

IMPACT OF AWARENESS IN CONTROLLING CRIMES AGAINST WOMEN IN INDIA: A MATHEMATICAL MODELING APPROACH WITH A CASE STUDY ON DELHI NCR

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ABSTRACT. The issue of crime against women in India is a noteworthy concern, that has garnered extensive attention and apprehension both domestically and globally. Despite India's advancements in women's rights and empowerment, there persists a troubling rate of reported crimes against women. In view of this, a non linear mathematical model has been developed to examine the effectiveness of media awareness programs in reducing crime against women. Five interacting variables are addressed in the modelling process namely, socially exposed group, susceptible, criminals, victims and media awareness programs. Our model demonstrates the existence of two equilibria, namely crime free equilibrium and the crime-persistent equilibrium. The findings of the analysis indicate the existence of crime free equilibria (CFE) is pervasive, while the occurrence of crime-persistent equilibria (CPE) is dependent upon the imposition of limitations on the parameters, leading to a criminality reproduction number (R_0^c) > 1 . The model analysis further indicates that the implementation of an adequate number of baseline media awareness program can successfully reduce crime against women within society. Sensitivity analysis and numerical simulation are conducted to study the impact of key parameters on the dynamics of crime against women. To validate the model, we undertake a case study with real crime data from Delhi (2012-2022), confirming the model's prediction alignment with observed patterns.

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Key words: mathematical model; epidemic approach; crime against women; stability analysis; media impact; sensitivity analysis; case study.

1. INTRODUCTION

A crime is defined as any activity that violates the laws and ordinances established by a government or other authoritative body. It's a broad term for a wide variety of behaviors that cause harm to people, groups, or society at large. With its large population and diversified cultural terrain, India is prone to a variety of criminal activities. To comprehend the nature and dynamics of crime in India, one must investigate its causes, prevalence, and societal impact. A well-written evaluation of the potential

benefits of mathematical modeling of crime may be found in [1,2]. Numerous research articles have been conducted on crime's root causes and their potential solutions [3–8]. Campbell and Ormerod [9] formulated a mathematical model to examine the effects of social interaction among susceptible, non-susceptible, and active criminals. Incorporating the notion that with the aid of suitable counseling and law enforcement, a subset of the criminal population has been observed to abandon criminal activities and adopt a conventional lifestyle, Srivastav et al., [10] introduced a category, referred to as "recovered individuals," into their framework. People who associate with criminals typically develop similar traits [11,12]. These interactions encourage non-susceptible people to commit crimes, leading to the corruption control model by Athithan et al., [13]. This study found the implementation of psychological pressure via media and advertisements resulted in a higher rate of self-cure and subsequently led to a decrease in corruption within the society. Another crime control strategy was adopted by Shukla et al., [14], who studied the impact of technology on reducing crime. It is observed that a rise in technological advancement leads to a reduction in the equilibrium density of criminal activity. Zhao et al., [15] presented a mathematical model to analyze the relationship between poverty and crime and investigated the efficacy of government measures in reducing criminal activity. Mishra [16] then presented a model describing the impact of police in combating crime and explored police force reduces crime by reducing the immigration of criminals and by detaining criminals. In continuation, Mebratie et. al., [17] extended their model by incorporating the impact of media coverage, informal learning, and moral/religious activity and concluded that these variables speed up the elimination of criminal activity. Recently, Divya et. al., [18] presented an optimal control model showing impact of media in reducing violence against women.

Out of all, crimes against women have received the most attention. It covers a wide variety of crimes, including sexual assault, domestic violence, harassment, dowry-related crimes, acid assaults, female infanticide, and trafficking. Women's crime in India is a widespread issue that affects women of various ages, castes, faiths, and socioeconomic situations. These crimes against women are distressingly prevalent, as revealed by NCRB [19]. As per the statistics, the incidence of criminal cases against women has increased from 79,037 in 1992 to 3,71,503 in 2020. In the year 2021, there were 4,28,278 reported incidents of crimes against women, indicating a 15.3 % increase from the preceding year. Furthermore, the incidence of criminal activity per one million female individuals has exhibited a rise from 56.5 in 2020 to 64.5 in 2021. The category 'Cruelty by husband or his relatives' accounted for 31.8% of all crimes committed against women, followed by 'assault on women with intent to outrage her modesty' (20.8%), 'kidnapping and abduction of women' (17.6%), and 'rape' (7.4%). Keep in mind that these numbers only include reported cases; there might be a considerable number of unreported cases.

Such crimes against women hinder their education, and employment and also damage trust and societal development. Physical and psychological trauma can cause long-term mental health concerns in victims. The Indian government has made efforts to combat violence against women by enacting the Criminal Law (Amendment) Act of 2013, which toughened penalties for sexual offenses and established specialized courts to speed up sexual assault trials in India. The Beti Bachao, Beti Padhao campaign, and One Stop Centres (Sakhi) help survivors of violence. Despite attempts to combat gender-based violence, Indian women still confront numerous crimes and prejudice. Creating a culture where women are valued, safe, and free to reach their full potential is essential. To end violence against women in India, continued awareness campaigns and stronger law enforcement are essential.

In the context of crime against women in India, a media-based awareness initiative can serve as a crucial mechanism for augmenting awareness and transforming attitudes. Hence, we have embraced the impact of media awareness programs in our research. It is assumed that by implementing awareness programs, socially exposed group will get more cautious and hence become less vulnerable to crime. Moreover, using efficacious messaging and narrative techniques, a media awareness initiative has the potential to alter attitudes and actions that foster violence against women. By promoting empathy, respect, consent, and gender equality, the program can influence individuals to reject violence and discriminatory practices.

The subsequent sections of the paper are structured in the following manner: The formulation of the model is presented in Section 2. The existence and stability analysis of the endemic equilibrium is conducted in Section 3 and Section 4 respectively. Section 5 is dedicated to the discussion of sensitivity analysis. Section 6 conducts numerical simulations and discussion. Section 7 contains the paper's conclusion.

2. FORMULATION OF MATHEMATICAL MODEL

Let the total population $N(t)$ be divided into four distinct compartments, namely, socially exposed group $X(t)$ in the region under consideration, susceptible women population (which are likely to become a victim) $Y(t)$, criminals $C(t)$ and women victims $V(t)$. The term "socially exposed group" refers to those in the general population who are not directly involved in crime dynamics but can be influenced — either by becoming susceptible women, committing crimes, or returning from criminal activity.

Women are believed to be more likely to become victims, making them part of the susceptible class at a rate proportional to the socially exposed group, i.e., $\gamma_1 X$. When they interact with criminals, women from the susceptible class migrate to the victim class at a rate according to the density of susceptible women and the density of criminals, i.e., βCY . Additionally, it is presumable that socially exposed group commit crimes while under the influence of criminals and join the criminal class at

a rate proportional to the density of both the socially exposed group and the criminals, i.e., γ_2XC . Additionally, there is a chance that criminals will stop committing crimes and move to the socially exposed group at a rate proportionate to the criminal population, i.e., γ_3C . Finally, it is assumed that the natural mortality rate μ for each class is proportional to the density of that class.

The model integrated a new dynamic variable, $M(t)$, to represent the number of media-implemented awareness programs that inform the public of ongoing criminal activity and effective crime prevention measures. It is believed that these programs can alter individuals' attitudes toward criminal behavior. We anticipate that the rate of awareness program implementation is proportional to the number of criminals. In addition, it is believed that the number of awareness programs diminishes at a rate of $\eta_0(M - M_0)$ due to inefficiency, where M_0 represents the baseline number of awareness programs in the absence of crime.

Moreover, it is assumed that awareness programs induce behavioral changes in individuals which limits the probability of susceptible getting influenced by criminals. This can reduce the crime rate by a fraction $(1 - \frac{pM(t)}{K_1+M(t)})$, with a saturating function $\frac{M(t)}{K_1+M(t)}$ depending on the number of awareness programs ' M '. Here, the effectiveness of awareness campaigns is constrained by the constant K_1 , while $p \in [0, 1]$ assesses the change in the criminal's behavior towards committing the crime. Further, susceptible individuals start taking precautions (such as avoiding going to crime hotspots or carrying safety gear) to reduce their likelihood of becoming victims. These precautions have the potential to lower the susceptibility by a fraction $(1 - \frac{qM(t)}{K_2+M(t)})$, where $q \in [0, 1]$ gauges the behavior change of susceptible individuals and constant K_2 limits the impact of awareness campaigns. The model's schematic flow diagram is depicted in Figure 1.

