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# FOUR NEW CONCEPTS OF IUP-IDEALS: TRANSMITTED, REFLECTED, RESONANT, AND DOMINANT IUP-IDEALS

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Abstract. In this paper, we introduce four new types of IUP-ideals—transmitted, reflected, resonant, and dominant—that enrich the structural framework of IUP-algebras. We provide formal definitions for each class and investigate their fundamental properties and mutual relationships. Furthermore, we examine their behavior under IUP-homomorphisms and establish necessary and sufficient conditions for a subset to belong to each type. Our main results include a complete characterization of transmitted and reflected IUP-ideals, proofs of equivalence conditions between resonant and dominant ideals under specific algebraic constraints, and the identification of closure properties preserved under homomorphic images. These findings deepen the ideal theory of IUP-algebras and offer new directions for algebraic reasoning in logic-based systems.

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## 1. Introduction

Algebraic structures are among the most fundamental concepts in mathematics and are closely related to everyday life. They can be regarded as the universal language of mathematics. Common examples of algebraic structures include groups, rings, fields, lattices, vector spaces, and many others. Moreover, there are more general algebraic structures that incorporate new areas of knowledge, such as topological groups, Lie groups, Hilbert spaces, and others. Algebraic structures constitute a significant

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area of research in mathematics. One such area is logical algebra, which has long attracted researchers' interest. For instance, among the abstract algebras that have recently attracted researchers' attention is the IUP-algebra, which will be discussed in the following paragraph.

Independent UP-algebra, commonly referred to as IUP-algebra [3], was introduced by Iampan et al. in 2022 as a novel structure in logical algebra. His foundational work defined key subsets—namely, IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals—and examined their interrelationships and structural properties. He also introduced the notion of homomorphism in IUP-algebras, thereby laying the groundwork for subsequent theoretical development. Since then, IUP-algebras have drawn increasing interest. In 2023, Chanmanee et al. [2] extended the theory to direct and weak direct products of IUP-algebras, including key results on (anti-)IUP-homomorphisms. A complementary study by the same authors [1] examined the structural behavior of external direct products in dual IUP-algebras, thereby enriching the algebraic framework for modular construction. In 2024, Kuntama et al. [5] integrated fuzzy set theory into IUP-algebras, introducing fuzzy IUP-subalgebras, ideals, filters, and strong ideals. This direction was further pursued by Suayngam et al. [8, 9, 11], who applied Fermatean fuzzy sets, intuitionistic fuzzy sets, and neutrosophic sets to IUP-algebras, defining generalized subsets and exploring their relationships. In 2025, Suayngam et al. [10] introduced Pythagorean fuzzy IUP-algebras and studied their structural properties, including level subsets. / A significant advancement was made with the development of Pythagorean neutrosophic IUP-algebras [7], which fused these uncertainty frameworks into a unified system of subalgebras, ideals, filters, and strong ideals. In the same year, Inthachot et al. [4] introduced bipolar fuzzy sets into the IUP-algebraic setting, enabling dual-valued modeling of both affirming and opposing tendencies—relevant to logic programming and bipolar decision models. Lastly, Suayngam et al. [6] synthesized these developments into intuitionistic neutrosophic IUP-algebras, offering a comprehensive algebraic system for representing complex uncertainty. Collectively, these contributions have significantly enriched the IUP-algebraic framework, bridging classical algebraic systems with fuzzy and neutrosophic logic, and offering a versatile foundation for future theoretical and applied research.

Building on previous studies of IUP-algebras and their extensions to other concepts, the present research introduces new types of special subsets of IUP-algebras. As a result, this study proposes four novel subsets: transmitted IUP-ideals, reflected IUP-ideals, resonant IUP-ideals, and dominant IUP-ideals. Furthermore, we investigate the relationships among all special subsets within IUP-algebras and apply the concept of homomorphisms to these four subsets to explore additional properties.

This paper is organized into three main parts. The first part, Preliminaries, reviews the fundamental concepts of IUP-algebras required to study the newly introduced special subsets. This includes the definition of IUP-algebras, their distinctive properties, previously established subsets such as IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals, and the concept of homomorphisms in

IUP-algebras. The second part, Main Results, introduces four new special subsets: transmitted IUP-ideal, reflected IUP-ideal, resonant IUP-ideal, and dominant IUP-ideal. We present their definitions, investigate specific properties, provide illustrative examples, and establish relationships among these subsets. Additionally, we apply the concept of homomorphisms to these newly defined subsets to derive further properties. The final part summarizes the research findings and suggests further studies and expansions of this research.

## 2. Preliminaries

This section outlines essential definitions, notations, and fundamental results that form the basis for the main developments in subsequent sections. Unless stated otherwise, all algebras and mappings considered herein are assumed to be defined within the framework of IUP-algebras. For completeness and clarity, we briefly recall key concepts and results related to IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals, as well as relevant properties that will be frequently employed throughout this paper.

**Definition 2.1.** [3] *An algebra $X = (X, \star, 0)$ of type $(2, 0)$ is called an IUP-algebra, where $X$ is a nonempty set, $\star$ is a binary operation on $X$, and $0$ is a fixed element of $X$ if it satisfies the following axioms:*

$$(\forall x \in X)(0 \star x = x) \tag{IUP-1}$$

$$(\forall x \in X)(x \star x = 0) \tag{IUP-2}$$

$$(\forall x, y, z \in X)((x \star y) \star (x \star z) = y \star z) \tag{IUP-3}$$

For simplicity, we will refer to $X$ as the IUP-algebra $X = (X, \star, 0)$ unless stated otherwise.

