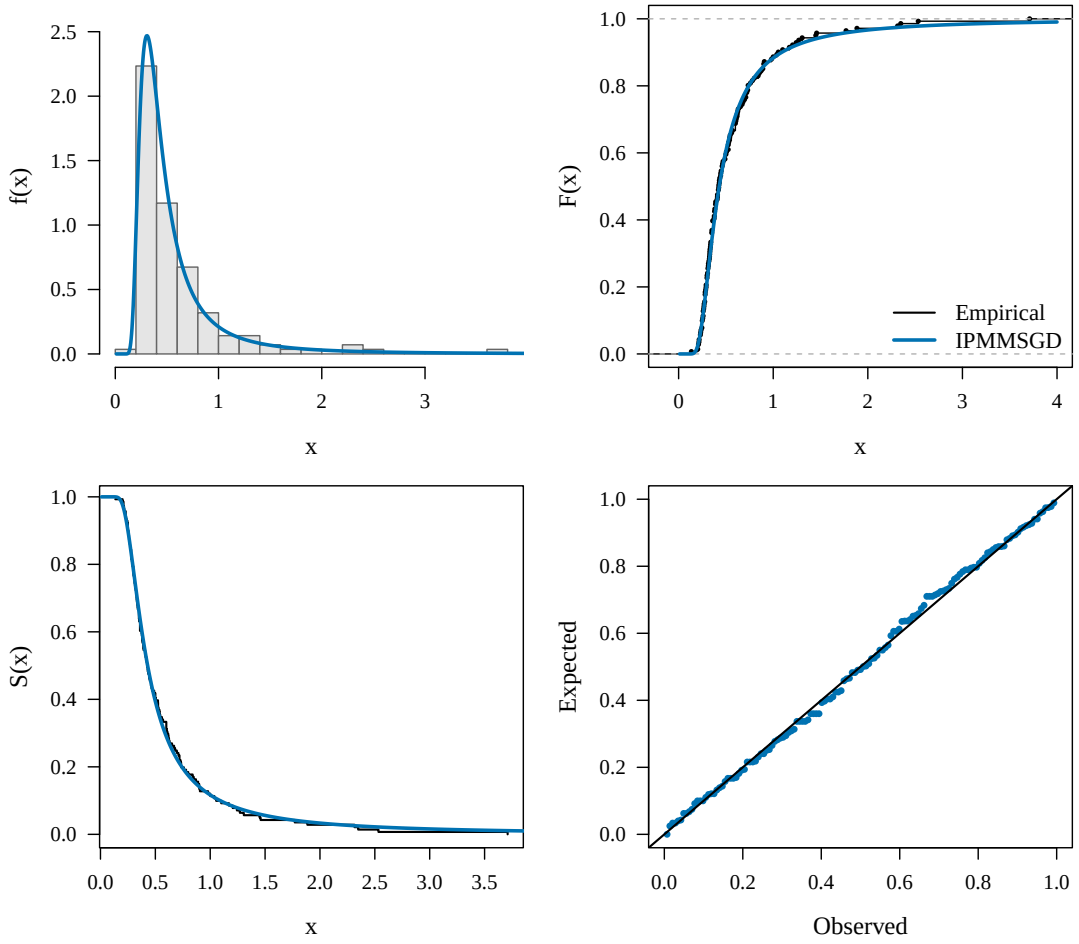


FIGURE 4. The fitted PDF, CDF, SF, and P-P plots for IPMMSGD based on the real dataset.

TABLE 12. Some descriptive statistics for the dataset (skewness and kurtosis are bias-adjusted sample estimates; kurtosis is reported as excess kurtosis, i.e., comparable to $K-3$)

Mean	**Variance**	**Skewness**	**Kurtosis**
0.5912	0.2439	3.2182	13.8258

TABLE 13. Data analysis results for the real dataset

	IPMMSG	**WB**	**GP**	**LY**	**GL**	**FW**
ℓ	-12.1238	-50.7890	-65.2817	-64.2037	-34.9075	-46.8866
AIC	28.2476	105.5780	134.5634	130.4073	73.8150	97.7732
BIC	34.1451	111.4756	140.4609	133.3561	79.7125	103.6707
CAIC	28.3345	105.6650	134.6503	130.4361	73.9020	97.8602
HQIC	30.6441	107.9746	136.9599	131.6056	76.2116	100.1698
KS	0.0435	0.1610	0.2598	0.2672	0.1276	0.2290
KS p**-val**	0.9521	0.0013	0.0000	0.0000	0.0203	0.0000
AD	0.1921	6.4109	11.1670	11.6303	4.0202	13.3381
AD p**-val**	0.9924	0.0006	0.0000	0.0000	0.0086	0.0000
CvM	0.0241	1.0335	1.9299	2.0395	0.6885	2.7714
CvM p**-val**	0.9916	0.0020	0.0000	0.0000	0.0135	0.0000
p_1	1.8125	1.4382	1.4867	2.2173	3.4420	0.7100
p_2	0.2926	0.6602	0.2376		4.0137	0.5034
se for p_1	0.1237	0.0813	0.1669	0.1512	0.5489	0.0540
se for p_2	0.0490	0.0412	0.1256		0.3286	0.0410

6. CONCLUSION

We presented a novel two-parameter distribution called the inverse power modified mixture symmetric gamma distribution (IPMMSGD), which is appropriate for modelling skewed data. Its mathematical characteristics are derived. Twelve traditional techniques are used to estimate the model parameters. The performance of the various estimators is investigated and compared through a simulation exercise. By examining the ordering performance of the different estimation techniques based on partial and total ranks, we also offer a recommendation for selecting the optimal approach for parameter estimation, which is highly relevant to applied statisticians and engineers. The maximum product of spacings approach performs better than the other estimation techniques, according to the simulation. Therefore, we can verify that this estimation method is the best for the IPMMSGD. Real-world data on the lengths of 141 major North American rivers are used to demonstrate the model's applicability, and the IPMMSGD provided the best fit among the six distributions considered.

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Data Availability Statement. The data that supports the findings of this study are available within the article.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

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