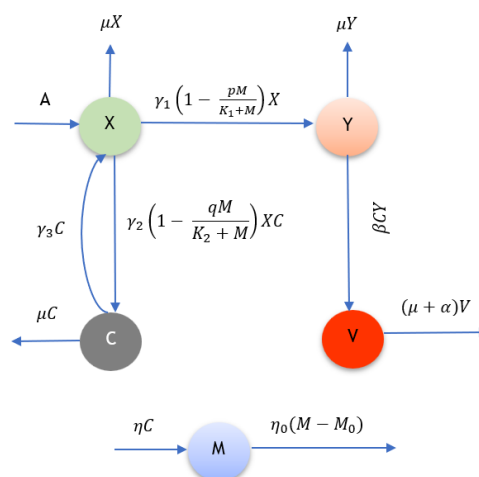


FIGURE 1. Schematic diagram to represent the flow among different compartments for the proposed model (1)

The following mathematical model is proposed based on all of the aforementioned assumptions:

$$\begin{aligned}
 \frac{dX(t)}{dt} &= A - \gamma_1 \left(1 - \frac{pM(t)}{K_1 + M(t)}\right) X(t) - \gamma_2 \left(1 - \frac{qM(t)}{K_2 + M(t)}\right) X(t)C(t) + \gamma_3 C(t) - \mu X(t), \\
 \frac{dY(t)}{dt} &= \gamma_1 \left(1 - \frac{pM(t)}{K_1 + M(t)}\right) X(t) - \beta C(t)Y(t) - \mu Y(t), \\
 \frac{dC(t)}{dt} &= \gamma_2 \left(1 - \frac{qM(t)}{K_2 + M(t)}\right) X(t)C(t) - \gamma_3 C(t) - \mu C(t), \\
 \frac{dV(t)}{dt} &= \beta C(t)Y(t) - (\mu + \alpha)V(t), \\
 \frac{dM(t)}{dt} &= \eta C(t) - \eta_0(M(t) - M_0),
 \end{aligned} \tag{1}$$

where $X(0) \geq 0, Y(0) \geq 0, C(0) \geq 0, V(0) \geq 0, M(0) = M_0 > 0$.

Since $N(t) = X(t) + Y(t) + C(t) + V(t)$, rewriting the above model:

$$\begin{aligned}
 \frac{dN(t)}{dt} &= A - \mu N(t) - \alpha V(t), \\
 \frac{dY(t)}{dt} &= \gamma_1 \left(1 - \frac{pM(t)}{K_1 + M(t)}\right) (N(t) - Y(t) - C(t) - V(t)) - \beta C(t)Y(t) - \mu Y(t), \\
 \frac{dC(t)}{dt} &= \gamma_2 \left(1 - \frac{qM(t)}{K_2 + M(t)}\right) (N(t) - Y(t) - C(t) - V(t))C(t) - \gamma_3 C(t) - \mu C(t), \\
 \frac{dV(t)}{dt} &= \beta C(t)Y(t) - (\mu + \alpha)V(t), \\
 \frac{dM(t)}{dt} &= \eta C(t) - \eta_0(M(t) - M_0).
 \end{aligned} \tag{2}$$

For the positive solution of model (2), the region of attraction [20] is given by the set:

$$\begin{aligned}
 \Omega = \left\{ (N, Y, C, V, M) \in \mathbb{R}_+^5 : 0 \leq Y \leq N \leq \frac{A}{\mu}, 0 \leq C \leq \frac{A}{\mu} \left(\frac{\gamma_2}{\gamma_3 + \mu}\right) = C_r, \right. \\
 \left. 0 \leq V \leq \frac{\beta C_r A}{\mu(\mu + \alpha)} = V_k, 0 \leq M \leq \frac{\eta C_r + M_0}{\eta_0} \right\}.
 \end{aligned} \tag{3}$$

3. EQUILIBRIUM ANALYSIS

The two non-negative equilibria of the model system (2) mentioned above are as follows:

- (i) Crime free equilibrium (CFE) $E_0 \left(\frac{A}{\mu}, \frac{\gamma_1 \left(1 - \frac{pM_0}{K_1 + M_0}\right) A}{\mu \left(\gamma_1 \left(1 - \frac{pM_0}{K_1 + M_0}\right) + \mu\right)}, 0, 0, 0 \right)$ always exists.
- (ii) crime-persistent equilibrium $E^*(N^*, Y^*, C^*, V^*, M^*)$ exists provided

$$R_0^c = \frac{A\gamma_2 \left(1 - \frac{qM_0}{K_2 + M_0}\right)}{\left(\gamma_1 \left(1 - \frac{pM_0}{K_1 + M_0}\right) + \mu\right)(\gamma_3 + \mu)} > 1, \tag{4}$$

where R_0^c is defined as criminality reproduction number (CRN).

Existence of Equilibria: The crime-free equilibrium E_0 is straightforward to identify. Next, for crime-persistent equilibrium to exist, set the model system (2) to zero:

$$A - \mu N - \alpha V = 0, \quad (5)$$

$$\gamma_1 \left(1 - \frac{pM}{K_1 + M} \right) (N - Y - C - V) - \beta CY - \mu Y = 0, \quad (6)$$

$$\gamma_2 \left(1 - \frac{qM}{K_2 + M} \right) (N - Y - C - V)C - \gamma_3 C - \mu C = 0, \quad (7)$$

$$\beta CY - (\mu + \alpha)V = 0 \quad (8)$$

$$\eta C - \eta_0(M - M_0) = 0 \quad (9)$$

From (5), (8) and (9) we get

$$N = \frac{A - \alpha V}{\mu}, \quad (10)$$

$$V = \frac{\beta CY}{(\mu + \alpha)}, \quad (11)$$

$$M = \frac{\eta C + \eta_0 M_0}{\eta_0}, \quad (12)$$

Equation (7) gives

$$N - Y - C - V = \frac{\gamma_3 + \mu}{\gamma_2 \left(1 - \frac{qM}{K_2 + M} \right)}. \quad (13)$$

Using (13) in (6), we get

$$\gamma_1 \left(1 - \frac{pM}{K_1 + M} \right) \left[\frac{\gamma_3 + \mu}{\gamma_2 \left(1 - \frac{qM}{K_2 + M} \right)} \right] - \beta CY - \mu Y = 0, \quad (14)$$

which implies

$$Y = \frac{\gamma_1 \left(1 - \frac{pM}{K_1 + M} \right) \left(\frac{\gamma_3 + \mu}{\gamma_2 \left(1 - \frac{qM}{K_2 + M} \right)} \right)}{(\beta C + \mu)} = g(C). \quad (15)$$

Using (10), (11), (12) and (15) in (13), we get

$$F(c) = \frac{A}{\mu} - \frac{\alpha \beta C g(C)}{\mu(\mu + \alpha)} - g(C) - C - \frac{\beta C g(C)}{\mu + \alpha} - \frac{\gamma_3 + \mu}{\gamma_2 \left(1 - \frac{qM}{K_2 + M} \right)} = 0. \quad (16)$$

It can be easily noted from equation (16) that

$$(i) F(0) = \frac{A}{\mu} - \frac{(\gamma_3 + \mu) \left(\gamma_1 \left(1 - \frac{pM_0}{K_1 + M_0} \right) + \mu \right)}{\mu \gamma_2 \left(1 - \frac{qM_0}{K_2 + M_0} \right)} > 0 \text{ (say)},$$

$$(ii) F\left(\frac{A}{\mu}\right) = -\frac{\alpha\beta\frac{A}{\mu}g\left(\frac{A}{\mu}\right)}{\mu(\mu+\alpha)} - g\left(\frac{A}{\mu}\right) - \frac{A}{\mu} - \frac{\beta\frac{A}{\mu}g(C)}{\mu+\alpha} - \frac{\gamma_3+\mu}{\gamma_2\left(1-\frac{qM}{K_2+M}\right)} < 0.$$

Thus, $F(C^*) = 0$ for a unique C^* in $(0, C_r)$ provided $R_0^c > 1$. Hence, equilibria $E^*(N^*, Y^*, C^*, V^*, M^*)$ exists, whenever condition (4) is satisfied.

4. STABILITY ANALYSIS

In this section, we will discuss the stability analysis crime free equilibrium E_0 and crime-persistent equilibrium E^* . Theorem 1 provides the necessary conditions for the local stability of both the equilibria, E_0 and E^* . In Theorem 2, we examine the global stability aspects of the crime-persistent equilibria E^* .

4.1. Local Stability.

Theorem 1. (i) The crime free equilibrium (CFE) E_0 always exists and is unstable if

$$R_0^c > 1.$$

(ii) The crime-persistent equilibrium E^* whenever exists, is stable if Routh-Hurwitz criteria is satisfied.