**Example 2.1.** *Let $X = \{0, 1, 2, 3, 4, 5\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 |
| 1 | 3 | 0 | 5 | 1 | 2 | 4 |
| 2 | 5 | 2 | 0 | 4 | 1 | 3 |
| 3 | 1 | 3 | 4 | 0 | 5 | 2 |
| 4 | 4 | 5 | 3 | 2 | 0 | 1 |
| 5 | 2 | 4 | 1 | 5 | 3 | 0 |

*then $X = (X, \star, 0)$ is an IUP-algebra.*

**Example 2.2.** [3] *Let $(G, \bullet, e)$ be a group with the identity element $e$ in which each element is its own inverse. Under this condition, $(G, \bullet, e)$ inherently satisfies the axioms of an IUP-algebra.*

**Example 2.3.** [3] *Let $X$ be a set, and let $\mathcal{P}(X)$ denote its power set. As shown in Example 2.2, $(\mathcal{P}(X), \triangle, \emptyset)$ forms an IUP-algebra, where $\triangle$ represents the symmetric difference between sets.*

**Example 2.4.** [3] *Let $(G, \cdot, e)$ be a group with the identity element $e$. Define a binary operation $\bullet$ on $G$ by:*

$$(\forall x, y \in G)(x \bullet y = y \cdot x^{-1}) \tag{2.1}$$

*Then $(G, \bullet, e)$ is an IUP-algebra.*

**Proposition 2.1.** [3] *In an IUP-algebra $X = (X, \star, 0)$, the following assertions are valid:*

$$(\forall x, y \in X)((x \star 0) \star (x \star y) = y) \tag{2.2}$$

$$(\forall x \in X)((x \star 0) \star (x \star 0) = 0) \tag{2.3}$$

$$(\forall x, y \in X)((x \star y) \star 0 = y \star x) \tag{2.4}$$

$$(\forall x \in X)((x \star 0) \star 0 = x) \tag{2.5}$$

$$(\forall x, y \in X)(x \star ((x \star 0) \star y) = y) \tag{2.6}$$

$$(\forall x, y \in X)(((x \star 0) \star y) \star x = y \star 0) \tag{2.7}$$

$$(\forall x, y, z \in X)(x \star y = x \star z \Leftrightarrow y = z) \tag{2.8}$$

$$(\forall x, y \in X)(x \star y = 0 \Leftrightarrow x = y) \tag{2.9}$$

$$(\forall x \in X)(x \star 0 = 0 \Leftrightarrow x = 0) \tag{2.10}$$

$$(\forall x, y, z \in X)(y \star x = z \star x \Leftrightarrow y = z) \tag{2.11}$$

$$(\forall x, y \in X)(x \star y = y \Rightarrow x = 0) \tag{2.12}$$

$$(\forall x, y, z \in X)((x \star y) \star 0 = (z \star y) \star (z \star x)) \tag{2.13}$$

$$(\forall x, y, z \in X)(x \star y = 0 \Leftrightarrow (z \star x) \star (z \star y) = 0) \tag{2.14}$$

$$(\forall x, y, z \in X)(x \star y = 0 \Leftrightarrow (x \star z) \star (y \star z) = 0) \tag{2.15}$$

$$\text{the right and the left cancellation laws hold} \tag{2.16}$$

Within IUP-algebras, four fundamental subsets stand out: IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals. These subsets form a critical framework that deepens our understanding and facilitates the application of IUP-algebras across different mathematical contexts.

**Definition 2.2.** [3] *A nonempty subset $S$ of $X$ is called*

$(i)$ *an IUP-subalgebra of $X$ if it satisfies the following condition:*

$$(\forall x, y \in S)(x \star y \in S) \tag{2.17}$$

$(ii)$ *an IUP-filter of $X$ if it satisfies the following conditions:*

$$\text{the constant } 0 \text{ of } X \text{ is in } S \tag{2.18}$$

$$(\forall x, y \in X)(x \star y \in S, x \in S \Rightarrow y \in S) \tag{2.19}$$

$(iii)$ *an IUP-ideal of $X$ if it satisfies the condition (2.18) and the following condition:*

$$(\forall x, y, z \in X)(x \star (y \star z) \in S, y \in S \Rightarrow x \star z \in S) \tag{2.20}$$

$(iv)$ *a strong IUP-ideal of $X$ if it satisfies the following condition:*

$$(\forall x, y \in X)(y \in S \Rightarrow x \star y \in S) \tag{2.21}$$

**Theorem 2.1.** [3] *A nonempty subset $S$ of an IUP-algebra $X = (X, \star, 0)$ is a strong IUP-ideal of $X$ if and only if $S = X$.*

**Theorem 2.2.** [3] *Let $X = (X, \star_X, 0_X)$ and $Y = (Y, \star_Y, 0_Y)$ be IUP-algebras. A function $f : X \to Y$ is called a homomorphism if it satisfies the following condition:*

$$(\forall x, y \in X)(f(x \star_X y)) = f(x) \star_Y f(y) \tag{2.22}$$

**Proposition 2.2.** [3] *Let $X = (X, \star_X, 0_X)$ and $Y = (Y, \star_Y, 0_Y)$ be IUP-algebras, and $f : X \to Y$ be a homomorphism. Then the following assertions are valid:*

(1) $f(0_X) = 0_Y$,
(2) $(\forall x, y \in X)(f(x \star_X y) = 0_Y) \Leftrightarrow f(x) = f(y)$,
(3) $(\forall x \in X)(f(x) = 0_Y \Leftrightarrow f(x \star_X 0_X) = 0_Y$,
(4) $(\forall x, y \in X)(x \star_X y = 0_X \Leftrightarrow f(x) = f(y))$, *and*
(5) $f^{-1}(\{0_Y\}) = \{0_X\}$ *if and only if $f$ is injective.*

According to [3], IUP-filters represent a unifying concept encompassing both IUP-ideals and IUP-subalgebras. These two subsets, IUP-ideals and IUP-subalgebras, are generalizations of strong IUP-ideals. Particularly, in an IUP-algebra $X$, strong IUP-ideals are equivalent to the entire algebra $X$ itself. This hierarchical relationship among these subsets is visually represented in Figure 1, illustrating the structure of special subsets within an IUP-algebra $X$.

## 3. Transmitted, Reflected, Resonant, and Dominant IUP-Ideals

The theory of IUP-ideals plays a central role in understanding the structural behavior of IUP-algebras. While prior studies have established foundational definitions and properties of IUP-ideals and strong IUP-ideals, further exploration of ideal-like subsets remains an open and promising direction.

In this section, we propose four novel classes of IUP-ideals—namely, transmitted, reflected, resonant, and dominant IUP-ideals—each capturing distinct structural phenomena within the IUP-algebraic

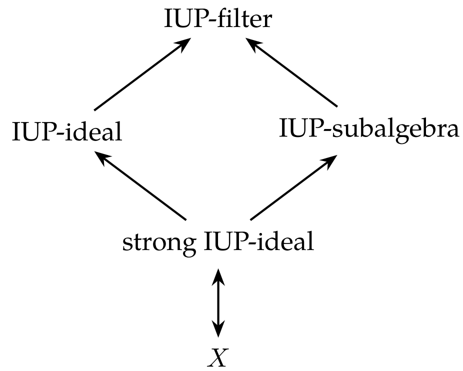


FIGURE 1. Structure of the four concepts of subsets in IUP-algebras

setting. These new concepts are defined by refined conditions on interactions among elements and on subset stability, offering a deeper lens for analyzing the ideal structure of IUP-algebras.

We begin by providing formal definitions of each class, followed by key properties, illustrative examples, and results that clarify their roles and interrelations. Throughout, we highlight how these new subsets extend and enrich the existing ideal theory within the IUP-algebra framework.

**Definition 3.1.** *A nonempty subset $S$ of $X$ is called a transmitted IUP-ideal of $X$ if it satisfies the condition* (2.18) *and the following condition:*

$$(\forall x, y, z \in X)((x \star y) \star z \in S, y \in S \Rightarrow x \star z \in S) \tag{3.1}$$

**Example 3.1.** *Let $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 0 | 3 | 2 | 5 | 4 | 7 | 6 |
| 2 | 2 | 6 | 0 | 4 | 3 | 7 | 1 | 5 |
| 3 | 6 | 2 | 4 | 0 | 7 | 3 | 5 | 1 |
| 4 | 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 5 | 4 | 7 | 6 | 1 | 0 | 3 | 2 |
| 6 | 3 | 7 | 1 | 5 | 2 | 6 | 0 | 4 |
| 7 | 7 | 3 | 5 | 1 | 6 | 2 | 4 | 0 |

*Then $X = (X, \star, 0)$ is an IUP-algebra. Let $S = \{0, 3, 5, 6\}$. Hence, $S$ is a transmitted IUP-ideal of $X$.*

**Definition 3.2.** *A nonempty subset $S$ of $X$ is called a reflected IUP-ideal of $X$ if it satisfies the condition* (2.18) *and the following condition:*

$$(\forall x, y, z \in X)((x \star y) \star z \in S, z \in S \Rightarrow x \star (y \star (y \star x)) \in S) \tag{3.2}$$

**Example 3.2.** *Let $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 0 | 3 | 2 | 5 | 4 | 7 | 6 |
| 2 | 2 | 3 | 0 | 1 | 6 | 7 | 4 | 5 |
| 3 | 3 | 2 | 1 | 0 | 7 | 6 | 5 | 4 |
| 4 | 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 5 | 4 | 7 | 6 | 1 | 0 | 3 | 2 |
| 6 | 6 | 7 | 4 | 5 | 2 | 3 | 0 | 1 |
| 7 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

*then $X = (X, \star, 0)$ is an IUP-algebra. Let $S = \{0, 3, 6, 7\}$. Hence, $S$ is a reflected IUP-ideal of $X$.*

**Definition 3.3.** *A nonempty subset $S$ of $X$ is called a resonant IUP-ideal of $X$ if it satisfies the condition* (2.18) *and the following condition:*

$$(\forall x, y \in X)(x \star y \in S \Rightarrow x \star ((y \star x) \star x) \in S) \tag{3.3}$$

**Example 3.3.** *Let $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 0 | 3 | 2 | 5 | 4 | 7 | 6 |
| 2 | 2 | 3 | 0 | 1 | 6 | 7 | 4 | 5 |
| 3 | 3 | 2 | 1 | 0 | 7 | 6 | 5 | 4 |
| 4 | 6 | 7 | 4 | 5 | 0 | 1 | 2 | 3 |
| 5 | 7 | 6 | 5 | 4 | 1 | 0 | 3 | 2 |
| 6 | 4 | 5 | 6 | 7 | 2 | 3 | 0 | 1 |
| 7 | 5 | 4 | 7 | 6 | 3 | 2 | 1 | 0 |

*then $X = (X, \star, 0)$ is an IUP-algebra. Let $S = \{0, 4, 6, 7\}$. Hence, $S$ is a resonant IUP-ideal of $X$.*

**Definition 3.4.** *A nonempty subset $S$ of $X$ is called a dominant IUP-ideal of $X$ if it satisfies the condition* (2.18) *and the following condition:*

$$(\forall x, y \in X)(x \star y \in S \Rightarrow (x \star (x \star y)) \star x \in S) \tag{3.4}$$

**Example 3.4.** *Let $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 0 | 3 | 2 | 5 | 4 | 7 | 6 |
| 2 | 7 | 3 | 0 | 4 | 6 | 2 | 1 | 5 |
| 3 | 3 | 7 | 4 | 0 | 2 | 6 | 5 | 1 |
| 4 | 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 5 | 4 | 7 | 6 | 1 | 0 | 3 | 2 |
| 6 | 6 | 2 | 1 | 5 | 7 | 3 | 0 | 4 |
| 7 | 2 | 6 | 5 | 1 | 3 | 7 | 4 | 0 |

*then* $X = (X, \star, 0)$ *is an IUP-algebra. Let* $S = \{0, 1, 2, 4, 5, 7\}$*. Hence,* $S$ *is a dominant IUP-ideal of* $X$*.*

**Theorem 3.1.** *Every strong IUP-ideal of* $X$ *is a transmitted IUP-ideal of* $X$*.*

*Proof.* It follows straightforwardly from Theorem 2.1. □

**Example 3.5.** *Let* $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ *be a set with a binary operation* $\star$ *defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 0 | 3 | 1 | 5 | 7 | 4 | 6 |
| 2 | 1 | 3 | 0 | 2 | 6 | 4 | 7 | 5 |
| 3 | 3 | 2 | 1 | 0 | 7 | 6 | 5 | 4 |
| 4 | 4 | 5 | 6 | 7 | 0 | 1 | 2 | 3 |
| 5 | 5 | 7 | 4 | 6 | 2 | 0 | 3 | 1 |
| 6 | 6 | 4 | 7 | 5 | 1 | 3 | 0 | 2 |
| 7 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