Proof. The Jacobian matrix J for the model system (2) is calculated as follows:

$$J = \begin{pmatrix} -\mu & 0 & 0 & -\phi & 0 \\ a_{21} & -a_{22} & -a_{23} & -a_{21} & a_{25} \\ a_{31} & -a_{31} & a_{33} & -a_{31} & a_{35} \\ 0 & \beta C & \beta Y & -(\mu + \alpha) & 0 \\ 0 & 0 & \eta & 0 & -\eta_0 \end{pmatrix}.$$

where ,

$$a_{21} = \gamma_1\left(1 - \frac{pM}{K_1+M}\right), a_{22} = \gamma_1\left(1 - \frac{pM}{K_1+M}\right) + \beta C + \mu, a_{23} = \gamma_1\left(1 - \frac{pM}{K_1+M}\right) + \beta Y, a_{25} = \gamma_1\frac{K_1p}{(K_1+M)^2}(N - Y - C - V), a_{31} = \gamma_2\left(1 - \frac{qM}{K_2+M}\right)C, a_{33} = \gamma_2\left(1 - \frac{qM}{K_2+M}\right)(N - Y - V - 2C) - \gamma_3 - \mu, a_{35} = \gamma_1\frac{K_2q}{(K_2+M)^2}(N - Y - C - V)C.$$

Let the matrix J at E_0 and E^* be defined as J_0 and J^* . Then, J_0 is given by

$$J_0 = \begin{pmatrix} -\mu & 0 & 0 & -\alpha & 0 \\ a_{21}^o & -a_{22}^o & -a_{23}^o & -a_{21}^o & -a_{25}^o \\ 0 & 0 & a_{33}^o & 0 & 0 \\ 0 & 0 & a_{43}^o & -(\mu + \alpha) & o \\ 0 & 0 & \eta & 0 & -\eta_0 \end{pmatrix}.$$

where,

$$a_{21}^o = \gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right), a_{22}^o = \gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right) + \mu, a_{23}^o = \gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right) + \beta \left(\frac{\gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right) A}{\mu(\mu + \gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right))}\right), a_{25}^o = \gamma_1 \frac{K_1 p}{(K_1+M_0)^2} \left(\frac{A}{\mu} - \left(\frac{\gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right) A}{\mu(\mu + \gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right))}\right)\right), a_{33}^o = \gamma_2 \left(\frac{A}{\mu} - \left(\frac{\gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right) A}{\mu(\mu + \gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right))}\right)\right) - \gamma_3 - \mu, a_{43}^o = \beta \left(\frac{\gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right) A}{\mu(\mu + \gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right))}\right)$$

Two eigen values of the above matrix are $-\gamma_1 \left(1 - \frac{pM_0}{K_1+M_0}\right) - \mu$ and $-\mu$. The other three eigen values can be obtained from the submatrix

$$\begin{pmatrix} a_{33}^o & 0 & 0 \\ a_{43}^o & -(\mu + \alpha) & 0 \\ \eta & 0 & -\eta_0 \end{pmatrix}.$$

Clearly, third and fourth eigen value are $-(\mu + \alpha)$ and $-\eta_0$ and the last eigen value a_{33}^o is positive if $R_0^c > 1$, which make the system unstable and existence of crime-persistent equilibria is justified.

Further, J^* is given by

$$J^* = \begin{pmatrix} -\mu & 0 & 0 & -\alpha & 0 \\ a_{21}^* & -a_{22}^* & -a_{23}^* & -a_{24}^* & a_{25}^* \\ a_{31}^* & -a_{31}^* & a_{33}^* & -a_{31}^* & a_{35}^* \\ 0 & \beta C^* & \beta Y^* & -(\mu + \alpha) & 0 \\ 0 & 0 & \eta & 0 & -\eta_0 \end{pmatrix},$$

where,

$$a_{21}^* = \gamma_1 \left(1 - \frac{pM^*}{K_1+M^*}\right), a_{22}^* = \gamma_1 \left(1 - \frac{pM^*}{K_1+M^*}\right) + \beta C^* + \mu, a_{23}^* = \gamma_1 \left(1 - \frac{pM^*}{K_1+M^*}\right) + \beta Y^*, a_{25}^* = \gamma_1 \frac{K_1 p}{(K_1+M^*)^2} (N^* - Y^* - C^* - V^*), a_{31}^* = \gamma_2 \left(1 - \frac{qM^*}{K_2+M^*}\right) C^*, a_{33}^* = \gamma_2 \left(1 - \frac{qM^*}{K_2+M^*}\right) (N^* - Y^* - V^* - 2C^*) - \gamma_3 - \mu, a_{35}^* = \gamma_2 \frac{K_2 q}{(K_2+M^*)^2} (N^* - Y^* - C^* - V^*) C^*.$$

The characteristic polynomial of J^* is given by

$$P(\lambda) = \lambda^5 + p_1 \lambda^4 + p_2 \lambda^3 + p_3 \lambda^2 + p_4 \lambda + p_5 = 0, \quad (17)$$

where

$$p_1 = 2\mu + a_{22}^* - a_{33}^* + \alpha + \eta_0.$$

$$p_2 = 2\mu a_{22}^* + \alpha(a_{22}^* - a_{33}^*) - 2\mu a_{33}^* - a_{22}^* a_{33}^* - a_{24}^* a_{32}^* + \mu(\mu + \alpha) + a_{24}^* \beta C^* + a_{22}^* \eta_0 + a_{33}^* \eta_0 + a_{34}^* \beta Y^* + \eta_0(2\mu + \alpha) - a_{35}^* \eta$$

$$p_3 = \mu^2(a_{33}^* - a_{22}^* - \eta_0) - \mu\alpha(a_{22}^* - a_{33}^*) + 2\mu(a_{22}^* a_{33}^* + a_{23}^* a_{32}^*) + \alpha(a_{22}^* a_{33}^* + a_{23}^* a_{32}^* - a_{21}^* \beta C^* - \mu\eta_0 - a_{31}^* \beta Y^*) - \mu(a_{24}^* \beta C^* - a_{34}^* \beta C^*) + \eta_0(2\mu a_{33}^* - \alpha a_{33}^* - 2\mu a_{22}^* - \alpha a_{22}^* - a_{24}^* \beta C^* - a_{34}^* \beta Y^* + a_{22}^* a_{33}^* + a_{23}^* a_{32}^*) + \eta(2\mu a_{35}^* + \alpha a_{35}^* + a_{22}^* a_{35}^* - a_{25}^* a_{32}^*) + \beta C^*(a_{23}^* a_{34}^* + a_{24}^* a_{33}^*)$$

$$+\beta Y^*(a_{24}^*a_{32}^* - a_{22}^*a_{34}^*).$$

$$\begin{aligned} p_4 = & \mu^2(a_{22}^*a_{33}^* + a_{23}^*a_{32}^* - a_{22}^*\eta_0) + a^2(a_{33}^*\eta_0 + a_{35}^*\eta) - \alpha a_{21}^*\beta C^*\eta_0 + \mu(\alpha a_{35}^*\eta - a_{24}^*\beta C^*\eta_0) + \\ & (2\mu\eta + \alpha\eta)(a_{22}^*a_{35}^* - a_{25}^*a_{32}^*) - \alpha a_{31}^*\beta Y^*\eta_0 - \mu a_{34}^*\beta Y^*\eta_0 + \eta_0\beta C^*(a_{23}^*a_{34}^* + a_{24}^*a_{33}^*) + \\ & \eta_0\beta Y^*(a_{24}^*a_{32}^* - a_{22}^*a_{34}^*) + \eta\beta C^*(a_{24}^*a_{34}^* - a_{25}^*a_{34}^*) + \mu\alpha(a_{22}^*a_{33}^* + a_{23}^*a_{32}^* - a_{22}^*\eta_0 + a_{33}^*\eta_0) + \\ & \alpha\beta C^*(a_{21}^*a_{33}^* + a_{23}^*a_{31}^*) + \alpha\beta Y^*(a_{21}^*a_{32}^* - a_{22}^*a_{31}^*) + \mu\beta C^*(a_{23}^*a_{34}^* + a_{24}^*a_{33}^*) + \\ & \mu\beta Y^*(a_{24}^*a_{32}^* - a_{22}^*a_{34}^*) + (2\mu + \alpha)\eta_0(a_{22}^*a_{33}^* + a_{23}^*a_{32}^*). \end{aligned}$$

$$\begin{aligned} p_5 = & -\eta_0(\mu^2 + \mu)(a_{22}^*a_{33}^* + a_{23}^*a_{32}^*) - \eta(\mu^2 + \mu)(a_{22}^*a_{35}^* - a_{25}^*a_{32}^*) - \alpha\eta_0\beta Y^*(a_{22}^*a_{31}^* - a_{21}^*a_{32}^*) - \\ & \alpha\eta_0\beta C^*(a_{21}^*a_{33}^* + a_{23}^*a_{31}^*) + \mu\eta_0\beta Y^*(a_{22}^*a_{34}^* - a_{24}^*a_{32}^*) - \mu\eta_0\beta C^*(a_{23}^*a_{34}^* + a_{24}^*a_{33}^*) - \\ & \alpha\eta\beta C^*(a_{21}^*a_{35}^* - a_{25}^*a_{31}^*) - \mu\eta\beta C^*(a_{24}^*a_{35}^* - a_{25}^*a_{34}^*). \end{aligned}$$

According to R.H criteria, E^* is locally stable if $p_5 > 0$, $p_1p_2 - p_3 > 0$, $p_3(p_1p_2 - p_3) - p_1(p_1p_4 - p_5) > 0$ and $p_4(p_3(p_1p_2 - p_3) - p_1(p_1p_4 - p_5)) - p_5(p_2(p_1p_2 - p_3) - (p_1p_4 - p_5)) > 0$.