*Then* $X$ *is an IUP-algebra. Let* $S = \{0, 1, 2, 3\}$*. Then* $S$ *is a transmitted IUP-ideal of* $X$*. Since* $0 \in S$ *but* $4 \star 0 = 4 \notin S$*. Hence,* $S$ *is not a strong IUP-ideal of* $X$*.*

**Lemma 3.1.** *If* $S$ *is a transmitted IUP-ideal of* $X$*, then it satisfies the following condition:*

$$(\forall x, y \in X)(y \in S \Rightarrow x \star (y \star x) \in S) \tag{3.5}$$

*Proof.* Assume that $S$ is a transmitted IUP-ideal of $X$. Let $x, y \in X$ be such that $y \in S$. By the assumption and (2.4), $((x \star y) \star 0) \star (y \star x) = (y \star x) \star (y \star x) = 0 \in S$. By the assumption, $(x \star y) \star (y \star x) \in S$. Thus, by the assumption, $x \star (y \star x) \in S$. Hence, $S$ satisfies the condition (3.5). □

**Theorem 3.2.** *Every transmitted IUP-ideal of* $X$ *is an IUP-subalgebra of* $X$*.*

*Proof.* Assume that $S$ is a transmitted IUP-ideal of $X$. Let $x, y \in S$. By (IUP-1) and (IUP-2), $(x \star x) \star y = 0 \star y = y \in S$. Thus, by the assumption, $x \star y \in S$. Hence, $S$ is an IUP-subalgebra of $X$. □

**Example 3.6.** *Let $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 0 | 3 | 2 | 5 | 4 | 7 | 6 |
| 2 | 4 | 5 | 0 | 1 | 6 | 7 | 3 | 2 |
| 3 | 5 | 4 | 1 | 0 | 7 | 6 | 2 | 3 |
| 4 | 2 | 3 | 7 | 6 | 0 | 1 | 4 | 5 |
| 5 | 3 | 2 | 6 | 7 | 1 | 0 | 5 | 4 |
| 6 | 7 | 6 | 5 | 4 | 2 | 3 | 0 | 1 |
| 7 | 6 | 7 | 4 | 5 | 3 | 2 | 1 | 0 |

*Then $X$ is an IUP-algebra. Let $S = \{0, 1\}$. Then $S$ is an IUP-subalgebra of $X$. Since $(2 \star 1) \star 4 = 5 \star 4 = 1 \in S$ and $1 \in S$ but $2 \star 4 = 6 \notin S$. Hence, $S$ is not a transmitted IUP-ideal of $X$.*

**Theorem 3.3.** *Every transmitted IUP-ideal of $X$ is an IUP-ideal of $X$.*

*Proof.* Assume that $S$ is a transmitted IUP-ideal of $X$. Then $S$ satisfies the condition (2.18). Let $x, y, z \in X$ be such that $x \star (y \star z) \in S$ and $y \in S$. By (2.5), $((x \star (y \star z)) \star 0) \star 0 \in S$. By the assumption, $(x \star (y \star z)) \star 0 \in S$. By (2.4), $(y \star z) \star x \in S$. Then

$$((z \star x) \star (z \star (y \star z))) \star 0 = (z \star (y \star z)) \star (z \star x) \quad \text{(by (2.4))}$$

$$= (y \star z) \star x. \quad \text{(by (IUP-3))}$$

By Lemma 3.1, $z \star (y \star z) \in S$. By the assumption, $x \star z = (z \star x) \star 0 \in S$. Hence, $S$ is an IUP-ideal of $X$. □

**Example 3.7.** *Let $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 0 | 3 | 1 | 5 | 7 | 4 | 6 |
| 2 | 1 | 3 | 0 | 2 | 6 | 4 | 7 | 5 |
| 3 | 3 | 2 | 1 | 0 | 7 | 6 | 5 | 4 |
| 4 | 4 | 6 | 5 | 7 | 0 | 2 | 1 | 3 |
| 5 | 6 | 7 | 4 | 5 | 1 | 0 | 3 | 2 |
| 6 | 5 | 4 | 7 | 6 | 2 | 3 | 0 | 1 |
| 7 | 7 | 5 | 6 | 4 | 3 | 1 | 2 | 0 |

*Then $X$ is an IUP-algebra. Let $S = \{0, 7\}$. Then $S$ is an IUP-ideal of $X$. Since $(2 \star 0) \star 5 = 1 \star 5 = 7 \in S$ and $0 \in S$ but $2 \star 5 = 4 \notin S$. Hence, $S$ is not a transmitted IUP-ideal of $X$.*

**Theorem 3.4.** *Every transmitted IUP-ideal of $X$ is a dominant IUP-ideal of $X$.*

*Proof.* Assume that $S$ is a transmitted IUP-ideal of $X$. Then $S$ satisfies the condition (2.18). Let $x, y \in X$ be such that $x \star y \in S$. By Lemma 3.1 and (2.5), $((x \star ((x \star y) \star x)) \star 0) \star 0 = x \star ((x \star y) \star x) \in S$. By the assumption, $(x \star ((x \star y) \star x)) \star 0 \in S$. Then

$$
\begin{aligned}
((x \star (x \star y)) \star 0) \star x &= ((x \star y) \star x) \star x && \text{(by (2.4))} \\
&= (x \star ((x \star y) \star x)) \star 0. && \text{(by (2.4))}
\end{aligned}
$$

By the assumption, $(x \star (x \star y)) \star x \in S$. Hence, $S$ is a dominant IUP-ideal of $X$. □

**Theorem 3.5.** *Every IUP-ideal of $X$ is a resonant IUP-ideal of $X$.*

*Proof.* Assume that $S$ is an IUP-ideal of $X$. Then $S$ satisfies the condition (2.18). Let $x, y \in X$ be such that $x \star y \in S$. Then

$$
\begin{aligned}
x \star ((x \star y) \star ((y \star x) \star x)) &= x \star (((y \star x) \star 0) \star ((y \star x) \star x)) && \text{(by (2.4))} \\
&= x \star (0 \star x) && \text{(by (IUP-3))} \\
&= x \star x && \text{(by (IUP-1))} \\
&= 0. && \text{(by (IUP-2))}
\end{aligned}
$$