□

4.2. Global Stability.

Theorem 2. *The crime-persistent equilibrium E^* exists whenever $R_0^c > 1$ and is nonlinearly asymptotically stable if the following conditions are fulfilled:*

$$\max\{A, B, C, D, E\} < m_1 < \frac{4}{35} \frac{\mu(\mu + \alpha)}{\alpha^2} \min\{F, G\}, \quad (18)$$

$$\max\{H, I\} < m_2 < \min\{J, K, L, M, N, O\}, \quad (19)$$

$$\max\{P, Q\} < m_3 < \min\{F, G\}, \quad (20)$$

$$\frac{9\gamma_2qK_2^2\left(\frac{A}{\mu}\right)^2}{\eta_0(K_2 + M_0)^2(K_2 + M^*)^2\left(1 - \frac{qM^*}{K_2 + M^*}\right)} < \frac{1}{9}\eta_0 \frac{\gamma_2\left(1 - \frac{qM^*}{K_2 + M^*}\right)}{\eta^2}, \quad (21)$$

where

$$A = 15 \frac{\gamma_2\mu}{\left(1 - \frac{qM^*}{K_2 + M^*}\right)}, B = 15 \frac{\gamma_2^2q^2(M^*)^2\mu}{(K_2 + M^*)^2\left(1 - \frac{qM^*}{K_2 + M^*}\right)}, C = \frac{4}{35} \frac{\mu(\mu + \alpha)}{\alpha^2} \min\{F, G\},$$

$$D = \frac{55}{4} \frac{\gamma_1^2}{\mu(\gamma_1\left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu)} \max\{H, I\},$$

$$E = \frac{55}{4} \frac{\gamma_1^2\left(\frac{pM^*}{K_1 + M^*}\right)^2}{\mu} (\gamma_1\left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu) \max\{H, I\}, F = \frac{4}{77} \frac{(\gamma_1\left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu)(\mu + \alpha)}{\beta^2(C^*)^2},$$

$$G = \frac{4}{77} \frac{\gamma_2\left(1 - \frac{qM^*}{K_2 + M^*}\right)}{\beta^2\left(\frac{A}{\mu}\right)^2(\mu + \alpha)}, H = \frac{132}{4} \frac{\gamma_2}{[\gamma_1\left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu]\left(1 - \frac{qM^*}{K_2 + M^*}\right)},$$

$$I = \frac{132}{4} \frac{\gamma_2q^2(M^*)^2}{(K_2 + M^*)^2[\gamma_1\left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu]\left(1 - \frac{qM^*}{K_2 + M^*}\right)}, J = \frac{1}{33\gamma_1^2} (\gamma_1\left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu)\gamma_2\left(1 - \frac{qM^*}{K_2 + M^*}\right),$$

$$\begin{aligned}
K &= \frac{1}{33} \frac{(K_1+M^*)^2}{\gamma_1^2 p^2 (M^*)^2} (\gamma_1 (1 - \frac{pM^*}{K_1+M^*}) + \beta C^* + \mu) \gamma_2 (1 - \frac{qM^*}{K_2+M^*}), L = \frac{1}{33} \frac{1}{\beta^2 Y^2} (\gamma_1 (1 - \frac{pM^*}{K_1+M^*}) + \beta C^* + \mu) \gamma_2 (1 - \frac{qM^*}{K_2+M^*}), \\
M &= \frac{4}{77} (\gamma_1 (1 - \frac{pM^*}{K_1+M^*}) + \beta C^* + \mu) (\mu + \alpha) \min\{F, G\}, \\
N &= \frac{4}{77} \frac{(K_1+M^*)^2}{\gamma_1^2 p^2 (M^*)^2} (\gamma_1 (1 - \frac{pM^*}{K_1+M^*}) + \beta C^* + \mu) (\mu + \alpha) \min\{F, G\}, \\
O &= \frac{1}{297} \frac{(K_1+M^*)^2 (K_1+M_0)^2 \eta_0^2}{\gamma_1^2 p^2 (\frac{A}{\mu})^2 (K_1)^2 \eta^2} (\gamma_1 (1 - \frac{pM^*}{(K_1+M^*)^2} + \beta C^* + \mu) \gamma_2 (1 - \frac{qM^*}{K_2+M^*}), \\
P &= \frac{77}{4} \frac{\gamma_2}{(\mu+\alpha)(1-\frac{qM^*}{K_2+M^*})}, Q = \frac{77}{4} \frac{\gamma_2 q^2 (M^*)^2}{(\mu+\alpha)(K_2+M^*)^2(1-\frac{qM^*}{K_2+M^*})}.
\end{aligned}$$

Proof. In order to determine the global stability of crime-persistent equilibrium E^* , a positive definite function is considered as follows:

$$W = \frac{m_1}{2} (N - N^*)^2 + \frac{m_2}{2} (Y - Y^*)^2 + \left((C - C^*) - C^* \log \frac{C}{C^*} \right) + \frac{m_3}{2} (V - V^*)^2 + \frac{m_4}{2} (M - M^*)^2,$$

where m_1, m_2, m_3 and m_4 are positive constants to be determined. On differentiating W w.r.t t along the solutions of model system (2), we get

$$\begin{aligned}
\frac{dW}{dt} &= -m_1 \mu (N - N^*)^2 - m_2 (\gamma_1 (1 - \frac{pM^*}{K_1 + M^*}) + \mu + \beta C^*) (Y - Y^*)^2 \\
&\quad - \left(\frac{\gamma_2 (1 - \frac{qM^*}{K_2 + M^*}) (C + C^* - N^* + Y^* + V^*) + \gamma_3 + \mu}{C} + \frac{\gamma_2 q M^*}{K_2 + M^*} \right) (C - C^*)^2 \\
&\quad - m_3 (\mu + \alpha) (V - V^*)^2 - m_4 \eta_0 (M - M^*)^2 + m_2 \gamma_1 (1 - \frac{pM^*}{K_1 + M^*}) (N - N^*) (Y - Y^*) \\
&\quad + \gamma_2 (1 - \frac{qM^*}{K_2 + M^*}) (N - N^*) (C - C^*) - m_1 \alpha (N - N^*) (V - V^*) \\
&\quad - \gamma_1 (1 - \frac{pM^*}{K_1 + M^*}) m_2 (Y - Y^*) (C - C^*) - \gamma_2 (1 - \frac{qM^*}{K_2 + M^*}) (Y - Y^*) (C - C^*) \\
&\quad - m_2 \beta Y (Y - Y^*) (C - C^*) - \gamma_1 m_2 (1 - \frac{pM^*}{K_1 + M^*}) (Y - Y^*) (V - V^*) \\
&\quad + \beta m_3 C^* (Y - Y^*) (V - V^*) - \gamma_2 (1 - \frac{qM^*}{K_2 + M^*}) (V - V^*) (C - C^*) \\
&\quad + \beta m_3 Y (V - V^*) (C - C^*) + m_4 \eta (C - C^*) (M - M^*) \\
&\quad - m_2 \gamma_1 p K_1 \frac{(N - Y - C - V)}{(K_1 + M^*) (K_1 + M)} (M - M^*) (Y - Y^*) \\
&\quad - \gamma_2 q K_2 \frac{(N - Y - C - V)}{(K_2 + M^*) (K_2 + M)} (M - M^*) (C - C^*).
\end{aligned}$$

Now, $\frac{dW}{dt}$ is negative definite provided m_1, m_2 and m_3 satisfy the following inequalities:

$$\begin{aligned}
\gamma_1^2 m_2 &< \frac{4}{55} m_1 \mu (\gamma_1 (1 - \frac{pM^*}{K_1 + M^*}) + \beta C^* + \mu), \\
m_2 (\gamma_1 (1 - \frac{pM^*}{K_1 + M^*})^2 &< \frac{4}{55} m_1 \mu (\gamma_1 (1 - \frac{pM^*}{K_1 + M^*}) + \beta C^* + \mu), \\
m_1 \alpha^2 &< \frac{4}{35} m_3 \mu (\mu + \alpha), \\
\gamma_2 &< \frac{4}{60} m_1 (1 - \frac{pM^*}{K_1 + M^*}) \mu,
\end{aligned}$$