By the assumption, $x \star ((y \star x) \star x) \in S$. Hence, $S$ is a resonant IUP-ideal of $X$. □

**Example 3.8.** *Let $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 2 | 0 | 3 | 1 | 5 | 7 | 4 | 6 |
| 2 | 1 | 3 | 0 | 2 | 6 | 4 | 7 | 5 |
| 3 | 3 | 2 | 1 | 0 | 7 | 6 | 5 | 4 |
| 4 | 5 | 4 | 7 | 6 | 0 | 2 | 1 | 3 |
| 5 | 4 | 6 | 5 | 7 | 1 | 0 | 3 | 2 |
| 6 | 7 | 5 | 6 | 4 | 2 | 3 | 0 | 1 |
| 7 | 6 | 7 | 4 | 5 | 3 | 1 | 2 | 0 |

*Then $X$ is an IUP-algebra. Let $S = \{0, 1, 4, 5, 6\}$. Then $S$ is a resonant IUP-ideal of $X$. Since $2 \star (6 \star 1) = 2 \star 5 = 4 \in S$ and $6 \in S$ but $2 \star 1 = 3 \notin S$. Hence, $S$ is not an IUP-ideal of $X$.*

**Lemma 3.2.** *If $S$ is a dominant IUP-ideal of $X$, then it satisfies the following condition:*

$$(\forall x, y \in X)(x \star y \in S \Rightarrow (y \star 0) \star x \in S) \tag{3.6}$$

*Proof.* Assume that $S$ is a dominant IUP-ideal of $X$. Then $S$ satisfies the condition (2.18). Let $x, y \in X$ be such that $x \star y \in S$. By (IUP-3), $((y \star x) \star x) \star ((y \star x) \star y) \in S$. By the assumption, $(((y \star x) \star x) \star (((y \star x) \star x) \star ((y \star x) \star y))) \star ((y \star x) \star x) \in S$. Then

$$\begin{aligned}
(y \star 0) \star x &= ((y \star x) \star (y \star 0)) \star ((y \star x) \star x) && \text{(by (IUP-3))}\\
&= (x \star 0) \star ((y \star x) \star x) && \text{(by (IUP-3))}\\
&= (((y \star x) \star x) \star ((y \star x) \star 0)) \star ((y \star x) \star x) && \text{(by (IUP-3))}\\
&= (((y \star x) \star x) \star (x \star y)) \star ((y \star x) \star x) && \text{(by (2.4))}\\
&= (((y \star x) \star x) \star (((y \star x) \star x) \star ((y \star x) \star y))) \star ((y \star x) \star x). && \text{(by (IUP-3))}
\end{aligned}$$

Hence, $(y \star 0) \star x \in S$.

□

**Lemma 3.3.** *If $S$ is a reflected IUP-ideal of $X$, then it satisfies the following condition:*

$$(\forall x \in X)(x \star (x \star 0) \in S) \tag{3.7}$$

*Proof.* Assume that $S$ is a reflected IUP-ideal of $X$. Then $S$ satisfies the condition (2.18). Let $x \in X$. By (IUP-2), $(x \star x) \star 0 = 0 \star 0 = 0 \in S$. By (IUP-2) and the assumption, $x \star (x \star 0) = x \star (x \star (x \star x)) \in S$. Hence, $x \star (x \star 0) \in S$.

□

**Lemma 3.4.** *A nonempty subset $S$ is a reflected IUP-ideal of $X$ if and only if it satisfies the following condition:*

$$(\forall x, y \in X)(x \star y \in S \Rightarrow x \star (y \star (y \star x)) \in S). \tag{3.8}$$

*Proof.* Assume that a nonempty subset $S$ is a reflected IUP-ideal of $X$. Then $S$ satisfies the condition (2.18). Let $x, y \in X$ be such that $x \star y \in S$. By (IUP-2), $(x \star y) \star (x \star y) = 0 \in S$. By the assumption, $x \star (y \star (y \star x)) \in S$. Hence, if it satisfies the following condition (3.8).

Conversely, a nonempty subset $S$ satisfies the following condition (3.8). Let $x \in S$. By (2.5), $(x \star 0) \star 0 = x \in S$. By the assumption, $(x \star 0) \star (0 \star (0 \star (x \star 0))) \in S$. Then

$$\begin{aligned}
0 &= (x \star 0) \star (x \star 0) && \text{(by (IUP-2))}\\
&= (x \star 0) \star (0 \star (x \star 0)) && \text{(by (IUP-1))}\\
&= (x \star 0) \star (0 \star (0 \star (x \star 0))). && \text{(by (IUP-1))}
\end{aligned}$$

Thus, $S$ satisfies the condition (2.18). Let $x, y, z \in X$ be such that $(x \star y) \star z \in S$ and $z \in S$. Then

$$
\begin{aligned}
x \star (y \star (y \star x)) &= (0 \star x) \star (((y \star 0) \star 0) \star (((y \star 0) \star 0) \star x)) && \text{(by (IUP-1) and (2.5))} \\
&= (((y \star 0) \star 0) \star ((y \star 0) \star x)) \star (((y \star 0) \star 0) \star (((y \star 0) \star 0) \star x)) && \text{(by (IUP-3))} \\
&= ((y \star 0) \star x) \star (((y \star 0) \star 0) \star x) && \text{(by (IUP-3))} \\
&= (0 \star ((y \star 0) \star x)) \star (((y \star 0) \star 0) \star x) && \text{(by (IUP-1))} \\
&= (((y \star 0) \star 0) \star ((y \star 0) \star ((y \star 0) \star x))) \star (((y \star 0) \star 0) \star x) && \text{(by (IUP-3))} \\
&= ((y \star 0) \star ((y \star 0) \star x)) \star x && \text{(by (IUP-3))} \\
&= (((y \star 0) \star x) \star ((y \star 0) \star ((y \star 0) \star x))) \star (((y \star 0) \star x) \star x) && \text{(by (IUP-3))} \\
&= (x \star ((y \star 0) \star x)) \star (((y \star 0) \star x) \star x) && \text{(by (IUP-3))} \\
&= (x \star ((y \star 0) \star x)) \star ((x \star ((y \star 0) \star x)) \star 0). && \text{(by (2.4))}
\end{aligned}
$$