$$\begin{aligned}
\gamma_2 \frac{q^2(M^*)^2}{(K_2 + M^*)^2} &< \frac{4}{60} m_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) \mu, \\
m_2 \gamma_1^2 &< \frac{1}{33} \gamma_2 \left(\gamma_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu\right) \left(1 - \frac{qM^*}{K_2 + M^*}\right), \\
m_2 \gamma_1^2 p^2 \frac{(M^*)^2}{(K_1 + M^*)^2} &< \frac{1}{33} \gamma_2 \left(\gamma_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu\right) \left(1 - \frac{qM^*}{K_2 + M^*}\right), \\
m_2 \beta^2 Y^2 &< \frac{1}{33} \gamma_2 \left(\gamma_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu\right) \left(1 - \frac{qM^*}{K_2 + M^*}\right), \\
\gamma_2^2 &< \frac{1}{33} m_2 \gamma_2 \left(\gamma_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu\right) \left(1 - \frac{qM^*}{K_2 + M^*}\right), \\
\gamma_2^2 q^2 \frac{(M^*)^2}{(K_2 + M^*)^2} &< \frac{1}{33} m_2 \gamma_2 \left(\gamma_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu\right) \left(1 - \frac{qM^*}{K_2 + M^*}\right), \\
\gamma_1^2 m_2 &< \frac{4}{77} m_3 (\alpha + \mu) \left(\gamma_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu\right), \\
\gamma_1^2 m_2 p^2 \frac{(M^*)^2}{(K_1 + M^*)^2} &< \frac{4}{77} m_3 (\alpha + \mu) \left(\gamma_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu\right), \\
m_3 \beta^2 (C^*)^2 &< \frac{4}{77} m_2 (\alpha + \mu) \left(\gamma_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu\right), \\
m_2 \gamma_1^2 p^2 \frac{\left(\frac{A}{\mu}\right)^2 K_1^2}{(K_1 + M^*)^2 (K_1 + M_0)^2} &< \frac{4}{33} \eta_0 m_4 \left(\gamma_1 \left(1 - \frac{pM^*}{K_1 + M^*}\right) + \beta C^* + \mu\right), \\
\gamma_2 &< \frac{4}{77} m_3 (\mu + \alpha) \left(1 - \frac{qM^*}{K_2 + M^*}\right), \\
\gamma_2 q^2 \frac{(M^*)^2}{(K_2 + M^*)^2} &< \frac{4}{77} m_3 (\mu + \alpha) \left(1 - \frac{qM^*}{K_2 + M^*}\right), \\
m_3 \beta^2 \left(\frac{A}{\mu}\right)^2 &< \frac{4}{77} (\mu + \alpha) \gamma_2 \left(1 - \frac{qM^*}{K_2 + M^*}\right), \\
\gamma_2 q^2 \frac{\left(\frac{A}{\mu}\right)^2 K_2^2}{(K_2 + M^*)^2 (K_2 + M_0)^2} &< \frac{1}{9} \eta_0 m_4 \left(1 - \frac{qM^*}{K_2 + M^*}\right), \\
\eta^2 m_4 &< \frac{1}{9} \eta_0 \gamma_2 \left(1 - \frac{qM^*}{K_2 + M^*}\right).
\end{aligned}$$

From the above inequalities, we may easily choose positive m_1, m_2, m_3 and m_4 provided condition (18),(19),(20) and (21) are satisfied respectively. \square

5. SENSITIVITY ANALYSIS OF R_0^c : INTERPRETATION OF R_0^c VERSUS MODEL PARAMETERS

Sensitivity analysis assists us in identifying the most influential parameters and enhances the understanding and credibility of mathematical models by measuring the impact of input variations. About a specific parameter P , the normalized forward sensitivity index [21] of R_0 is differentiable and is defined by:

$$\Gamma_P^{R_0} = \frac{P}{R_0} \frac{\delta R_0}{\delta P} \quad (22)$$

To examine the impact of these parameters, we calculated the normalised forward sensitivity indices of the criminality reproduction number, R_0^c with respect to parameter $A, \gamma_1, \gamma_2, \gamma_3, \mu$ and M_0 and have displayed the results in Table 1.

Negative signs in table 1 indicate that parameters are inversely proportional to R_0^c , while positive sign parameters are directly proportional to R_0^c . Model parameters whose sensitivity index values are close to -1 or 1 indicate that a change in their magnitude has a significant effect on increasing or decreasing the size of R_0^c , respectively. It can be noted that R_0^c is sensitive to parameteres γ_1, γ_3 and M_0 , where 10% increase in γ_1, γ_3 and M_0 decreases R_0^c by 9.9%, 9.9 % and 1.8% respectively. The impact of several important model parameters on R_0^c is depicted visually in Figures 2 – 5. It can be observed that the most effective methods for lowering R_0^c are raising awareness among socially exposed group, increasing crime quitting rate, and the baseline number of media awareness programs.

TABLE 1. Sensitivity indices of R_0^c evaluated at parameter values.

| Parameters | Sign | Sensitivity indices |
|-----------------------------|------|---------------------|
| $\Gamma_A^{R_0^c}$ | + | 1 |
| $\Gamma_{\gamma_1}^{R_0^c}$ | - | 0.99 |
| $\Gamma_{\gamma_2}^{R_0^c}$ | + | 1 |
| $\Gamma_{\gamma_3}^{R_0^c}$ | - | 0.99 |
| $\Gamma_{\mu}^{R_0^c}$ | - | 0.00052 |
| $\Gamma_{M_0}^{R_0^c}$ | - | 0.181 |

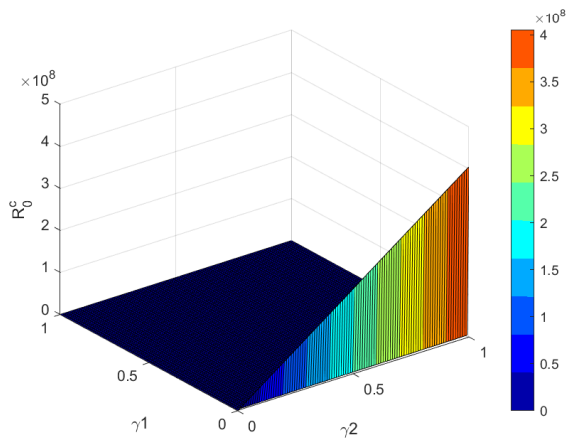


FIGURE 2. Influence of parameter γ_1 and γ_2 on R_0^c .

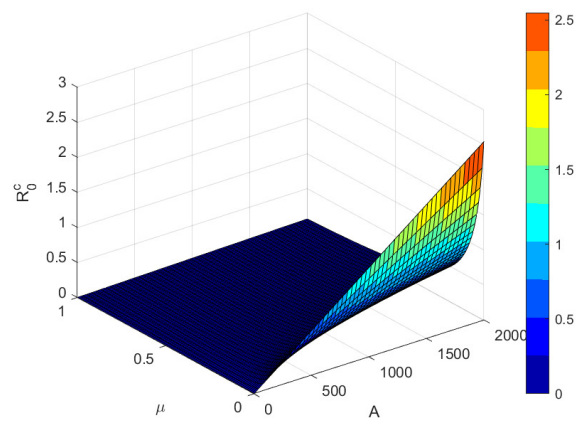


FIGURE 3. Influence of parameter A and μ on R_0^c .

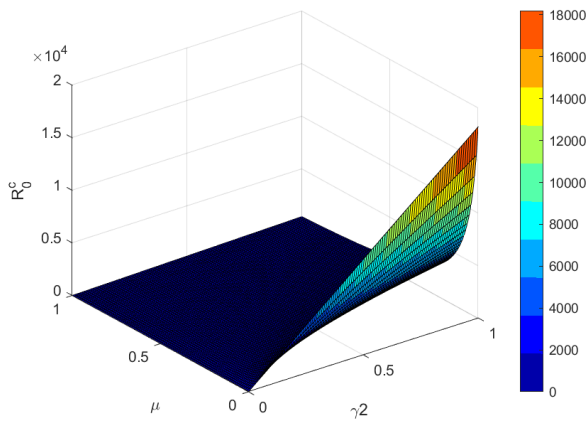


FIGURE 4. Influence of parameter γ_2 and μ on R_0^c .

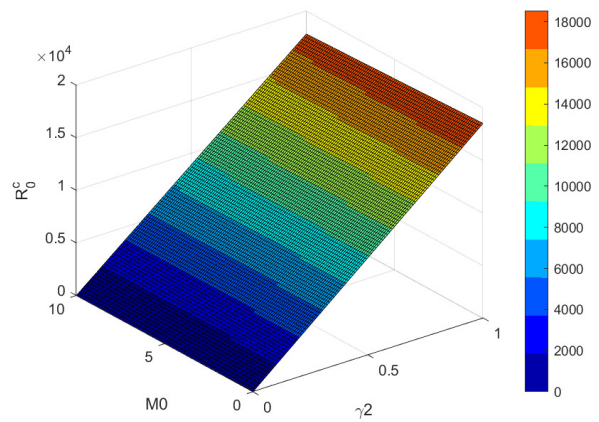


FIGURE 5. Influence of parameter γ_2 and M_0 on R_0^c .

6. NUMERICAL SIMULATION AND DISCUSSION

Here, we demonstrate the graphical findings and discuss the model’s numerical solution using the MATLAB program. First, we compute the value of natural death rate (i.e. μ) of the population. The average life duration in India is 70.42 years [22], thus $\mu = 0.00003891$ per day. The remaining parameters values, which are shown in the following table 2, were assumed.

TABLE 2. Fitted or estimated parameters for present model.

| Parameters | Description | Baseline value (per year) |
|------------|---|---------------------------|
| A | Immigration Rate | 1000 |
| μ | Natural death rate | 0.00003891 |
| γ_1 | Rate of flow from socially exposed group to susceptible | 0.9 |
| γ_2 | Crime committing rate of socially exposed group | 0.00007 |
| γ_3 | Conversion rate of criminals to socially exposed group | 0.06 |
| β | Rate at which susceptible are getting victimised under the influence of criminals | 0.000000003 |
| α | Death rate of victims | 0.02 |
| η | rate at which new awareness programs are controlling crime against women | 0.000032 |
| η_0 | rate at which the media awareness becomes less effective or out-dated | 0.005 |
| p | rate of implementing media campaigns | 0.4 |
| q | rate of fading media campaigns | 0.6 |
| K_1 | Constant which limits the effect of media | 50 |
| K_2 | Constant which limits the effect of media | 50 |
| M_0 | Baseline media programs present before crime | 5 |

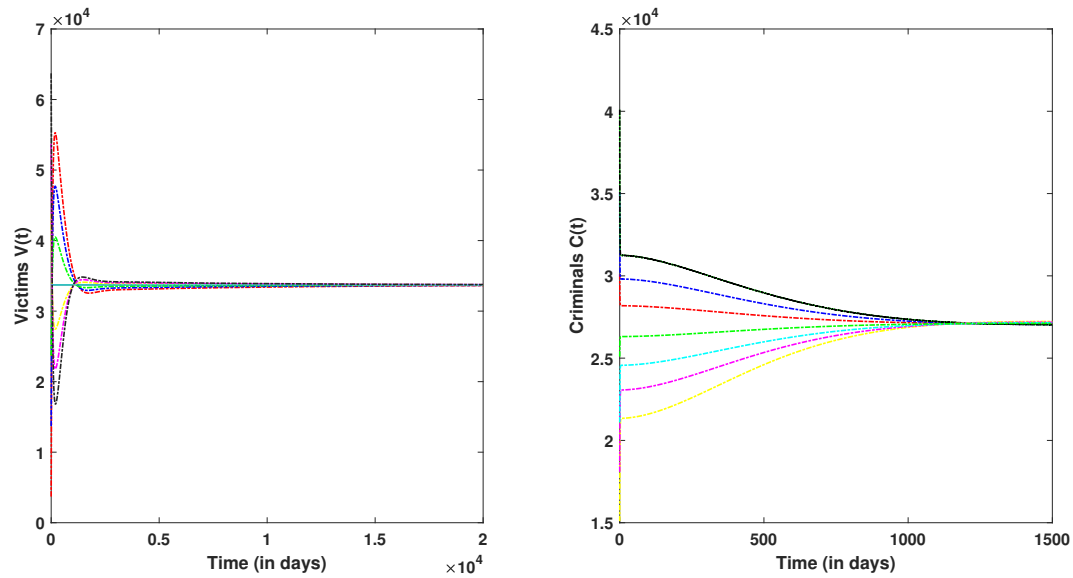
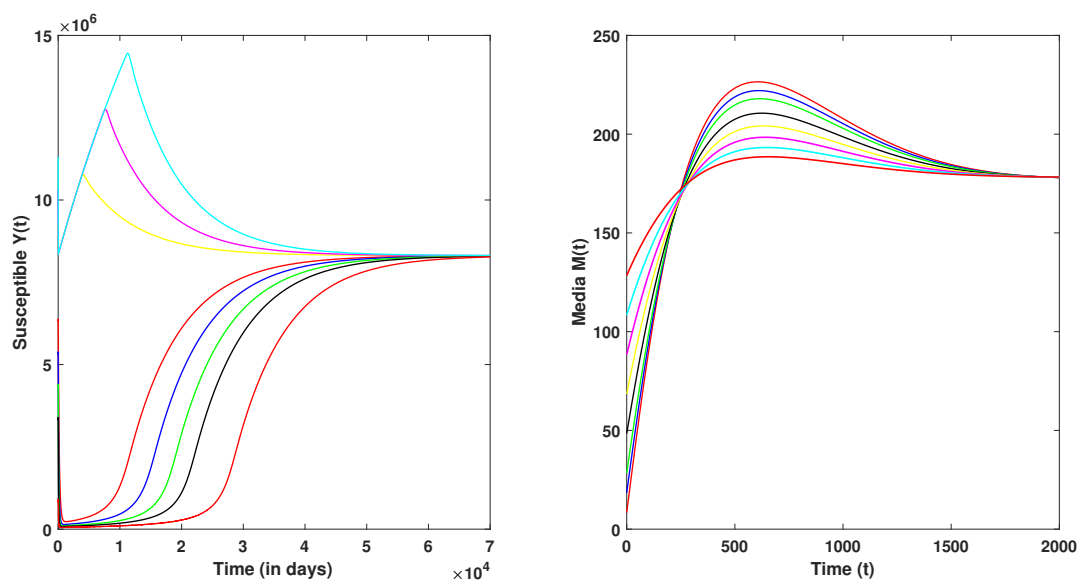


FIGURE 6. Non-linear Stability of Y^* , C^* , V^* and M^* with various initial starts.

From the resultant set of parameters, crime-persistent equilibrium E^* is guaranteed to occur and is nonlinearly stable. The equilibrium values of coordinates of E^* are $N^* = 8372830.4929$, $Y^* = 8310410.094474$, $C^* = 27095.5279$, $V^* = 33710.658$ and $M^* = 178.4113$. The eigenvalues of the Jacobian matrix that correspond to the E^* for the aforementioned set of parameters are -0.00012 , $-0.0025-0.002i$, $-0.0025+0.002i$, -0.02 , and -1.626 . Since all eigenvalues are negative, Routh Hurwitz condition demonstrates that E^* is locally asymptotically stable. Figure 14 represents the non linear stability of E^* with different initial values of Y^* , C^* , V^* and M^* .



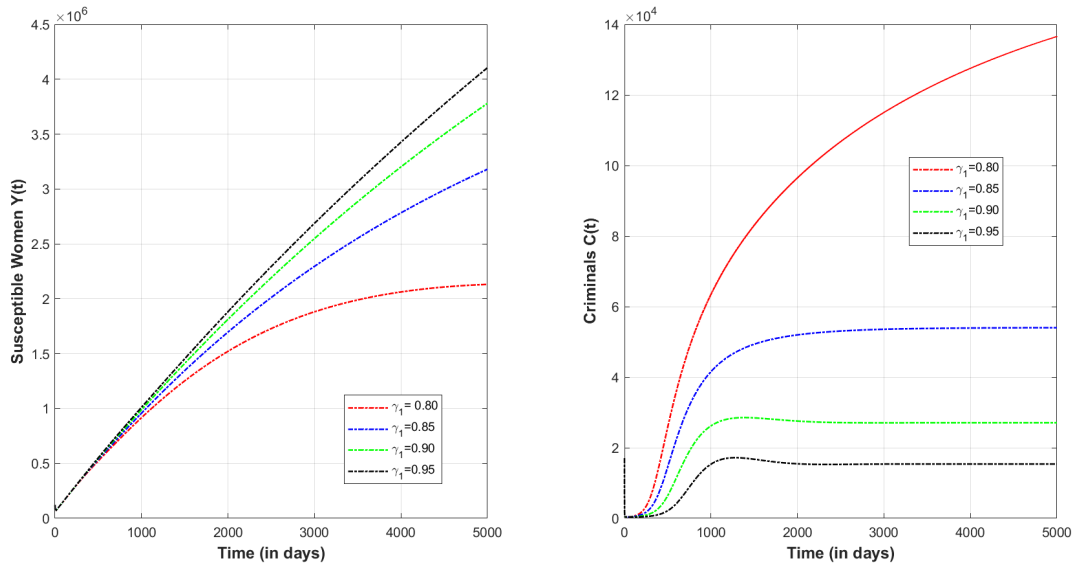


FIGURE 7. Variation in susceptible women population $Y(t)$ and criminals $C(t)$ population with variation in γ_1 .

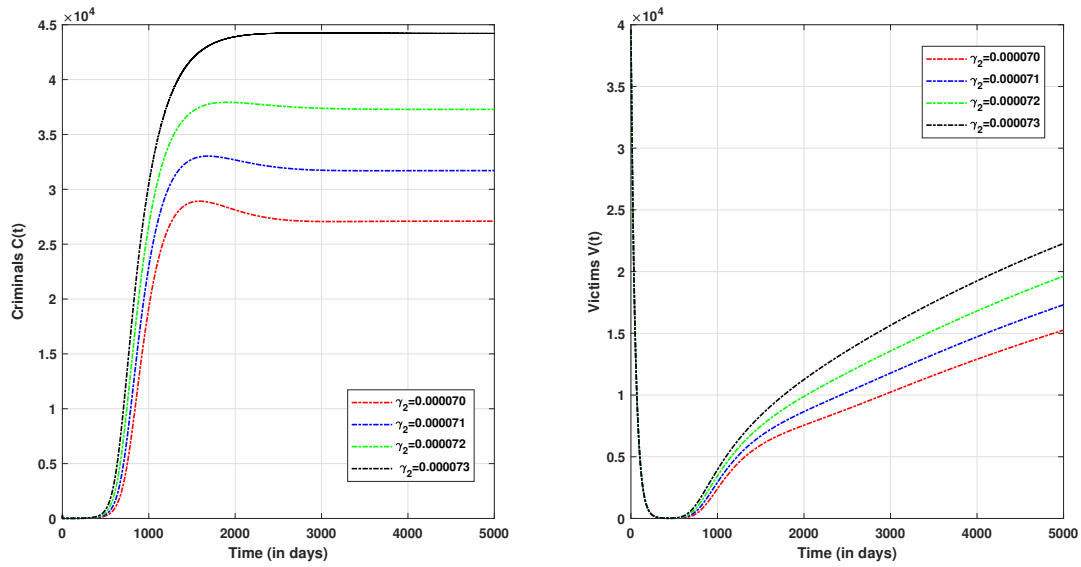


FIGURE 8. Variation in criminals $C(t)$ and victims $V(t)$ population with variation in γ_2 .