By Lemma 3.3, $x \star (y \star (y \star x)) \in S$. Hence, $S$ is a reflected IUP-ideal of $X$. □

**Lemma 3.5.** *If $S$ is a dominant IUP-ideal of $X$, then it satisfies the following condition:*

$$(\forall x, y \in X)(x \star y \in S \Rightarrow y \star x \in S) \tag{3.9}$$

*Proof.* Assume that $S$ is a dominant IUP-ideal of $X$. Let $x, y \in X$ be such that $x \star y \in S$. By (IUP-1), $0 \star (x \star y) = x \star y \in S$. Then

$$
\begin{aligned}
y \star x &= (x \star y) \star 0 && \text{(by (2.4))} \\
&= (0 \star (x \star y)) \star 0 && \text{(by (IUP-1))} \\
&= (0 \star (0 \star (x \star y))) \star 0. && \text{(by (IUP-1))}
\end{aligned}
$$

By the assumption, $y \star x \in S$. Hence, $S$ satisfies the condition (3.9). □

**Theorem 3.6.** *Every dominant IUP-ideal of $X$ is a resonant IUP-ideal of $X$.*

*Proof.* Assume that $S$ is a dominant IUP-ideal of $X$. Then $S$ satisfies the condition (2.18). Let $x, y \in X$ be such that $x \star y \in S$. Then

$$
\begin{aligned}
(x \star 0) \star ((y \star x) \star x) &= ((y \star x) \star (y \star 0)) \star ((y \star x) \star x) && \text{(by (IUP-3))} \\
&= (y \star 0) \star x. && \text{(by (IUP-3))}
\end{aligned}
$$

By Lemma 3.2, $(x \star 0) \star ((y \star x) \star x) = (y \star 0) \star x \in S$. By (3.9), $((y \star x) \star x) \star (x \star 0) \in S$. By (2.5) and Lemma 3.2, $x \star ((y \star x) \star x) = ((x \star 0) \star 0) \star ((y \star x) \star x) \in S$. Hence, $S$ is a resonant IUP-ideal of $X$. □

**Example 3.9.** *Let $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 0 | 3 | 2 | 5 | 4 | 7 | 6 |
| 2 | 5 | 4 | 0 | 1 | 6 | 7 | 2 | 3 |
| 3 | 4 | 5 | 1 | 0 | 7 | 6 | 3 | 2 |
| 4 | 3 | 2 | 7 | 6 | 0 | 1 | 5 | 4 |
| 5 | 2 | 3 | 6 | 7 | 1 | 0 | 4 | 5 |
| 6 | 7 | 6 | 5 | 4 | 2 | 3 | 0 | 1 |
| 7 | 6 | 7 | 4 | 5 | 3 | 2 | 1 | 0 |

*Then $X$ is an IUP-algebra. Let $S = \{0, 2, 3, 5\}$. Then $S$ is a resonant IUP-ideal of $X$. Since $6 \star 4 = 2 \in S$ but $(6 \star (6 \star 4)) \star 6 = (6 \star 2) \star 6 = 5 \star 6 = 4 \notin S$. Hence, $S$ is not a dominant IUP-ideal of $X$.*

**Theorem 3.7.** *Every dominant IUP-ideal of $X$ is a reflected IUP-ideal of $X$.*

*Proof.* Assume that $S$ is a dominant IUP-ideal of $X$. Then $S$ satisfies the condition (2.18). Let $x, y \in X$ be such that $x \star y \in S$. Then by (IUP-2), $y \star y = 0 \in S$. By (IUP-3) and Lemma 3.2, $(x \star (y \star 0)) \star (x \star y) = (y \star 0) \star y \in S$. By (2.4) and Lemma 3.2, $(y \star x) \star (x \star (y \star 0)) = ((x \star y) \star 0) \star (x \star (y \star 0)) \in S$. By Lemma 3.2, $((x \star (y \star 0)) \star 0) \star (y \star x) \in S$. Then

$$
\begin{aligned}
x \star (y \star (y \star x)) &= (y \star ((y \star 0) \star x)) \star (y \star (y \star x)) && \text{(by (2.6))} \\
&= ((y \star 0) \star x) \star (y \star x) && \text{(by (IUP-3))} \\
&= ((x \star (y \star 0)) \star (x \star x)) \star (y \star x) && \text{(by (IUP-3))} \\
&= ((x \star (y \star 0)) \star 0) \star (y \star x). && \text{(by (IUP-2))}
\end{aligned}
$$

Thus, $x \star (y \star (y \star x)) \in S$. Hence, $S$ is a reflected IUP-ideal of $X$. □

**Example 3.10.** *Let $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a set with a binary operation $\star$ defined by the following Cayley table:*

| $\star$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 0 | 3 | 2 | 5 | 4 | 7 | 6 |
| 2 | 3 | 2 | 0 | 1 | 6 | 7 | 5 | 4 |
| 3 | 2 | 3 | 1 | 0 | 7 | 6 | 4 | 5 |
| 4 | 4 | 5 | 7 | 6 | 0 | 1 | 3 | 2 |
| 5 | 5 | 4 | 6 | 7 | 1 | 0 | 2 | 3 |
| 6 | 7 | 6 | 5 | 4 | 2 | 3 | 0 | 1 |
| 7 | 6 | 7 | 4 | 5 | 3 | 2 | 1 | 0 |

*Then $X$ is an IUP-algebra. Let $S = \{0, 1, 3, 4, 5, 6\}$. Then $S$ is a reflected IUP-ideal of $X$. Since $5 \star 7 = 3 \in S$ but $(5 \star (5 \star 7)) \star 5 = (5 \star 3) \star 5 = 7 \star 5 = 2 \notin S$. Hence, $S$ is not a dominant IUP-ideal of $X$.*

Based on the aforementioned relational structure, we obtain the following relational diagram for the IUP-algebra $X$, which elegantly captures the intricate interconnections among its underlying components.