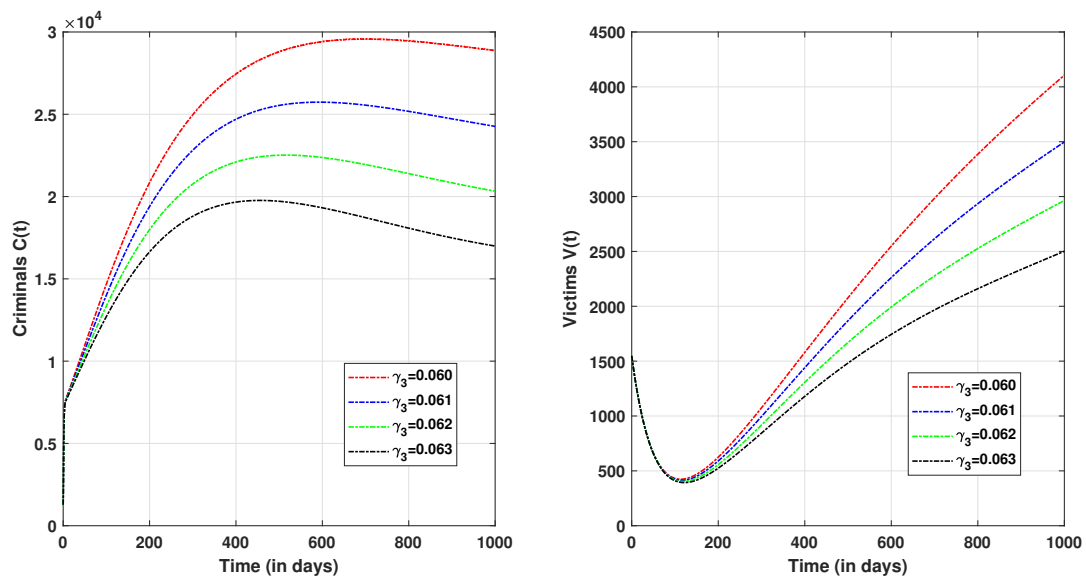


FIGURE 9. Variation in criminals $C(t)$ and victims $V(t)$ with variation in γ_3 .

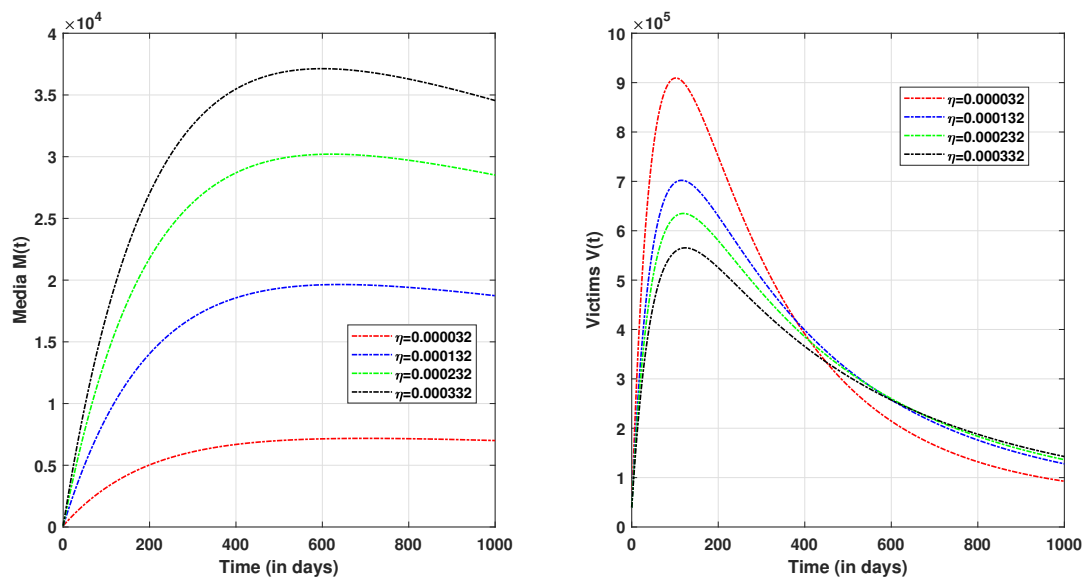


FIGURE 10. Variation in media $M(t)$ and victims $V(t)$ population with variation in η .

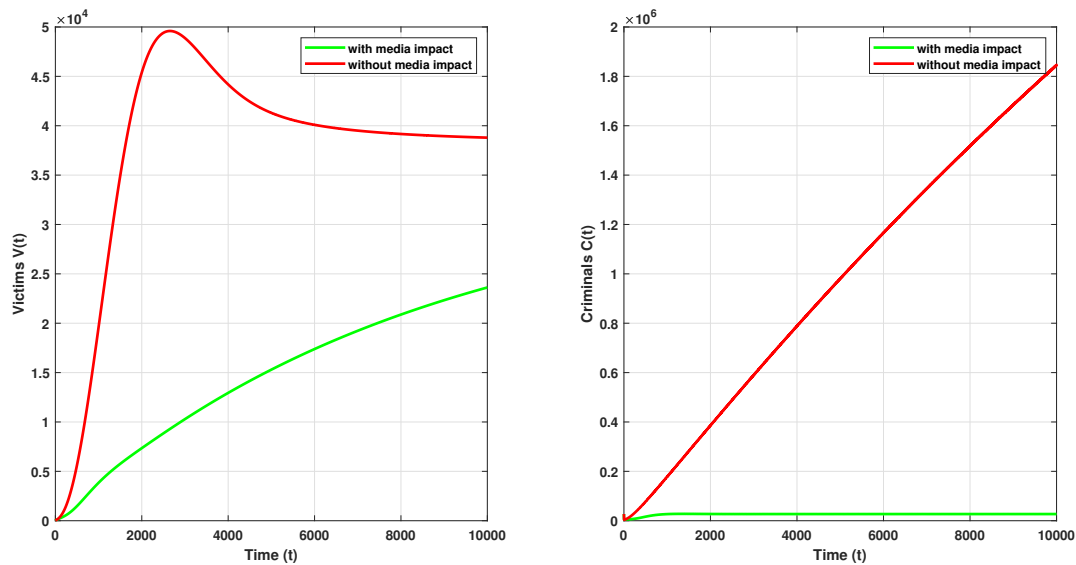


FIGURE 11. Victims $V(t)$ and criminal $C(t)$ population with and without media impact.

For the aforementioned parameters, model (2) is solved for various variables and the result is depicted graphically. Figure 7 showed how susceptible women $Y(t)$ and criminals $C(t)$ behaved w.r.t time ' t ' for various values of γ_1 . The graph demonstrated that the number of susceptible women rises and criminals decreases as γ_1 grows in value. This can be explained by the fact that the increase in conversion rate of socially exposed group to susceptible increases the susceptible, which results in implementing more media awareness activities, hence reducing criminals. The variation of criminals $C(t)$ and victims $V(t)$ with respect to time ' t ' for different values of γ_2 is shown in Figure 8. This graph makes it clear that as crime committing rate of socially exposed group increases, both criminals and victims increases. Figure 9 depicted the fluctuation of criminals $C(t)$ and victims $V(t)$ over time ' t ' for various values of γ_3 . According to the graph, criminals and victims show fall in values as conversion rate of socially exposed group to criminals decreases. The behaviour of media $M(t)$ and victims $V(t)$ with respect to time ' t ' for various values of η is illustrated in Figure 10. It is evident from the figure that media awareness $Y(t)$ increases while victims $V(t)$ decreases with increase in awareness programs. Figure 11 illustrates the behaviour of victims $V(t)$ and criminals $C(t)$ in the presence and absence of media awareness programs. It shows that victims $V(t)$ and criminals $C(t)$ decrease due to impact of media. This collectively imply that prevalence of crime can be controlled by decreasing the value of γ_2 i.e., decreasing the interaction between criminals and socially exposed group and by increasing the value of γ_1 and γ_3 i.e., with the increase in susceptible and criminals, media awareness programs will be implemented and will induce behavioural changes in individuals to prevent them from committing crime.

Further, table 3 represent the changes in equilibrium number of criminals (C^*), victims (V^*) and

criminality reproduction number (R_0^c) for different values of baseline number of media awareness program (M_0). It shows that the equilibrium number of criminals and victims, as well as the value of R_0^c decreases as M_0 increases. Here, it's worth noting that when $M_0 = 200$, the criminality reproduction number $R_0^c (= 0.99)$ falls below unity, thus eliminating crime and hence victims from the system (i.e. $C^* = 0$ and $V^* = 0$).

TABLE 3. Variation of C^* , V^* and R_0^c for different values of M_0 .

| M_0 | 0 | 5 | 10 | 50 | 100 | 150 | 200 |
|---------|-------|-------|-------|-------|-------|-------|------|
| C^* | 27867 | 27095 | 26323 | 20145 | 12422 | 4700 | 0 |
| V^* | 34014 | 33710 | 33394 | 30331 | 24400 | 13271 | 0 |
| R_0^c | 1.29 | 1.27 | 1.24 | 1.13 | 1.05 | 1.01 | 0.99 |

7. CASE STUDY APPLICATION - DELHI NCR

To determine the proposed model's practical effectiveness, we compared the model's simulated victims to empirical data on crime rates against women in Delhi from 2012 to 2022. This region has continuously recorded the highest number of crimes against women in India, making it an ideal test case for assessing the model's robustness and policy implications. The NCRB 2022 report states that 14,158 incidences of crimes against women, like as rape, assault, kidnapping, and domestic abuse, occurred in Delhi alone. These figures account for more than 30% of all such incidents in the largest cities in India. Awareness initiatives like the Bell Bajao, Beti Bachao Beti Padhao, and frequent public service announcements on radio, television, and internet platforms have all been part of the region's awareness campaigns.

To make meaningful comparisons, the model output was rescaled using linear regression to match the amplitude and scale of the real crime rate data. Figure. 12 illustrates that the real data and model projection shows a similar rising trend until 2016 and then a gradual decrease, which corresponds to the influence of increased media awareness initiatives after 2013 (e.g., public campaigns, legal reforms). The model accurately captures the flattening of victim growth post 2016, which is consistent with real-world efforts to reduce crime through social and institutional reforms. This supports the model's central mechanism i.e. media awareness campaigns reduce effective victimization rates by altering population dynamics.

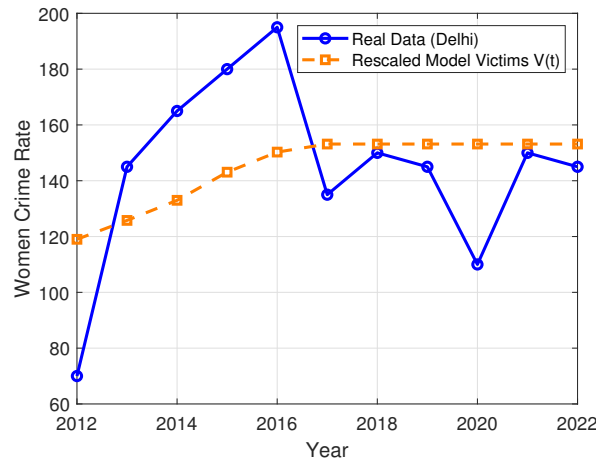


FIGURE 12. Model Victims $V(t)$ against real crime data (Delhi, 2012-2022).

The close alignment of the rescaled model predictions and real data demonstrates that the model accurately reflects both the timing and character of crime evolution as media influence varies. This strengthens the model's credibility as a policy-relevant instrument for simulating the impact of awareness-based interventions on crime dynamics. The model's behavior under "with media impact" conditions is strikingly similar to the reported transition from high crime development to stabilization in Delhi, validating the theoretical framework and its practical utility.

Moreover, we simulated model's dynamics using real-world data from the National Capital Region (NCR), particularly Delhi. Considering the parameters given in table 2, two scenarios were compared using simulations conducted between 2012 and 2025:

- Scenario 1: Low Awareness (2012-2014), where $\eta = 0.000032$, represents minimal awareness outreach.
- Scenario 2: Enhanced Awareness (2015-2025) where $\eta = 0.000041$, reflects growth in national and regional campaigns after 2015 (e.g., public participation and changes following the Nirbhaya incident).

The results represented by Figures. 13, 14 depicts that the victim count increased more quickly in the low-awareness scenario, suggesting inadequate prevention. The growing rate of victims decreased dramatically under increased public awareness. Whereas, under the influence of media, criminals decline more rapidly due to behavioral changes and deterrence.

The Delhi NCR scenario emphasizes the importance of media awareness in reducing crimes against women. The case study demonstrates how consistent and scalable media efforts—through TV, radio, schools, digital media, and public campaigns—can greatly curb the occurrence of crime and aid in achieving long-term safety objectives.

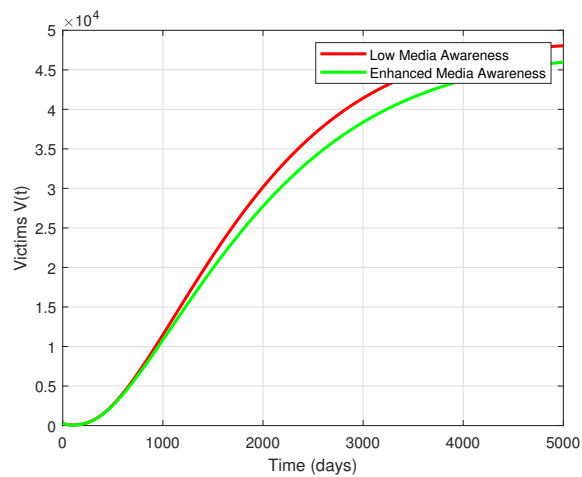


FIGURE 13. Impact of Media Awareness on Victim Population

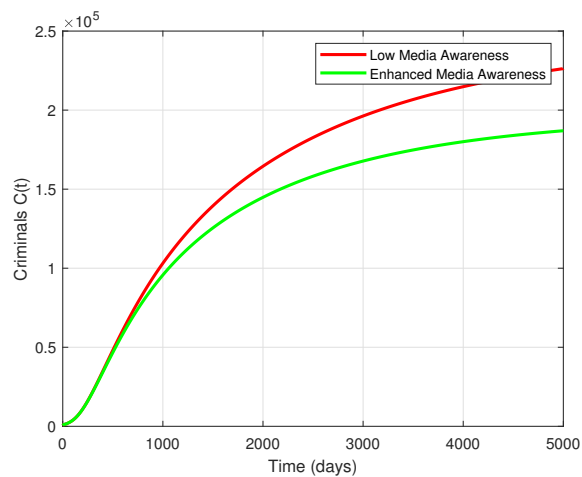


FIGURE 14. Impact of Media Awareness on Criminal Population

8. CONCLUSION

Crime against women in India has been widely recognized as an alarming issue in recent years. The occurrence of these offenses poses enormous risks to the safety, dignity, and well-being of women, and reflects ingrained social, cultural, and systemic problems. It is believed that criminal behavior is contagious, like an epidemic, and spreads through peer pressure. Thus, an epidemic modeling approach can be readily applied to model the dynamics and control of crime in society. Also, Media awareness programs are supposed to bring behavioral changes in criminals or precautionary changes in victims which could limit the crime or victims respectively. Thus, the impact of media awareness programs is examined and an additional compartment is incorporated. Considering this, a nonlinear mathematical model is presented in this article to investigate the spread of crime against women and its

prevention strategies. The proposed model has two equilibria namely crime-free equilibrium (CFE) and crime-persistent equilibrium (CPE). When $R_0^c < 1$, the CFE always exists and is locally asymptotically stable. Whereas, whenever the value of R_0^c surpasses unity, CFE becomes unstable, resulting in the existence of CPE. Under certain conditions, the CPE is locally and nonlinearly stable. The suggested model undergoes sensitivity analysis to identify model factors that impact the basic reproduction number and crime transmission. Analysis suggests that crime and hence victims are reduced in the presence of awareness programs. Also, the system needs to maintain some baseline media programs that raise awareness among people as it has been observed that increasing baseline media awareness programs could eradicate crime against women. The application of the suggested model to real-world data from Delhi gives persuasive proof of its practical utility. Using a case study of reported crime rates against women in Delhi from 2012 to 2022, the model demonstrated its ability to mimic the observed trend of growing victimization followed by stabilization. This pattern coincided with the chronology of increasing media awareness and public interventions, confirming the model's central assumption that awareness campaigns can have a considerable impact on crime dynamics. The case study not only verifies the model's theoretical validity, but also highlights its potential as a decision-support tool for policymakers seeking to design and evaluate media-based crime reduction programs in urban areas.

Authors' Contributions. All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Conflicts of Interest. The authors declare that there are no conflicts of interest regarding the publication of this paper.

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