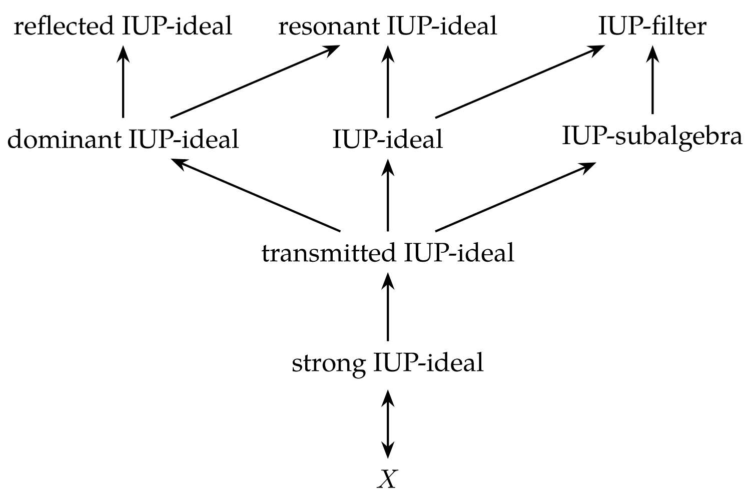


Figure 2. Structure of the eight new concepts of subsets in IUP-algebras

**Theorem 3.8.** *Let $X = (X, \star_X, 0_X)$ and $Y = (Y, \star_Y, 0_Y)$ be IUP-algebras, and $f : X \to Y$ be a homomorphism. Then the following assertions are valid:*

(1) *if $S$ is a nonempty subset of $X$ such that $f^{-1}(f(S))$ is a transmitted IUP-ideal of $X$,then the image $f(S)$ is a transmitted IUP-ideal of $f(X)$,*
(2) *if $S$ is a transmitted IUP-ideal of $X$ and $f$ is injective, then the image $f(S)$ is a transmitted IUP-ideal of $f(X)$,*
(3) *if $S$ is a transmitted IUP-ideal of $Y$, then the inverse image $f^{-1}(S)$ is a transmitted IUP-ideal of $X$,*
(4) *if $S$ is a nonempty subset of $X$ such that $f^{-1}(f(S))$ is a reflected IUP-ideal of $X$,then the image $f(S)$ is a reflected IUP-ideal of $f(X)$,*
(5) *if $S$ is a reflected IUP-ideal of $X$ and $f$ is injective, then the image $f(S)$ is a reflected IUP-ideal of $f(X)$,*
(6) *if $S$ is a reflected IUP-ideal of $Y$, then the inverse image $f^{-1}(S)$ is a reflected IUP-ideal of $X$,*
(7) *if $S$ is a nonempty subset of $X$ such that $f^{-1}(f(S))$ is a resonant IUP-ideal of $X$,then the image $f(S)$ is a resonant IUP-ideal of $f(X)$,*
(8) *if $S$ is a resonant IUP-ideal of $X$ and $f$ is injective, then the image $f(S)$ is a resonant IUP-ideal of $f(X)$,*
(9) *if $S$ is a resonant IUP-ideal of $Y$, then the inverse image $f^{-1}(S)$ is a resonant IUP-ideal of $X$,*
(10) *if $S$ is a nonempty subset of $X$ such that $f^{-1}(f(S))$ is a dominant IUP-ideal of $X$,then the image $f(S)$ is a dominant IUP-ideal of $f(X)$,*

(11) *if $S$ is a dominant IUP-ideal of $X$ and $f$ is injective, then the image $f(S)$ is a dominant IUP-ideal of $f(X)$,*

(12) *if $S$ is a dominant IUP-ideal of $Y$, then the inverse image $f^{-1}(S)$ is a dominant IUP-ideal of $X$.*

*Proof.* (1) Assume that $S$ is a nonempty subset of $X$ such that $f^{-1}(f(S))$ is a transmitted IUP-ideal of $X$. By (2.18) and Proposition 2.2 (1), $0_Y = f(0_X) \in f(S)$. Let $f(x), f(y), f(z) \in f(X)$ be such that $(f(x) \star_Y f(y)) \star_Y f(z), f(y) \in f(S)$. Then $f((x \star_X y) \star_X z) \in f(S)$. Thus, $(x \star_X y) \star_X z, y \in f^{-1}(f(S))$. By (3.1), $x \star_X z \in f^{-1}(f(S))$. Thus, $f(x) \star_Y f(z) = f(x \star_X z) \in f(S)$. Hence, $f(S)$ is a transmitted IUP-ideal of $f(X)$.

(2) Assume that $S$ is a transmitted IUP-ideal of $X$. Since $f$ is injective, we have $f^{-1}(f(S)) = S$ is a transmitted IUP-ideal of $X$. Hence, by (1), $f(S)$ is a transmitted IUP-ideal of $f(X)$.

(3) Assume that $S$ is a transmitted IUP-ideal of $Y$. By (2.18) and Proposition 2.2 (1), $f(0_X) = 0_Y \in S$. Thus, $0_X \in f^{-1}(S)$. Let $x, y, z \in X$ be such that $(x \star_X y) \star_X z, y \in f^{-1}(S)$. Then $(f(x) \star_Y f(y)) \star_Y f(z) = f((x \star_X y) \star_X z) \in S$ and $f(y) \in S$. By (3.1), $f(x \star_X z) = f(x) \star_Y f(z) \in S$. Thus, $x \star_X z \in f^{-1}(S)$. Hence, $f^{-1}(S)$ is a transmitted IUP-ideal of $X$.

(4) Assume that $S$ is a nonempty subset of $X$ such that $f^{-1}(f(S))$ is a reflected IUP-ideal of $X$. By (2.18) and Proposition 2.2 (1), $0_Y = f(0_X) \in f(S)$. Let $f(x), f(y), f(z) \in f(X)$ be such that $(f(x) \star_Y f(y)) \star_Y f(z) \in f(S)$ and $f(z) \in f(S)$. Then $f((x \star_X y) \star_X z) \in f(S)$. Thus, $(x \star_X y) \star_X z, z \in f^{-1}(f(S))$. By (3.2), $x \star_X (y \star_X (y \star_X x)) \in f^{-1}(f(S))$. Thus, $f(x) \star_Y (f(y) \star_Y (f(y) \star_Y f(x))) = f(x) \star_Y (f(y) \star_Y f(y \star_X x)) = f(x) \star_Y f(y \star_X (y \star_X x)) = f(x \star_X (y \star_X (y \star_X x))) \in f(S)$. Hence, $f(S)$ is a reflected IUP-ideal of $f(X)$.

(5) Assume that $S$ is a reflected IUP-ideal of $X$. Since $f$ is injective, we have $f^{-1}(f(S)) = S$ is a reflected IUP-ideal of $X$. Hence, by (4), $f(S)$ is a reflected IUP-ideal of $f(X)$.

(6) Assume that $S$ is a reflected IUP-ideal of $Y$. By (2.18) and Proposition 2.2 (1), $f(0_X) = 0_Y \in S$. Thus, $0_X \in f^{-1}(S)$. Let $x, y, z \in X$ be such that $(x \star_X y) \star_X z \in f^{-1}(S)$ and $z \in f^{-1}(S)$. Then $(f(x) \star_Y f(y)) \star_Y f(z) = f((x \star_X y) \star_X z) \in S$ and $f(z) \in S$. By (3.2), $f(x \star_X (y \star_X (y \star_X x))) = f(x) \star_Y (f(y) \star_Y (f(y) \star_Y f(x))) \in S$. Thus, $x \star_X (y \star_X (y \star_X x)) \in f^{-1}(S)$. Hence, $f^{-1}(S)$ is a reflected IUP-ideal of $X$.

(7) Assume that $S$ is a nonempty subset of $X$ such that $f^{-1}(f(S))$ is a resonant IUP-ideal of $X$. By (2.18) and Proposition 2.2 (1), $0_Y = f(0_X) \in f(S)$. Let $f(x), f(y) \in f(X)$ be such that $f(x) \star_Y f(y) \in f(S)$. Then $f(x \star_X y) \in f(S)$. Thus, $x \star_X y \in f^{-1}(f(S))$. By (3.3), $x \star_X ((y \star_X x) \star_X x) \in f^{-1}(f(S))$. Thus, $f(x) \star_Y ((f(y) \star_Y f(x)) \star_Y f(x)) = f(x) \star_Y (f(y \star_X x) \star_Y f(x)) = f(x) \star_Y f((y \star_X x) \star_X x) = f(x \star_X ((y \star_X x) \star_X x)) \in f(S)$. Hence, $f(S)$ is a resonant IUP-ideal of $f(X)$.

(8) Assume that $S$ is a resonant IUP-ideal of $X$. Since $f$ is injective, we have $f^{-1}(f(S)) = S$ is a resonant IUP-ideal of $X$. Hence, by (7), $f(S)$ is a resonant IUP-ideal of $f(X)$.

(9) Assume that $S$ is a resonant IUP-ideal of $Y$. By (2.18) and Proposition 2.2 (1), $f(0_X) = 0_Y \in S$. Thus, $0_X \in f^{-1}(S)$. Let $x, y \in X$ be such that $x \star_X y \in f^{-1}(S)$. Then $f(x) \star_Y f(y) = f(x \star_X y) \in S$. By (3.3), $f(x \star_X ((y \star_X x) \star_X x)) = f(x) \star_Y ((f(y) \star_Y f(x)) \star_Y f(x)) \in S$. Thus, $x \star_X ((y \star_X x) \star_X x) \in f^{-1}(S)$. Hence, $f^{-1}(S)$ is a resonant IUP-ideal of $X$.

(10) Assume that $S$ is a nonempty subset of $X$ such that $f^{-1}(f(S))$ is a dominant IUP-ideal of $X$. By (2.18) and Proposition 2.2 (1), $0_Y = f(0_X) \in f(S)$. Let $f(x), f(y) \in f(X)$ be such that $f(x) \star_Y f(y) \in f(S)$. Then $f(x \star_X y) \in f(S)$. Thus, $x \star_X y \in f^{-1}(f(S))$. By (3.4), $(x \star_X (x \star_X y)) \star_X x \in f^{-1}(f(S))$. Thus, $(f(x) \star_Y (f(x) \star_Y f(y))) \star_Y f(x) = (f(x) \star_Y f(x \star_X y)) \star_Y f(x) = f(x \star_X (x \star_X y)) \star_Y f(x) = f((x \star_X (x \star_X y)) \star_X x) \in f(S)$. Hence, $f(S)$ is a dominant IUP-ideal of $f(X)$.

(11) Assume that $S$ is a dominant IUP-ideal of $X$. Since $f$ is injective, we have $f^{-1}(f(S)) = S$ is a dominant IUP-ideal of $X$. Hence, by (10), $f(S)$ is a dominant IUP-ideal of $f(X)$.

(12) Assume that $S$ is a dominant IUP-ideal of $Y$. By (2.18) and Proposition 2.2 (1), $f(0_X) = 0_Y \in S$. Thus, $0_X \in f^{-1}(S)$. Let $x, y \in X$ be such that $x \star_X y \in f^{-1}(S)$. Then $f(x) \star_Y f(y) = f(x \star_X y) \in S$. By (3.4), $f((x \star_X (x \star_X y)) \star_X x) = (f(x) \star_Y (f(x) \star_Y f(y))) \star_Y f(x) \in S$. Thus, $(x \star_X (x \star_X y)) \star_X x \in f^{-1}(S)$. Hence, $f^{-1}(S)$ is a dominant IUP-ideal of $X$. □

## 4. Conclusion

This study expands the theory of IUP-algebras by introducing and characterizing four novel types of IUP-ideals: transmitted, reflected, resonant, and dominant. Through formal analysis of their defining properties and interrelations, along with their behavior under IUP-homomorphisms, the results offer a deeper structural understanding and enhance the foundational framework of IUP-ideal theory.

Future work may explore integrating these new subsets with other uncertainty models or algebraic systems. Such extensions hold promise for advancing theoretical developments and fostering applications in areas such as logic-based reasoning, machine learning, and multi-criteria decision analysis.

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